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The analytic fuzzy I-convergent of $\chi^{2I(F)}_{f(\Delta,p)}$ space defined by modulus

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Abstract. The aim of this paper is to introduce and study a new concept of the analytic fuzzy ideal convergent double sequences defined by modulus and also some topological properties of the resulting sequence spaces of fuzzy numbers were examined.

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1. Introduction

The concept of fuzzy sets and fuzzy set operations were first introduced by Zadeh [27], fuzzy logic has become an important area of research in various branches of Mathematics such as metric and topological spaces, theory of functions, approximation theory etc. Subsequently several authors have discussed various aspects of the theory and applications of fuzzy sets. The concept of fuzziness has been applied in various fields such as Statistics, Cybernetics, Artificial intelligence, Operation research, Decision making, Agriculture, Weather forecasting, Quantum physics. Similarity relations of fuzzy orderings, fuzzy measures of fuzzy events, fuzzy mathematical programming etc.

Let (x_{mn}) be a double sequence of real or complex numbers. Then the series $\sum_{m,n=1}^{\infty} x_{mn}$ is called a double series. The double series $\sum_{m,n=1}^{\infty} x_{mn}$is said to be convergent if and only if the double sequence $(S_{mn})$is convergent, where

$$S_{mn} = \sum_{i,j=1}^{m,n} x_{ij} (m, n = 1, 2, 3, ...) \ .$$

We denote w^2 as the class of all complex double sequences (x_{mn}). A sequence $x = (x_{mn})$is said to be double analytic if

$$sup_{mn} |x_{mn}|^{1/(m+n)} < \infty.$$

The vector space of all prime sense double analytic sequences are usually denoted by Λ^2. A sequence $x = (x_{mn})$ is called double gai sequence if

$$((m+n)!\, |x_{mn}|)^{1/(m+n)} \to 0 \text{ as } m, n \to \infty.$$

The vector space of all prime sense double gai sequences are usually denoted by χ^2. The space Λ^2 is a metric space with the metric

$$d(x,y) = sup_{mn} \left\{ |x_{mn} - y_{mn}|^{1/(m+n)} : m, n : 1, 2, 3, ... \right\}, \tag{1.1}$$

for all $x = \{x_{mn}\} \text{and}\, y = \{y_{mn}\}\, in\, \Lambda^2$. The space χ^2 is a metric space with the metric

$$d(x,y) = sup_{mn} \left\{ ((m+n)!\, |x_{mn} - y_{mn}|)^{1/(m+n)} : m, n : 1, 2, 3, ... \right\}, \tag{1.2}$$

for all $x = \{x_{mn}\} \text{and}\, y = \{y_{mn}\}\, in\, \chi^2$.

Consider a double sequence $x = (x_{ij})$. The $(m,n)^{th}$ section $x^{[m,n]}$ of the sequence is defined by $x^{[m,n]} = \sum_{i,j=0}^{m,n} x_{ij}\delta_{ij}$ for all $m, n \in \mathbb{N}$,

$$\delta_{mn} = \begin{pmatrix} 0, & 0, & ...0, & 0, & ... \\ 0, & 0, & ...0, & 0, & ... \\ . & & & & \\ . & & & & \\ . & & & & \\ 0, & 0, & ...1, & 0, & ... \\ 0, & 0, & ...0, & 0, & ... \end{pmatrix}$$

with 1 in the $(m,n)^{th}$ position and zero other wise. An FK-space(or a metric space) X is said to have AK property if (δ_{mn}) is a Schauder basis for X. Or equivalently $x^{[m,n]} \to x$. We need the following inequality in the sequel of the paper:

Lemma 1.1. *For $a, b \geq 0$ and $0 < p < 1$, we have*

$$(a+b)^p \leq a^p + b^p.$$

Some initial works on double sequence spaces is found in Bromwich [2]. Later on it was investigated by Hardy [6], Moricz [9], Moricz and Rhoades [12], Basarir and Solankan [1], Tripathy [18], Colak and Turkmenoglu [3], Turkmenoglu [26], and many others. Tripathy and Dutta [20], introduced andinvestigated different types of fuzzy real valued double sequence spaces. Generalizing the concept of ordinary convergence for real sequences Kostyrko et al.[8] introduced the concept of ideal convergence which is a generalization of statistical convergence, by using the ideal I of the subsets of the set of natural numbers. For more details of this concept we refer to M.Mursaleen and S.A.Mohiuddine [14, 15, 16], S.A.Mohiuddine et al. [11, 10] and references therein.

Throughout the article Λ^2, χ^2 denote the spaces of analytic and gai sequences respectively and Λ_F^{2I} and χ_F^{2I} denote the classes of $I-$ analytic and $I-$gai fuzzy real valued double sequences, respectively.

The notion of difference sequence spaces (for single sequences) was introduced by Kizmaz [7] as follows

$$Z(\Delta) = \{x = (x_k) \in w : (\Delta x_k) \in Z\}$$

for $Z = c, c_0$ and ℓ_∞, where $\Delta x_k = x_k - x_{k+1}$ for all $k \in \mathbb{N}$. Here w, c, c_0 and ℓ_∞ denote the classes of all, convergent, null and bounded scalar valued single sequences, respectively. The above spaces are Banach spaces normed by

$$\|x\| = |x_1| + sup_{k \geq 1} |\Delta x_k| .$$

Later on the notion was further investigated by many others. The notion of difference double sequence was introduced by Tripathy and Sarma [19] as follows

$$Z(\Delta) = \left\{x = (x_{mn}) \in w^2 : (\Delta x_{mn}) \in Z\right\}$$

where $Z = \Lambda^2$ and χ^2, respectively. $\Delta x_{mn} = (x_{mn} - x_{mn+1}) - (x_{m+1n} - x_{m+1n+1}) = x_{mn} - x_{mn+1} - x_{m+1n} + x_{m+1n+1}$ for all $m, n \in \mathbb{N}$. Further generalized this notion and introduced the following notion. For $m, n \geq 1$,

$$Z\left(\Delta_\gamma^\mu\right) = \left\{x = (x_{mn} : \left(\Delta_\gamma^\mu x_{mn}\right) \in Z\right\} \text{ for } Z = \Lambda^2 \text{ and } \chi^2.$$

An Orlicz function is a function $M : [0, \infty) \to [0, \infty)$ which is continuous, non-decreasing and convex with $M(0) = 0$, $M(x) > 0$, for $x > 0$ and $M(x) \to \infty$ as $x \to \infty$. If convexity of Orlicz function M is replaced by $M(x+y) \leq M(x) + M(y)$, then this function is called modulus function.

Remark 1.2. An Modulus function satisfies the inequality $M(\lambda x) \leq \lambda M(x)$ for all λ with $0 < \lambda < 1$.

2. Preliminaries

Let X be a non-empty set, then a family of sets $I \subset 2^X$ (the class of all subsets of X) is called an ideal if and only if for each $A, B \in I$, we have $A \bigcup B \in I$ and for each $A \in I$ and each each $B \subset A$, we have $B \in I$. A non-empty family of sets $F \subset 2^X$ is a filter on X if and only if $\phi \notin F$, for each $A, B \in F$, we have $A \bigcap B \in F$ and each $A \in F$ and each $A \subset B$, we have $B \in F$. An ideal I is called non-trivial ideal if $I \neq \phi$ and $X \notin I$. Clearly $I \subset 2^X$ is a non-trivial ideal if $F = F(I) = \{X/A : A \in I\}$ is a filter on X. A non-trivial ideal $I \subset 2^X$ is called admissible if and only if $\{\{x\} : x \in X\} \subset I$. Further details on ideals of 2^X can be found in Kostyrko, et. al.[8]. The notion was further investigated by Salat, et. al. and others. Throughout the ideals of 2^N and $2^{N \times N}$ will be denoted by I and I_2 respectively.

A fuzzy real number X is a fuzzy set on R, a mapping $X : R \to L (= [0, 1])$ associating each real number t with its grade of membership $X(t)$. The $\alpha-$ level set of a fuzzy real number $X, 0 < \alpha < 1$ denoted by $[X]^\alpha$ is defined as $[X]^\alpha = \{t \in R : X(t) \geq \alpha\}$. A fuzzy real number X is called convex if $X(t) \geq X(s) \wedge X(r) = min(X(s), X(r))$, where $s < t < r$. If there exists $t_0 \in R$ such that $X(t_0) = 1$, then the fuzzy real number X is called normal. A fuzzy real X is said to be upper semi-continuous if for each $\epsilon > 0, X^{-1}([0, a + \epsilon))$, for all $a \in L$ is open in the usual topology of R. The set of all upper semi continuous, normal convex fuzzy number is denoted by $L(R)$.

Throughout a fuzzy real valued double sequence is denoted by (X_{mn}) i.e a double infinite array of fuzzy real number X_{mn} for all $m, n \in \mathbb{N}$.

Every real number r can express as a fuzzy real number $\overline{r}$ as follows:

$$\overline{r} = \begin{cases} 1, & \text{if } t = r; \\ 0, & \text{otherwise} \end{cases}$$

Let D be the set of all closed bounded intervals $X = \left[X^L, X^R\right]$. Then $X \leq Y$ if and only if $X^L \leq Y^L$ and $X^R \leq Y^R$. Also

$$d(X, Y) = max\left(\left|X^L - Y^L\right|, \left|X^R - Y^R\right|\right).$$

Then (D, d) is a complete metric space. Let $\overline{d} : L(R) \times L(R) \to R$ be defined by

$$\overline{d}(X, Y) = sup_{0 \leq \alpha \leq 1} d\left([X]^\alpha, [Y]^\alpha\right) \text{ for } X, Y \in L(R).$$

Then $\overline{d}$ defined a metric on $L(R)$.

Definition 2.1 ([4]). Let A denote a four dimensional summability method that maps the complex double sequences x into the double sequence. Ax where the $mn - th$ term to Ax is as follows

$$(Ax)_{k,\ell} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{k\ell}^{mn} x_{mn}.$$

In [5] Hardy presented the notion of regularity of two dimensional matrix transformations. The definition is as follows: a two dimensional matrix transformation is said to be regular if it maps every convergent sequence into a convergent sequence with the same limit. In addition, to the numerous theorems characterizing regularity. Hardy also presented the Silvermann-Toeplitz characterization of regularity following this work Robison in 1926 presented a four dimensional analog of regularity for double sequences in which he added an additional assumption of boundedness. This assumption was made because a double sequence which is $P-$convergent is not necessarily bounded along these same lines, Robison and Hamiltion presented a Silverman-Toeplitz type multidimensional characterization of regularity in [17] and [4]. The definition of regularity for four dimensional matrices will be stated next, followed by the Robison-Hamilton characterization of the regularity of four dimensional matrices.

Definition 2.2 ([13]). A double sequences (X_{mn}) is said to be convergent in Pringsheim's sense to the fuzzy real number X, if for every $\epsilon > 0$, there exists $n_0 = n_0(\epsilon), k_0 = k_0(\epsilon) \in N$ such that $\overline{d}(X_{mn}, X) < \epsilon$ for all $n \geq n_0$, $k \geq k_0$.

Definition 2.3 ([23]). A double sequence (X_{mn}) is said to be $I-$convergent to the fuzzy number X_0, if for all $\epsilon > 0$, the set $\left\{(n, k) \in N^2 : \overline{d}(X_{mn}, X_0) \geq \epsilon\right\} \in I_2$. We write $I_2 - lim X_{mn} = X_0$. [see [23]]

Definition 2.4 ([24]). A fuzzy real-valued double sequence space E^F is said to be solid of $(Y_{mn}) \in E^F$ whenever $(X_{mn}) \in E^F$ and $|Y_{mn}| \leq |X_{mn}|$ for all $m, n \in \mathbb{N}$.

Let $K = \{(m_i, n_i) : i \in \mathbb{N}; m_1 < m_2 < m_3 \cdots and\, n_1 < n_2 < n_3 \cdots\} \subseteq \mathbb{N} \times \mathbb{N}$ and E^F be a double sequence space. A $K-$ step space of E^F is a sequence space $\lambda_K^E = \left\{(x_{m_i n_i}) \in w^{2F} : (x_{mn}) \in E^F\right\}$.

A canonical pre-image of a sequence $(x_{m_i n_i}) \in E^F$ is a sequence (Y_{mn}) defined as follows:

$$Y_{mn} = \begin{cases} X_{mn}, & \text{if } (m, n) \in K, \\ \overline{0}, & \text{otherwise.} \end{cases}$$

A canonical pre-image of a step space λ_K^E is a set of canonical pre-images of all elements in λ_K^E.

Definition 2.5 ([25]). A double sequence E^F is said to be monotone if E^F contains the canonical pre-image of all its step spaces.

Definition 2.6 ([22]). A double sequence E^F is said to be symmetric if $\left(X_{\pi(m),\pi(n)}\right) \in E^F$, whenever $(X_{mn}) \in E^F$, where π is a permutation of $N \times N$.

Definition 2.7 ([23]). A double sequence E^F is said to be sequence algebra if $(X_{mn} \otimes Y_{mn}) \in E^F$, whenever $(X_{mn}), (Y_{mn}) \in E^F$.

Definition 2.8 ([16]). A double sequence E^F is said to be convergence free if $(Y_{mn}) \in E^F$, whenever $(X_{mn}) \in E^F$ and $X_{mn} = \overline{0}$ implies $Y_{mn} = \overline{0}$.

Let (X_{mn}) be a double sequence of fuzzy numbers and (p_{mn}) be a double sequence of analytic strictly positive real numbers such that $0 < p_{mn} \le supp_{mn} < \infty$. We introduce the following sequence spaces:

$$\chi^{2I(F)}_{f(\Delta,p)} = \Big\{ X = (X_{mn}) : \Big\{ (m,n) \in \mathbb{N} \times \mathbb{N} : f\left[\overline{d}\left(\left((m+n)!\Delta X_{mn} \right)^{1/(m+n)}, \overline{0} \right) \right]^{p_{mn}} \ge \epsilon \Big\} \in I_2, \Big\}$$

for every $\epsilon > 0$.

$$\Lambda^{2F}_{f(\Delta,p)} = \left\{ X = (X_{mn}) : sup_{mn} f\left[\overline{d}\left(\left(\Delta X_{mn} \right)^{1/(m+n)}, \overline{0} \right) \right]^{p_{mn}} < \infty \right\}.$$

Also we write $\Lambda^{2I(F)}_{f(\Delta,p)} = \chi^{2I(F)}_{f(\Delta,p)} \bigcap \Lambda^{2F}_{f(\Delta,p)}$

Lemma 2.9 ([19]). *If a sequence E^F is solid, then it is monotone. See* [12], p. 53.

3. Major section

Proposition 3.1. *Let $p = (p_{mn})$ be a double sequence of analytic strictly positive numbers. Then $\Lambda^{2I(F)}_{f(\Delta,p)}$ and $\Lambda^{2F}_{f(\Delta,p)}$ are linear spaces.*

Proof. This is the proof of Proposition 3.1, is easy. Therefore omit the proof. □

Proposition 3.2. *Let the double sequence $p = (p_{mn})$ be analytic. Then $\chi^{2I(F)}_{f(\Delta,p)} \subseteq \Lambda^{2F}_{f(\Delta,p)}$ and the inclusion are strict.*

Proof. This is the proof of Proposition 3.2, the inclusion $\chi^{2I(F)}_{f(\Delta,p)} \subseteq \Lambda^{2F}_{f(\Delta,p)}$ is obvious. For establishing that the inclusion is proper, consider the following example.

Example: We prove the result for the case $\chi^{2I(F)}_{f(\Delta,p)} \subseteq \Lambda^{2F}_{f(\Delta,p)}$, the other case similar. Let $f(\Delta X) = \Delta X$. Let the sequence ΔX_{mn} be defined by for $m > n$,

$$\Delta X_{mn}(t) = \begin{cases} \frac{(mt-m-1)^{(m+n)}(m-1)^{-(m+n)}}{(m+n)!}, & \text{for } 1 + \frac{1}{m} \le t \le 2, \\ \frac{(3-t)^{(m+n)}}{(m+n)!}, & \text{for } 2 < t \le 3, \\ 0, & \text{otherwise.} \end{cases}$$

and for $m < n$

$$\Delta X_{mn}(t) = \begin{cases} \frac{(mt-1)^{(m+n)}(m-1)^{-(m+n)}}{(m+n)!}, & \text{for } \frac{1}{m} \le t \le 1, \\ \frac{(-t+2)^{(m+n)}}{(m+n)!}, & \text{for } 1 \le t \le 2, \\ 0, & \text{otherwise.} \end{cases}$$

Then, $(\Delta X_{mn}) \in \Lambda^{2F}_{f(\Delta,p)}$ but $(\Delta X_{mn}) \notin \chi^{2I(F)}_{f(\Delta,p)}$. □

Proposition 3.3. *The class of sequence* $\Lambda^{2I(F)}_{f(\Delta,p)}$ *is solid and also monotone.*

Proof. This is the proof of Proposition 3.3, let $(X_{mn}),(Y_{mn}) \in \Lambda^{2I(F)}_{f(\Delta,p)}$ be such that $|Y_{mn}| \leq |X_{mn}| \Rightarrow \overline{d}\left(Y_{mn},\overline{0}\right) \leq \overline{d}\left(X_{mn},\overline{0}\right)$ for each $m,n \in \mathbb{N}$. Let $\epsilon > 0$ be given. Then the solidness of $\Lambda^{2I(F)}_{f(\Delta,p)}$ follows from the following relation:
$\left\{(m,n) \in \mathbb{N}\times\mathbb{N} : f\left[\overline{d}\left(((m+n)!\Delta X_{mn})^{1/(m+n)},\overline{0}\right)\right]^{p_{mn}} \geq \epsilon\right\} \supseteq$
$\left\{(m,n) \in \mathbb{N}\times\mathbb{N} : f\left[\overline{d}\left(((m+n)!\Delta Y_{mn})^{1/(m+n)},\overline{0}\right)\right]^{p_{mn}} \geq \epsilon\right\}.$
Also by Lemma 2.9, it follows that the space $\Lambda^{2I(F)}_{f(\Delta,p)}$ is monotone. □

Proposition 3.4. *The class of sequence* $\Lambda^{2I(F)}_{f(\Delta,p)}$ *is sequence algebra.*

Proof. This is the proof of Proposition 3.4, let $(X_{mn}),(Y_{mn}) \in \Lambda^{2I(F)}_{f(\Delta,p)}$ and $0 < \epsilon < 1$. Then the result follows from the following inclusion relation:
$\left\{(m,n) \in \mathbb{N}\times\mathbb{N} : f\left[\overline{d}\left(((m+n)!\left(\Delta X_{mn}\otimes\Delta Y_{mn}\right))^{1/(m+n)},\overline{0}\right)\right]^{p_{mn}} < \epsilon\right\} \supseteq$
$\left\{(m,n) \in \mathbb{N}\times\mathbb{N} : f\left[\overline{d}\left(((m+n)!\Delta X_{mn})^{1/(m+n)},\overline{0}\right)\right]^{p_{mn}} < \epsilon\right\}$
$\bigcap\left\{(m,n) \in \mathbb{N}\times\mathbb{N} : f\left[\overline{d}\left(((m+n)!\Delta Y_{mn})^{1/m+n},\overline{0}\right)\right]^{p_{mn}} < \epsilon\right\}.$
Similarly we can prove the result for other cases. □

Proposition 3.5. *The class of sequence* $\Lambda^{2I(F)}_{f(\Delta,p)}$ *is complete metric space with respect to the metric* ρ *defined by*

$$\rho(X,Y) = sup_m f\left[\overline{d}\left(((m+1)!\left(X_{m1},Y_{m1}\right))^{1/m+1},\overline{0}\right)\right]^{p_{mn}}$$
$$+sup_n f\left[\overline{d}\left(((1+n)!\left(X_{1n},Y_{1n}\right))^{1/1+n},\overline{0}\right)\right]^{p_{mn}}$$
$$+sup_{mn} f\left[\overline{d}\left(((m+n)!\left(\Delta X_{mn},\Delta Y_{mn}\right))^{1/m+n},\overline{0}\right)\right]^{p_{mn}}$$

where $X=(X_{mn}), Y=(Y_{mn}) \in \Lambda^{2I(F)}_{f(\Delta,p)}$ *and* $\Delta X_{mn} = (X_{mn} - X_{mn+1}) - (X_{m+1n} - X_{m+1n+1}) = X_{mn} - X_{mn+1} - X_{m+1n} + X_{m+1n+1}$ *for all* $m,n \in \mathbb{N}$.

Proposition 3.6. *The class of sequence* $\Lambda^{2I(F)}_{f(\Delta,p)}$ *is nowhere dense subsets of* $\Lambda^{2F}_{f(\Delta,p)}$.

Proof. This is the proof of Proposition 3.6, by Proposition 3.1, the sequence space $\Lambda^{2I(F)}_{f(\Delta,p)}$ are proper subspace of $\Lambda^{2F}_{f(\Delta,p)}$. Hence by proposition 3.5 the result follows. □

4. Conclusions

In this article are introduce fuzzy $I-$ convergent χ^2_Δ space defined by modulus function and disucss some topogological properties.

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