Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 6, No. 2, (September 2013), pp. 415–424
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr

Decision-making approach with entropy weight based on intuitionistic fuzzy soft set

Yong Wei Yang, Ting Qian

Received 28 October 2012; Revised 30 December 2012; Accepted 18 January 2013

Abstract. This paper analyzes the limitations of Jiang et al. approach to intuitionistic fuzzy soft sets based decision making, then a new entropy measure based on the degree of intuitionistic fuzziness is presented to calculate the weights of criteria of alternatives. By level soft sets and score functions, a novel decision-making approach with entropy weight is proposed. The method considers not only the given condition, but also the numbers of criteria of alternatives satisfied the given condition. Finally, examples are given showing that its practicality and effectiveness.

2010 AMS Classification: 03E72, 54A40

Keywords: Intuitionistic fuzzy soft set, Decision making, Entropy, Score function, Level soft set.

Corresponding Author: Yong Wei Yang (yangyw2010@126.com)

1. Introduction

Molodtsov [18] introduced the concept of soft sets as a new mathematical tool to deal with complex systems involving uncertain or not clearly defined objets and he also pointed out several directions for the applications of soft sets. At present, soft set theory has received much attention in the field of algebraic structures such as groups [1], semigroups [3, 20], hemirings [24] and BL-algebras [27]. Jun et al. applied soft theory to BCK/BCI-algebras [10, 11] and d-algebras[12], respectively, and investigated their properties.

Since Maji et al. [15] introduced the notion of fuzzy soft sets, as a generalization of the standard soft sets, some researchers have shown great interest in fuzzy soft sets [4, 25, 26]. Maji et al. presented a fuzzy soft set theoretic approach of object recognition from an imprecise multi observer data [19] and a neutrosophic soft set theoretic approach towards the a multiobserver decision making problem [17], respectively. Based on grey relational analysis, Kong et al. [13] proposed a new algorithm whose bases are multiple. Following the research of Feng et al. [6, 7], Jiang et al. [9] considered level soft sets of intuitionistic fuzzy soft sets, and extended Feng's adjustable decision making to the setting of intuitionistic fuzzy soft sets.

In this paper, we first analyzes Jiang et al. approach to intuitionistic fuzzy soft sets based decision making, then point out its limitations. In order to calculate the weights of criteria, we present a new entropy measure based on the degree of fuzziness and intuitionism, then give a example to show its validity. Using the scores of alternatives at certain level, we propose a novel decision-making approach with entropy weight based on intuitionistic fuzzy soft sets. The new approach is an adjustable method based on Jiang et al.'s and it considers not only the given condition, but also the numbers of criteria of alternatives satisfied the given condition. The new method can be successfully applied in some decision making problems and some examples are given showing its practicality and effectiveness.

2. Preliminaries

In the section, we will recall some relevant notions which will be used in the paper.

Definition 2.1 ([2])**.** An intuitionistic fuzzy set (IFS, for short) $\tilde{A}$ in X is given by

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x))|x \in X\}$$

where $\mu_{\tilde{A}} : X \to [0,1]$ and $\nu_{\tilde{A}} : X \to [0,1]$ with the condition: $0 \le \mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) \le 1, \forall x \in X$.

The numbers $\mu_{\tilde{A}}(x)$ and $\nu_{\tilde{A}}(x)$ represent, respectively, the membership degree and non-membership degree of the element $x \in X$ to the set $\tilde{A}$. We denote the set of all the IFSs in X as $\mathcal{IF}(X)$.

$\pi_{\tilde{A}(x)} = 1 - \mu_{\tilde{A}}(x) - \nu_{\tilde{A}}(x)$ for any $x \in X$ is called the degree of indeterminacy of x to IFS $\tilde{A}$. It is obvious that $0 \le \pi_{\tilde{A}}(x) \le 1, x \in X$. For the sake of simplicity, we call $\alpha = (\mu_\alpha, \nu_\alpha)$ intuitionistic fuzzy number (IFN), where $\mu_\alpha \in [0,1]$ and $\nu_\alpha \in [0,1]$.

The operations of IFS [2] are defined as follows, for every $\tilde{A}, \tilde{B} \in \mathcal{IF}(X)$

- $\tilde{A} \le \tilde{B}$ if and only if $\mu_{\tilde{A}}(x) \le \mu_{\tilde{B}}(x)$ and $\nu_{\tilde{A}}(x) \ge \nu_{\tilde{B}}(x)$ for all $x \in X$.
- $\tilde{A} = \tilde{B}$ if and only if $\tilde{A} \le \tilde{B}$ and $\tilde{A} \ge \tilde{B}$.
- The complementary of IFS $\tilde{A}$ is $\tilde{A}^c(x) = \{(x, \nu_{\tilde{A}}(x), \mu_{\tilde{A}}(x))|x \in X\}$.
- $\tilde{A} \cap \tilde{B} = \{(x, min(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)), max(\nu_{\tilde{A}}(x), \nu_{\tilde{B}}(x)))|x \in X\}$.
- $\tilde{A} \cup \tilde{B} = \{(x, max(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)), min(\nu_{\tilde{A}}(x), \nu_{\tilde{B}}(x)))|x \in X\}$.

In the following, we will briefly review some concepts related to soft sets. Through the whole paper, let U be an initial universe of objects and E be a set of parameters related to the objects in U. Let $\mathcal{P}(U)$ denote the power set of U and $A \subseteq U$. Then Molodtsov defined the notion of soft sets as follows.

Definition 2.2 ([18])**.** . A pair (F, A) is called a soft set over U where F is a mapping given by $F : A \to \mathcal{P}(U)$.

Through the combination of the theories of intuitionistic fuzzy set and soft set, Maji et al. introduced the concept of intuitionistic fuzzy soft sets.

Definition 2.3 ([16])**.** A pair (F, A) is called an intuitionistic fuzzy soft set over U, where F is a mapping given by $F : A \to \mathcal{IF}(U)$.

Note that the concept of level soft sets in fuzzy soft set theory was initiated by Feng et al. [6]. Based on the concept of level soft sets of fuzzy soft sets, Jiang et al. defined level soft sets of intuitionistic fuzzy soft sets as follows.

Definition 2.4 ([9])**.** Let $\varpi = (F, A)$ be an intuitionistic fuzzy soft set over the universe U, where $A \subseteq E$ and E be a set of parameters. For $s, t \in [0, 1]$, the (s, t)-level soft set of ϖ is a crisp soft set $L(\varpi; s, t) = (F_{(s,t)}, A)$ defined by

$$F_{(s,t)}(\varepsilon) = L(F(\varepsilon); s, t) = \{x \in U | \mu_{F(\varepsilon)}(x) \geq s \text{ and } \nu_{F(\varepsilon)}(x) \leq t\}$$

for all $\varepsilon \in A$.

Definition 2.5 ([9])**.** Let $\varpi = (F, A)$ be an intuitionistic fuzzy soft set over the universe U, where $A \subseteq E$ and E be a set of parameters. Let $\lambda : A \to [0, 1] \times [0, 1]$ be an intuitionistic fuzzy set in A which is called a threshold intuitionistic fuzzy set. The level soft set of ϖ with respect to λ is a crisp soft set $L(\varpi; \lambda) = (F_\lambda, A)$ defined by $F_\lambda(\varepsilon) = L(F(\varepsilon); \lambda(\varepsilon)) = \{x \in U | \mu_{F(\varepsilon)}(x) \geq \mu_{\lambda(\varepsilon)}(x) \text{ and } \nu_{F(\varepsilon)}(x) \leq \nu_{\lambda(\varepsilon)}(x)\}$ for all $\varepsilon \in A$.

For real-life application of intuitionistic fuzzy soft sets based on decision making, the threshold intuitionistic fuzzy soft set λ is in advance chosen by decision makers.

3. The analysis of Jiang et al. decision making approach

In the section, we first show Jiang et al. approach to intuitionistic fuzzy soft sets based decision making in [9], and point out its limitations.

Jiang et al. proposed an adjustable approach to intuitionistic fuzzy soft sets based decision making as follows.

Algorithm 1.

(1) Input the (result) intuitionistic fuzzy soft set $\varpi = (F, A)$.
(2) Input a threshold intuitionistic fuzzy set $\lambda : A \to [0, 1] \times [0, 1]$ for decision making.
(3) Compute the level soft set $L(\varpi; \lambda)$.
(4) Present the level soft set $L(\varpi; \lambda)$ in tabular form and compute the choice value c_i of x_i, $\forall i$.
(5) The optimal decision is to select x_k if $c_k = max_i c_i$.
(6) If k has more than one value, then any one of x_k may be chosen.

In order to describe the basic idea of the above algorithm, let us consider the following example. Some of it is quoted from [7, 9].

Example 3.1. Let us consider an intuitionistic fuzzy soft set $\varpi = (F, A)$ which describes the "attractiveness of houses" that Mr. X is considering for purchase. Suppose there are six houses in the domain $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$ under consideration, and $A = \{\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5, \varepsilon_6\}$ is a set of decision parameters. The $\varepsilon_i (i = 1, 2, 3, 4, 5, 6)$ stand for the parameters "modern", "cheap", "beautiful", "large", "convenient traffic" and "in green surroundings", respectively. Table 1 gives the tabular representation of the intuitionistic fuzzy soft set $\varpi = (F, A)$.

If we choose $\lambda = topbottom_\varpi$ where $topbottom_\varpi : A \to [0, 1] \times [0, 1]$ defined by $\mu_{topbottom_\varpi}(\varepsilon) = max\{\mu_{F(\varepsilon)}(x) | x \in U\}$ and $\nu_{topbottom_\varpi}(\varepsilon) = min\{\mu_{F(\varepsilon)}(x) | x \in U\}$

for all $\varepsilon \in A$, then $topbottom_{\varpi} = \{(\varepsilon_1, 0.8, 0), (\varepsilon_2, 0.8, 0.1), (\varepsilon_3, 0.8, 0), (\varepsilon_4, 0.7, 0.1), (\varepsilon_5, 0.8, 0.1), (\varepsilon_6, 0.7, 0.1)\}$.

By using Algorithm 1, we obtain the level soft set $L(\varpi; topbottom_{\varpi})$ with the choice values with tabular representation as in Table 2.

From Table 2, it is easy to see that the maximum choice value is $c_5 = c_6 = 4$, hence h_5 and h_6 are the optimal alternatives.

Table 1

Intuitionistic fuzzy soft set $\varpi = (F, A)$

U	ε_1	ε_2	ε_3	ε_4	ε_5	ε_6
h_1	(0.3, 0.5)	(0.6, 0.3)	(0.6, 0.3)	(0.6, 0.3)	(0.3, 0.5)	(0.5, 0.2)
h_2	(0.8, 0.0)	(0.8, 0.1)	(0.8, 0.0)	(0.6, 0.2)	(0.4, 0.1)	(0.6, 0.1)
h_3	(0.5, 0.4)	(0.5, 0.4)	(0.2, 0.6)	(0.2, 0.6)	(0.5, 0.3)	(0.2, 0.1)
h_4	(0.2, 0.7)	(0.2, 0.6)	(0.0, 0.9)	(0.0, 0.9)	(0.2, 0.4)	(0.1, 0.7)
h_5	(0.7, 0.0)	(0.8, 0.1)	(0.8, 0.0)	(0.7, 0.1)	(0.5, 0.3)	(0.7, 0.1)
h_6	(0.8, 0.0)	(0.8, 0.1)	(0.5, 0.4)	(0.7, 0.1)	(0.8, 0.1)	(0.5, 0.1)

Table 2

Level soft set $L(\varpi; topbottom_{\varpi})$ with choice values

U	ε_1	ε_2	ε_3	ε_4	ε_5	ε_6	Choice value (c_i)
h_1	0	0	0	0	0	0	$c_1 = 0$
h_2	1	1	1	0	0	0	$c_2 = 3$
h_3	0	0	0	0	0	0	$c_3 = 0$
h_4	0	0	0	0	0	0	$c_4 = 0$
h_5	0	1	1	1	0	1	$c_5 = 4$
h_6	1	1	0	1	1	0	$c_6 = 4$

From the above example, it can see that level soft sets of intuitionistic fuzzy soft sets serve as bridges between intuitionistic fuzzy soft sets and crisp soft sets, and the choice value of an alternative in the level soft set represents the number of criteria satisfied by the alternatives at certain level of membership degrees. Algorithm 1 usually choose the alternatives which satisfy the condition at utmost, but not the best one under the situation. There are two optimal alternatives in Example 3.1, but we don't know which one is better. Thus, in decision making problems, we should consider not only the difference of criteria of an alternative, but also the entirety of criteria of an alternative.

4. A new approach based on entropy weight

In order to propose a new approach to intuitionistic fuzzy soft set based decision making, we introduce the following notions.

It is well known that for fuzzy sets entropy is a measure of fuzziness, while the notion of IFSs is a generalization of fuzzy sets, IFSs entropy is expected a measure of intuitionistic fuzziness [22]. By intuition judgement we realize that some of axiomatic requirements of an intuitionistic fuzzy entropy ignore the influence of the degree of indeterminacy. If $\mu_{\tilde{A}}(x) = \nu_{\tilde{A}}(x) = 0$ for all $x \in X$, we can not get any information of the object, the entropy of the IFS depends solely on the intuitionistic component, since the degree of intuitionistic fuzziness is zero, therefore the entropy value should be maximum. The greater of the degree of indeterminacy, the greater of the value of IFSs entropy. Hence, the entropy measure should consider the influence of the degrees of intuitionistic fuzziness and indeterminacy. Inspired by [28], we can give a modified definition of intuitionistic fuzzy entropy as follows.

Definition 4.1. A real-valued function $H : IFS(X) \to [0,1]$ is called an entropy for IFSs, if it satisfies the following axiomatic requirements, let $\tilde{A}, \tilde{B} \in IF(X)$:

(1) $H(\tilde{A}) = 0$ iff $\tilde{A}$ is a crisp set.
(2) $H(\tilde{A}) = 1$ iff $\mu_{\tilde{A}}(x) = \nu_{\tilde{A}}(x) = 0, \pi_{\tilde{A}}(x) = 1$ for any $x \in X$.
(3) If $\pi_{\tilde{A}}(x) \geq \pi_{\tilde{B}}(x)$ and $|\mu_{\tilde{A}}(x) - \nu_{\tilde{A}}(x)| = |\mu_{\tilde{B}}(x) - \nu_{\tilde{B}}(x)|$ for all $x \in X$, then $H(\tilde{A}) \geq H(\tilde{B})$.
(4) If $|\mu_{\tilde{A}}(x) - \nu_{\tilde{A}}(x)| \leq |\mu_{\tilde{B}}(x) - \nu_{\tilde{B}}(x)|$ and $\pi_{\tilde{A}}(x) = \pi_{\tilde{B}}(x)$ for all $x \in X$, then $H(\tilde{A}) \geq H(\tilde{B})$.
(5) $H(\tilde{A}) = H(\tilde{A}^c)$.

Then according to Definition 4.1, we can give a new formula of entropy measure.

Theorem 4.2. *Let $\tilde{A}$ be an intuitionistic fuzzy set in the universe of discourse $X = \{x_1, x_2, \cdots, x_n\}$, then*

$$H_Y(\tilde{A}) = \frac{1}{n}\sum_{i=1}^{n} \frac{1-(\mu_{\tilde{A}}(x_i)-\nu_{\tilde{A}}(x_i))^2}{1+\mu_{\tilde{A}}(x_i)+\nu_{\tilde{A}}(x_i)}.$$

is entropy measure of the intuitionistic fuzzy set $\tilde{A}$ which satisfies the requirement conditions of Definition 4.1.

Proof. Let $\tilde{A}, \tilde{B} \in IF(X)$.

(1) Suppose that $H_Y(\tilde{A}) = 0$. Since $\frac{1-(\mu_{\tilde{A}}(x)-\nu_{\tilde{A}}(x))^2}{1+\mu_{\tilde{A}}(x)+\nu_{\tilde{A}}(x)} \geq 0$ for all $x \in X$, hence $\mu_{\tilde{A}}(x) = 0, \nu_{\tilde{A}}(x) = 1$ or $\mu_{\tilde{A}}(x) = 1, \nu_{\tilde{A}}(x) = 0$, thus $\tilde{A}$ is a crisp set. The converse is obvious.
(2) Assume that $H_Y(\tilde{A}) = 1$, then $\frac{1-(\mu_{\tilde{A}}(x)-\nu_{\tilde{A}}(x))^2}{1+\mu_{\tilde{A}}(x)+\nu_{\tilde{A}}(x)} = 1$ for all $x \in X$, hence $\mu_{\tilde{A}}(x) - \nu_{\tilde{A}}(x) = 0$ and $\mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) = 0$, so $\mu_{\tilde{A}}(x) = \nu_{\tilde{A}}(x) = 0$ and $\pi_{\tilde{A}}(x) = 1$. The converse is obvious.
(3) If $\pi_{\tilde{A}}(x) \geq \pi_{\tilde{B}}(x)$ and $|\mu_{\tilde{A}}(x) - \nu_{\tilde{A}}(x)| = |\mu_{\tilde{B}}(x) - \nu_{\tilde{B}}(x)|$ for all $x \in X$, then $\mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) \leq \mu_{\tilde{B}}(x) + \nu_{\tilde{B}}(x)$, hence $\frac{1-(\mu_{\tilde{A}}(x)-\nu_{\tilde{A}}(x))^2}{1+\mu_{\tilde{A}}(x)+\nu_{\tilde{A}}(x)} \geq \frac{1-(\mu_{\tilde{B}}(x)-\nu_{\tilde{B}}(x))^2}{1+\mu_{\tilde{B}}(x)+\nu_{\tilde{B}}(x)}$, so $H(\tilde{A}) \geq H(\tilde{B})$.
(4) If $|\mu_{\tilde{A}}(x) - \nu_{\tilde{A}}(x)| \leq |\mu_{\tilde{B}}(x) - \nu_{\tilde{B}}(x)|$ and $\pi_{\tilde{A}}(x) = \pi_{\tilde{B}}(x)$ for all $x \in X$, then $\frac{1-(\mu_{\tilde{A}}(x)-\nu_{\tilde{A}}(x))^2}{1+\mu_{\tilde{A}}(x)+\nu_{\tilde{A}}(x)} \geq \frac{1-(\mu_{\tilde{B}}(x)-\nu_{\tilde{B}}(x))^2}{1+\mu_{\tilde{B}}(x)+\nu_{\tilde{B}}(x)}$, so $H(\tilde{A}) \geq H(\tilde{B})$.
(5) Obviously.

□

To show our entropy measure is effective, we cite some researchers' entropy measures of the intuitionistic fuzzy set $\tilde{A}$ of n elements as follows.

Szmidt and Kacprzyk [21] defined entropy measure of $\tilde{A}$ as

$$H_S(\tilde{A}) = \frac{1}{n}\sum_{i=1}^{n} \frac{maxCount(\tilde{A}_i \cap \tilde{A}_i^c)}{maxCount(\tilde{A}_i \cup \tilde{A}_i^c)}.$$

where $\tilde{A}_i = (x_i, \mu_{\tilde{A}}(x_i), \nu_{\tilde{A}}(x_i))$ and $maxCount(\tilde{A}_i) = \mu_{\tilde{A}}(x_i) + \pi_{\tilde{A}(x)}$

Li et al. [14] defined a new formula of entropy measure of $\tilde{A}$ as

$$H_L(\tilde{A}) = \frac{\sum_{i=1}^{n}(min(\mu_{\tilde{A}}(x_i), \nu_{\tilde{A}}(x_i)))}{\sum_{i=1}^{n}(max(\mu_{\tilde{A}}(x_i), \nu_{\tilde{A}}(x_i)))}.$$

Vlachos and Sergiadis [22] defined entropy measure of IFSs which is the extension of nonprobability entropy for fuzzy sets as

$$H_V(\tilde{A}) = -\frac{1}{nln2}\sum_{i=1}^{n}\left[\mu_{\tilde{A}}(x_i)ln\frac{\mu_{\tilde{A}}(x_i)}{\mu_{\tilde{A}}(x_i)+\nu_{\tilde{A}}(x_i)} + \nu_{\tilde{A}}(x_i)ln\frac{\nu_{\tilde{A}}(x_i)}{\mu_{\tilde{A}}(x_i)+\nu_{\tilde{A}}(x_i)} - \pi_{\tilde{A}}(x_i)ln2\right].$$

Example 4.3. Let $\tilde{A}_1 = \{(x, 0.3, 0.0)\}$, $\tilde{A}_2 = \{(x, 0.5, 0.0)\}$, $\tilde{A}_3 = \{(x, 0.6, 0.4)\}$, $\tilde{A}_4 = \{(x, 0.1, 0.8)\}$, $\tilde{A}_5 = \{(x, 0.4, 0.4)\}$, $\tilde{A}_6 = \{(x, 0.5, 0.5)\}$ be intuitionistic fuzzy sets in the set $X = \{x\}$. We calculate the entropies of the IFSs $\tilde{A}_i$, $i = 1, 2, 3, 4, 5, 6$ and obtain the entropy values of IFSs $\tilde{A}_i$ with tabular representation as in Table 3.

From Table 3, we see that the entropy values of IFSs $\tilde{A}_1$ and $\tilde{A}_2$ are equal to H_L, they are contradictory with our intuitive judgment. The entropy values of IFSs $\tilde{A}_3$ and $\tilde{A}_4$ calculated by H_V are very larger than the ones calculated by other formulas, therefore it is not efficient to measure the entropy of IFS. For $\tilde{A}_5$, $\mu_{\tilde{A}_5} = \nu_{\tilde{A}_5} = 0.4$ and $\pi_{\tilde{A}_5} = 0.2$, then the degrees of intuitionistic fuzziness and indeterminacy exist; for $\tilde{A}_6$, $\mu_{\tilde{A}_6} = \nu_{\tilde{A}_6} = 0.5$ and $\pi_{\tilde{A}_6} = 0$, only the degree of intuitionistic fuzziness exists, while the entropy values calculated by the previous formulas are the same, thus they can not describe the difference between IFSs $\tilde{A}_5$ and $\tilde{A}_6$. Our formula of entropy measure is very efficient to measure the information of the IFSs $\tilde{A}_i$.

Table 3

Tabular representation of the entropy values of IFSs $\tilde{A}_i$

i	1	2	3	4	5	6
$\tilde{A}_i$	$(0.3, 0.0)$	$(0.5, 0.0)$	$(0.6, 0.4)$	$(0.1, 0.8)$	$(0.4, 0.4)$	$(0.5, 0.5)$
$H_S(\tilde{A}_i)$	0.6000	0.5000	0.6667	0.2222	1.0000	1.0000
$H_L(\tilde{A}_i)$	0	0	0.6667	0.1250	1.0000	1.0000
$H_V(\tilde{A}_i)$	0.6000	0.5000	0.9710	0.5529	1.0000	1.0000
$H_Y(\tilde{A}_i)$	0.6000	0.5000	0.4800	0.2684	0.5556	0.5000

Since the ordered relation in IFS is not total, and not any two intuitionistic fuzzy sets are comparable. While score function can solve the problem by converting IFNs into real numbers. In the following, we define a new score function of IFS with the influence of the degree of indeterminacy.

Definition 4.4. Let $\alpha = (\mu_\alpha, \nu_\alpha)$ be a IFN, then the score function of α is:

$$S(\alpha) = \mu_\alpha - \nu_\alpha + \frac{2(\mu_\alpha - \nu_\alpha)}{1+\mu_\alpha+\nu_\alpha}\pi_\alpha.$$

By extending some concepts of [5], we introduce some basic notions as follows. From now on, $|A|$ is the cardinality of A and $N_A = \{1, 2, \cdots, |A|\}$.

Definition 4.5. Let $\varpi = (F, A)$ be an intuitionistic fuzzy soft set over the universe U, where $A \subseteq E$ and λ be a threshold intuitionistic fuzzy set. The function $\delta_{L(\varpi;\lambda)} : U \to N$ is defined by

$$\delta_{L(\varpi;\lambda)}(x) = \sum_{\varepsilon \in A} \chi_{F_\lambda(\varepsilon)}(x)$$

is called the choice value function of ϖ respect to λ, where $\chi_{F_\lambda(\varepsilon)}$ denotes the characteristic function of $F_\lambda(\varepsilon)$.

Definition 4.6. Let $\varpi = (F, A)$ be an intuitionistic fuzzy soft set over the universe U, where $A \subseteq E$ and λ be a threshold intuitionistic fuzzy set. The choice value soft set of ϖ respect to λ is a soft set $\Gamma(L(\varpi;\lambda)) = (\kappa_{L(\varpi;\lambda)}, N_A)$ over U, where $\kappa_{L(\varpi;\lambda)} : N_A \to \mathcal{P}(\mathcal{U})$ is given by

$$\kappa_{L(\varpi;\lambda)}(n) = \{x \in U | \delta_{L(\varpi;\lambda)}(x) \geq n\}.$$

Let's denote $\Upsilon_{L(\varpi;\lambda)} = max\{\delta_{L(\varpi;\lambda)}(x) | x \in U\}$ as the choice value rank of the level soft set of ϖ respect to λ.

Let $\varpi = (F, A)$ be intuitionistic fuzzy soft set over $U = \{x_1, x_2, , \cdots, x_n\}$ where U is a set of alternatives and $A = \{\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_m\}$ is a set of criteria. Assume that the criteria are dependent to each other. If the value of $F(\varepsilon_j)(x_i)$ is denoted by $\alpha_{ij} = (\mu_{ij}, \nu_{ij})$, then the value of the ith alternative is $\alpha_i = \{\alpha_{ij} | j = 1, 2, \cdots, m\}$, $i = 1, 2, \cdots, n$. We can establish an exact model of entropy weights for determining weight ω_j of the jth criteria $\varepsilon_j \in A$ as follows:

$$\omega_j = \frac{1 - H(F(\varepsilon_j))}{m - \Sigma_{k=1}^m H(F(\varepsilon_k))},$$

then $\omega = (\omega_1, \omega_2, \cdots, \omega_m)$ is a set of criteria weights.

Now we show our adjustable approach as follows:

Algorithm 2.

(1) Input the (result) intuitionistic fuzzy soft set $\varpi = (F, A)$.
(2) Input a threshold intuitionistic fuzzy set $\lambda : A \to [0, 1] \times [0, 1]$ for decision making.
(3) Compute the level soft set $L(\varpi; \lambda)$.
(4) Present the level soft set $L(\varpi; \lambda)$ in tabular form and compute the choice value c_i of x_i, $\forall i$.
(5) If $|\kappa_{L(\varpi;\lambda)}(\Upsilon_{L(\varpi;\lambda)})| = 1$, then the optimal decision is to select x_k where $x_k \in \kappa_{L(\varpi;\lambda)}(\Upsilon_{L(\varpi;\lambda)})$, else if $|\kappa_{L(\varpi;\lambda)}(\Upsilon_{L(\varpi;\lambda)})| \geq 2$, then go to 6.
(6) Compute the entropy weight ω_j of each parameter $\varepsilon_j \in A$.
(7) Calculate the score $S(x_i)$ of alternative $x_i \in \kappa_{L(\varpi;\lambda)}(\gamma)$:

$$S(x_i) = \sum_{j=1}^m \omega_j S(\alpha_{ij}).$$

Then the optimal decision is to select x_k if

$$S(x_k) = max\{S(x_i) | x_i \in \kappa_{L(\varpi;\lambda)}(\gamma)\}.$$

It is see that our proposed algorithm is an adjustable one for Jiang et al.'s. Compared with Jiang et al. algorithm, our algorithm emphasizes how to choose the best one from the alternatives which has the largest numbers of criteria satisfied by the alternatives at certain level of membership degrees.

5. Example illustration

To illustrate the basic idea of Algorithm 2, we apply it to the following examples.

Example 5.1. Let us reconsider Example 3.1, if we deal with this problem by threshold intuitionistic fuzzy set $\lambda = topbottom_\varpi$, then we obtain the level soft set $L(\varpi; topbottom_\varpi)$ with choice values with tabular representation as in Table 2. We have $\Upsilon_{L(\varpi;\lambda)} = 5$, $|\kappa_{L(\varpi;topbottom_{\varpi})}(5)| = 2$ and $h_5, h_6 \in \kappa_{L(\varpi;topbottom_{\varpi})}(5)$. By calculating the set of criteria weights

$$\omega = (0.1805, 0.1747, 0.1890, 0.1770, 0.1371, 0.1418),$$

we get $S(h_5) = 0.7581$ and $S(h_6) = 0.6534$. So the optimal decision is to select h_5.

Example 5.2. Let us suppose there is an investment company, which wants to invest a sum of money in the best option (adapted from [8]). Let us consider an intuitionistic fuzzy soft set $\mathcal{G} = (F, A)$ which describes the "attractiveness of companies" that the investment company is considering for investment. Suppose there are possible five alternative companies in the domain $U = \{u_1, u_2, u_3, u_4, u_5\}$: u_1 is a car company, u_2 is a food company, u_3 is a computer company, u_4 is a arms company, u_5 is a TV company. The investment company must take a decision according to criteria set $A = \{a_1, a_2, a_3, a_4\}$, where a_1 is the risk analysis, a_2 is the growth analysis, a_3 is the social-political impact analysis, a_4 is the environment analysis. The five possible alternatives u_i $(i = 1, 2, 3, 4, 5)$ are to be evaluated using the intuitionistic fuzzy information by the decision maker under the above criteria, as listed in Table 4.

If we deal with the problem with threshold intuitionistic fuzzy set $\lambda = mid_{\mathcal{G}}$ where $mid_{\mathcal{G}} : A \to [0,1] \times [0,1]$ defined by $\mu_{mid_{\mathcal{G}}}(a) = \frac{1}{|U|}\sum_{u \in U} \mu_{F(a)}(u)$ and $\nu_{mid_{\mathcal{G}}}(a) = \frac{1}{|U|}\sum_{u \in U} \nu_{F(a)}(u)$, thus we obtain the level soft set $L(\mathcal{G}; mid_{\mathcal{G}})$ with choice values with tabular as in Table 5.

Thus $\Upsilon_{L(mid_{\mathcal{G}};\lambda)} = 3$, then $|\kappa_{L(\mathcal{G};mid_{\mathcal{G}})}(3)| = 1$ and $u_5 \in \kappa_{L(\mathcal{G};mid_{\mathcal{G}})}(3)$. Hence the optimal decision is to select u_5. The result is consistent with [23].

Table 4

Intuitionistic fuzzy soft set $\mathcal{G} = (F, A)$

U	a_1	a_2	a_3	a_4
u_1	$(0.5, 0.4)$	$(0.6, 0.3)$	$(0.3, 0.6)$	$(0.6, 0.2)$
u_2	$(0.7, 0.3)$	$(0.7, 0.2)$	$(0.7, 0.2)$	$(0.4, 0.5)$
u_3	$(0.5, 0.4)$	$(0.5, 0.4)$	$(0.5, 0.3)$	$(0.4, 0.3)$
u_4	$(0.8, 0.1)$	$(0.6, 0.3)$	$(0.3, 0.4)$	$(0.2, 0.6)$
u_5	$(0.6, 0.2)$	$(0.8, 0.1)$	$(0.7, 0.1)$	$(0.7, 0.3)$

Table 5

Level soft set $\mathcal{G} = (F, A)$ with choice values

U	a_1	a_2	a_3	a_4	Choice value (c_i)
u_1	0	0	0	1	$c_1 = 1$
u_2	0	1	1	0	$c_2 = 2$
u_3	0	0	1	0	$c_3 = 1$
u_4	1	0	0	0	$c_4 = 1$
u_5	0	1	1	1	$c_5 = 3$

6. Conclusions

In the paper, through analyzing Jiang et al. approach, we proposed a novel decision-making approach with entropy weight based on intuitionistic fuzzy soft sets. We computed the weight of each parameter by our new entropy measure and chose the final optimal decisions based on the scores of alternatives at certain level. To extend this work, one can apply the entropy measure of IFSs to other practical applications or discuss how to cope with the weights of parameters.

Acknowledgements. The authors are thankful to the anonymous referees for their valuable comments and constructive suggestions which greatly improved the quality of this paper.

References

[1] H. Aktaş and N. Çağman, Soft sets and soft groups, Inform. Sci. 177 (2007) 2726–2735.
[2] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20 (1986) 87–96.
[3] T. Changphas and B. Thongkam, On soft Γ-semigroups, Ann. Fuzzy Math. Inform. 4(2) (2012) 217–223.
[4] F. Feng, Soft rough sets applied to multicriteria group decision making, Ann. Fuzzy Math. Inform. 2(1) (2011) 69–80.
[5] F. Feng, Y. Li and N. Çağman, Generalized uni-int decision making schemes based on choice value soft sets, European J. Oper. Res. 220 (2012) 162–170.
[6] F. Feng, Y. B. Jun, X. Liu and L. Li, An adjustable approach to fuzzy soft set based decision making, J. Comput. Appl. Math. 234 (2010) 10–20.
[7] F. Feng, Y. Li and V. Leoreanu-Fotea, Application of level soft sets in decision making based on interval-valued fuzzy soft sets, Comput. Math. Appl. 60 (2010) 1756–1767.
[8] F. Herrera, E. Herrera-Viedma and L. Martnez, A fusion approach for managing multi-granularity linguistic term sets in decision making, Fuzzy Sets and Systems 114 (2000) 43–58.
[9] Y. Jiang, Y. Tang and Q. Chen, An adjustable approach to intuitionistic fuzzy soft sets based decision making, Appl. Math. Model. 35 (2011) 824–836.
[10] Y. B. Jun and C. H. Park, Applications of soft sets in ideal theory of BCK/BCI-algebras, Inform. Sci. 178 (2008) 2466–2475.
[11] Y. B. Jun, K. J. Lee and E. H. Roh, Intersectional soft BCK/BCI-ideals, Ann. Fuzzy Math. Inform. 4(1) (2012) 1–7.
[12] Y. B. Jun, K. J. Lee and C. H. Park, Soft set theory applied to ideals in d-algebras, Comput. Math. Appl. 57 (2009) 367–378.
[13] Z. Kong, L. Wang and Z. Wu, Application of fuzzy soft set in decision making problems based on grey theory, J. Comput. Appl. Math. 236 (2011) 1521–1530.
[14] J. Li, G. Deng, H. Li and W. Zeng, The relationship between similarity measure and entropy of intuitionistic fuzzy sets, Inform. Sci. 188 (2012) 314–321.
[15] P. K. Maji, R. Biswas and A. R. Roy, Fuzzy soft sets, J. Fuzzy Math. 9(3) (2001) 589–602.
[16] P. K. Maji, A. R. Roy and R. Biswas, On intuitionistic fuzzy soft sets, J. Fuzzy Math. 12 (2004) 669–683.
[17] P. K. Maji, A neutrosophic soft set approach to a decision making problem, Ann. Fuzzy Math. Inform. 3(2) (2012) 313–319.
[18] D. Molodtsov, Soft set theory-first results, Comput. Math. Appl. 37 (1999) 19–31.
[19] A. R. Roy and P. K. Maji, A fuzzy soft set theoretic approach to decision making problems, J. Comput. Appl. Math. 203 (2007) 412–418.
[20] M. Shabir and A. Ahmad, On soft ternary semigroups, Ann. Fuzzy Math. Inform. 3(1) (2012) 39–59.
[21] E. Szmidt and J. Kacprzyk, Entropy for intuitionistic fuzzy sets, Fuzzy Sets and Systems 118 (2001) 467–477.

[22] I. K. Vlachos and G. D. Sergiadis, Intuitionistic fuzzy information-Applications to pattern recognition, Pattern Recogn. Lett. 28 (2007) 197–206.
[23] G. W. Wei, Gray relation alanalysis method for intuitionistic fuzzy multiple attribute decision making, Expert Syst. Appl. 38 (2011) 11671–11677.
[24] H. Wu and J. Zhan, The characterizations of some kinds of soft hemirings, Ann. Fuzzy Math. Inform. 3(2)(2012) 267–280.
[25] Z. Xiao, K. Gong and Y. Zou, A combined forecasting approach based on fuzzy soft sets, J. Comput. Appl. Math. 228 (2009) 326–333.
[26] Y. Yang, X. Tan and C. Meng, The multi-fuzzy soft set and its application in decision making, Appl. Math. Model. 37 (2013) 4915–4923.
[27] J. Zhan and Y. B. Jun, Soft BL-algebras based on fuzzy sets, Comput. Math. Appl. 59 (2010) 2037–2046.
[28] L. Zhang, X. Xin and Q. Liu, Algebra properties and fuzzy entropy of vague sets (in chinese), Computer Engineering and Applications 46 (2010) 32–35.

Yong Wei Yang (yangyw2010@126.com)
Department of Mathematics, Northwest University, Xi'an, 710127, China

Ting Qian (qiant2000@126.com)
Department of Mathematics, Northwest University, Xi'an, 710127, China