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Contrast removal fuzzy operator in image processing

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ABSTRACT. In the classical image processing paradigm, the fundamental idea of image enhancement is to produce a new image such that it exposes information for then the original image. Fuzzy logic methods are one of the valuable and frequently used techniques among many other image enhancement approaches. In this paper, a method for image enhancement based on the Contrast Removal Fuzzy Operator is established. Experiments were performed using grayscale and colour images.

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Keywords: Image enhancement, Contrast Removal Fuzzy Operator, Over-contrast image, Over-bright image, low-contrast image.

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1. INTRODUCTION

Image enhancement plays a fundamental role in image processing applications where human beings (the expert) make decisions depending on the image information. Type of image enhancement include noise reduction, edge enhancement and contrast enhancement. Enhancement may be used to restore an image that has suffered some kind of deterioration or to enhance certain features of an image. In image processing applications, one has to use expert knowledge to overcome the difficulties (e.g. object recognition, scene analysis). Fuzzy set theory and fuzzy logic are powerful tools to represent and process human knowledge in the form of fuzzy if-then rules. The difficulties in image processing arise because the data/tasks/results are uncertain. This uncertainty, however, is not always due to the randomness but due to the ambiguity and vagueness.

Image enhancement methods were categorized by Gonzalez and Woods [4] into two broad classes: Transform domain methods and spatial domain methods. The techniques in the first category are based on modifying the frequency transform of an image. However, computing a two dimensional transform for a large array (an image) is a very time consuming task even the fast transformation techniques [10] (Lee, 1980) are not suitable for real time processing. The techniques in the second category directly operate on the pixels. Contrast enhancement is one of the important image enhancement techniques in spatial domain. Besides two popular methods: histogram equalization and histogram specifications [4, 5] (Gonzalez and Woods, 1992), we may mention a few important spatial domain methods such as an iterative histogram modification of gray images [3] (Gauch, 1992), an efficient adaptive neighborhood histogram equalization [13] (Mukherjee and Chatterji, 1995) and Gabor's technique [12] (Lindenbaum et al., 1994). Adaptive neighborhood histogram method achieves better identification of different gray level histogram in the locality of every pixel. Lindenbaum et al. (1994) have used Gabor's technique for image enhancement, edge detection and segmentation. They have suggested a method for image deblurring based on directional sensitive filters. Because of poor and non-uniform lighting conditions of the object and non-linearity of the imaging system, vagueness is introduced in the acquired image. This vagueness in an image appears in the form of imprecise boundaries and colour values. Fuzzy set [16] (Zadeh, 1973) offers a problems solving tool between the precision of classical mathematics and the inherent imprecision of the real world. In 1991, Zimmermann [9] initiated that the imprecision in an image contained within colour value can be handled using fuzzy sets. The notations like "good contrast" or "sharp boundaries", "light red", "dark green" etc. used in image enhancement by fuzzy logic are termed as linguistic hedges. These hedges can be perceived qualitatively by the human reasoning. As they lack in crisp and exhaustive quantification, they may not be understood by a machine. To overcome this limitation to a large extent, fuzzy logic tools empower a machine to mimic human reasoning. In the fuzzy framework of image enhancement and smoothing, two contributions merit an elaboration. The first one deals with 'IF..THEN. ELSE' fuzzy rules [15] (Russo and Ramponi, 1995) for image enhancement. Here, a set of neighbourhood pixels form the antecedent part of the rule and the pixel to be enhanced is changed by the consequent part of the rule. These fuzzy rules give directives much similar to human-like reasoning. The second one proposes a rule based filtering [2] (Choi and Krishnapuram, 1997) in which different filter classes are devised on the basis of compatibility with the neighbourhood. We now discuss another kind of approaches where some pixel properties like, gray tone, or colour intensity is modeled into a fuzzy set using a membership function. In these approaches, an image can be considered as an array of fuzzy singletons [14] (Pal and King, 1981) having a membership value that denotes the degree of some image property in the range [0, 1]. An intensification operator globally modifies the membership function. The approach by Hauli and Yang [8] (1989) describes an efficient enhancement based on fuzzy relaxation technique. Different orders of fuzzy membership functions and different statistics are attempted to improve the enhancement speed and quality respectively. In 1997, Hanmandlu et al. have proposed a Gaussian type of fuzzification function [6] that contains a single fuzzifier and a new intensification operator called NINT. Fuzzifier is obtained by maximizing the fuzzy contrast and the parameter is obtained by minimizing the entropy. Note that the above works have been confined to the enhancement of gray images only. Cheng and Huijuan [1] in 2000 have proposed a fuzzy logic approach to contrast enhancement. Then Hanmandlu, Jha and Sharma [7] in 2003 described colour image enhancement by fuzzy intensification. Gonzalez and Woods in 2002 described Histogram Equalization (HE) and Kim et al. in 2001 described Partially Overlapped Sub-Block Histogram Equalization (POSHE) for image contrast enhancement. Leung [11] in 2005 presented a method based on Generalized Fuzzy Operator, pre-processed with HE and POSHE for low contrast and low illumination images. In 2006 Leung [16] described a new method in contrast correction based on the generalized fuzzy operator with least square method. However, all these methods do not work on images with overbright and over-contrast conditions. In this paper, we propose a new approach for image enhancement based on the Contrast Removal Fuzzy Operator (CRFO). This method gives better performance in image enhancement of extremely over-bright, over-contrast, low- contrast and also in low-illumination condition.

This paper is presented as follows: In section 2, background of the study is cited. In Section 3 image enhancement using our new operator and algorithm are shown. In section 4, applications of the proposed method on low- contrast, low-illumination, over-contrast and over-bright real images are given. A comparative study is also done with three other methods. Lastly conclusion of the paper is given.

2. Preliminaries

In this section the preliminaries necessary for further study are cited.

Definition 2.1 ([14]). An image G of size $m \times n$ and L gray levels k ranging between 0 to L - 1 can be consider as an array of fuzzy singletones, each having a value of membership denoting its degree of brightness relative to some brightness lavels. For an image G, we can write in the notion of fuzzy sets as

$$G = \{ \langle G_k(i,j), \mu_{G_k}(i,j) \rangle | i = 1, 2, \dots, m, j = 1, 2, \dots, n \}$$
(1)

where $G_k(i,j)$ is the value of G at position (i,j) at any gray level k, $\mu_{G_k}(i,j)$ denotes the degree of brightness possessed by the gray level intensity $G_k(i,j)$ of the (i,j)th pixel.

Example 2.2. In figure 1(ii) we have given a gray scale image of size 4×4 pixels.

Definition 2.3 ([11]). A Generalized Fuzzy Set (GFS), S, in the region, R, is defined as

$$S = \int \frac{\mu_S(x)}{x}, x \in R \tag{2}$$

here $\mu_S(x) \in [-1,1]$ is called the Generalized Membership Function (GMF) of S on R. When $\mu_S(x) \in [-1,0]$, the GMF of x in S is independent on R. When $\mu_S(x) \in (0,1]$, the GMF of x in S is dependent on R. When $\mu_S(x) = 0$, the GMF is the fuzzy bound point function (FBF) in S. If $\mu_S(x) = -1$ or $\mu_S(x) = +1$, we can say that x is full independent to R and full dependent to R respectively. If R is finite and only includes finite elements, $R = \{x_1, x_2, ... x_n\}$, then the GFS, S can be defined by

$$S = \bigcup_{i=1}^{n} \frac{\mu_S(x_i)}{x_i} \tag{3}$$

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0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.	7	0.8	0.9	1.0
(i)											
			0.	.1 0	.6 0	.4 (0.8				
			0.	.0 0	.3 1	.0	0.7				
			0.	.4 0	.6 0	.7	0.0				
		A =	о. Г	.2 0	.8 0	.5 (0.6	ı			
(ii)											

FIGURE 1. (i) Degree of brightness in different gray levels. (ii) Gray scale image as a fuzzy set.

Definition 2.4 ([11]). Let p_i have some properties in the grades, where $x_i \in R$ (i = 1, 2, ..N), and $p_i \in [-1, 1]$. Then, the function set, P, consisting of p_i is called a Generalized Property Set (GPS) in R. If a 2-D gray image, $X = (x_{ij})$, with an $M \times N$ matrix, based on Eq.(3), X can be written as

$$X = \bigcup_{i=1}^{M} \bigcup_{j=1}^{N} \frac{p_{ij}}{x_{ij}}$$

$$\tag{4}$$

where $p_{ij}/x_{ij}, -1 \le p_{ij} \le 1$, denotes the grade, p_{ij} , that may have some properties of every element (i, j) in image X.

Definition 2.5 ([11]). A Generalized Fuzzy Operator (*GFO*) as a function of the GFS, $\mu_S(x)$, and then generate another fuzzy set, $\mu_{S'}(x)$, i.e.

$$\mu_{S'}(x) = GFO\left[\mu_S(x)\right] \tag{5}$$

$$\mu_{S'}(x) = \begin{cases} \sqrt[\beta]{1 - [1 + \mu_S(x)]^\beta}, & -1 \le \mu_S(x) < 0\\ [\mu_S(x)]^\beta, & 0 \le \mu_S(x) < r\\ \sqrt[\beta]{1 - \alpha[1 + \mu_S(x)]^\beta}, & r \le \mu_S(x) \le 1 \end{cases}$$

where β can range from 1 to infinity.

3. Image enhancement using Contrast Removal Fuzzy Operator

Image enhancement suppresses the disturbances such as noise, blurring, geometrical distortions, and illumination corrections. Whenever the properties of an image cannot be numerically quantified, fuzzy image enhancement techniques take a major role. Fuzzy image enhancement is based on gray level mapping into a fuzzy plane, using a membership transformation function.

3.1. Contrast Removal Fuzzy Operator (CRFO). In a low-contrast image most of the gray level lies in the lower luminance range (typically under 0.5 on a [0; 1] scale) and fewer gray level lies in the upper luminance range. A higher contrast in an image can be achieved by darkening the gray level in the lower luminance range (typically under 0.5 on a [0; 1] scale) and brightening the ones in the upper luminance range. Moreover in the case of over-bright images all gray level lies in the upper luminance range (typically above 0.5 on a [0; 1] scale) and fewer gray level lies in the upper luminance range. Enhancements of such images are very difficult task.

We define Contrast Removal Fuzzy Operator (CRFO) for enhancement of extremely over-bright, over-contrast, low-contrast and also low-illumination images. A mathematical expression of such operator for an image G at any gray level k may be denoted as, $CRFO:[0, 1] \rightarrow [0, 1]$ and expressed as

$$CRFO(\mu_{G_k}(i,j)) = \mu_{G_k}^{/}(i,j) = \{1 - e^{-\alpha \mu_{G_k}^{2}(i,j)}, 0 \le \mu_{G_k}(i,j) \le 1$$
(6)

where $\mu_{G_k}(i, j)$ denotes the degree of brightness possessed by the gray level intensity $G_k(i, j)$ of the (i, j) th pixel, $\mu'_{G_k}(i, j)$ is the modified membership value of $\mu_{G_k}(i, j)$ by CRFO and α is an intensification parameter which can range from 1 to infinity. Then we generate new gray level $G'_k(i, j)$ for $G_k(i, j)$ by the following way

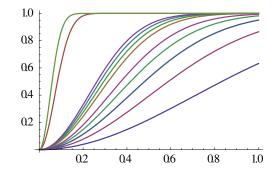


FIGURE 2. Graph of CRFO for $\alpha = 1, 2, ..., 10, 100, 200$ from bottom to top respectively.

$$G_{k}^{\prime}(i,j) = (L-1)\left(1 - \left(1 - \mu_{G_{k}}^{\prime}(i,j)^{\frac{\sigma}{\tau}}\right)\right)$$
(7)

where L is the highest gray level of the image, $\sigma > 0$ and $\tau \ge 1$ are arbitrary parameters.

The CRFO operator distribute membership values from lower luminance (for the case of low-contrast images) to the interval [0, 1] and upper luminance (for the case of over-bright images) to the interval [0, 1]. Fig. 2 represents the graph of CRFO for different value of α . It is easy to see from the Fig. 2 that, as the value of parameter α increases degree of brightness of image increases and as α decreases the degree of brightness also decreases.

This method uses the CRFO to reduce the fuzziness from the image that results in an image enhancement with good quality improvement. For the enhancement of gray level images by this method we need computer algorithm, which is formulated as follows:

Algorithm1:

Step 01. Input the given image G of size $m \times n$ with L gray levels k ranging between 0 to L - 1.

Step 02. Define the membership function.

$$u_{G_{k}}(i,j) = \frac{G_{k}(i,j)}{L-1} | i = 1, 2, ..., m; j = 1, 2, ..., n$$

Step 03. Set the parameter α .

Step 04. Set the CRFO in parametric form.

$$CRFO(\mu_{G_k}(i,j)) = \mu'_{G_k}(i,j) = \{1 - e^{-\alpha\mu'_{G_k}(i,j)}, 0 \le \mu_{G_k}(i,j) \le 1\}$$

Step 05. Set the parameter $\sigma > 0$ and $\tau \ge 1$.

Step 06. Generate new gray levels.

$$G_{k}^{/}(i,j) = (L-1)\left(1 - \left(1 - \mu_{G_{k}}^{/}(i,j)^{\frac{\sigma}{\tau}}\right)\right)$$

Step 07. Enhanced image is obtained using contrast removal fuzzy operator.

Step 08. Analysis is made on all the images and the same is displayed.

Color image enhancement is a very difficult task in image processing. Any unique algorithm is certainly never established to take all kind of images. In the case of colour images each colour channel red (r), green (g) and blue (b) are modified by the Eqs. 6-7 separately. For the enhancement of colour images by this method, we need computer algorithm, which is formulated as follows:

Algorithm 2:

Step 01. Input the given image G of size $m \times n$ with L colour levels k ranging between 0 to L - 1.

Step 02. Define the membership function for each of the colour red (r), green (g) and blue (b).

$$\begin{split} \mu_{G_k^r}\left(i,j\right) &= \frac{G_k^r\left(i,j\right)}{L-1} | i = 1, 2, ..., m; j = 1, 2, ..., n\\ \mu_{G_k^g}\left(i,j\right) &= \frac{G_k^g\left(i,j\right)}{L-1} | i = 1, 2, ..., m; j = 1, 2, ..., n\\ \mu_{G_k^b}\left(i,j\right) &= \frac{G_k^b\left(i,j\right)}{L-1} | i = 1, 2, ..., m; j = 1, 2, ..., n \end{split}$$

Step 03. Set the parameter α .

Step 04. Set the CRFO in parametric form for each colour red (r), green (g) and blue (b).

$$CRFO\left(\mu_{G_{k}^{r}}\left(i,j\right)\right) = \mu_{G_{k}^{r}}^{/}\left(i,j\right) = \{1 - e^{-\alpha\mu_{G_{k}^{p}}^{2}\left(i,j\right)}, 0 \le \mu_{G_{k}^{r}}\left(i,j\right) \le 1$$
$$CRFO\left(\mu_{G_{k}^{g}}\left(i,j\right)\right) = \mu_{G_{k}^{g}}^{/}\left(i,j\right) = \{1 - e^{-\alpha\mu_{G_{k}^{p}}^{2}\left(i,j\right)}, 0 \le \mu_{G_{k}^{g}}\left(i,j\right) \le 1$$
$$CRFO\left(\mu_{G_{k}^{b}}\left(i,j\right)\right) = \mu_{G_{k}^{b}}^{/}\left(i,j\right) = \{1 - e^{-\alpha\mu_{G_{k}^{b}}^{2}\left(i,j\right)}, 0 \le \mu_{G_{k}^{b}}\left(i,j\right) \le 1$$

Step 05. Set the parameter $\sigma > 0$ and $\tau \ge 1$.

Step 06. Generate new colour levels for each colour channel red (r), green (g) and blue (b).

$$\begin{split} G_{k}^{r\,\prime}\left(i,j\right) &= (L-1)\left(1 - \left(1 - \mu_{G_{k}^{r}}^{\prime}\left(i,j\right)^{\frac{\sigma}{\tau}}\right)\right)\\ G_{k}^{g\,\prime}\left(i,j\right) &= (L-1)\left(1 - \left(1 - \mu_{G_{k}^{g}}^{\prime}\left(i,j\right)^{\frac{\sigma}{\tau}}\right)\right)\\ G_{k}^{b\,\prime}\left(i,j\right) &= (L-1)\left(1 - \left(1 - \mu_{G_{k}^{b}}^{\prime}\left(i,j\right)^{\frac{\sigma}{\tau}}\right)\right) \end{split}$$

Step 07. Enhanced colour image is obtained using contrast removal fuzzy operator.

Step 08. Analysis is made on all the images and the same is displayed.

4. Result and discussion

Enhancement algorithms are used to enhance contrast, to reduce over-brightness and to remove fuzziness from the image. Enhancement improves the quality of the image and generally provides a clearer image for a human observer. The images taken for analysis are shown in Fig. 3, Fig. 4 and in Fig. 5.

4.1. Enhancement of gray scale images by the CRFO. Contrast within an image is the measure of difference between the grav levels in an image. The greater the contrast, the greater is the distinction between gray levels in the image. Images of high contrast have either all black or all white regions. There is only very little gray in the image. Low contrast images have lots of similar gray levels in the image and very few black or white regions. High-contrast images can be thought of as crisp and low-contrast ones are completely fuzzy. We used two gray scale images, one image coffee.jpg $(300 \times 300 \text{ pixels})$ another document image $(248 \times 191 \text{ pixels})$ [11] in low-illumination, low- contrast, over-contrast and over-bright conditions which are shown in Fig.3(a), 3(e), 3(i), 3(m), 3(r), 3(v). The member histogram of each original and resultant image is used to analyze the amount of enhancement achieved. It can be seen from Fig. 3(c) that, membership values of Fig. 3(a) lies in [0.3, 0.6], hence low in contrast. Fig. 3(b) shows the contrast improvement of original image by the proposed method. The proposed method distributed membership values from [0.3, 0.6] to [0, 1], which can be seen from Fig. 3(d). In the case of over-contrast image Fig. 3(e), 3(m), it is very difficult to detect every character from the image by human eye, especially in the case of document image. But after enhanced by our method it is easy to recognize different characters, which are shown in Fig. 3(f), 3(n). Now consider the Fig. 3(i), 3(v) which are in over- bright condition. It is another difficult task to detect characters from such kind of image. From the membership histogram Fig. 3(k), 3(x) it is to be noted that all the membership values lies in the interval [0.8, 1]. Over-brightness is reduced by the proposed method, which can be seen from Fig. 3(j), 3(l), 3(w) and 3(y).

4.2. Comparative study. To show the comparison between the proposed method and other commonly used method we have consider two image, one document image(248 × 191 pixels) another IC chip $(313 \times 159 \text{ pixels})$ from [11]. Fig. 4(a), 4(b), 4(k) 4(l) shows the images are in low-contrast and in low-illumination condition. Fig.4(c), 4(m), show the result of HE alone, Fig.4(d), 4(n) show the result of Generalized Fuzzy Operator with HE defined in [11], Fig.4 (e), 4(p) show the result of Generalized Fuzzy Operator with POSHE defined in [11]. Fig. 4(f), 4(q) shows effectiveness of the proposed method in comparison to the other three methods.

4.3. Enhancement of colour images by the CRFO. Colour image enhancement is a difficult task in image processing. We used here four real colour images, Fig. 5(a) Mahek. jpg (351×4486 pixels) and Fig. 5(e) Mahek. jpg captured by a digital camera in a low illumination environment, Fig. 5(i) Lena. jpg (512×512 pixels) in over-brightness condition, Fig.5(m) Mosque. jpg (400×267 pixels) in low-contrast condition. Fig. 5 shows the effectiveness of the proposed method in different condition of colour images.

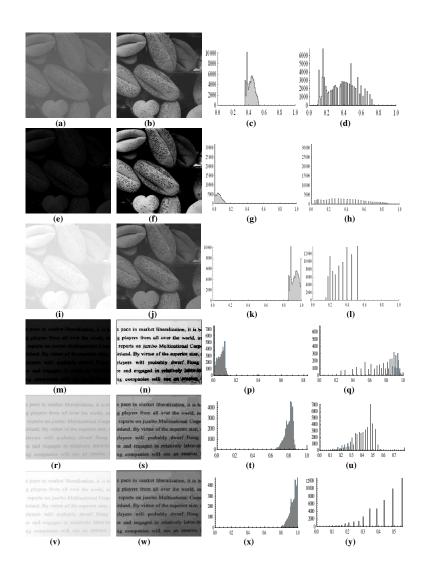


FIGURE 3. Original image : (a), (e), (i), (m), (r), (v); Enhanced by the proposed method at : (b) $\alpha = 12$, $\sigma = 18$ and $\tau = 2$ (f) $\alpha = 100$, $\sigma = 2$ and $\tau = 3$ (j) $\alpha = 4$, $\sigma = 100$ and $\tau = 3$ (n) $\alpha = 160$, $\sigma = 1$ and $\tau = 4$ (s) $\alpha = 4$, $\sigma = 35$ and $\tau = 3$ (w) $\alpha = 4$, $\sigma = 60$ and $\tau = 2$; Membership histogram before enhancement: (c), (g), (k), (p), (t), (x) and after enhancement: (d), (h), (l), (q), (u), (y).

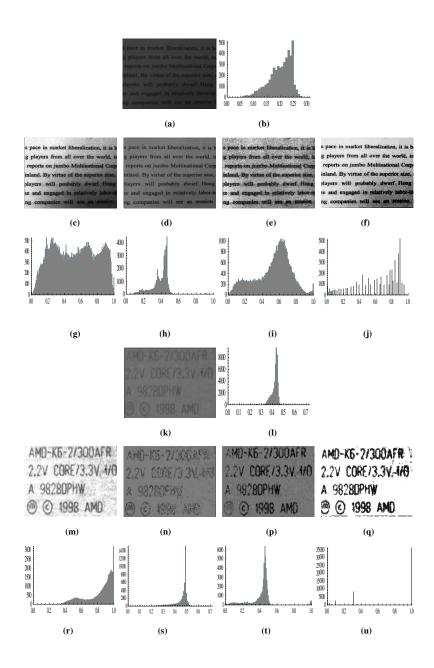


FIGURE 4. Original image: (a), (k); Result by HE: (c), (m); Result of Generalized Fuzzy Operator with HE: (d), (n); Result of Generalized Fuzzy Operator with POSHE: (e), (p); Result of proposed method at parameter: (f) $\alpha = 70$, $\sigma = 25$ and $\tau = 4$; (q) $\alpha = 37$, $\sigma = 600$ and $\tau = 2$.

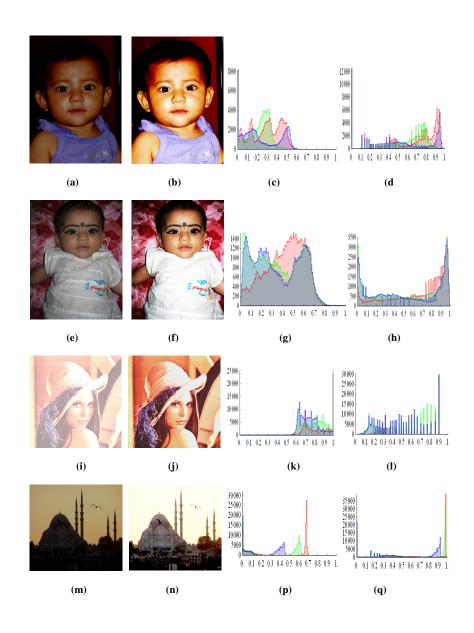


FIGURE 5. Original image : (a), (e), (i), (m); Enhanced by the proposed method at parameter: (b) $\alpha = 8$, $\sigma = 3$ and $\tau = 5$ (f) $\alpha = 8$, $\sigma = 2$ and $\tau = 2$ (j) $\alpha = 5$, $\sigma = 25$ and $\tau = 2$ (n) $\alpha = 8$, $\sigma = 1$ and $\tau = 3$; Membership histogram before enhancement:(c), (g), (k), (p); Membership histogram after enhancement: (d), (h), (i), (q). 411

5. Conclusions

In this paper we have proposed image enhancement method based on Contrast Removal Fuzzy Operator (CRFO). Enhancement algorithm is used to reduce overcontrast, over-brightness of image and to improve the contrast of low-contrast, lowillumination images. Generally provides a clearer image for a human observer. However more experimental results are required in order to establish a reliable comparison.

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