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Somewhat fuzzy faintly pre-$\mathcal{I}$-continuous functions

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Abstract. Recently, El-Naschie has shown that the notion of fuzzy topology may be relevant to quantam particle physics in connection with string theory and E-infinite space time theory. In this paper, the concept of somewhat fuzzy faintly pre-$\mathcal{I}$-continuous functions and somewhat fuzzy faintly pre-$\mathcal{I}$-open functions. Some characterizations and interesting properties of these functions are discussed.

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1. Introduction

The concept of fuzzy set was introduced by Zadeh [11]. Based on the concept of fuzzy sets, Chang [1] introduced and developed the concept of fuzzy ideal topological spaces. In [7] Mahmoud and in [10], Sarkar indepentently presented some of the ideal concepts in the fuzzy trend and studied many of their properties. The concept of fuzzy topology may be relevent to quantum particle physics purticularly in connection with stong theory and E-infinite theory [2, 3, 4, 5] . Recently, Nasef and Hatir [9] introduced the concepts of fuzzy pre-$\mathcal{I}$-open sets and fuzzy pre-$\mathcal{I}$-continuity in fuzzy ideal topological spaces. In the present paper, to introduce and study the concept of somewhat fuzzy faintly pre-$\mathcal{I}$-continuous functions and somewhat fuzzy faintly pre-$\mathcal{I}$-open functions in fuzzy ideal topological spaces.

2. Preliminaries

Throughtout this paper, (X, τ) always means fuzzy topolocial space in the sense of Chang [1]. Let A be a fuzzy subset of X. The fuzzy closure of A is denoted by $Cl(A)$ and is defined as $Cl(A) = \wedge\{B|B \geq A$, B is a fuzzy closed subset of $(X, \tau)\}$. Also, the fuzzy interior of A is denoted by $Int(A) = \vee\{B|B \leq A$, B is a fuzzy open subset of $(X, \tau)\}$. A nonempty collection of fuzzy sets I of a set X is called a fuzzy ideal [7, 10] if and only if (i) $A \in \mathcal{I}$ and $A \leq B$, then $B \in \mathcal{I}$, (ii) if $A \in \mathcal{I}$ and $B \in \mathcal{I}$, then $A \vee B \in \mathcal{I}$. The triple $(X, \tau, \mathcal{I})$ means a fuzzy ideal topological space with a fuzzy ideal $\mathcal{I}$ and fuzzy topology τ. For $(X, \tau, \mathcal{I})$, the fuzzy local function of a fuzzy subset A of X with respect to τ and $\mathcal{I}$ denoted by $A^*(\tau, \mathcal{I})$ (briefly, A^*) and is defined $A^*(\tau, \mathcal{I}) = \vee\{x \in X : A \wedge U \notin \mathcal{I} foreveryU \in \tau\}$. While A^* is the union of the fuzzy points such that if $U \in \tau$ and $E \in \mathcal{I}$, then there is atleast one $y \in X$ for which $U(y) + A(y) - 1 > E(y)$. Also, for a fuzzy set A of $(X, \tau, \mathcal{I})$, $Cl^*(A) = A \vee A^*$. A fuzzy subset A of $(X, \tau, \mathcal{I})$ is said to be fuzzy pre-$\mathcal{I}$-open [9] if $A \leq Int(Cl^*(A))$. The complement of fuzzy pre-$\mathcal{I}$-open set is called fuzzy pre-$\mathcal{I}$-closed. The family of all fuzzy pre-$\mathcal{I}$-open (resp. fuzzy pre-$\mathcal{I}$-closed) subsets of $(X, \tau, \mathcal{I})$ is denoted by $FP\mathcal{I}O(X)$ (resp. $FP\mathcal{I}C(X)$). The fuzzy pre-$\mathcal{I}$-closure and fuzzy pre-$\mathcal{I}$-interior of a fuzzy set A are respectively, denoted by $p\mathcal{I}Cl(A)$ and $p\mathcal{I}Int(A)$ and is defined as $p\mathcal{I}Cl(A) = \wedge\{B|A \leq B, B \leq FP\mathcal{I}C(X)\}$ and $p\mathcal{I}Int(A) = \vee\{B|B \leq A, B \in FP\mathcal{I}O(X)\}$. A fuzzy set A is said to be fuzzy pre-$\mathcal{I}$-closed (resp. fuzzy pre-$\mathcal{I}$-open) if and only if $p\mathcal{I}Cl(A) = A$ (resp. $p\mathcal{I}Int(A) = A$). Clearly, $p\mathcal{I}Cl(1 - A) = 1 - p\mathcal{I}Int(A)$ and $p\mathcal{I}Int(1 - A) = 1 - p\mathcal{I}Cl(A)$. A fuzzy set A in (X, τ) is called proper if $A \neq 0$ and $A \neq 1$. A fuzzy point x_p in X is a fuzzy set in X defined by

$$x_p(y) = \begin{cases} p, \quad p \in (0, 1], & for\ y = x, y \in X \\ 0, & for\ y = x, y \in X \end{cases}$$

and x and p are respectively, called the support and the value of x_p. A fuzzy point x_p is said to be quasi-coincident [6] with α, denoted by $x_p \text{q} \alpha$ if and only of $p > (1 - \alpha)$ or $p + \alpha(x) > 1$. A fuzzy subset μ in a fuzzy topological space X is said to be a q-neighbourhood [6] for a fuzzy point x_p if and only if there exists a fuzzy open subset β such that $x_\alpha \text{q} \beta \leq \mu$. A fuzzy set μ is called fuzzy θ-open [8] if and only if $\mu = Int_\theta(\mu)$, where $Int_\theta(\mu) = \vee\ \{x_p \in X$, for some open q-neighbourhood $\beta of x_p, Cl(\beta) \leq \mu\}$. The complement of fuzzy θ-open set is called fuzzy θ-closed [8].

3. Somewhat fuzzy faintly pre-$\mathcal{I}$-continuous functions

Definition 3.1. A function $f : (X, \tau) \to (Y, \sigma)$ is called somewhat fuzzy faintly pre-$\mathcal{I}$-continuous if for every fuzzy θ-open set A in Y such that $f^{-1}(A) \neq 0$, there exists a fuzzy pre-$\mathcal{I}$-open set $B \neq 0$ in (X, τ) such that $B \leq f^{-1}(A)$.

Definition 3.2. A function $f : (X, \tau) \to (Y, \sigma)$ is called fuzzy faintly continuous [8] if for $f^{-1}(\mu)$ is fuzzy open in X for every fuzzy θ-open set μ in Y.

It is clear that every fuzzy faintly continuous function is somewhat fuzzy faintly pre-$\mathcal{I}$-continuous but the converse is not true as the following example shows.

Example 3.3. Let μ_1, μ_2, μ_3 be fuzzy sets on $I = [0,1]$ defined as follows:

$$\mu_1(x) = \begin{cases} 0, & 0 \leq x \leq \frac{1}{2} \\ 2x-1, & \frac{1}{2} \leq x \leq 1, \end{cases}$$

$$\mu_2(x) = \begin{cases} 1, & 0 \leq x \leq \frac{1}{4} \\ -4x+2, & \frac{1}{4} \leq x \leq \frac{1}{2} \\ 0, & \frac{1}{2} \leq x \leq 1, \end{cases}$$

$$\mu_3(x) = \begin{cases} x, & 0 \leq x \leq \frac{1}{4} \\ 1, & \frac{1}{4} \leq x \leq 1. \end{cases}$$

Clearly $\tau_1 = \{0, \mu_2, 1\}$, $\tau_2 = \{0, \mu_1, \mu_2, \mu_1 \vee \mu_2, 1\}$ are two topologies on I and $\mathcal{I} = \{\varnothing\}$. Let $f : (I, \tau_1, \mathcal{I}) \to (I, \tau_2, \mathcal{I})$ be defined as follows $f(x) = x$, for each x in I. Then the fuzzy sets μ_3 and 1 are both fuzzy pre-$\mathcal{I}$-open sets of $(X, \tau_1, \mathcal{I})$ and also they are the only fuzzy θ-open sets of $(I, \tau_2, \mathcal{I})$. Then, f is somewhat fuzzy pre-$\mathcal{I}$-continuous but not fuzzy faintly pre-$\mathcal{I}$-continuous, since $f^{-1}(\mu_3) = \mu_3 \notin \tau_1$.

Theorem 3.4. *Let $(X, \tau, \mathcal{I})$ and $(Y, \sigma, \mathcal{I})$ be any two fuzzy ideal topological spaces. Let $f : (X, \tau) \to (Y, \sigma)$ be a function. Then the following are equivalent:*

(i) *f is somewhat fuzzy faintly pre-$\mathcal{I}$-continuous.*

(ii) *If A is a fuzzy θ-closed set of Y such that $f^{-1}(A) \neq 1$, then there exists a proper fuzzy pre-$\mathcal{I}$-closed set B of X such that $B \geq f^{-1}(A)$.*

(iii) *If A is a fuzzy pre-$\mathcal{I}$-dense set, then $f(A)$ is a fuzzy θ-dense set in Y.*

Proof. (i) $\Rightarrow$ (ii): Suppose f is somewhat fuzzy faintly pre-$\mathcal{I}$-continuous and A is any fuzzy θ-closed set in Y such that $f^{-1}(A) \neq 1$. Therefore, clearly $1 - A$ is a fuzzy θ-open set and $f^{-1}(1 - A) = 1 - f^{-1}(A) \neq 0$. But by (i), there exists a fuzzy pre-$\mathcal{I}$-open set B in (X, τ) such that $B \neq 0$ and $B \leq f^{-1}(1 - A)$. Therefore, $1 - B \geq 1 - f^{-1}(1 - A) = 1 - (1 - f^{-1}(A)) = f^{-1}(A)$. Put $1 - B = C$. Clearly, μ is proper fuzzy pre-$\mathcal{I}$-closed set and $B \geq f^{-1}(A)$.

(ii) $\Rightarrow$ (iii): Let A be a fuzzy pre-$\mathcal{I}$-dense set in X and suppose $f(A)$ is not fuzzy θ-dense in Y. Then there exists a fuzzy θ-closed set, say, B such that $f(A) < B < 1$. Now, $B < 1 \Rightarrow f^{-1}(B) \neq 1$. Then by (ii), there exists a proper fuzzy pre-$\mathcal{I}$-closed set C in $(X, \tau, \mathcal{I})$ such that $C \geq f^{-1}(B)$. But by (i), $f^{-1}(B) > f^{-1}(f(A)) \geq A$, that is, $C > A$. This implies that there exists a proper fuzzy pre-$\mathcal{I}$-closed set C such that $C > A$, which is a contradiction, since A is fuzzy pre-$\mathcal{I}$-dense.

(iii) $\Rightarrow$ (i): Let A be any fuzzy θ-open set in (Y, σ) and suppose $f^{-1}(A) \neq 0$ and hence $A \neq 0$. Suppose $p\mathcal{I}Int(f^{-1}(A)) = 0$. Then $p\mathcal{I}Cl(1 - f^{-1}(A)) = 1 - p\mathcal{I}Int(f^{-1}(A)) = 1 - 0 = 1$. This means that $1 - f^{-1}(A)$ is a fuzzy pre-$\mathcal{I}$-dense set in X. By (iii), $f(1 - f^{-1}(A))$ is a fuzzy θ-dense in Y. That is, $Cl_\theta(f(1 - f^{-1}(A))) = 1$, but $f(1 - f^{-1}(A)) = f(f^{-1}(1 - A)) \leq 1 - A = 1$, since $A \neq 0$. Since $1 - A$ is fuzzy θ-closed and $f(1 - f^{-1}(A)) \leq 1 - A$, $Cl_\theta(f(f^{-1}(A))) \leq 1 - A$. That is, $1 \leq 1 - A \Rightarrow A \leq 0$ and hence $A = 0$, which is a contradiction to the fact that $A \neq 0$. Therefore, we must have $p\mathcal{I}Int(f^{-1}(A)) \neq 0$. This means that, there exists a fuzzy pre-$\mathcal{I}$-open set B in $(X, \tau, \mathcal{I})$ such that $0 \neq B \leq f^{-1}(A)$ and consequently f is somewhat fuzzy faintly pre-$\mathcal{I}$-continuous. □

Proposition 3.5. *Let $(X, \tau, \mathcal{I})$ and $(Y, \sigma, \mathcal{I})$ be fuzzy ideal topological spaces and $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{I})$ be somewhat fuzzy faintly pre-$\mathcal{I}$-continuous. Let U be a* *subset of X such that $\mathcal{X}_A \wedge B \neq 0$ for some $0 \neq B$ in $(X, \tau, \mathcal{I})$. Let $\tau_{|A}$ be the induced fuzzy topology on X. Then $f_{|A} : (A, \tau_{|A}) \to (Y, \sigma, \mathcal{I})$ is somewhat fuzzy faintly pre-$\mathcal{I}$-continuous.*

Proof. Suppose A is a fuzzy θ-open set in $(Y, \sigma, \mathcal{I})$ such that $f^{-1}(A) \neq 0$. Since f is somewhat fuzzy faintly pre-$\mathcal{I}$-continuous, there exists a fuzzy pre-$\mathcal{I}$-open set B in $(X, \tau, \mathcal{I})$ such that $B \neq 0$ and $B \leq f^{-1}(A)$. But $B_{|U} \in \tau_{|U}$ and $B_{|U} \neq 0$, since $\mathcal{X}_A \cap B \neq 0$ for all $B \in \tau$. Now $(f_{|U})^{-1}(A)(x) = A|(f_{|U}(x)) = Af(x) > B(x) = B|U(x)$ for $x \in U$. Also B is a fuzzy pre-$\mathcal{I}$-open set in $(X, \tau, \mathcal{I})$ implies $B_{|U}$ is fuzzy pre-$\mathcal{I}$-open in $(U, \tau_{|U})$ and $B_{|U} < (f_{|U})^{-1}(A)$. □

Definition 3.6. Let τ and τ^* be any two fuzzy topologies for X. We say that τ^* is pre-$\mathcal{I}$-equivalent (θ-equivalent) to τ if $A \neq 0$ is a fuzzy pre-$\mathcal{I}$-open (θ-open) set in (X, τ), then there is a fuzzy pre-$\mathcal{I}$-open set B in (X, τ^*) such that $B \neq 0$ and $B \leq A$.

Proposition 3.7. *If $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{I})$ is a somewat fuzzy faintly pre-$\mathcal{I}$-continuous function and τ^* is a fuzzy topology pre-$\mathcal{I}$-equivalent to τ, then $f : (X, \tau^*) \to (Y, \sigma, \mathcal{I})$ is somewhat fuzzy faintly pre-$\mathcal{I}$-continuous.*

Proof. Let A be a fuzzy θ-open set in Y such that $f^{-1}(A) \neq 0$. Since $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{I})$ is somewhat fuzzy faintly pre-$\mathcal{I}$-continuous, there is a fuzzy pre-$\mathcal{I}$-open set B in $(X, \tau, \mathcal{I})$ and $B \neq 0$ such that $B \leq f^{-1}(A)$ since $\tau^*\gamma$-equivalent to τ, there exists a fuzzy pre-$\mathcal{I}$-open set B^* such that B^* in (X, τ^*) and $B^* \neq 0$ and $B^* < B$. But $B < f^{-1}(A)$ implies $B^* \leq f^{-1}(A)$. This means $f : (X, \tau^*) \to (Y, \sigma, \mathcal{I})$ is somewhat fuzzy faintly pre-$\mathcal{I}$-continuous. □

Proposition 3.8. *If $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{I})$ is a somewhat fuzzy faintly pre-$\mathcal{I}$-continuous and σ^* is a fuzzy topology θ-equivalent to σ, then $f : (X, \tau, \mathcal{I}) \to (Y, \sigma^*)$ is a somewhat fuzzy faintly pre-$\mathcal{I}$-continuous.*

Proof. Let A be a fuzzy θ-open set in Y such that $f^{-1}(A) \neq 0$. Since σ^* is θ-equivalent to σ, there exists a fuzzy θ-open set A^* in $(Y, \sigma, \mathcal{I})$ such that $0 \neq A^* \leq A$. Now, $0 \neq f^{-1}(A^*) \leq f^{-1}(A)$. Since f is somewhat fuzzy faintly pre-$\mathcal{I}$-continuous from $(X, \tau, \mathcal{I})$ to $(Y, \sigma, \mathcal{I})$, there exists a fuzzy pre-$\mathcal{I}$-open in $(X, \tau, \mathcal{I})$, say, B such that $B \neq 0$ and $B \leq f^{-1}(A^*)$. This means $B \leq f^{-1}(A)$ and so f is somewhat fuzzy faintly pre-$\mathcal{I}$-continuous function from $(X, \tau, \mathcal{I})$ to (Y, σ^*). □

Definition 3.9. A fuzzy deal topological space $(X, \tau, \mathcal{I})$ is said to be fuzzy separable (fuzzy pre-$\mathcal{I}$-separable) if there exists a fuzzy θ-dense (fuzzy pre-$\mathcal{I}$-dense) set A in X such that $A \neq 0$ for atmost countably many points of X.

Proposition 3.10. *If $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{I})$ is a somewhat fuzzy faintly pre-$\mathcal{I}$-continuous function and if X is fuzzy pre-$\mathcal{I}$-separable, then Y is fuzzy θ-separable.*

Proof. Since X is fuzzy pre-$\mathcal{I}$-separable, there exists a fuzzy pre-$\mathcal{I}$-dense set A such that $A \neq 0$ for almost countably many points of X. Also since f is somewhat fuzzy faintly pre-$\mathcal{I}$-continuous, it follows by Theorem 3.4 that $f(A)$ is fuzzy θ-dense in $(Y, \sigma, \mathcal{I})$ and since $A \neq 0$ and A is pre-$\mathcal{I}$-dense for atmost countably many points, it follows that $f(A) \neq 0$ for atmost countably many points. Thus, we find that $(Y, \sigma, \mathcal{I})$ is θ-separable. □

4. Somewhat fuzzy faintly pre-$\mathcal{I}$-open functions

Definition 4.1. A function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{I})$ is called somewhat fuzzy faintly pre-$\mathcal{I}$-open if and only if for any fuzzy pre-$\mathcal{I}$-open set A, $A \neq 0$ in $(X, \tau, \mathcal{I})$ implies that there exists a fuzzy θ-open set B in $(Y, \sigma, \mathcal{I})$ such that $B \neq 0$ and $B < f(A)$, that is $Int_\theta(f(A)) \neq 0$.

Proposition 4.2. *If $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{I})$ and $g : (Y, \sigma, \mathcal{I}) \to (Z, \gamma, \mathcal{I})$ are somewhat fuzzy faintly pre-$\mathcal{I}$-open functions. Then $g \circ f$: $(X, \tau, \mathcal{I}) \to (Z, \gamma, \mathcal{I})$ is somewhat fuzzy faintly pre-$\mathcal{I}$-open.*

Proof. Let A be a fuzzy pre-$\mathcal{I}$-open set in X. Since f is somewhat fuzzy faintly pre-$\mathcal{I}$-open, then there exists a fuzzy θ-open set B in Y such that $B \leq f(A)$. Now, $Int_\theta(f(A)) \in \sigma$ and since g is somewhat fuzzy faintly pre-$\mathcal{I}$-open, then there exists a fuzzy θ-open set C in $(Z, \gamma, \mathcal{I})$ such that $C < g(Int(f(A)))$. But $g(Int(f(A))) < g(f(A))$. Thus, we find that there exists a fuzzy θ-open set C in $(Z, \gamma, \mathcal{I})$ such that $C < (g \circ f)(A)$. This proves $g \circ f$ is somewhat fuzzy faintly pre-$\mathcal{I}$-open. □

Theorem 4.3. *For a surjective function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{I})$ the following are equivalent:*

(i) *f is somewhat fuzzy pre-$\mathcal{I}$-open.*
(ii) *If A is a fuzzy pre-$\mathcal{I}$-closed set in X such that $f(A) \neq 1$, then there exists a fuzzy θ-closed set B in Y such that $B \neq 1$ and $B > f(A)$.*

Proof. (i) ⇒ (ii): Let A be a fuzzy pre-$\mathcal{I}$-closed set in X such that $f(A) \neq 1$. Then $1 - A$ is a fuzzy pre-$\mathcal{I}$-open set such that $f(1 - A) = 1 - f(A) \neq 0$. Since f is somewhat fuzzy faintly pre-$\mathcal{I}$-open, there exists a fuzzy θ-open set B in Y such that $B \neq 0$ and $B \leq f(1 - A)$. Now $1 - B$ is fuzzy θ-closed set in Y such that $1 - B \neq 1$ an $B < f(1 - A)$. Put $1 - B = A$. Then $B > 1 - f(1 - A)= f(A)$.

(ii) ⇒ (i): Let A be a fuzzy pre-$\mathcal{I}$-open of X such that $A \neq 0$. Then $1 - A$ is fuzzy pre-$\mathcal{I}$-closed and $1 - A \neq 1$, $f(1 - A) = 1 - f(A) \neq 1$. Hence by (ii), there exists a fuzzy θ-closed set B in Y such that $B \neq 1$ and $B > f(1 - A) = 1 - f(A)$, that is, $f(A) > 1 - B$ and let $1 - B = C$. Clearly, C is a fuzzy θ-open set of Y such that $C < f(A)$ and $C \neq 0$. Hence f is somewhat fuzzy faintly pre-$\mathcal{I}$-open. □

Theorem 4.4. *For a surjective function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{I})$, the following are equivalent:*

(i) *f is somewhat fuzzy faintly pre-$\mathcal{I}$-open;*
(ii) *If A is a fuzzy θ-dense set of Y, then $f^{-1}(A)$ is fuzzy pre-$\mathcal{I}$-dense set in X.*

Proof. (i) ⇒ (ii): Suppose A is fuzzy θ-dense set of Y. We want to show that $f^{-1}(A)$ is fuzzy pre-$\mathcal{I}$-dense in X. Suppose not, then there exists a fuzzy pre-$\mathcal{I}$-closed set B in X such that $f^{-1}(B) < B < 1$. Now $A = f(f^{-1}(B)) < f(B) < f(1)$. Since f is somewhat fuzzy faintly pre-$\mathcal{I}$-open by Theorem 4.3, there exists a fuzzy θ-closed set C in Y such that $f(B) < C$. Thus, we find $A < f(B) < C < 1$, which is a contradiction to our hypothesis that A is fuzzy θ-dense in X. Hence $f^{-1}(A)$ must be fuzzy pre-$\mathcal{I}$-dense set.

(ii) ⇒ (i): Suppose $f^{-1}(A)$ is fuzzy pre-$\mathcal{I}$-dense in X where A is fuzzy θ-dense set in Y. We want to show that f is somewhat fuzzy faintly pre-$\mathcal{I}$-open. Assume $A \in \tau$

and $A \neq 0$ be a fuzzy pre-$\mathcal{I}$-open set in (X, τ). We have to show that $Int_\theta(f(A)) \neq 0$. Suppose not, then $Int_\theta(f(A)) = 0$ whenever $A \in \tau$. Then $Cl_\theta(1 - f(A)) = 1 - Int_\theta(f(A)) = 1 - 0 = 1$. That is, $1 - f(A)$ is fuzzy θ-dense in Y. Therefore by assumption $f^{-1}(1 - f(A))$ is fuzzy pre-$\mathcal{I}$-dense in X. Therefore, $1 = p\mathcal{I}Cl(f^{-1}(1 - f(A))) = p\mathcal{I}Cl(1 - A) = 1 - A$. This shows that $A = 0$, which is a contradiction and so $p\mathcal{I}Int(f(A)) \neq 0$. □

Definition 4.5. A fuzzy ideal topological space $(X, \tau, \mathcal{I})$ is called a fuzzy D_γ-space (D_θ-space) if every nonzero fuzzy pre-$\mathcal{I}$-open (fuzzy θ-open) set in X is fuzzy pre-$\mathcal{I}$-dense (fuzzy θ-dense) in X.

Proposition 4.6. *If $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{I})$ is a somewhat fuzzy faintly pre-$\mathcal{I}$-continuous and (X, τ) is a fuzzy D_γ-space, then Y is a fuzzy D_θ-space.*

Proof. Let A be a nonzero fuzzy θ-open set in Y. We want to show that A is fuzzy θ-dense in Y. Suppose not, then there exists a fuzzy θ-closed set B in Y such that $A < B < 1$. Therefore, $f^{-1}(A) < f^{-1}(B) < f^{-1}(1) = 1$. Since $A \neq 0$, $f^{-1}(A) \neq 0$ and since f is somewhat fuzzy faintly pre-$\mathcal{I}$-continuous, $0 \neq p\mathcal{I}Int(f^{-1}(A)) < f^{-1}(B) < p\mathcal{I}Cl(f^{-1}(\mu)) < 1$. This is contradiction to the assumption that $(X, \tau, \mathcal{I})$ i sa fuzzy D_γ-space and hence Y is a fuzzy D_θ-space. □

Proposition 4.7. *Let $(X_1, \tau_1, \mathcal{I})$, $(X_2, \tau_2, \mathcal{I})$, $(Y_1, \sigma_1, \mathcal{I})$ and $(Y_2, \sigma_2, \mathcal{I})$ be fuzzy ideal topological spaces such that Y_1 is product related to Y_2 and X_1 is product related to X_2. Then the product $f_1 \times f_2 : X_1 \times X_2 \to Y_1 \times Y_2$ of somewhat fuzzy faintly pre-$\mathcal{I}$-continuous functions $f_1 : (X, \tau_1, \mathcal{I}) \to (Y_1, \sigma_1, \mathcal{I})$ and $f_2 : (X_2, \tau_2, \mathcal{I}) \to (Y, \sigma_2, \mathcal{I})$ is somewhat fuzzy faintly pre-$\mathcal{I}$-continuous.*

Proof. Let $A = \vee(A_i \times A_j)$, where A_i's and A_j's are fuzzy θ-open sets of Y_1 and Y_2, respectively be a fuzzy θ-open set of $Y_1 \times Y_2$. Then, we have $(f_1 \times f_2)^{-1}(A) = \vee(f_1^{-1}(A_i) \times f_2^{-1}(A_j))$. Since f_1 and f_2 are somewhat fuzzy faintly pre-$\mathcal{I}$-continuous, there exist fuzzy pre-$\mathcal{I}$-open sets B_i in X_i, fuzzy pre-$\mathcal{I}$-open set B_j in X_2 such that $B_i \leq f_1^{-1}(A_i)$; $B_j \leq f_2^{-1}(A_j)$. Therefore, $(f_1 \times f_2)^{-1}(A) \leq \vee(B_i \times B_j)$. Since X_1 and X_2 are product related, $\vee(B_i \times B_j)$ is fuzzy pre-$\mathcal{I}$-open. Hence $f_1 \times f_2$ is somewhat fuzzy faintly pre-$\mathcal{I}$-continuous. □

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