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On intuitionistic fuzzy semi-α-irresolute functions

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Abstract. In this paper the concept of intuitionistic fuzzy semi-α-irresolute functions are introduced and studied. Besides giving characterizations of these functions, several interesting properties of these functions are also given. We also study relationship between this function with other existing functions.

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1. Introduction

Ever since the introduction of fuzzy sets by L.A.Zadeh [13], the fuzzy concept has invaded almost all branches of mathematics. The concept of fuzzy topological spaces was introduced and developed by C.L.Chang [2]. Atanassov[1] introduced the notion of intuitionistic fuzzy sets, Coker [3] introduced the intuitionistic fuzzy topological spaces. In this paper we have introduced the concept of intuitionistic fuzzy semi-α-irresolute functions and studied their properties. Also we have given characterizations of intuitionistic fuzzy semi-α-irresolute functions. We also study relationship between this function with other existing functions.

2. Preliminaries

Definition 2.1 ([1])**.** Let X be a nonempty fixed set and I the closed interval [0,1]. An intuitionistic fuzzy set (IFS) A is an object of the following form

$$A = \{<\text{x}, \mu_A(\text{x}), \nu_A(\text{x})>; \text{x}\in\text{X}\}$$

where the mappings $\mu_A(x) : X \to$I and $\nu_A(x) : X \to$I denote the degree of membership(namely) $\mu_A(x)$ and the degree of nonmembership(namely) $\nu_A(x)$ for each element $x \in X$ to the set A respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each x∈X.

Definition 2.2 ([1]). Let A and B are IFSs of the form A = {<x, μ_A(x), ν_A(x)>; x∈X} and B = {<x, μ_B(x), ν_B(x)>; x∈X}.Then

(a) A ⊆ B if and only if μ_A(x) ≤ μ_B(x) and ν_A(x) ≥ ν_B(x);
(b) $\bar{A}$(or A^c) = {<x, ν_A(x), μ_A(x)>; x∈X};
(c) A ∩ B = {<x, μ_A(x) ∧ μ_B(x), ν_A(x) ∨ ν_B(x)>; x∈X};
(d) A ∪ B = {<x, μ_A(x) ∨ μ_B(x), ν_A(x)∧ ν_B(x) >; x∈X}.

We will use the notation A= {<x, μ_A, ν_A>; x∈X} instead of

A= {<x, μ_A(x), ν_A(x)>; x∈X}.

Definition 2.3 ([3]). $0_\sim$ = {<x,0,1>; x∈X} and $1_\sim$ = {<x,1,0>; x∈X}.

Let α, $\beta \in$ [0,1] such that $\alpha + \beta \leq 1$. An intuitionistic fuzzy point (IFP)$p_{(\alpha,\beta)}$ is intuitionistic fuzzy set defined by $p_{(\alpha,\beta)}(\mathrm{x}) = \begin{cases} (\alpha,\beta) & \text{if } x = p, \\ (0,1) & \text{otherwise} \end{cases}$

Definition 2.4 ([9]). Let $p_{(\alpha,\beta)}$ be an IFP in IFTS X. An IFS A in X is called an intuitionistic fuzzy neighborhood (IFN) of $p_{(\alpha,\beta)}$ if there exists an IFOS B in X such that $p_{(\alpha,\beta)}$ ∈B ⊆A.

Let X and Y are two non-empty sets and f:(X,τ) → (Y,σ) be a function. If B = {<y, μ_B(y), ν_B(y)>;y∈Y} is an IFS in Y, then the pre-image of B under f is denoted and defined by $f^{-1}(B)$ = {<x, $f^{-1}(\mu_B$(x)), $f^{-1}(\nu_B$(x))>; x∈X } Since μ_B(x) , ν_B(x) are fuzzy sets, we explain that $f^{-1}(\mu_B$(x)) = μ_B(x)(f(x))

Definition 2.5 ([3]). An intuitionstic fuzzy topology (IFT) in Coker's sense on a nonempty set X is a family τ of intuitionistic fuzzy sets in X satisfying the following axioms:

(i) $0_\sim$, $1_\sim \in \tau$;
(ii) $G_1 \cap G_2 \in \tau$, for any G_1 ,$G_2 \in \tau$;
(iii) $\cup\, G_i \in \tau$ for any arbitrary family {G_i ; i ∈ J } ⊆ τ.

In this paper by (X,τ) or simply by X we will denote the intuitionistic fuzzy topological space (IFTS). Each IFS which belongs to τ is called an intuitionistic fuzzy open set (IFOS) in X. The complement $\bar{A}$ of an IFOS A in X is called an intuitionstic fuzzy closed set (IFCS) in X.

Definition 2.6 ([3]). Let (X,τ) be an IFTS and A = {<x, μ_A(x) , ν_A(x)>; x∈X } be an IFS in X.Then the intuitionistic fuzzy interior and intuitionstic fuzzy closure of A are defined by

(i) cl(A) = $\bigcap$ {C:C is an IFCS in X and C ⊇ A};
(ii) int(A) = $\bigcup$ {D:D is an IFOS in X and D ⊆ A};

It can be also shown that cl(A) is an IFCS, int(A) is an IFOS in X and A is an IFCS in X if and only if cl(A) = A ; A is an IFOS in X if and only int(A) = A.

Proposition 2.7. *Let (X,τ) be an IFTS and A,B be IFSs in X. Then the following properties hold:*

(i) $cl\overline{A} = \overline{(int(A))}$, $int\overline{(A)} = \overline{(cl(A))}$;
(ii) *int(A) ⊆A ⊆cl(A)*.[3]

Definition 2.8 ([5]). An IFS A in an IFTS X is called an intuitionistic fuzzy pre open set (IFPOS) if A$\subseteq$ int(clA). The complement of an IFPOS A in IFTS X is called an intuitionistic fuzzy pre closed (IFPCS) inX.

Definition 2.9 ([5]). An IFS A in an IFTS X is called an intuitionistic fuzzy α-open set (IFαOS) if and only if A $\subseteq$ int(cl(intA)). The complement of an IFαOS A in X is called intuitionistic fuzzy α-closed(IFαCS) in X.

Definition 2.10 ([5]). An IFS A in an IFTS X is called an intuitionistic fuzzy semi open set(IFSOS) if and only if A $\subseteq$ cl(int(A)). The complement of an IFSOS A in X is called intuitionistic fuzzy semi closed(IFSCS) in X.

Definition 2.11 ([8]). An IFS A in an IFTS X is called an intuitionistic fuzzy β-open set (IFβOS)(otherwise called as intuitionisitic fuzzy semi pre open set) if and only if A $\subseteq$ cl(int(clA)). The complement of an IFβOS A in X is called intuitionistic fuzzy β-closed(IFβCS) in X.

Definition 2.12 ([5, 8]). Let f be a mapping from an IFTS X into an IFTS Y. The mapping f is called:

- (i) intuitionistic fuzzy continuous if and only if $f^{-1}(B)$ is an IFOS in X, for each IFOS B in Y;
- (ii) intuitionistic fuzzy α-continuous if and only if $f^{-1}(B)$ is an IFαOS in X, for each IFOS B in Y;
- (iii) intuitionistic fuzzy pre continuous if and only if $f^{-1}(B)$ is an IFPOS in X, for each IFOS B in Y;
- (iv) intuitionistic fuzzy semi contiunous if and only if $f^{-1}(B)$ is an IFSOS in X, for each IFOS B in Y;
- (v) intuitionistic fuzzy β-contiunous if and only if $f^{-1}(B)$ is an IFβOS in X, for each IFOS B in Y.

Definition 2.13 ([12]). Let (X,τ) be an IFTS and A = $\{<$x, μ_A(x), ν_A(x)$>$; x$\in$X$\}$ be an IFS in X.Then the intuitionstic fuzzy α-closure and intuitionistic fuzzy α-interior of A are defined by

- (i) αcl(A) = $\bigcap$ {C:C is an IFαCS in X and C $\supseteq$ A};
- (ii) αint(A) = $\bigcup$ {D:D is an IFαOS in X and D $\subseteq$ A}.

Definition 2.14 ([10]). Let f be a mapping from an IFTS of X into an IFTS of Y. The mapping f is called intuitionistic fuzzy strongly α-continuous if and only if $f^{-1}(B)$ is an IFαOS in X, for each IFSOS B in Y.

Definition 2.15 ([7]). A function f:(X,τ)$\rightarrow$(Y,σ) from a intuitionistic fuzzy topological space (X,τ) to another intuitionistic fuzzy topological space (Y,σ) is said to be intuitionistic fuzzy irresolute if $f^{-1}(B)$ is an IFSOS in (X,τ) for each IFSOS B in (Y,σ).

Definition 2.16 ([10]). A function f:(X,τ)$\rightarrow$(Y,σ) from a intuitionistic fuzzy topological space (X,τ) to another intuitionistic fuzzy topological space (Y,σ) is said to be intuitionistic fuzzy α-irresolute if $f^{-1}(B)$ is an IFαOS in (X,τ) for each IFαOS B in (Y,σ).

Definition 2.17 ([11])**.** A function f:(X,τ)$\rightarrow$(Y,σ) from a intuitionistic fuzzy topological space (X,τ) to another intuitionistic fuzzy topological space (Y,σ) is said to be intuitionistic fuzzy pre-α-irresolute if $f^{-1}(B)$ is an IFPOS in (X,τ) for each IFαOS B in (Y,σ).

3. Intuitionistic fuzzy semi-α-irresolute functions

Definition 3.1. A function f:(X,τ)$\rightarrow$(Y,σ) from a intuitionistic fuzzy topological space (X,τ) to another intuitionistic fuzzy topological space (Y,σ) is said to be intuitionistic fuzzy semi-α-irresolute if $f^{-1}(B)$ is an IFSOS in (X,τ) for each IFαOS B in (Y,σ).

$$\begin{array}{ccccc} & \text{IFstrongly } \alpha\text{-continuous} & & \text{IF}\alpha\text{-irresolute} & \\ & & \searrow \quad \swarrow & & \\ & & \text{IF semi-}\alpha\text{-irresolute} & & \\ & \nearrow & \downarrow & \searrow & \\ \text{IF irresolute} & & \text{IF semi continuous} & & \text{IF } \beta\text{-continuous} \end{array}$$

Proposition 3.2. *Every intuitionistic fuzzy α-irresolute function is an intuitionistic fuzzy semi-α-irresolute function.*

Proof. Follows from the definitions. However, the converse of the above Proposition 3.2 is need not be true, as shown by the following example. □

Example 3.3. Let X = $\{a, b\}$, Y= $\{c, d\}$, τ ={$0_\sim$, $1_\sim$,A} , σ={$0_\sim$, $1_\sim$, B} where A = {<x,$(\frac{a}{0.3},\frac{b}{0.4})$,$(\frac{a}{0.6},\frac{b}{0.5})$>; x∈X}, B = {<y,$(\frac{c}{0.4},\frac{d}{0.5})$,$(\frac{c}{0.5},\frac{d}{0.5})$>; y∈Y}. Define an intuitionistic fuzzy mapping f:(X,τ)$\rightarrow$(Y,σ) by $f(a) = d, f(b) = c$. B is an IFαOS in (Y,σ), since B $\subseteq$ int(cl(int(B)))=B. $f^{-1}(B)$ = {<x,$(\frac{a}{0.5},\frac{b}{0.4})$,$(\frac{a}{0.5},\frac{b}{0.5})$>; x∈X}. And cl(int$f^{-1}(B)$) = A^c. Thus $f^{-1}(B) \subseteq$ cl(int$f^{-1}(B)$). Hence $f^{-1}(B)$ is IFSOS in X, which implies f is IF semi-α-irresolute. B is an IFαOS in (Y,σ), and int(cl(int$f^{-1}(B)$)) = A. So, $f^{-1}(B) \nsubseteq$ int(cl(int$f^{-1}(B)$)). Thus $f^{-1}(B)$ is not IFαOS in X. Hence f is not IF α-irresolute.

Proposition 3.4. *Every intuitionistic fuzzy semi-α-irresolute is an intuitionistic fuzzy semi continuous.*

Proof. Follows from the definitions. However the converse of the above Proposition 3.4 is need not be true, in general as shown by the following example. □

Example 3.5. Let X = {a,b}, Y = {c,d}, τ ={$0_\sim$, $1_\sim$, A}, σ={$0_\sim$, $1_\sim$, B} where A = {<x,$(\frac{a}{0.3},\frac{b}{0.4})$,$(\frac{a}{0.6},\frac{b}{0.5})$>; x∈X}, B = {<y,$(\frac{c}{0.4},\frac{d}{0.5})$,$(\frac{c}{0.5},\frac{d}{0.3})$>; y∈Y}, C = {<y,$(\frac{c}{0.4},\frac{d}{0.6})$,$(\frac{c}{0.5},\frac{d}{0.2})$>; y∈Y}. Define an intuitionistic fuzzy mapping f:(X,τ)$\rightarrow$(Y,σ) by $f(a) = d, f(b) = c$. B is an IFOS in (Y, σ) .$f^{-1}(B)$ = {<x, $(\frac{a}{0.5}, \frac{b}{0.4})$, $(\frac{a}{0.3}, \frac{b}{0.5})$>; x∈X} is an IFSOS in (X,τ), since cl(nt$f^{-1}(B)$) = A^c. and $f^{-1}(B) \subseteq$ cl(int$f^{-1}(B)$). Hence f is an IF semi continuous. C is an IFS in Y. Also C $\subseteq$ int(cl(intC))= $1_\sim$ which implies C is an IFαOS in Y. $f^{-1}(C)$ = {<x, $(\frac{a}{0.6}, \frac{b}{0.4})$, $(\frac{a}{0.2}, \frac{b}{0.5})$>; x∈X} cl(int$f^{-1}(C)$)= A^c. Hence $f^{-1}(C) \nsubseteq$ cl(int$f^{-1}(C)$) which implies $f^{-1}(C)$ is not IFSOS in X. Thus f is not IF semi-α-irresolute function.

Proposition 3.6. *Every intuitionistic fuzzy strongly α-continuous is an IF semi-α-irresolute function.*

Proof. Follows from the definitions. However the converse of the above Proposition 3.6 is need not be true, as shown by the following example. ☐

Example 3.7. Let X = $\{a, b\}, Y = \{c, d\}$, τ =$\{0_\sim$, $1_\sim$, A$\}$, σ=$\{0_\sim$, $1_\sim$, B$\}$ where
A = $\{<x,(\frac{a}{0.2},\frac{b}{0.4}),(\frac{a}{0.3},\frac{b}{0.4})>; x\in X\}$,
B = $\{<y,(\frac{c}{0.4},\frac{d}{0.2}),(\frac{c}{0.4},\frac{d}{0.3})>; y\in Y\}$,
C = $\{<y,(\frac{c}{0.4},\frac{d}{0.3}),(\frac{c}{0.4},\frac{d}{0.2})>; y\in Y\}$.
Define an intuitionistic fuzzy mapping f:(X,τ)$\rightarrow$(Y,σ) by $f(a) = d, f(b) = c$. B is an IFOS in (Y, σ). And B is an IFαOS in Y, since B $\subseteq$ int(cl(intB))) = B. $f^{-1}(B) = \{<x, (\frac{a}{0.2},\frac{b}{0.4}), (\frac{a}{0.3},\frac{b}{0.4})>; x\in X\}$, and cl(int$f^{-1}(B)) = A^c$. Thus $f^{-1}(B) \subseteq$ cl(int$f^{-1}(B)$). Hence $f^{-1}(B)$ is IFSOS in X, which implies f is IF semi-α-irresolute. C is an IFS in Y. cl(intC) = B^c. Hence C $\subseteq$ cl(intC). Thus C is IFSOS in Y. $f^{-1}(C)$ = $\{<x, (\frac{a}{0.3},\frac{b}{0.4}), (\frac{a}{0.2},\frac{b}{0.4})>; x\in X\}$ And int(cl(int$f^{-1}(C)$)) = A. Since $f^{-1}(C) \nsubseteq$ int(cl(int$f^{-1}(C)$)), $f^{-1}(C)$ is not an IFαOS in X. Hence f is not IF strongly-α-continuous.

Proposition 3.8. *Every intuitionistic fuzzy semi-α-irresolute is an intuitionistic fuzzy β continuous.*

Proof. Follows from the definitions. However the converse of the above Proposition 3.8 is need not be true, in general as shown by the following example. ☐

Example 3.9. Let X = $\{a, b, c\} = Y$, τ =$\{0_\sim$, $1_\sim$, A$\}$, σ=$\{0_\sim$, $1_\sim$, B$\}$ where
A = $\{<x, (\frac{a}{0.7},\frac{b}{0.5},\frac{c}{0.3}), (\frac{a}{0.1},\frac{b}{0.4}, \frac{c}{0.2})>; x\in X\}$,
B = $\{<y, (\frac{a}{0.1},\frac{b}{0.3},\frac{c}{0.4}), (\frac{a}{0.2},\frac{b}{0.4}, \frac{c}{0.6})>; y\in Y\}$
Define an intuitionistic fuzzy mapping f:(X,τ)$\rightarrow$(Y,σ) by $f(a) = b, f(b) = c, f(c) = a$. B is an IFOS in (Y,$\sigma$). And $f^{-1}(B) = \{<x, (\frac{a}{0.3},\frac{b}{0.4},\frac{c}{0.1}), (\frac{a}{0.4}, \frac{b}{0.6},\frac{c}{0.2})>; x\in X\}$, cl(int(cl$f^{-1}(B)$)) = $1_\sim$. Since $f^{-1}(B) \subseteq$ cl(int(cl$f^{-1}(B)$)), $f^{-1}(B)$ is an IFβOS in X. Thus f is IF β-continuous. B is an IFαOS in (Y,σ), since B $\subseteq$ int(cl(int(B)))=B. And cl(int$f^{-1}(B)$) = $0_\sim$. So $f^{-1}(B) \nsubseteq$ cl(int$f^{-1}(B)$). Hence $f^{-1}(B)$ is not IFSOS in X . So f is not IF semi-α-irresolute function.

Proposition 3.10. *Every intuitionistic fuzzy irresolute function is an intuitionistic fuzzy semi-α-irresolute function.*

Proof. Follows from the definitions. However, the converse of the above Proposition 3.10 is need not be true, as shown by the following example. ☐

Example 3.11. Let

X = $\{a, b\}$, Y= $\{c, d\}$, τ =$\{0_\sim$, $1_\sim$, A, B, A$\cup$B, A$\cap$B$\}$, σ=$\{0_\sim$, $1_\sim$, C$\}$

where
A = $\{<x,(\frac{a}{0.2},\frac{b}{0.2}),(\frac{a}{0.5},\frac{b}{0.4})>; x\in X\}$,
B = $\{<x,(\frac{c}{0.4},\frac{d}{0.4}),(\frac{c}{0.6},\frac{d}{0.6})>; x\in X\}$,
C = $\{<y,(\frac{c}{0.2},\frac{d}{0.4}),(\frac{c}{0.6},\frac{d}{0.6})>; y\in Y\}$.
D = $\{<y,(\frac{c}{0.3},\frac{d}{0.4}),(\frac{c}{0.3},\frac{d}{0.5})>; y\in Y\}$.
Define an intuitionistic fuzzy mapping f:(X,τ)$\rightarrow$(Y,σ) by $f(a) = c, f(b) = d$. C ia an IFαOS in (Y,σ), since C $\subseteq$ int(cl(int(C)))=C. $f^{-1}(C) = \{<x,(\frac{a}{0.2},\frac{b}{0.4}),(\frac{a}{0.6},\frac{b}{0.6})>;$ $x\in X\}$. And cl(int$f^{-1}(C)$) = $(A \cup B)^c$. Thus $f^{-1}(C) \subseteq$ cl(int$f^{-1}(C)$). Hence

$f^{-1}(C)$ is IFSOS in X, which implies f is IF semi-α-irresolute. D ia an IFS in Y, and cl(int(D)) = C^c. Hence D $\subseteq$ cl(int(D)). Thus D is an IFSOS in Y. $f^{-1}(D)$ = $\{$<x,$(\frac{a}{0.3},\frac{b}{0.4}),(\frac{a}{0.3},\frac{b}{0.5})$>; x$\in$X$\}$. And cl(int$f^{-1}(D)$)= $(A \cup B)^c$. So, $f^{-1}(D) \nsubseteq$ cl(int$f^{-1}(B)$). Thus $f^{-1}(B)$ is not IFSOS in X. Hence f is not IF irresolute.

Remark 3.12. Intuitionistic fuzzy semi-α-irresolute function and intuitionistic fuzzy pre-α-irresolute function are independant of each other.

Example 3.13. Let X = $\{a, b\}$, Y= $\{c, d\}$, τ =$\{0_\sim$, $1_\sim$,A$\}$, σ=$\{0_\sim$, $1_\sim$, B$\}$ where
A = $\{$<x,$(\frac{a}{0.3},\frac{b}{0.4}),(\frac{a}{0.6},\frac{b}{0.5})$>; x$\in$X$\}$,
B = $\{$<y,$(\frac{c}{0.4},\frac{d}{0.5}),(\frac{c}{0.5},\frac{d}{0.5})$>; y$\in$Y$\}$.
Define an intuitionistic fuzzy mapping f:(X,τ)$\to$(Y,σ) by $f(a) = d, f(b) = c$. B ia an IFαOS in (Y,σ), since B $\subseteq$ int(cl(int(B)))=B. $f^{-1}(B)$ = $\{$<x,$(\frac{a}{0.5},\frac{b}{0.4}),(\frac{a}{0.5},\frac{b}{0.5})$>; x$\in$X$\}$. And cl(int$f^{-1}(B)$) = A^c. Thus $f^{-1}(B) \subseteq$ cl(int$f^{-1}(B)$), and $f^{-1}(B)$ is IFSOS in X. Hence f is IF semi-α-irresolute. B is an IFαOS in (Y,σ), and int(cl$f^{-1}(B)$) = A. So, $f^{-1}(B) \nsubseteq$ int(cl$f^{-1}(B)$). Thus $f^{-1}(B)$ is not IFPOS in X. Hence f is not IF pre-α-irresolute.

Example 3.14. Let X = $\{a, b, c\} = Y$, τ =$\{0_\sim$, $1_\sim$, A$\}$, σ=$\{0_\sim$, $1_\sim$, B$\}$ where
A = $\{$<x, $(\frac{a}{0.7},\frac{b}{0.5},\frac{c}{0.3}),(\frac{a}{0.1},\frac{b}{0.4}, \frac{c}{0.2})$>; x$\in$X$\}$,
B = $\{$<y, $(\frac{a}{0.1},\frac{b}{0.3},\frac{c}{0.4}),(\frac{a}{0.2},\frac{b}{0.4}, \frac{c}{0.6})$>; y$\in$Y$\}$
Define an intuitionistic fuzzy mapping f:(X,τ)$\to$(Y,σ) by $f(a) = b, f(b) = c, f(c) = a$. B is an IF$\alpha$OS in Y. And $f^{-1}(B)$ = $\{$<x, $(\frac{a}{0.3},\frac{b}{0.4},\frac{c}{0.1}),(\frac{a}{0.4},\frac{b}{0.6}, \frac{c}{0.2})$>; x$\in$X$\}$, int(cl$f^{-1}(B)$) = $1_\sim$. Since $f^{-1}(B) \subseteq$ int(cl$f^{-1}(B)$), $f^{-1}(B)$ is an IFPOS in X. Thus f is IF pre-α-irresolute. B is an IFαOS in Y. And cl(int$f^{-1}(B)$))= $0_\sim$. So $f^{-1}(B) \nsubseteq$ cl(int$f^{-1}(B)$). Hence $f^{-1}(B)$ is not IFSOS in X . So f is not IF semi-α-irresolute function.

Theorem 3.15. *If f is a function from a IFTS (X,τ) to another IFTS(Y,σ) then the following are equivalent.*
(a) *f is IF semi-α-irresolute.*
(b) *f^{-1}(B) $\subseteq$ cl(intf^{-1}(B)) for every IFαOS B in Y.*
(c) *f^{-1}(C) is IF semi closed in X for every IFα-closed set C in Y.*
(d) *int(clf^{-1}(D)))$\subseteq$ f^{-1}(αcl(D)) for every IFS D of Y.*
(e) *f(int(cl(E))) $\subseteq$ αcl(f(E)) for every IFS E of X.*

Proof. (a) $\Rightarrow$ (b) Let B be IFαOS in Y.By (a), f^{-1}(B) is a IF semi open in X. Therefore, f^{-1}(B) $\subseteq$ cl(intf^{-1}(B)). Hence (a) $\Rightarrow$ (b) is proved.

(b) $\Rightarrow$ (c) Let C be any IFαCS in Y. Then $\bar{\text{C}}$ be IFαOS in Y. By (b), $f^{-1}(\bar{\text{C}}) \subseteq$ cl(int$f^{-1}(\bar{\text{C}})$). But $\overline{f^{-1}(C)} \subseteq$ cl(int$(\overline{f^{-1}(C)})$) = cl$(\overline{clf^{-1}(C)})$ = $\overline{int(clf^{-1}(C)}$
$\Rightarrow \overline{f^{-1}(C)} \subseteq \overline{int(clf^{-1}(C)}$
$\Rightarrow$ int(cl f^{-1}(C)) $\subseteq$ f^{-1}(C)
$\Rightarrow$ f^{-1}(C) is IF semi closed in X.
Hence (b)$\Rightarrow$(c) is proved.

(c) $\Rightarrow$ (d) Let D be IFS in Y. Then αcl(D) is a IFα-closed in Y. By(c) f^{-1}(αcl(D)) is IF semi closed in X. Then int(clf^{-1}(αcl(D)) $\subseteq$ f^{-1}(αcl(D)). Thus we have

$$\text{int}(\text{cl}f^{-1}(\text{D})) \subseteq f^{-1}(\alpha\text{cl}(\text{D})).$$

Hence (c)⇒(d) is proved.

(d) ⇒ (e) Let E be an IFS in X. Then, int(cl(E)) = int(cl(f^{-1}(f(E)))) ⊆ int(cl(f^{-1} (αcl(f(E))))) ⊆ f^{-1}(αcl(f(E))). Therefore, int(cl(E)) ⊆ f^{-1}(αcl(f(E)). We get f(int(cl(E))) ⊆ αcl(f(E)) Thus (d)⇒(e) is proved.

(e) ⇒ (a) Let B be IFα-open set in Y. Then $f^{-1}(\overline{B}) = \overline{f^{-1}(B)}$ is an IFS in X. By (e), $f(\text{int}(\text{cl}(f^{-1}(\overline{B}))) \subseteq \alpha\text{cl}(f(f^{-1}(\overline{B}))) \subseteq \alpha\text{cl}(\overline{B}) = \overline{\alpha int B} = \overline{B}$. It implies $f(\text{int}(\text{cl}(f^{-1}(\overline{B}))) \subseteq \overline{B}$ ⇢ ①

Consider, $\overline{cl(int(f^{-1}(B)))} = \text{int}(\overline{int(f^{-1}(B))}) = \text{int}(\text{cl}(\overline{(f^{-1}(B))})) = \text{int}(\text{cl}((f^{-1}(\overline{B}))$ $\subseteq (f^{-1}(\text{f}(\text{int}(\text{cl}((f^{-1}(\overline{B})))$ ⇢ ②

By ①, ②, $\overline{cl(int(f^{-1}(B)))} \subseteq f^{-1}(f(\text{int}(\text{cl}((f^{-1}(\overline{B}))))) \subseteq f^{-1}(\overline{B})) = \overline{f^{-1}(B)}$. Hence f^{-1}(B) ⊆ cl(int(f^{-1}(B)). Thus f^{-1}(B) is an IF semi open in X. Therefore f is IF semi-α-irresolute. Hence (e)⇒(a) is proved. □

4. Properties of intuitionistic fuzzy semi-α-irresolute functions

The following four lemmas are given here for convenience of the reader.

Lemma 4.1 ([3]). *Let f:X→Y be a mapping, and A_α be a family of IF sets of Y. Then*

(a) $f^{-1}(\bigcup A_\alpha) = \bigcup f^{-1}(A_\alpha)$

(b) $f^{-1}(\bigcap A_\alpha) = \bigcap f^{-1}(A_\alpha)$

Lemma 4.2 ([6]). *Let f:X_i→Y_i be a mapping and A,B are IFS's of Y_1 and Y_2 respectively then* $(f_1 \times f_2)^{-1}(A \times B) = {f_1}^{-1}(A) \times {f_2}^{-1}(B)$

Lemma 4.3 ([6]). *Let g:X→X×Y be a graph of a mapping f:(X,τ)→(Y,σ). If A and B are IFS's of X and Y respectivly, then* $g^{-1}(1_\sim \times B) = (1_\sim \cap f^{-1}(B))$

Lemma 4.4 ([6]). *Let X and Y be intuitionistic fuzzy topological spaces, then (X,τ) is product related to (Y,σ) if for any IFS C in X, D in Y whenever $\bar{A} \not\supseteq C$, $\bar{B} \not\supseteq D$ implies $\bar{A} \times 1_\sim \bigcup 1_\sim \times \bar{B} \supseteq C \times D$ there exists $A_1 \in \tau$, $B_1 \in \sigma$ such that $\overline{A_1} \supseteq C$ and $\overline{B_1} \supseteq D$ and $\overline{A_1} \times 1_\sim \bigcup 1_\sim \times \overline{B_1} = \bar{A} \times 1_\sim \bigcup 1_\sim \times \bar{B}$.*

Lemma 4.5 ([10]). *Let X and Y be intuitionistic fuzzy topological spaces such that X is product related to Y. Then the product A×B of IFαOS A in X and a IFαOS B in Y is a IFαOS in fuzzy product spaces X×Y.*

Theorem 4.6. *Let f:X→Y be a function and assume that X is product related to Y. If the graph g:X→ X×Y of f is IF semi-α-irresolute then so is f.*

Proof. Let B be IFαOS in Y. Then by lemma 4.3

$$f^{-1}(\text{B}) = 1_\sim \cap f^{-1}(\text{B}) = g^{-1}(1_\sim \times \text{B}).$$

Now $1_\sim \times$ B is a IFαOS in X×Y. Since g is IF semi-α-irresolute then $g^{-1}(1_\sim \times \text{B})$ is IF semi open in X. Hence f^{-1}(B) is IF semi open in X. Thus f is IF semi-α-irresolute. □

Theorem 4.7. *If a function f:X→ ΠY_i is a IF semi-α-irresolute, then $P_i \circ f$:X→ Y_i is IF semi-α-irresolute, where P_i is the projection of ΠY_i onto Y.*

Proof. Let B_i be any IFαOS of Y_i. Since P_i is IF continuous and IFOS, it is IFαOS. Now P_i:$\Pi Y_i \to Y_i$; $P_i^{-1}(B_i)$ is IFαOS in Π Y_i. Therefore, P_i is IF α-irresolute function. Now $(P_i \circ f)^{-1}(B_i) = f^{-1}$ $(P_i^{-1}(B_i))$, since f is IF semi-α-irresolute and $P_i^{-1}(B_i)$ is IFαOS, f^{-1} $(P_i^{-1}(B_i))$ is IFSOS in X. Hence $(P_i \circ f)$ is IF semi-α-irresolute. □

Theorem 4.8. *If f_i:$X_i \to Y_i$, (i=1,2) are IF semi-α-irresolute and X_1 is product related to X_2 then $f_1 \times f_2$:$X_1 \times X_2 \to Y_1 \times Y_2$ is IF semi-α-irresolute.*

Proof. Let C = $\bigcup(A_i \times B_i)$ where A_i and B_i are IFαopen sets of Y_1 and Y_2 respectively. Since Y_1 is product related to Y_2 , by previous lemma(4.5), that C = $\bigcup(A_i \times B_i)$ is IFα-Open of $Y_1 \times Y_2$. Then by lemma (4.1) and (4.2) we have $(f_1 \times f_2)^{-1}$(C) = $(f_1 \times f_2)^{-1} \bigcup(A_i \times B_i) = \bigcup$ $(f_1{}^{-1}(A_i) \times f_2{}^{-1}(B_i))$. Since f_1 and f_2 are IF semi-α-irresolute, $(f_1 \times f_2)^{-1}$(C) is an IFSOS in $X_1 \times X_2$ and hence $f_1 \times f_2$ is IF semi-α-irresolute function. □

Definition 4.9 ([4])**.** Let (X,τ) be any IFTS and let A be any IFS in X. Then A is called IF dense set if clA = $1_\sim$ and A is called nowhere IF dense set if int(cl(A))= $0_\sim$.

Theorem 4.10. *If a function f:(X,τ)$\to$(Y,σ) is IF semi-α-irresolute, then $f^{-1}(A)$ is IF semi closed in X for any nowhere IF dense set A of Y.*

Proof. Let A be any nowhere IF dense set in Y. Then int(clA)= $0_\sim$. Now, $\overline{int(clA)}$ = $1_\sim$. $\Longrightarrow$ cl($\overline{cl(A)}$) = $1_\sim$ which implies cl(int($\overline{A}$)) = $1_\sim$. Since int$1_\sim$ = $1_\sim$, int(cl(int($\overline{A}$)) = int$1_\sim$ = $1_\sim$. Hence $\overline{A} \subseteq$ int(cl(int($\overline{A}$))) = $1_\sim$. Then $\overline{A}$ is IFαOS in Y. Since f is IF semi-α-irresolute, $f^{-1}(\overline{A})$ is IF semi open set in X. Hence f^{-1}(A) is IFSCS in X. □

Theorem 4.11. *A mapping f:X$\to$Y from an IFTS X into an IFTS Y is IF semi α-irresolute if and only if for each IFP $p_{(\alpha,\beta)}$ in X and IFαOS B in Y such that f($p_{(\alpha,\beta)}$)$\in$B , there exists an IFSOS A in X such that $p_{(\alpha,\beta)}$ $\in$A and f(A) $\subseteq$ B.*

Proof. Let f be any IF semi α-irresolute mapping, $p_{(\alpha,\beta)}$ be an IFP in X and B be any IFαOS in Y such that $f(p_{(\alpha,\beta)})$ $\in$B. Then $p_{(\alpha,\beta)} \in f^{-1}$(B). Let A = f^{-1}(B). Then A is an IFSOS in X which containing IFP $p_{(\alpha,\beta)}$ and f(A) = $f(f^{-1}$(B))$\subseteq$ B.

Conversely, let B be an IFαOS in Y and $p_{(\alpha,\beta)}$ be IFP in X such that $p_{(\alpha,\beta)} \in$ f^{-1}(B). According to assumption there exists IFSOS A in X such that $p_{(\alpha,\beta)}$ $\in$A and f(A)$\subseteq$B. Hence $p_{(\alpha,\beta)}$ $\in$A $\subseteq f^{-1}$(B). We have $p_{(\alpha,\beta)}$ $\in$A $\subseteq$ cl(intA) $\subseteq$ cl(intf^{-1}(B)). Therefore, f^{-1}(B)$\subseteq$ cl(intf^{-1}(B)). So f is IF semi-α-irresolute mapping. □

Theorem 4.12. *The following hold for functions f:X$\to$Y and g:Y$\to$Z*

(i) *If f is IF semi-α-irresolute and g is IF α-irresolute then g$\circ f$ is IF semi-α-irresolute.*
(ii) *If f is IF semi-α-irresolute and g is IF α-continuous then g$\circ f$ is IF semi continuous.*
(iii) *If f is IF irresolute and g is IF semi-α-irresolute then g$\circ f$ is IF semi-α-irresolute .*

Proof. (i) Let B be an IFαOS in Z. Since g is IF α-irresolute, g^{-1}(B) is an IFαOS in Y. Now $(g \circ f)^{-1}$(B) $= f^{-1}(g^{-1}(B))$. Since f is IF semi-α-irresolute, $f^{-1}(g^{-1}(B))$ is IFSOS in X. Hence $g \circ f$ is IF semi-α-irresolute.

(ii) Let B be IFOS in Z. Since g is IF α-continuous, g^{-1}(B) is an IFαOS in Y. Now $(g \circ f)^{-1}$(B) $= f^{-1}(g^{-1}(B))$. Since f is IF semi-α-irresolute, $f^{-1}(g^{-1}(B))$ is IFSOS in X which implies $g \circ f$ is IF semi continuous.

(iii) Let B be an IFαOS in Z. Since g is IF semi-α-irresolute, g^{-1}(B) is an IFSOS in Y. Now $(g \circ f)^{-1}$(B) $= f^{-1}(g^{-1}(B))$. Since f is IF irresolute, $f^{-1}(g^{-1}(B))$ is IFSOS in X. Hence $g \circ f$ is IF semi-α-irresolute. □

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