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# Soft linear operators in soft normed linear spaces

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Abstract. In the present paper an idea of soft linear operator over soft linear spaces has been introduced and some basic properties of such operators are studied. Spaces of continuous soft linear operators are investigated and lastly the inverse of a soft linear operator is defined and its properties are studied.

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## 1. Introduction

Molodtsov [17] initiated the theory of soft sets as a new mathematical tool for dealing with uncertainties. He has shown several applications of this theory in solving many practical problems in economics, engineering, social science, medical science, etc. Research works in soft set theory and its applications in various fields have been progressing rapidly since Maji et al. ([14], [15]) introduced several operations on soft sets and applied it to decision making problems. In the line of reduction and addition of parameters of soft sets some works have been done by Chen [3], Pei and Miao [18] , Kong et al. [13] , Zou and Xiao [21]. Aktas and Cagman [1] introduced the notion of soft group and discussed various properties. Jun ([11], [12]) investigated soft BCK/BCI – algebras and its application in ideal theory. Feng et al. [9] worked on soft semirings, soft ideals and idealistic soft semirings. Ali et al. [2] and Shabir and Irfan Ali ([2], [19]) studied soft semigroups and soft ideals over a semi group which characterize generalized fuzzy ideals and fuzzy ideals with thresholds of a semigroup. The idea of soft topological spaces was first given by M. Shabir, M. Naz [20] and mappings between soft sets were described by P. Majumdar, S. K. Samanta [16]. Feng et al. [10] worked on soft sets combined with fuzzy sets and rough sets. Recently in ([4], [5]) we have introduced a notion of soft real sets, soft real numbers, soft complex sets, soft complex numbers and some of their basic properties have been investigated. Some applications of soft real sets and soft real numbers have been presented in real life problems. In ([6], [7], [8]) we introduced the concepts of 'soft metric' over an absolute soft set and 'soft norm', 'soft inner product' over 'soft linear spaces'. Various properties of 'soft metric spaces', 'soft linear spaces', 'soft normed linear spaces' and 'soft inner product spaces' have been investigated with examples and counter examples.

In fact, in this paper we have introduced a notion of soft linear operator over soft linear spaces and studied some of their properties. In section 2 some preliminary results are given. In section 3, a notion of 'soft linear operator' over a 'soft linear space' is given and some properties of such operators are studied. It has been shown that every bounded soft linear operator is also a continuous soft linear operator; the converse of which holds under some further considerations. In section 4, the spaces of continuous soft linear operators are studied. In that section it has been shown that the set of all continuous soft linear operators can be identified as to form an absolute soft vector spece $L(\check{X}, \check{Y})$. It has been also established that $L(\check{X}, \check{Y})$ is a soft normed linear space with respect to a suitable soft norm and with the completeness of $\check{Y}$ it is a soft Banach space under certain restrictions. In section 5, inverse of a soft linear operator is defined and its various properties are also studied. Section 6 concludes the paper.

## 2. Preliminaries

**Definition 2.1** ([17])**.** Let $U$ be an universe and $E$ be a set of parameters. Let $\mathcal{P}(U)$ denote the power set of $U$ and $A$ be a non-empty subset of $E$. A pair $(F, A)$ is called a soft set over $U$, where $F$ is a mapping given by $F : A \to \mathcal{P}(U)$. In other words, a soft set over $U$ is a parametrized family of subsets of the universe $U$. For $\varepsilon \in A$, $F(\varepsilon)$ may be considered as the set of $\varepsilon$ − approximate elements of the soft set $(F, A)$.

**Definition 2.2** ([10])**.** For two soft sets $(F, A)$ and $(G, B)$ over a common universe $U$, we say that $(F, A)$ is a soft subset of $(G, B)$ if

(1) $A \subseteq B$ and
(2) for all $e \in A$, $F(e) \subseteq G(e)$. We write $(F, A)\tilde{\subset} (G, B)$.

$(F, A)$ is said to be a soft superset of $(G, B)$, if $(G, B)$ is a soft subset of $(F, A)$. We denote it by$(F, A)\tilde{\supset} (G, B)$.

**Definition 2.3** ([10])**.** Two soft sets $(F, A)$ and $(G, B)$ over a common universe $U$ are said to be equal if $(F, A)$ is a soft subset of $(G, B)$ and $(G, B)$ is a soft subset of $(F, A)$.

**Definition 2.4** ([15])**.** The union of two soft sets $(F, A)$ and $(G, B)$ over the common universe $U$ is the soft set
$(H, C)$ , where $C = A \cup B$ and for all $e \in C$,

$$H(e) = \begin{cases} F(e) & if\ e \in A - B \\ G(e) & if\ e \in B - A \\ F(e) \cup G(e) & if\ e \in A \cap B. \end{cases}$$

We express it as $(F, A)\,\tilde{\cup}\,(G, B) = (H, C)$.

The following definition of intersection of two soft sets is given as that of the bi-intersection in [9].

**Definition 2.5** ([9])**.** The intersection of two soft sets $(F, A)$ and $(G, B)$ over the common universe $U$ is the soft set $(H, C)$, where $C = A \cap B$ and for all $e \in C$, $H(e) = F(e) \cap G(e)$. We write $(F, A) \widetilde{\cap} (G, B) = (H, C)$.

Let $X$ be an initialuniversal set and $A$ be the non-empty set of parameters. In the above definitions the set of parameters may vary from soft set to soft set, but in our considerations, throught this paper all soft sets have the same set of parameters $A$. The above definitions are also valid for these type of soft sets as a particular case of those definitions.

**Definition 2.6** ([10])**.** The complement of a soft set $(F, A)$ is denoted by $(F, A)^c = (F^c, A)$, where $F^c : A \to \mathcal{P}(U)$ is a mapping given by $F^c(\alpha) = U - F(\alpha)$, for all $\alpha \in A$.

**Definition 2.7** ([15])**.** A soft set $(F, A)$ over $U$ is said to be an *absolute* soft set denoted by $\check{U}$ if for all $\varepsilon \in A$, $F(\varepsilon) = U$.

**Definition 2.8** ([15])**.** A soft set $(F, A)$ over $U$ is said to be a *null* soft set denoted by $\Phi$ if for all $\varepsilon \in A$, $F(\varepsilon) = \varnothing$.

**Definition 2.9** ([20])**.** The difference $(H, A)$ of two soft sets $(F, A)$ and $(G, A)$ over $X$, denoted by $(F, A) \setminus (G, A)$, is defined by $H(e) = F(e) \setminus G(e)$ for all $e \in A$.

**Proposition 2.10** ([20])**.** *Let $(F, A)$ and $(G, A)$ be two soft sets over $X$. Then*

(i) $((F, A) \widetilde{\cup} (G, A))^c = (F, A)^c \widetilde{\cap} (G, A)^c$
(ii) $((F, A) \widetilde{\cap} (G, A))^c = (F, A)^c \widetilde{\cup} (G, A)^c$.

**Definition 2.11** ([4])**.** Let $X$ *be* a non-empty set and $A$ be a non-empty parameter set. Then a function $\varepsilon : A \to X$ is said to be a soft element of $X$. A soft element $\varepsilon$ of $X$ is said to belongs to a soft set $B$ of $X$, which is denoted by $\varepsilon \widetilde{\in} B$, if $\varepsilon(e) \in A(e)$, $\forall e \in A$. Thus for a soft set A of $X$ with respect to the index set $A$, we have $B(e) = \{\varepsilon(e), \varepsilon \widetilde{\in} B\}$, $e \in A$.

It is to be noted that every singleton soft set (a soft set $(F, A)$ for which $F(e)$ is a singleton set, $\forall e \in A$) can be identified with a soft element by simply identifying the singleton set with the element that it contains $\forall e \in A$.

**Definition 2.12** ([4])**.** Let $R$ be the set of real numbers and $\mathfrak{B}(R)$ the collection of all non-empty bounded subsets of $R$ and $A$ taken as a set of parameters. Then a mapping $F : A \to \mathfrak{B}(R)$ is called a *soft real set*. It is denoted by $(F, A)$. If specifically $(F, A)$ is a singleton soft set, then after identifying $(F, A)$ with the corresponding soft element, it will be called a *soft real number*.

The set of all soft real numbers is denoted by $\mathbb{R}(A)$ and the set of all non-negative soft real numbers by $\mathbb{R}(A)^*$.

**Definition 2.13** ([5])**.** Let $C$ be the set of complex numbers and $\wp(C)$be the collection of all non-empty bounded subsets of the set of complex numbers.$A$ be a set of parameters. Then a mapping

$$F : A \to \wp(C)$$

is called a soft complex set. It is denoted by $(F, A)$.

If in particular $(F, A)$ is a singleton soft set, then identifying $(F, A)$ with the corresponding soft element, it will be called a soft complex number.

The set of all soft complex numbers is denoted by $\mathbb{C}(A)$.

**Definition 2.14** ([5])**.** Let $(F, A)$ be a soft complex set. Then the complex conjugate of $(F, A)$ is denoted by $(\overline{F}, A)$and is defined by $\overline{F}(\lambda) = \{\overline{z} : z \in F(\lambda)\}$, $\forall \lambda \in A$ , where $\overline{z}$ is complex conjugate of the ordinary complex number $z$. The complex conjugate of a soft complex number $(F, A)$ is $\overline{F}(\lambda) = \overline{z}; z \in F(\lambda), \forall \lambda \in A$.

**Definition 2.15** ([5])**.** Let $(F, A), (G, A) \in \mathbb{C}(A)$. Then the sum, difference, product and division are defined by

$(F + G)(\lambda) = z + w; z \in F(\lambda),\ w \in G(\lambda), \forall \lambda \in A;$
$(F - G)(\lambda) = z - w; z \in F(\lambda),\ w \in G(\lambda), \forall \lambda \in A;$
$(FG)(\lambda) = zw; z \in F(\lambda),\ w \in G(\lambda), \forall \lambda \in A;$
$(F/G)(\lambda) = z/w; z \in F(\lambda),\ w \in G(\lambda), \forall \lambda \in A$; provided $G(\lambda) \neq \varnothing$, $\forall \lambda \in A$.

**Definition 2.16** ([5])**.** Let $(F, A)$ be a soft complex number. Then the modulus of $(F, A)$ is denoted by $(|F|, A)$and is defined by $|F|(\lambda) = |z|; z \in F(\lambda), \forall \lambda \in A$, where $z$ is an ordinary complex number.

Since the modulus of each ordinary complex number is a non-negative real number and by definition of soft real numbers it follows that $(|F|, A)$is a non-negative soft real number for every soft complex number $(F, A)$.

Let $X$ be a non-empty set. Let $\check{X}$ be the absolute soft set i.e., $F(\lambda) = X, \forall \lambda \in A$, where $(F, A) = \check{X}$. Let $\mathcal{S}(\check{X})$ be the collection all soft sets $(F, A)$ over $X$ for which $F(\lambda) \neq \varnothing$, for all $\lambda \in A$ together with the null soft set $\Phi$.

Let $(F, A)(\neq \Phi) \in \mathcal{S}(\check{X})$, then the collection of all soft elements of $(F, A)$ will be denoted by $SE(F, A)$. For a collection $\mathfrak{B}$ of soft elements of $\check{X}$, the soft set generated by $\mathfrak{B}$ is denoted by $SS(\mathfrak{B})$.

**Definition 2.17** ([6])**.** A mapping $d : SE(\check{X}) \times SE(\check{X}) \to \mathbb{R}(A)^*$ , is said to be a *soft metric* on the soft set $\check{X}$ if $d$ satisfies the following conditions:

(M1). $d(\tilde{x}, \tilde{y}) \widetilde{\geq} \overline{0}$, for all $\tilde{x}, \tilde{y} \widetilde{\in} \check{X}$.
(M2). $d(\tilde{x}, \tilde{y}) = \overline{0}$, if and only if $\tilde{x} = \tilde{y}$.
(M3). $d(\tilde{x}, \tilde{y}) = d(\tilde{y}, \tilde{x})$ for all $\tilde{x}, \tilde{y} \widetilde{\in} \check{X}$.
(M4). For all $\tilde{x}, \tilde{y}, \tilde{z} \widetilde{\in} \check{X}$, $d(\tilde{x}, \tilde{z}) \widetilde{\leq} d(\tilde{x}, \tilde{y}) + d(\tilde{y}, \tilde{z})$

The soft set $\check{X}$ with a soft metric $d$ on $\check{X}$ is said to be a *soft metric space* and is denoted by $(\check{X}, d, A)$ or $(\check{X}, d)$.

**Definition 2.18** ([7])**.** Let $V$ be a vector space over a field $K$ and let $A$ be a parameter set. Let $G$ be a soft set over $(V, A)$. Now $G$ is said to be a soft vector space or soft linear space of $V$ over $K$ if $G(\lambda)$ is a vector subspace of $V$, $\forall \lambda \in A$.

**Definition 2.19** ([7])**.** Let $F$ be a soft vector space of $V$ over $K$. Let $G : A \to \wp(V)$ be a soft set over $(V, A)$. Then $G$ is said to be a soft vector subspace of $F$ if

(i) for each $\lambda \in A$, $G(\lambda)$ is a vector subspace of $V$ over $K$ and
(ii) $F(\lambda) \supseteq G(\lambda)$, $\forall \lambda \in A$.

**Definition 2.20** ([7]). Let $G$ be a soft vector space of $V$ over $K$. Then a soft element of $G$ is said to be a soft vector of $G$. In a similar manner a soft element of the soft set $(K, A)$ is said to be a soft scalar, K being the scalar field.

**Definition 2.21** ([7]). Let $\tilde{x}, \tilde{y}$ be soft vectors of $G$ and $\tilde{k}$ be a soft scalar. Then the addition $\tilde{x} + \tilde{y}$ of $\tilde{x}, \tilde{y}$ and scalar multiplication $\tilde{k}.\ \tilde{x}$ of $\tilde{k}$ and $\tilde{x}$ are defined by $(\tilde{x} + \tilde{y})(\lambda) = \tilde{x}(\lambda) + \tilde{y}(\lambda)$, $\left(\tilde{k}.\ \tilde{x}\right)(\lambda) = \tilde{k}(\lambda).\tilde{x}(\lambda)$, $\forall \lambda \in A$. Obviously, $\tilde{x} + \tilde{y}$, $\tilde{k}.\ \tilde{x}$ are soft vectors of $G$.

**Definition 2.22** ([7]). Let $\check{X}$ be the absolute soft vector space i.e., $\check{X}(\lambda) = X$, $\forall \lambda \in A$. Then a mapping $\|.\| : SE(\check{X}) \to \mathbb{R}(A)^*$ is said to be a soft norm on the soft vector space $\check{X}$ if $\|.\|$ satisfies the following conditions:

(N1). $\|\tilde{x}\| \tilde{\geq} \overline{0}$, for all $\tilde{x} \tilde{\in} \check{X}$;
(N2). $\|\tilde{x}\| = \overline{0}$ if and only if $\tilde{x} = \Theta$;
(N3). $\|\tilde{\alpha}.\tilde{x}\| = |\tilde{\alpha}| \|\tilde{x}\|$ for all $\tilde{x} \tilde{\in} \check{X}$ and for every soft scalar $\tilde{\alpha}$;
(N4). For all $\tilde{x}, \tilde{y} \tilde{\in} \check{X}$, $\|\tilde{x} + \tilde{y}\| \tilde{\leq} \|\tilde{x}\| + \|\tilde{y}\|$.

The soft vector space $\check{X}$ with a soft norm $\|.\|$ on $\check{X}$ is said to be a soft normed linear space and is denoted by $(\check{X}, \|.\|, A)$ or $(\check{X}, \|.\|)$. (N1), (N2), (N3) and (N4) are said to be soft norm axioms.

**Theorem 2.23** ([7]). *Suppose a soft norm $\|.\|$ satisfies the condition*
*(N5). For $\xi \in X$, and $\lambda \in A$, $\{\|\tilde{x}\|(\lambda) : \tilde{x}(\lambda) = \xi\}$ is a singleton set.*
*Then for each $\lambda \in A$, the mapping $\|.\|_\lambda : X \to R^+$ defined by $\|\xi\|_\lambda = \|\tilde{x}\|(\lambda)$, for all $\xi \in X$ and $\tilde{x} \tilde{\in} \check{X}$ such that $\tilde{x}(\lambda) = \xi$, is a norm on $X$.*

**Definition 2.24** ([7]). A sequence of soft elements $\{\tilde{x}_n\}$ in a soft normed linear space $(\check{X}, \|.\|, A)$ is said to be convergent and converges to a soft element $\tilde{x}$ if $\|\tilde{x}_n - \tilde{x}\| \to \overline{0}$ as $n \to \infty$. This means for every $\tilde{\varepsilon} \tilde{>} \overline{0}$, chosen arbitrarily, $\exists$ a natural number $N = N(\tilde{\varepsilon})$, such that $\overline{0} \tilde{\leq} \|\tilde{x}_n - \tilde{x}\| \tilde{<} \tilde{\varepsilon}$, whenever $n > N$.
i.e., $n > N \implies \tilde{x}_n \in B(\tilde{x}, \tilde{\varepsilon})$. We denote this by $\tilde{x}_n \to \tilde{x}$ as $n \to \infty$ or by $\lim_{n\to\infty} \tilde{x}_n = \tilde{x}$. $\tilde{x}$ is said to be the limit of the sequence $\tilde{x}_n$ as $n \to \infty$.

**Definition 2.25** ([7]). A sequence $\{\tilde{x}_n\}$ of soft elements in a soft normed linear space $(\check{X}, \|.\|, A)$ is said to be a Cauchy sequence in $\check{X}$ if corresponding to every $\tilde{\varepsilon} \tilde{>} \overline{0}$, $\exists\ m \in N$ such that $\|\tilde{x}_i - \tilde{x}_j\| \tilde{\leq} \tilde{\varepsilon}$, $\forall\ i, j \geq m$ i.e., $\|\tilde{x}_i - \tilde{x}_j\| \to \overline{0}$ as $i, j \to \infty$.

**Definition 2.26** ([7]). Let $(\check{X}, \|.\|, A)$ be a soft normed linear space. Then $\check{X}$ is said to be complete if every Cauchy sequence in $\check{X}$ converges to a soft element of $\check{X}$ . Every complete soft normed linear space is called a soft Banach Space.

**Theorem 2.27** ([7]). *Every Cauchy sequence in $\mathbb{R}(A)$, where $A$ is a finite set of parameters, is convergent, i.e., the set of all soft real numbers with its usual modulus soft norm with respect to a finite set of parameters, is a soft Banach space.*

**Proposition 2.28** ([7]). *A set $S = \{\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_n\}$ of soft vectors in a soft vector space $G$ over $V$ is linearly independent if and only if the sets*

$$S(\lambda) = \{\tilde{\alpha}_1(\lambda), \tilde{\alpha}_2(\lambda), \ldots, \tilde{\alpha}_n(\lambda)\}$$

*are linearly independent in $V$, $\forall \lambda \in A$.*

**Proposition 2.29** ([7]). *A set $S = \{\widetilde{\alpha}_1, \widetilde{\alpha}_2, \ldots, \widetilde{\alpha}_n\}$ of soft vectors in a soft vector space $G$ over $V$ is linearly dependent if and only if the sets*

$$S(\lambda) = \{\widetilde{\alpha}_1(\lambda), \widetilde{\alpha}_2(\lambda), \ldots, \widetilde{\alpha}_n(\lambda)\},$$

*are linearly dependent in $V$ for some $\lambda \in A$.*

**Definition 2.30** ([7]). A soft linear space $\check{X}$ is said to be of finite dimensional if there is a finite set of linearly independent soft vectors in $\check{X}$ which also generates $\check{X}$, i.e., any soft element of $\check{X}$ can be expressed as a linear combination of those linearly independent soft vectors.

The set of those linearly independent soft vectors is said to be the basis for $\check{X}$ and the number of soft vectors of the basis is called the dimension of $\check{X}$.

**Lemma 2.31** ([7]). *Let $\tilde{x}_1, \tilde{x}_2, \ldots.., \tilde{x}_n$ be a linearly independent set of soft vectors in a soft linear space $\check{X}$, satisfying (N5). Then there is a soft real number $\tilde{c}\tilde{>}\overline{0}$ such that for every set of soft scalars $\widetilde{\alpha}_1, \widetilde{\alpha}_2, \ldots.., \widetilde{\alpha}_n$ we have*

$$\|\widetilde{\alpha}_1\tilde{x}_1 + \widetilde{\alpha}_2\tilde{x}_2 + \cdots + \widetilde{\alpha}_n\tilde{x}_n\| \tilde{\geq} \tilde{c}\left(|\widetilde{\alpha}_1| + |\widetilde{\alpha}_2| + \cdots + |\widetilde{\alpha}_n|\right).$$

**Theorem 2.32** ([7]). *Every finite dimensional soft normed linear space which satisfies (N5) and have a finite set of parameters, is complete.*

**Theorem 2.33** ([7]). *Every Cauchy sequence in $\mathbb{R}(A)$, where $A$ is a finite set of parameters, is convergent, i.e., the set of all soft real numbers with respect to a finite set of parameters and with its usual modulus soft norm is a soft Banach space.*

**Definition 2.34** ([8]). Let $\check{X}$ be the absolute soft vector space i.e., $\check{X}(\lambda) = X$, $\forall \lambda \in A$. Then a mapping $\langle . \rangle : SE(\check{X}) \times SE(\check{X}) \to \mathbb{C}(A)$ is said to be a soft inner product on the soft vector space $\check{X}$ if $\langle . \rangle$ satisfies the following conditions:

(I1). $\langle \tilde{x}, \tilde{x} \rangle \tilde{\geq} \overline{0}$, for all $\tilde{x} \tilde{\in} \check{X}$ and $\langle \tilde{x}, \tilde{x} \rangle = \overline{0}$ if and only if $\tilde{x} = \Theta$;

(I2). $\langle \tilde{x}, \tilde{y} \rangle = \overline{\langle \tilde{y}, \tilde{x} \rangle}$ where bar denote the complex conjugate of soft complex numbers;

(I3). $\langle \widetilde{\alpha}.\tilde{x}, \tilde{y} \rangle = \widetilde{\alpha}. \langle \tilde{x}, \tilde{y} \rangle$ for all $\tilde{x}, \tilde{y} \tilde{\in} \check{X}$ and for every soft scalar $\widetilde{\alpha}$;

(I4). For all $\tilde{x}, \tilde{y} \tilde{\in} \check{X}$, $\langle \tilde{x} + \tilde{y}, \tilde{z} \rangle = \langle \tilde{x}, \tilde{z} \rangle + \langle \tilde{y}, \tilde{z} \rangle$.

The soft vector space $\check{X}$ with a soft inner product $\langle . \rangle$ on $\check{X}$ is said to be a soft inner product space and is denoted by $(\check{X}, \langle . \rangle, A)$ or $(\check{X}, \langle . \rangle)$. (I1), (I2), (I3) and (I4) are said to be soft inner product axioms.

**Theorem 2.35** ([8]). *(Decomposition Theorem). Suppose a soft inner product $\langle . \rangle$ satisfies the condition*

*(I5). For $(\xi, \eta) \in X \times X$ and $\lambda \in A$,*

$$\left\{\langle \tilde{x}, \tilde{y} \rangle(\lambda) : \tilde{x}, \tilde{y} \tilde{\in} \check{X} \text{ such that } \tilde{x}(\lambda) = \xi, \tilde{y}(\lambda) = \eta\right\}$$

*is a singleton set. Then for each $\lambda \in A$, the mapping $\langle . \rangle_\lambda : X \times X \to \mathbb{C}$ defined by $\langle \xi, \eta \rangle_\lambda = \langle \tilde{x}, \tilde{y} \rangle(\lambda)$, for all $(\xi, \eta) \in X \times X$, and $\tilde{x}, \tilde{y} \tilde{\in} \check{X}$ such that $\tilde{x}(\lambda) = \xi, \tilde{y}(\lambda) = \eta$, is an inner product on $X$.*

**Theorem 2.36** ([8]). *(Schwarz inequality). Let $(\check{X}, \langle . \rangle, A)$ be a soft inner product space satisfying (I5). Let $\tilde{x}$, $\tilde{y}$, $\tilde{\in} \check{X}$. Then $|\langle \tilde{x}, \tilde{y} \rangle| \tilde{\leq} \|\tilde{x}\| . \|\tilde{y}\|$.*

**Theorem 2.37** ([8]). *Let $(\check{X}, \langle . \rangle, A)$ be a soft inner product space satisfying (I5). Let us define $\|.\| : \check{X} \to R(A)^*$ by $\|\tilde{x}\| = \sqrt{\langle \tilde{x}, \tilde{x} \rangle}$, for all $\tilde{x} \tilde{\in} \check{X}$. Then $\|.\|$ is a soft norm on $\check{X}$ satisfying (N5).*

We now establish the following lemma regarding dimension of soft vector spaces.

**Lemma 2.38.** *The dimension of an absolute soft vector space $\check{X}$ is $n$ if and only if dimension of $\check{X}(\lambda) = X$ is $n$ for each $\lambda \in A$.*

*Proof.* Let dimension of $\check{X}$ be $n$. Then there is $n$ linearly independent soft vectors $S = \{\widetilde{\alpha}_1, \widetilde{\alpha}_2, \ldots, \widetilde{\alpha}_n\}$ which also generates $\check{X}$. Then by Proposition 2.28, $S(\lambda) = \{\widetilde{\alpha}_1(\lambda), \widetilde{\alpha}_2(\lambda), \ldots, \widetilde{\alpha}_n(\lambda)\}$ is linearly independent, $\forall\lambda \in A$. Again since $S$ generates $G$, $S(\lambda)$ generates $X$, $\forall\lambda \in A$. Thus $S(\lambda)$ form a basis for $X$, $\forall\lambda \in A$. Hence dimension of $X$ is $n$ for each $\lambda \in A$.

Conversely, let dimension of $X$ be $n$ for each $\lambda \in A$. Then for each $\lambda \in A$, there are $n$ linearly independent vectors $S_\lambda = \{\alpha_{\lambda 1}, \alpha_{\lambda 2}, \alpha_{\lambda 3}, \ldots\ldots, \alpha_{\lambda n}\}$ which also generates $X$. Let us consider soft vectors $S = \{\widetilde{\alpha}_1, \widetilde{\alpha}_2, \ldots, \widetilde{\alpha}_n\}$ by $\widetilde{\alpha}_i(\lambda) = \alpha_{\lambda i}$, for each $\lambda \in A$, and for each $i = 1, 2, \ldots, n$. Then $S(\lambda) = S_\lambda$, $\forall\lambda \in A$ and hence by Proposition 2.28, it follows that $S = \{\widetilde{\alpha}_1, \widetilde{\alpha}_2, \ldots, \widetilde{\alpha}_n\}$ is linearly independent. Again since $S(\lambda) = S_\lambda$ generates $X$, $\forall\lambda \in A$; obviously $S$ generates $\check{X}$. Hence $S = \{\widetilde{\alpha}_1, \widetilde{\alpha}_2, \ldots, \widetilde{\alpha}_n\}$ form a basis for $\check{X}$ and dimension of $\check{X}$ is $n$. □

## 3. Soft linear operators

Let $X$, $Y$ be two vector spaces over a field $K$, $A$ be a non-empty set of parameters. Let $\check{X}$, $\check{Y}$ be the corresponding absolute soft vector spaces i.e., $\check{X}(\lambda) = X$, $\check{Y}(\lambda) = Y$, $\forall\lambda \in A$. We use the notation $\tilde{x}$, $\tilde{y}$, $\tilde{z}$ to denote soft vectors of a soft vector space.

**Definition 3.1.** Let $T : SE(\check{X}) \to SE(\check{Y})$ be an operator. Then $T$ is said to be soft linear if

(L1). $T$ is additive, i.e., $T(\tilde{x}_1 + \tilde{x}_2) = T(\tilde{x}_1) + T(\tilde{x}_2)$ for every soft elements $\tilde{x}_1, \tilde{x}_2 \widetilde{\in} \check{X}$,

(L2). $T$ is homogeneous, i.e. for every soft scalar $\tilde{c}$, $T(\tilde{c}\tilde{x}) = \tilde{c}T(\tilde{x})$, for every soft element $\tilde{x} \widetilde{\in} \check{X}$.

The properties (L1) and (L2) can be put in a combined form $T(\tilde{c}_1\tilde{x}_1 + \tilde{c}_2\tilde{x}_2) = \tilde{c}_1 T(\tilde{x}_1) + \tilde{c}_2 T(\tilde{x}_2)$ for every soft elements $\tilde{x}_1, \tilde{x}_2 \widetilde{\in} \check{X}$ and every soft scalars $\tilde{c}_1$, $\tilde{c}_2$.

**Definition 3.2.** The operator $T : SE(\check{X}) \to SE(\check{Y})$ is said to be continuous at $\tilde{x}_0 \widetilde{\in} \check{X}$ if for every sequence $\{\tilde{x}_n\}$ of soft elements of $\check{X}$ with $\tilde{x}_n \to \tilde{x}_0$ as $n \to \infty$, we have $T(\tilde{x}_n) \to T(\tilde{x}_0)$ as $n \to \infty$ i.e., $\|\tilde{x}_n - \tilde{x}_0\| \to \overline{0}$ as $n \to \infty$ implies $\|T(\tilde{x}_n) - T(\tilde{x}_0)\| \to \overline{0}$ as $n \to \infty$. If $T$ is continuous at each soft element of $\check{X}$, then $T$ is said to be a continuous operator.

**Example 3.3.** If $\check{X}$ be a soft normed linear space. Then the *identity operator* $I : SE(\check{X}) \to SE(\check{X})$ such that $I(\tilde{x}) = \tilde{x}$, for every soft element $\tilde{x} \widetilde{\in} \check{X}$, is a continuous soft linear operator.

**Example 3.4.** If $\check{X}, \check{Y}$ be two soft normed linear spaces. Then the *zero operator* $O : SE(\check{X}) \to SE(\check{Y})$ such that $O(\tilde{x}) = \Theta$, for every soft element $\tilde{x} \widetilde{\in} G$. The zero operator is a continuous soft linear operator.

**Example 3.5.** Let $\mathbb{R}(A)$ be the set of all soft real numbers defined over the parameter set $A$. Let an operator $T : \mathbb{R}(A) \to \mathbb{R}(A)$ be defined by $T(\tilde{x}) = \overline{2}.\tilde{x}$, for every soft real number $\tilde{x}$. Then $T$ is a continuous soft linear operator.

**Theorem 3.6.** *Let $\check{X}$, $\check{Y}$ be two soft normed linear spaces. If $T : SE(\check{X}) \to SE(\check{Y})$ be a soft linear operator, then*

(i) $T(\tilde{x} - \tilde{y}) = T(\tilde{x}) - T(\tilde{y})$;
(ii) $T(\Theta) = \Theta$;
(iii) $T(-\tilde{x}) = -T(\tilde{x})$;
(iv) $T\left(\sum_{k=1}^{n} \tilde{c}_k \tilde{x}_k\right) = \sum_{k=1}^{n} \tilde{c}_k T(\tilde{x}_k)$, *$\tilde{c}_k$ are soft scalars.*

*Proof.* (i) Let $\tilde{x} = \tilde{y} + \tilde{z}$, then $\tilde{z} = \tilde{x} - \tilde{y}$ and we obtain

$$T(\tilde{x}) = T(\tilde{y} + \tilde{z}) = T(\tilde{y}) + T(\tilde{z}) = T(\tilde{y}) + T(\tilde{x} - \tilde{y}).$$

This implies $T(\tilde{x} - \tilde{y}) = T(\tilde{x}) - T(\tilde{y})$.

(ii) In (i), put $\tilde{x} = \tilde{y}$, then $T(\Theta) = \Theta$.

(iii) In (i), put $\tilde{x} = \Theta$, then $T(-\tilde{y}) = T(\Theta) - T(\tilde{y}) = -T(\tilde{y})$.

(iv) We prove this by induction. For $n = 1$ the result is obvious. We suppose that the result is true for $(n-1)$ i.e.,

$$T\left(\sum_{k=1}^{n-1} \tilde{c}_k \tilde{x}_k\right) = \sum_{k=1}^{n-1} \tilde{c}_k T(\tilde{x}_k).$$

Then $T\left(\sum_{k=1}^{n} \tilde{c}_k \tilde{x}_k\right) = T\left(\sum_{k=1}^{n-1} \tilde{c}_k \tilde{x}_k + \tilde{c}_n \tilde{x}_n\right) = T\left(\sum_{k=1}^{n-1} \tilde{c}_k \tilde{x}_k\right) + T(\tilde{c}_n \tilde{x}_n) = \sum_{k=1}^{n-1} \tilde{c}_k T(\tilde{x}_k) + \tilde{c}_n T(\tilde{x}_n) = \sum_{k=1}^{n} \tilde{c}_k T(\tilde{x}_k)$. □

**Theorem 3.7.** *Let $T : SE(\check{X}) \to SE(\check{Y})$ be a soft linear operator, where $\check{X}, \check{Y}$ are soft normed linear spaces. If $T$ is continuous at some soft element $\tilde{x}_0 \tilde{\in} \check{X}$ then $T$ is continuous at every soft element of $\check{X}$.*

*Proof.* Let $\tilde{x} \tilde{\in} \check{X}$ be an arbitrary soft element of and let $\tilde{x}_n \to \tilde{x}$ as $n \to \infty$. Then $\tilde{x}_n - \tilde{x} + \tilde{x}_0 \to \tilde{x}_0$ as $n \to \infty$. Since $T$ is continuous at $\tilde{x}_0$, we have

$$T(\tilde{x}_n - \tilde{x} + \tilde{x}_0) \to T(\tilde{x}_0).$$

But, $T(\tilde{x}_n - \tilde{x} + \tilde{x}_0) = T(\tilde{x}_n) - T(\tilde{x}) + T(\tilde{x}_0)$. Therefore,

$$\lim_{n\to\infty} T(\tilde{x}_n) - T(\tilde{x}) + T(\tilde{x}_0) = T(\tilde{x}_0), \text{ i.e., } \lim_{n\to\infty} T(\tilde{x}_n) = T(\tilde{x}).$$

This shows that $T$ is continuous at $\tilde{x}$. Since $\tilde{x}$ is an arbitrary soft element of $\check{X}$, $T$ is continuous at every soft element of $\check{X}$. □

**Definition 3.8.** Let $T : SE(\check{X}) \to SE(\check{Y})$ be a soft linear operator, where $\check{X}, \check{Y}$ are soft normed linear spaces. The operator $T$ is called bounded if there exists some positive soft real number $\tilde{M}$ such that for all $\tilde{x} \tilde{\in} \check{X}$, $\|T(\tilde{x})\| \tilde{\leq} \tilde{M} \|\tilde{x}\|$.

**Theorem 3.9.** *Let $T : SE(\check{X}) \to SE(\check{Y})$ be a soft linear operator, where $\check{X}, \check{Y}$ are soft normed linear spaces. If $T$ is bounded then $T$ is continuous.*

*Proof.* Suppose that $T$ is bounded. Then there exists a positive soft real number $\tilde{M}$ such that for all $\tilde{x} \tilde{\in} \check{X}$, $\|T(\tilde{x})\| \tilde{\leq} \tilde{M} \|\tilde{x}\|$. Let $\tilde{x}_n \to \tilde{x}$ as $n \to \infty$ i.e., $\|\tilde{x}_n - \tilde{x}\| \to \overline{0}$ as $n \to \infty$. Then

$$\|T(\tilde{x}_n) - T(\tilde{x})\| = \|T(\tilde{x}_n - \tilde{x})\| \tilde{\leq} \tilde{M} \|\tilde{x}_n - \tilde{x}\| \to \overline{0} \text{ as } n \to \infty.$$

Therefore $T(\tilde{x}_n) \to T(\tilde{x})$ and so $T$ is continuous at $\tilde{x}\widetilde{\in}\check{X}$. Since $\tilde{x}\widetilde{\in}\check{X}$ is arbitrary, $T$ is continuous. □

**Theorem 3.10.** *(Decomposition Theorem). Suppose a soft linear operator $T : SE(\check{X}) \to SE(\check{Y})$, where $\check{X}, \check{Y}$ are soft vector spaces, satisfies the condition (L3). For $\xi \in X$, and $\lambda \in A$,*

$$\left\{T\left(\tilde{x}\right)\left(\lambda\right) : \tilde{x} \;\widetilde{\in}\check{X} \text{ such that } \tilde{x}\left(\lambda\right) = \xi\right\}$$

*is a singleton set. Then for each $\lambda \in A$, the mapping $T_\lambda : X \to Y$ defined by $T_\lambda(\xi) = T\left(\tilde{x}\right)\left(\lambda\right)$, for all $\xi \in X$ and $\tilde{x}\widetilde{\in}\check{X}$ such that $\tilde{x}\left(\lambda\right) = \xi$, is a linear operator.*

*Proof.* Clearly $\forall\lambda \in A$, $T_\lambda : X \to Y$ is a rule that assigns an element of $Y$ corresponding to each element of $X$. Now the well defined property of $T_\lambda$ , $\forall\lambda \in A$ follows from the condition (L3). The conditions (L1), (L2) of $T$ resulted the linearity property of $T_\lambda$, $\forall\lambda \in A$. Thus the soft linear operator $T$ satisfying (L3) gives a parameterized family of crisp linear operators. With this point of view, it also follows that, a soft linear operator $T$ satisfying (L3), is a particular 'soft mapping' as defined by P. Majumdar et al. in [16], where $T : A \to (Y)^X$. □

The converse of the above theorem is also true. In this regard, we have the following theorem:

**Theorem 3.11.** *Let $\{T_\lambda : X \to Y,\ \lambda \in A\}$ be a family of crisp linear operators from the vector space $X$ to the vector space $Y$, and $\check{X}, \check{Y}$ be the corresponding absolute soft vector spaces. Then there exists a soft linear operator $T : SE(\check{X}) \to SE(\check{Y})$, defined by $T\left(\tilde{x}\right)\left(\lambda\right) = T_\lambda\left(\xi\right)$ if $\tilde{x}\left(\lambda\right) = \xi$, $\lambda \in A$; which satisfies (L3) and $T\left(\lambda\right) = T_\lambda$, for all $\lambda \in A$.*

*Proof.* Let $\tilde{x} \;\widetilde{\in}\check{X}$. We define $T : SE(\check{X}) \to SE(\check{Y})$, by $T\left(\tilde{x}\right)\left(\lambda\right) = T_\lambda\left(\xi\right)$ if $\tilde{x}\left(\lambda\right) = \xi$; for each $\lambda \in A$. Let $\tilde{x}_1,\ \tilde{x}_2 \;\widetilde{\in}\check{X}$, be any two soft elements, $\lambda \in A$ be arbitrary and $\tilde{x}_1\left(\lambda\right) = \xi$, $\tilde{x}_2\left(\lambda\right) = \eta$. Then $\xi, \eta \in X$ and we have

$$\begin{aligned} T\left(\tilde{x}_1 + \tilde{x}_2\right)\left(\lambda\right) &= T\left(\tilde{x}_1\left(\lambda\right) + \tilde{x}_2\left(\lambda\right)\right) \\ &= T_\lambda\left(\xi + \eta\right) = T_\lambda\left(\xi\right) + T_\lambda\left(\eta\right) \\ &= T\left(\tilde{x}_1\right)\left(\lambda\right) + T\left(\tilde{x}_2\right)\left(\lambda\right) \\ &= \left(T\left(\tilde{x}_1\right) + T\left(\tilde{x}_2\right)\right)\left(\lambda\right). \end{aligned}$$

This is true for all $\lambda \in A$ and for all $\tilde{x}_1,\ \tilde{x}_2$. Hence $T\left(\tilde{x}_1 + \tilde{x}_2\right) = T\left(\tilde{x}_1\right) + T\left(\tilde{x}_2\right)$. Again, for every soft scalar $\tilde{c}$, we have

$$\begin{aligned} T\left(\tilde{c}\tilde{x}\right)\left(\lambda\right) &= T\left(\tilde{c}\left(\lambda\right).\tilde{x}\left(\lambda\right)\right) \\ &= T_\lambda\left(\tilde{c}\left(\lambda\right).\xi\right) \\ &= \tilde{c}\left(\lambda\right).T_\lambda\left(\xi\right) \\ &= \tilde{c}\left(\lambda\right).T\left(\tilde{x}\right)\left(\lambda\right) \\ &= \left(\tilde{c}.T\left(\tilde{x}\right)\right)\left(\lambda\right). \end{aligned}$$

This is true for all $\lambda \in A$ and for all $\tilde{x}$. Hence $T\left(\tilde{c}\tilde{x}\right) = \tilde{c}T(\tilde{x})$. Thus the operator $T : SE(\check{X}) \to SE(\check{Y})$, is a soft linear operator. It is obvious from the above definition of the soft linear operator $T$ that, for $\xi \in X$, and $\lambda \in A$,

$$\left\{T(\tilde{x})(\lambda) : \tilde{x}\,\widetilde{\in}\check{X} \text{ such that } \tilde{x}(\lambda) = \xi\right\}$$

is a singleton set. Thus the operator $T : SE(\check{X}) \to SE(\check{Y})$, satisfies (L3). From the above theorem it is obvious that $T(\lambda) = T_\lambda$, for all $\lambda \in A$. □

**Theorem 3.12.** *Let $\check{X}$ and $\check{Y}$ be soft normed linear spaces which satisfy (N5) and $T : SE(\check{X}) \to SE(\check{Y})$ be a soft linear operator satisfying (L3). If $T$ is continuous then $T$ is bounded.*

*Proof.* If possible, assume that $T$ is not bounded. Then there exists a sequence $\{\tilde{x}_n\}$ of soft elements of $\check{X}$ such that for each $n = 1, 2, \ldots \ldots$, there exists at least one $\mu_n \in A$ such that

$$\|T(\tilde{x}_n)\|(\mu_n) > (\overline{n}.\|\tilde{x}_n\|)(\mu_n) \tag{3.1}$$

Clearly $\tilde{x}_n(\mu_n) \neq \theta$, for any $n$. Because, if $\tilde{x}_n(\mu_n) = \theta$, for some $\mu_n \in A$ and for some $n$, then since $\check{X}$ satisfies (N5), $\|\tilde{x}_n(\mu_n)\|_{\mu_n} = 0$ and since $T$ satisfies (L3), $T(\tilde{x}_n)(\mu_n) = T_{\mu_n}(\tilde{x}_n(\mu_n)) = \theta$, also since $\check{Y}$ satisfies (N5), $\|T(\tilde{x}_n)(\mu_n)\|_{\mu_n} = 0$. This contradicts (3.1). Let us consider a sequence $\{\tilde{y}_n\}$ of soft elements of $\check{X}$ such that for $n = 1, 2, \ldots \ldots$, $\tilde{y}_n(\lambda) = \tilde{x}_n(\mu_n)$, $\forall \lambda \in A$. Then $\|\tilde{y}_n\| \tilde{>} \overline{0}$. Let $\tilde{y}_n^{/} = \frac{\tilde{y}_n}{\overline{n}.\|\tilde{y}_n\|}$. Then $\left\|\tilde{y}_n^{/}\right\| = \frac{1}{\overline{n}} \to \overline{0}$. So $\tilde{y}_n^{/} \to \Theta$ as $n \to \infty$. Since $T$ is continuous at $\tilde{x} = \Theta$, $T\left(\tilde{y}_n^{/}\right) \to T(\Theta) = \Theta$ i.e., $\left\|T(\tilde{y}_n^{/})\right\| \to \overline{0}$ as $n \to \infty$. In particular,

$$\left\|T(\tilde{y}_n^{/})\right\|(\mu_n) = \left\|T(\tilde{y}_n^{/})(\mu_n)\right\|_{\mu_n} \to 0 \tag{3.2}$$

as $n \to \infty$. On the other hand,

$$\left\|T(\tilde{y}_n^{/})\right\| = \left\|T(\tfrac{\tilde{y}_n}{\overline{n}.\|\tilde{y}_n\|})\right\| = \left\|\tfrac{1}{\overline{n}.\|\tilde{y}_n\|}T(\tilde{y}_n)\right\| = \tfrac{1}{\overline{n}.\|\tilde{y}_n\|}\|T(\tilde{y}_n)\|$$

So, $\left\|T\left(\tilde{y}_n^{/}\right)\right\|(\mu_n) = \left(\frac{1}{\overline{n}.\|\tilde{y}_n\|}\right)(\mu_n).\|T(\tilde{y}_n)\|(\mu_n) = \left(\frac{1}{\overline{n}.\|\tilde{y}_n\|}\right)(\mu_n).\|[T(\tilde{y}_n)](\mu_n)\|_{\mu_n}$ $= \left(\frac{1}{\overline{n}.\|\tilde{y}_n\|}\right)(\mu_n)\|[T(\tilde{x}_n)](\mu_n)\|_{\mu_n} = \left(\frac{1}{\overline{n}.\|\tilde{y}_n\|}\right)(\mu_n).\|T(\tilde{x}_n)\|(\mu_n) >$ $\left[\left(\frac{1}{\overline{n}.\|\tilde{y}_n\|}\right)(\mu_n)\right].[(\overline{n}.\|\tilde{x}_n\|)(\mu_n)] = \left[\left(\frac{1}{\overline{n}.\|\tilde{y}_n\|}\right)(\overline{n}.\|\tilde{x}_n\|)\right](\mu_n) = \left(\frac{\|\tilde{x}_n\|}{\|\tilde{y}_n\|}\right)(\mu_n)$ $= \frac{\|\tilde{x}_n\|(\mu_n)}{\|\tilde{y}_n\|(\mu_n)} = \frac{\|\tilde{x}_n(\mu_n)\|_{\mu_n}}{\|\tilde{y}_n(\mu_n)\|_{\mu_n}} = \frac{\|\tilde{y}_n(\mu_n)\|_{\mu_n}}{\|\tilde{y}_n(\mu_n)\|_{\mu_n}} = 1$. Therefore,

$$\left\|T\left(\tilde{y}_n^{/}\right)\right\|(\mu_n) > 1, \tag{3.3}$$

for $n = 1, 2, \ldots$ The relations (3.2) and (3.3) are contradictory. Therefore $T$ must be bounded. □

**Theorem 3.13.** *Let $\check{X}$ and $\check{Y}$ be soft normed linear spaces which satisfy (N5) and $T : SE(\check{X}) \to SE(\check{Y})$ be a soft linear operator satisfying (L3). If $\check{X}$ is of finite dimension, then $T$ is bounded and hence continuous.*

*Proof.* Let the dimension of $\check{X}$ be $n$ and $\{\tilde{e}_1, \tilde{e}_2, \ldots\ldots, \tilde{e}_n\}$ be a basis of a soft linear space $\check{X}$. Let us consider a soft real number $\tilde{D}$ such that

$$\tilde{D}(\lambda) = \max\left\{\|T(\tilde{e}_1)\|(\lambda), \|T(\tilde{e}_2)\|(\lambda), \ldots, \|T(\tilde{e}_n)\|(\lambda)\right\},\ \forall \lambda \in A.$$

Let $\tilde{x} = \sum_{i=1}^{n} \widetilde{\gamma}_i \tilde{e}_i$ be any soft element of $\check{X}$, then because $T$ is soft linear,

$$\|T(\tilde{x})\| = \|T(\sum_{i=1}^{n} \widetilde{\gamma}_i \tilde{e}_i)\| = \|\sum_{i=1}^{n} \widetilde{\gamma}_i T(\tilde{e}_i)\| \,\widetilde{\leq} \sum_{i=1}^{n} |\widetilde{\gamma}_i| \, \|T(\tilde{e}_i)\| \,\widetilde{\leq} \tilde{D} \sum_{i=1}^{n} |\widetilde{\gamma}_i|.$$

By Lemma 2.31, there exists a soft real number $\tilde{c} \tilde{>} \overline{0}$, such that

$$\|\tilde{x}\| = \left\| \sum_{i=1}^{n} \widetilde{\gamma}_i \tilde{e}_i \right\| \widetilde{\geq} \tilde{c}. \sum_{i=1}^{n} |\widetilde{\gamma}_i|.$$

So, $\sum_{i=1}^{n} |\widetilde{\gamma}_i| \widetilde{\leq} \frac{1}{\tilde{c}}. \|\tilde{x}\|$ . Therefore, $\|T(\tilde{x})\| \,\widetilde{\leq} \tilde{D}. \, \frac{1}{\tilde{c}}. \|\tilde{x}\| = \tilde{M}. \, \|\tilde{x}\|$, where $\tilde{M} = \frac{\tilde{D}}{\tilde{c}}$. So, $T$ is bounded and hence continuous. □

## 4. Spaces of continuous soft linear operators

Throughout this section we shall assume that $\check{X}, \check{Y}$ are soft normed linear spaces and $S$, $T$ etc. are continuous soft linear operators each mapping $SE(\check{X})$ into $SE\left(\check{Y}\right)$.

**Definition 4.1.** Let $T$ be a bounded soft linear operator from $SE(\check{X})$ into $SE\left(\check{Y}\right)$. Then the norm of the operator $T$ denoted by $\|T\|$, is a soft real number defined as the following: For each $\lambda \in A$,

$$\|T\|(\lambda) = \inf\{t \in R; \|T(\tilde{x})\|(\lambda) \leq t. \|\tilde{x}\|(\lambda), \text{ for each } \tilde{x} \,\widetilde{\in} \check{X}\}.$$

**Theorem 4.2.** *Let $\check{X}, \check{Y}$ be soft normed linear spaces which satisfy (N5) and $T$ satisfy (L3). Then for each $\lambda \in A$, $\|T\|(\lambda) = \|T_\lambda\|_\lambda$, where $\|T_\lambda\|_\lambda$ is the norm of the linear operator $T_\lambda : X \to Y$.*

*Proof.* By definition of norm of bounded linear operators over crisp normed linear spaces we have,

$$\|T_\lambda\|_\lambda = \inf\{t \in R; \|T_\lambda(x)\|_\lambda \leq t.\|x\|_\lambda, \text{ for all } x \in X\}.$$

We shall now prove that for each $\lambda \in A$,

$$\{t \in R; \|T(\tilde{x})\|(\lambda) \leq t. \|\tilde{x}\|(\lambda), \text{ for all } \tilde{x} \,\widetilde{\in} \check{X}\} = \{t \in R; \|T_\lambda(x)\|_\lambda \leq t.\|x\|_\lambda, \text{ for all } x \in X\}.$$

Let $r \in \{t \in R; \|T(\tilde{x})\|(\lambda) \leq t. \|\tilde{x}\|(\lambda), \text{ for all } \tilde{x} \,\widetilde{\in} \check{X}\}$. Then $\|T(\tilde{x})\|(\lambda) \leq r. \|\tilde{x}\|(\lambda)$, for all $\tilde{x} \,\widetilde{\in} \check{X}$. Let $y \in X$. Choose an $\tilde{x} \,\widetilde{\in} \check{X}$ such that $\tilde{x}(\lambda) = y$. Then

$$\|T_\lambda(y)\|_\lambda = \|T_\lambda \tilde{x}(\lambda)\|_\lambda = \|T(\tilde{x})\|(\lambda) \leq r. \|\tilde{x}\|(\lambda) = r.\|\tilde{x}(\lambda)\|_\lambda = r.\|y\|_\lambda.$$

So, $r \in \{t \in R; \|T_\lambda(x)\|_\lambda \leq t.\|x\|_\lambda, \text{ for all } x \in X\}$. Thus

$$\begin{aligned} &\{t \in R; \|T(\tilde{x})\|(\lambda) \leq t. \|\tilde{x}\|(\lambda), \forall \tilde{x} \,\widetilde{\in} \check{X}\} \\ &\qquad \subset \{t \in R; \|T_\lambda(x)\|_\lambda \leq t.\|x\|_\lambda, \forall x \in X\} \end{aligned} \tag{4.1}$$

Conversely, let $s \in \{t \in R; \|T_\lambda(x)\|_\lambda \leq t.\|x\|_\lambda, \text{ for all } x \in X\}$. Then $\|T_\lambda(x)\|_\lambda \leq s.\|x\|_\lambda$, for all $x \in X$. Then for $\tilde{x} \widetilde{\in} \check{X}$, $\tilde{x}(\lambda) \in X$,

$$\|T(\tilde{x})\|(\lambda) = \|T_\lambda \tilde{x}(\lambda)\|_\lambda \leq s.\|\tilde{x}(\lambda)\|_\lambda = s. \|\tilde{x}\|(\lambda).$$

So, considering all the elements of $X$ we see that $\|T(\tilde{x})\|(\lambda) \leq s.\|\tilde{x}\|(\lambda)$, for all $\tilde{x} \tilde{\in} \check{X}$. So, $s \in \{t \in R; \|T(\tilde{x})\|(\lambda) \leq t.\|\tilde{x}\|(\lambda)$, for all $\tilde{x} \tilde{\in} \check{X}\}$. Thus

$$\begin{aligned}(4.2)\quad &\{t \in R; \|T_\lambda(x)\|_\lambda \leq t.\|x\|_\lambda, \forall x \in X\} \\ &\subset \{t \in R; \|T(\tilde{x})\|(\lambda) \leq t.\|\tilde{x}\|(\lambda), \forall \tilde{x} \tilde{\in} \check{X}\}\end{aligned}$$

From (4.1) and (4.2) it follows that

$$\begin{aligned}&\{t \in R; \|T(\tilde{x})\|(\lambda) \leq t.\|\tilde{x}\|(\lambda), \text{ for all } \tilde{x} \tilde{\in} \check{X}\} \\ &= \{t \in R; \|T_\lambda(x)\|_\lambda \leq t.\|x\|_\lambda, \text{ for all } x \in X\}.\end{aligned}$$

Then, For each $\lambda \in A$,

$$\begin{aligned}\|T\|(\lambda) &= \inf \{t \in R; ; \|T(\tilde{x})\|(\lambda) \leq t.\|\tilde{x}\|(\lambda), \text{ for all } \tilde{x} \tilde{\in} \check{X}\} \\ &= \inf \{t \in R; \|T_\lambda(x)\|_\lambda \leq t.\|x\|_\lambda, \text{ for all } x \in X\} = \|T_\lambda\|_\lambda.\end{aligned}$$

□

**Theorem 4.3.** $\|T(\tilde{x})\| \tilde{\leq} \|T\| \|\tilde{x}\|$, for all $\tilde{x} \tilde{\in} \check{X}$.

*Proof.* For arbitrary $\tilde{\varepsilon} \tilde{>} \bar{0}$, it follows from the definition that

$$(4.3)\qquad \|T(\tilde{x})\|(\lambda) \leq (\|T\|(\lambda) + \tilde{\varepsilon}(\lambda)).\|\tilde{x}\|(\lambda)$$

for each $\lambda \in A$, for all $\tilde{x} \tilde{\in} \check{X}$. If possible suppose that there exists $\tilde{x}_1 \tilde{\in} \check{X}$ and $\mu \in A$ such that $\|T(\tilde{x}_1)\|(\mu) > \|T\|(\mu).\|\tilde{x}_1\|(\mu)$. Then for some $\varepsilon_\mu > 0$,

$$\begin{aligned}\|T(\tilde{x}_1)\|(\mu) &> \|T\|(\mu).\|\tilde{x}_1\|(\mu) + \varepsilon_\mu \|\tilde{x}_1\|(\mu) = (\|T\|(\mu) + \varepsilon_\mu)\|\tilde{x}_1\|(\mu) \\ &= (\|T\|(\mu) + \tilde{\varepsilon}(\mu))\|\tilde{x}_1\|(\mu), \text{ taking } \tilde{\varepsilon} \tilde{>} \bar{0} \text{ with } \tilde{\varepsilon}(\mu) = \varepsilon_\mu.\end{aligned}$$

This contradicts (4.3). Hence $\|T(\tilde{x})\| \tilde{\leq} \|T\| \|\tilde{x}\|$, for all $\tilde{x} \tilde{\in} \check{X}$. □

**Lemma 4.4.** *Let $\check{X}$ and $\check{Y}$ be soft normed linear spaces which satisfy (N5) and $T : SE(\check{X}) \to SE(\check{Y})$ be a soft linear operator satisfying (L3). Then*

$$\{\|T_\lambda(x)\|_\lambda : x \in X, \|x\|_\lambda \leq 1\} = \left\{\|T(\tilde{x})\|(\lambda) : \tilde{x} \tilde{\in} \check{X}, \|\tilde{x}\| \tilde{\leq} \bar{1}\right\}.$$

*Proof.* Let $r \in \{\|T(\tilde{x})\|(\lambda) : \tilde{x} \tilde{\in} \check{X}, \|\tilde{x}\| \tilde{\leq} \bar{1}\}$. Then $\exists\ \tilde{x} \tilde{\in} \check{X}$ with $\|\tilde{x}\| \tilde{\leq} \bar{1}$ such that $\|T(\tilde{x})\|(\lambda) = r$. Now $\|\tilde{x}\| \tilde{\leq} \bar{1}$ implies $\|\tilde{x}\|(\lambda) = \|\tilde{x}(\lambda)\|_\lambda \leq 1$ and $r = \|T(\tilde{x})\|(\lambda) = \|T_\lambda \tilde{x}(\lambda)\|_\lambda$. So, $r \in \{\|T_\lambda(x)\|_\lambda : x \in X, \|x\|_\lambda \leq 1\}$. Thus

$$(4.4)\qquad \{\|T(\tilde{x})\|(\lambda) : \tilde{x} \tilde{\in} \check{X}, \|\tilde{x}\| \tilde{\leq} \bar{1}\} \subset \{\|T_\lambda(x)\|_\lambda : x \in X, \|x\|_\lambda \leq 1\}$$

Conversely, let $s \in \{\|T_\lambda(x)\|_\lambda : x \in X, \|x\|_\lambda \leq 1\}$. Then $\exists\ y \in X$ such that $\|T_\lambda(y)\|_\lambda = s$ and $\|y\|_\lambda \leq 1$. Since $SE(\check{X})$ is the collection of all soft elements of $\check{X}$ and $SS\left(SE(\check{X})\right) = \check{X}$, $\exists\ \tilde{x} \tilde{\in} \check{X}$ with $\tilde{x}(\lambda) = y$ such that $\|\tilde{x}\| \tilde{\leq} \bar{1}$ and $s = \|T_\lambda(y)\|_\lambda = \|T_\lambda \tilde{x}(\lambda)\|_\lambda = \|T(\tilde{x})\|(\lambda)$. So, $s \in \{\|T(\tilde{x})\|(\lambda) : \tilde{x} \tilde{\in} \check{X}, \|\tilde{x}\| \tilde{\leq} \bar{1}\}$. Thus

$$(4.5)\qquad \{\|T_\lambda(x)\|_\lambda : x \in X, \|x\|_\lambda \leq 1\} \subset \{\|T(\tilde{x})\|(\lambda) : \tilde{x} \tilde{\in} \check{X}, \|\tilde{x}\| \tilde{\leq} \bar{1}\}.$$

From (4.4) and (4.5) it follows that

$$\{\|T_\lambda(x)\|_\lambda : x \in X, \|x\|_\lambda \leq 1\} = \left\{\|T(\tilde{x})\|(\lambda) : \tilde{x} \tilde{\in} \check{X}, \|\tilde{x}\| \tilde{\leq} \bar{1}\right\}.$$

□

**Theorem 4.5.** *Let $\check{X}$ and $\check{Y}$ be soft normed linear spaces which satisfy (N5) and $T : SE(\check{X}) \to SE(\check{Y})$ be a soft linear operator satisfying (L3). Then*

(i) $\|T\|(\lambda) = sup \left\{ \|T(\tilde{x})\|(\lambda) : \|\tilde{x}\| \tilde{\leq} \bar{1} \right\} = \|T_\lambda\|_\lambda$, *for each* $\lambda \in A$;

(ii) $\|T\|(\lambda) = sup \left\{ \|T(\tilde{x})\|(\lambda) : \|\tilde{x}\| = \bar{1} \right\} = \|T_\lambda\|_\lambda$, *for each* $\lambda \in A$;

(iii) $\|T\|(\lambda) = sup \left\{ \frac{\|T(\tilde{x})\|}{\|\tilde{x}\|}(\lambda) : \|\tilde{x}\|(\mu) \neq 0, \text{ for all } \mu \in A \right\} = \|T_\lambda\|_\lambda$, *for each* $\lambda \in A$.

*Proof.* We prove only (i). The relations (ii) and (iii) can be proved similarly. Since $\check{X}, \check{Y}$ satisfy (N5) and $T$ satisfy (L3), we have for each $\lambda \in A$,

$$\|T\|(\lambda) = \|T_\lambda\|_\lambda = \sup \{ \|T_\lambda(x)\|_\lambda : \|x\|_\lambda \leq 1 \} = \sup \{ \|T(\tilde{x})\|(\lambda) : \|\tilde{x}\| \tilde{\leq} \bar{1} \}$$

by using Lemma 4.4. □

**Example 4.6.** For the identity operator $I : SE(\check{X}) \to SE(\check{Y})$, $\|I\| = \bar{1}$, and for the zero operator $O : SE(\check{X}) \to SE(\check{Y})$, $\|O\| = \bar{0}$.

**Theorem 4.7.** *Let $\check{X}$ and $\check{Y}$ be a soft normed linear spaces which satisfy (N5). Let $T : SE(\check{X}) \to SE(\check{Y})$ be a continuous soft linear operator satisfying (L3). Then $T_\lambda$ is continuous on $X$ for each $\lambda \in A$.*

*Proof.* Let $T$ be continuous. Then $T$ is continuous at each soft element of $\check{X}$. Let $\lambda \in A$, $x \in X$ be arbitrary. Let $\{x_n\}$ be any sequence in $X$ such that $x_n \to x$ as $n \to \infty$. Let us consider any soft element $\tilde{x} \tilde{\in} \check{X}$ such that $\tilde{x}(\lambda) = x$ and any sequence $\{\tilde{x}_n\}$ of soft elements of $\check{X}$ such that $\tilde{x}_n \to \tilde{x}$ as $n \to \infty$. We now construct a sequence $\{\tilde{y}_n\}$ of soft elements of $\check{X}$ such that $\tilde{y}_n(\mu) = x_n$ *if* $\mu = \lambda$ and $\tilde{y}_n(\mu) = \tilde{x}_n(\mu)$ *if* $\mu \in A \backslash \lambda$. Let $\tilde{\varepsilon} \tilde{>} \bar{0}$ be arbitrary, since $\tilde{x}_n \to \tilde{x}$ as $n \to \infty$ there exists a positive integer $N_1$ such that $\|\tilde{x}_n - \tilde{x}\| \tilde{<} \tilde{\varepsilon}$, for all $n \geq N_1$. Then $\|\tilde{x}_n - \tilde{x}\|(\mu) < \tilde{\varepsilon}(\mu)$, i.e., $\|\tilde{x}_n(\mu) - \tilde{x}(\mu)\|_\mu < \tilde{\varepsilon}(\mu)$, for all $\mu \in A$ and $n \geq N_1$. Then we have $\|\tilde{y}_n(\mu) - \tilde{x}(\mu)\|_\mu < \tilde{\varepsilon}(\mu)$, i.e., $\|\tilde{y}_n - \tilde{x}\|(\mu) < \tilde{\varepsilon}(\mu)$, for all $\mu \in A \backslash \lambda$ and $n \geq N_1$. Also, since $x_n \to x$ as $n \to \infty$, there exists a positive integer $N_2$ such that $\|x_n - x\|_\lambda \leq \tilde{\varepsilon}(\lambda)$, i.e., $\|\tilde{y}_n(\lambda) - \tilde{x}(\lambda)\|_\lambda < \tilde{\varepsilon}(\lambda)$ for all $n \geq N_2$. Let $N = \max? \{N_1, N_2\}$. Then $\|\tilde{y}_n(\mu) - \tilde{x}(\mu)\|_\mu < \tilde{\varepsilon}(\mu)$, i.e., $\|\tilde{y}_n - \tilde{x}\|(\mu) < \tilde{\varepsilon}(\mu)$, for all $\mu \in A$ and $n \geq N$. Thus $\|\tilde{y}_n - \tilde{x}\| \tilde{<} \tilde{\varepsilon}$, for all $n \geq N$. So, $\tilde{y}_n \to \tilde{x}$ as $n \to \infty$. Since $T$ is continuous, $T(\tilde{y}_n) \to T(\tilde{x})$ as $n \to \infty$. Let $\varepsilon > 0$ be arbitrary, then for any $\tilde{\varepsilon} \tilde{>} \bar{0}$ with $\tilde{\varepsilon}(\lambda) = \varepsilon$, there exists a positive integer $M$ such that $\|T(\tilde{y}_n) - T(\tilde{x})\| \tilde{<} \tilde{\varepsilon}$, for all $n \geq M$. So $\|T(\tilde{y}_n) - T(\tilde{x})\|(\mu) < \tilde{\varepsilon}(\mu)$, i.e., $\|T(\tilde{y}_n)(\mu) - T(\tilde{x})(\mu)\|_\mu < \tilde{\varepsilon}(\mu)$, for all $\mu \in A$ and $n \geq M$. Thus in particular, $\|T(\tilde{y}_n)(\lambda) - T(\tilde{x})(\lambda)\|_\lambda < \tilde{\varepsilon}(\lambda)$, i.e., $\|T(x_n) - T(x)\|_\lambda < \tilde{\varepsilon}(\lambda) = \varepsilon$ for all $n \geq M$. Proving that $T_\lambda$ is continuous at $x$. Since $\lambda \in A$, $x \in X$ are arbitrary, it follows that $T_\lambda$ is continuous on $X$ for each $\lambda \in A$. □

**Theorem 4.8.** *Let $\check{X}$ and $\check{Y}$ be a soft normed linear spaces which satisfy (N5) over a finite set of parameters $A$. Let $\{T_\lambda; \lambda \in A\}$ be a family of continuous linear operators such that $T_\lambda : X \to Y$ for each $\lambda$. Then the operator $T : SE(\check{X}) \to SE(\check{Y})$ defined by $T(\lambda) = T_\lambda$, $\forall \lambda \in A$; is a continuous soft linear operator satisfying (L3).*

*Proof.* Let $T_\lambda$ be continuous on $X$ for each $\lambda \in A$. Let us consider the operator $T : SE(\check{X}) \to SE(\check{Y})$ defined by $T(\lambda) = T_\lambda$, $\forall \lambda \in A$. From Theorem 3.11, it follows that $T$ is soft linear and $T$ satisfies (L3). Let us consider any soft element $\tilde{x} \tilde{\in} \check{X}$ and $\tilde{\varepsilon} \tilde{>} \bar{0}$ be arbitrary. Let us consider any sequence $\{\tilde{x}_n\}$ of soft elements of $\check{X}$ such that $\tilde{x}_n \to \tilde{x}$ as $n \to \infty$. Then there exists a positive integer $N_3$ such that $\|\tilde{x}_n - \tilde{x}\| \tilde{<} \tilde{\varepsilon}$, for all $n \geq N_3$. Then $\|\tilde{x}_n - \tilde{x}\|(\lambda) < \tilde{\varepsilon}(\lambda)$, i.e., $\|\tilde{x}_n(\lambda) - \tilde{x}(\lambda)\|_\lambda < \tilde{\varepsilon}(\lambda)$, for all $\lambda \in A$ and $n \geq N_3$. Thus $\tilde{x}_n(\lambda) \to \tilde{x}(\lambda)$ as $n \to \infty$ for each $\lambda \in A$. For each $\lambda \in A$, using continuity of $T_\lambda$, there exists a positive integers $N_\lambda$ such that $\|T(\tilde{x}_n)(\lambda) - T(\tilde{x})(\lambda)\|_\lambda < \tilde{\varepsilon}(\lambda)$, for all $n \geq N_\lambda$. Let $N = \max\{N_\lambda, \lambda \in A\}$, such $N$ exists since $A$ is finite. Then we have $\|T(\tilde{x}_n)(\lambda) - T(\tilde{x})(\lambda)\|_\lambda < \tilde{\varepsilon}(\lambda)$, i.e., $\|T(\tilde{x}_n) - T(\tilde{x})\|(\lambda) < \tilde{\varepsilon}(\lambda)$, for all $n \geq N$. Thus $\|T(\tilde{x}_n) - T(\tilde{x})\| \tilde{<} \tilde{\varepsilon}$, for all $n \geq N$. Therefore, $T$ is continuous at $\tilde{x}$. Since $\tilde{x} \tilde{\in} \check{X}$ is arbitrary, $T$ is continuous on $\check{X}$. □

**Remark 4.9.** (Soft linear space of operators). Let $\check{X}, \check{Y}$ be soft normed linear spaces satisfying (N5) over a finite set of parameters $A$. Consider the set $W$ of all continuous soft linear operators $S$, $T$ etc. which satisfy (L3) each mapping $SE(\check{X})$ into $SE(\check{Y})$. Then using Theorem 4.7, it follows that for each $\lambda \in A$, $S_\lambda$, $T_\lambda$ etc.are continuous linear operators on $X$. Let $W(\lambda) = \{T_\lambda(= T(\lambda)); T \in W\}$, for all $\lambda \in A$. Also using Theorem 4.7 and Theorem 4.8, it follows that, $W(\lambda)$ is the collection of all continuous linear operators on $X$. By the property of crisp linear operators it follows that $W(\lambda)$ forms a vector space for each $\lambda \in A$ with respect to the usual operations of addition and scalar multiplication of linear operators. It also follows that $W(\lambda)$ is identical with the set of all continuous linear operators on $X$ for all $\lambda \in A$. Thus the absolute soft set generated by $W(\lambda)$ form an absolute soft vector space. Hence $W$ can be interpreted as to form an absolute soft vector space. We shall denote this absolute soft linear (vector) space by $L(\check{X}, \check{Y})$.

**Proposition 4.10.** *Each element of* $SE(L(\check{X}, \check{Y}))$ *can be identified uniquely with a member of* $W$ *i.e., to a continuous soft linear operator* $T : SE(\check{X}) \to SE(\check{Y})$.

*Proof.* Let $\tilde{f} \in SE(L(\check{X}, \check{Y}))$. Then for each $\lambda \in A$, $\tilde{f}(\lambda)$ is a continuous linear operator from $X$ into $Y$. Then by Theorem 4.8, it follows that the operator $T : SE(\check{X}) \to SE(\check{Y})$ defined by $T(\lambda) = \tilde{f}(\lambda)$, $\forall \lambda \in A$; is a continuous soft linear operator satisfying (L3). Thus each element $\tilde{f}$ of $SE(L(\check{X}, \check{Y}))$ can be identified to a continuous soft linear operator $T : SE(\check{X}) \to SE(\check{Y})$, i.e. to a member of $W$. We now show that such $T$ is unique. If possible let $S$, $T$ be two such continuous soft linear operators. Then $S(\lambda) = \tilde{f}(\lambda)$, $\forall \lambda \in A$; and $T(\lambda) = \tilde{f}(\lambda)$, $\forall \lambda \in A$; i.e., $S(\lambda) = T(\lambda), \forall \lambda \in A$. Let $\tilde{x} \tilde{\in} \check{X}$ and $\lambda \in A$ be arbitrary. Then

$$(S(\tilde{x}))(\lambda) = S(\lambda)((\tilde{x})(\lambda)) = T(\lambda)((\tilde{x})(\lambda)) = (T(\tilde{x}))(\lambda).$$

This is true for all $\tilde{x} \tilde{\in} \check{X}$ and $\lambda \in A$. Hence $S = T$ and thus the identification is unique. □

**Theorem 4.11.** *If* $S$, $T$ *are continuous soft linear operators, then*

$$\|S + T\| \tilde{\leq} \|S\| + \|T\|.$$

*Proof.* For each $\tilde{x}\tilde{\in}\check{X}$, we have by Theorem 4.3,

$$\|(S+T)(\tilde{x})\| = \|S(\tilde{x})+T(\tilde{x})\| \tilde{\leq} \|S(\tilde{x})\| + \|T(\tilde{x})\|$$
$$\tilde{\leq} \|S\|\,\|\tilde{x}\| + \|T\|\,\|\tilde{x}\| = (\|S\|+\|T\|)\,\|\tilde{x}\|.$$

This implies that $\|S+T\| \tilde{\leq} \|S\| + \|T\|$. □

**Theorem 4.12.** *$L(\check{X},\check{Y})$ is a soft normed linear space where for $\tilde{f} \in SE\left(L\left(\check{X},\check{Y}\right)\right)$, we can identify $\tilde{f}$ to a unique $T \in W$ and $\left\|\tilde{f}\right\|$ is defined by*

$$\left\|\tilde{f}\right\|(\lambda) = \|T\|(\lambda) = sup\left\{\|T(\tilde{x})\|(\lambda) : \|\tilde{x}\| \tilde{\leq} \bar{1}\right\},$$

*for each $\lambda \in A$.*

*Proof.* Clearly $\left\|\tilde{f}\right\|$ is a mapping from $SE(L\left(\check{X},\check{Y}\right))$ to the set of all non-negative soft real numbers. So $\left\|\tilde{f}\right\|$ will be a soft norm if it satisfies soft norm axioms (N1) – (N4) of Definition 2.22.

(N1). Clearly $\left\|\tilde{f}\right\|(\lambda) = \|T\|(\lambda) \geq 0,\ \forall\lambda \in A$ i.e., $\left\|\tilde{f}\right\| \tilde{\geq} \bar{0}$.

(N2). If $\tilde{f}(\lambda) = O$, the null operator from $X$ to $Y$, for each $\lambda \in A$; then $T =$ the null soft linear operator and $\left\|\tilde{f}(\lambda)\right\| = \|T_\lambda\|_\lambda = 0$ and hence $\left\|\tilde{f}\right\| = \|T\| = \bar{0}$. Suppose that $\left\|\tilde{f}\right\| = \|T\| = \bar{0}$ i.e., $\left\|\tilde{f}\right\|(\lambda) = \|T\|(\lambda) = \sup\left\{\|T(\tilde{x})\|(\lambda) : \|\tilde{x}\| \tilde{\leq} \bar{1}\right\} = 0$, for each $\lambda \in A$. Since $\check{X},\check{Y}$ satisfy (N5) and $T$ satisfy (L3), we have for each $\lambda \in A$, $\|T\|(\lambda) = \|T_\lambda\|_\lambda = \sup\left\{\|T_\lambda(x)\|_\lambda : \|x\|_\lambda \leq 1\right\} = \sup\left\{\|T(\tilde{x})\|(\lambda) : \|\tilde{x}\| \tilde{\leq} \bar{1}\right\}$. [ Using Lemma 4.4] Thus we have $\|T_\lambda\|_\lambda = \sup\left\{\|T_\lambda(x)\|_\lambda : \|x\|_\lambda \leq 1\right\} = 0$, for each $\lambda \in A$. By the property of crisp operators it follows that $T_\lambda = O = T(\lambda)$, for each $\lambda \in A$. Hence we have $T =$ the null soft linear operator and hence $\tilde{f}(\lambda) = O$, the null linear operator on $X$, for each $\lambda \in A$.

(N3). If $\tilde{\gamma}$ be a soft scalar, then for each $\lambda \in A$,

$$\left\|\tilde{\gamma}\tilde{f}\right\|(\lambda) = \|\tilde{\gamma}T\|(\lambda) = \|(\tilde{\gamma}T)_\lambda\|_\lambda = \|\tilde{\gamma}(\lambda).T_\lambda\|_\lambda$$
$$= |\tilde{\gamma}|(\lambda).\|T_\lambda\|_\lambda = (|\tilde{\gamma}|\,\|T\|)(\lambda) = \left(|\tilde{\gamma}|\left\|\tilde{f}\right\|\right)(\lambda).$$

So, $\left\|\tilde{\gamma}\tilde{f}\right\| = |\tilde{\gamma}|\left\|\tilde{f}\right\|$.

(N4). We have $\left\|\tilde{f}+\tilde{g}\right\|(\lambda) = \|S+T\|(\lambda) \leq (\|S\|+\|T\|)(\lambda) = (\left\|\tilde{f}\right\| + \|\tilde{g}\|)(\lambda)$, for each $\lambda \in A$; where $\tilde{f},\tilde{g}$ are identified with $S$ and $T$ respectively. Thus, $\left\|\tilde{f}\right\|$ is a soft norm on $L(\check{X},\check{Y})$ and hence $L(\check{X},\check{Y})$ is a soft normed linear space. □

**Remark 4.13.** (Ring of operators). Let $\check{X}$ be a soft normed linear space satisfying (N5) over a finite set of parameters $A$. Consider the set $W$ of all continuous soft linear operators $S$, $T$ etc. which satisfy (L3) each mapping $SE(\check{X})$ into $SE\left(\check{X}\right)$. We define the product of two such continuous soft linear operators $S$ and $T$ by $(ST)(\tilde{x}) = S(T(\tilde{x}))$ for every $\tilde{x}\tilde{\in}\check{X}$. It is a matter of simple verification that $ST$ becomes a continuous soft linear operator $T : SE(\check{X}) \to SE\left(\check{X}\right)$. We write $AA = A^2$, $A^2A = A^3$, $A^3A = A^4$, and so on. It may be verified further that $(ST)V =$

$S(TV)$, $(S+T)V = SV + TV$, $V(S+T) = VS + VT$. Further, there exists an operator $I$, the identity operator such that for all soft linear continuous operators $S$, $SI = IS = S$. Therefore $W$ becomes a ring with identity element. Since the elements of $SE(L(\check{X}, \check{Y}))$ can be identified uniquely to the elements of $W$. Thus $SE(L(\check{X}, \check{Y}))$ can be interpreted as to form a ring with identity element.

**Theorem 4.14.** *For $S$, $T \in W$, $\|ST\| \tilde{\leq} \|S\| . \|T\|$.*

*Proof.* We have, $\|ST(\tilde{x})\| = \|S(T(\tilde{x}))\| \tilde{\leq} \|S\| . \|T(\tilde{x})\| \tilde{\leq} \|S\| . \|T\|(\tilde{x})$, for every $\tilde{x} \tilde{\in} \check{X}$. Therefore, $\|ST\| \tilde{\leq} \|S\| . \|T\|$. □

**Theorem 4.15.** *For any positive integer $n$, $\|S^n\| \tilde{\leq} \|S\|^n$.*

*Proof.* Put $S = T$ in Theorem 4.14, then $\|S^2\| \tilde{\leq} \|S\| . \|S\| = \|S\|^2$. Suppose that the result is true for $n = m$, i.e., $\|S^m\| \tilde{\leq} \|S\|^m$. Let $B = A^m$ in Theorem 4.14, then $\|SS^m\| \tilde{\leq} \|S\| . \|S^m\|$ i.e., $\|S^{m+1}\| \tilde{\leq} \|S\| . \|S\|^m = \|S\|^{m+1}$. Hence by induction the theorem is proved. □

**Definition 4.16.** Let $S_n$, $S \in W$. Then

$$\begin{aligned}\|S_n - S\|(\lambda) &= \sup\{\|(S_n - S)(\tilde{x})\|(\lambda) : \|\tilde{x}\| \tilde{\leq} \bar{1}\} \\ &= \sup\{\|(S_n(\tilde{x}) - S(\tilde{x}))\|(\lambda) : \|\tilde{x}\| \tilde{\leq} \bar{1}\},\end{aligned}$$

for each $\lambda \in A$. If $\|S_n - S\| \to \bar{0}$ as $n \to \infty$, then we say that the sequence of operators $\{S_n\}$ converges in norm to the operator $S$ and we write $S_n \to S$ (in norm).

**Definition 4.17.** Let $\tilde{f}_n, \tilde{f} \in L(\check{X}, \check{Y})$, then $\tilde{f}_n, \tilde{f}$ can be identified uniquely to $S_n$, $S \in W$. We define $\tilde{f}_n \to \tilde{f}$ (in norm) if $S_n \to S$ (in norm).

**Theorem 4.18.** *Let $\check{X}, \check{Y}$ be soft normed linear spaces which satisfy (N5) and having a finite set of parametes $A$. If $\check{Y}$ is a soft Banach space, then $L(\check{X}, \check{Y})$ is also a soft Banach space with respect to the above identification.*

*Proof.* Let $\{\tilde{f}_n\}$ be a Cauchy sequence in $L(\check{X}, \check{Y})$. Then $\{\tilde{f}_n\}$ can be a identified to a Cauchy sequence $\{S_n\}$ in $W$, i.e., let $\|S_n - S_m\| \to \bar{0}$ as $m, n \to \infty$. If $\tilde{x} \tilde{\in} \check{X}$, then we have

$$\|S_n(\tilde{x}) - S_m(\tilde{x})\| = \|(S_n - S_m)(\tilde{x})\| \tilde{\leq} \|(S_n - S_m)\| \|\tilde{x}\| \to \bar{0} \text{ as } m, n \to \infty.$$

Therefore $\{S_n(\tilde{x})\}$ is a Cauchy sequence of soft elements of $\check{Y}$ and since $\check{Y}$ is complete, $S_n(\tilde{x}) \to \tilde{y}$ (say) as $n \to \infty$. With every $\tilde{x} \tilde{\in} \check{X}$, we can associate $\tilde{y} \tilde{\in} \check{Y}$, thereby obtaining an operator $S$ defined by $S(\tilde{x}) = \tilde{y} (= \lim_{n\to\infty} S_n(\tilde{x}))$. We prove first that $S \in W$. We have

$$S(\tilde{x}_1 + \tilde{x}_2) = \lim_{n\to\infty} S_n(\tilde{x}_1 + \tilde{x}_2) = \lim_{n\to\infty} S_n(\tilde{x}_1) + S_n(\tilde{x}_2) = S(\tilde{x}_1) + S(\tilde{x}_2),$$

so $S$ is additive. If $\tilde{c}$ is a soft scalar, then

$$S(\tilde{c}\tilde{x}_1) = \lim_{n\to\infty} S_n(\tilde{c}\tilde{x}_1) = \tilde{c} \lim_{n\to\infty} S_n(\tilde{x}_1) = \tilde{c}.S(\tilde{x}_1).$$

So, $S$ is a soft linear operator. Now, $|\,\|S_n\| - \|S_m\|\,| \tilde{\leq} \|S_n - S_m\| \to \bar{0}$ as $m, n \to \infty$, so the sequence $\{\|S_n\|\}$ of non-negative soft real numbers is Cauchy and since the parameter set is finite, is convergent (By Theorem 2.27). Therefore the sequence is bounded, i.e., there exists a soft real number $\tilde{K}$ such that $\|S_n\| \tilde{\leq} \tilde{K}$ for $n = 1, 2, \ldots$ . Hence $\|S(\tilde{x})\| = \lim_{n\to\infty} \|S_n(\tilde{x})\| \tilde{\leq} \lim_{n\to\infty} \|S_n\| \|\tilde{x}\| \tilde{\leq} \tilde{K} \|\tilde{x}\|$, for all $\tilde{x} \tilde{\in} \check{X}$. Thus $S$ is bounded and hence is continuous. Therefore, $S \in W$. Now we show that $\|S_n - S\| \to \bar{0}$ as $n \to \infty$. Since $\{S_n\}$ is a Cauchy sequence, for $\tilde{\varepsilon} \tilde{>} \bar{0}$ arbitrary there exists $n_0$ such that $\|S_{n+p} - S_n\| \tilde{<} \tilde{\varepsilon}$ if $n \geq n_0$ and $p = 1, 2, \ldots\ldots$. So for all $\tilde{x} \tilde{\in} \check{X}$, $\|\tilde{x}\| \tilde{\leq} \bar{1}$, we have for $n \geq n_0$ and $p = 1, 2, \ldots\ldots$

$$\|S_{n+p}(\tilde{x}) - S_n(\tilde{x})\| \tilde{\leq} \|S_{n+p} - S_n\| \|\tilde{x}\| \tilde{\leq} \|S_{n+p} - S_n\| \tilde{<} \tilde{\varepsilon}.$$

Letting $p \to \infty$, we obtain $\|S(\tilde{x}) - S_n(\tilde{x})\| \tilde{<} \tilde{\varepsilon}$ if $n \geq n_0$ and $\|\tilde{x}\| \tilde{\leq} \bar{1}$. Therefore for $n \geq n_0$, $\|S - S_n\|(\lambda) = \sup \left\{ \|S(\tilde{x}) - S_n(\tilde{x})\|(\lambda) : \|\tilde{x}\| \tilde{\leq} \bar{1} \right\} < \tilde{\varepsilon}(\lambda)$, for every $\lambda \in A$. i.e., $\|S - S_n\| \tilde{<} \tilde{\varepsilon}$ i.e., $S_n \to S$. Hence by definition, $\tilde{f}_n \to \tilde{f}$ (in norm). Therefore $L(\check{X}, \check{Y})$ is a soft Banach space. □

**Definition 4.19.** Let $\check{X}$ be a soft Banach space and let the operations $(\tilde{x}, \tilde{y}) \to \tilde{x} + \tilde{y}$ $(\tilde{c}, \tilde{x}) \to \tilde{c}.\tilde{x}$ be defined on $SE(\check{X})$. If $SE(\check{X})$ together with these operations form a ring and is assumed to satisfy the condition $\|\tilde{x}.\tilde{y}\| \tilde{\leq} \|\tilde{x}\| . \|\tilde{y}\|$, $\forall\ \tilde{x}, \tilde{y} \in SE(\check{X})$. Then such a system is called a *soft Banach Algebra.*

**Example 4.20.** By Theorem 4.18, $L(\check{X}, \check{Y})$ is a soft Banach space and the elements of $SE(L(\check{X}, \check{Y}))$ can be identified uniquely to the elements of $W$ which is a ring with unity. We further notice that for any $S, T \in W$, $\|ST\| \tilde{\leq} \|S\| . \|T\|$. So, with respect to the above identification, $L(\check{X}, \check{Y})$ is a soft Banach Algebra.

## 5. Inverse of soft linear operators

Let $T : SE(\check{X}) \to SE(\check{Y})$ where $\check{X}, \check{Y}$ are soft normed linear spaces having the same underlying scalar field. $SE(\check{X})$ is the domain of the operator $T$. The set $\{T(\tilde{x}) : \tilde{x} \tilde{\in} \check{X}\}$ is the range of $T$. The domain of $T$ and the range of $T$ are denoted respectively by $D(T)$ and $R(T)$. Throughout this section we shall assume that $\check{X}, \check{Y}$ are soft normed linear spaces having the same underlying scalar field.

**Definition 5.1.** Let $T : SE(\check{X}) \to SE(\check{Y})$. The inverse of the operator $T$, denoted by $T^{-1}$, is said to exist if $T^{-1}(T(\tilde{x})) = \tilde{x}$ for all $\tilde{x} \tilde{\in} \check{X}$ i.e., $T^{-1}(T) = I$, the identity operator. Such an operator $T^{-1}$ clearly exists if and only if $T(\tilde{x}_1) = T(\tilde{x}_2)$ implies $\tilde{x}_1 = \tilde{x}_2$ for every $\tilde{x}_1, \tilde{x}_2 \tilde{\in} \check{X}$.

**Theorem 5.2.** *Let $T : SE(\check{X}) \to SE(\check{Y})$ be a soft linear operator. Then $T^{-1}$ exists if and only if $T(\tilde{x}) = \Theta$ implies $\tilde{x} = \Theta$.*

*Proof.* Suppose $T(\tilde{x}) = \Theta$ implies $\tilde{x} = \Theta$ and let $T(\tilde{x}_1) = T(\tilde{x}_2)$. Then because $T$ is soft linear, $T(\tilde{x}_1 - \tilde{x}_2) = T(\tilde{x}_1) - T(\tilde{x}_2) = \Theta$ and therefore $\tilde{x}_1 - \tilde{x}_2 = \Theta$ i.e., $\tilde{x}_1 = \tilde{x}_2$ and so $T^{-1}$ exists.

Conversely, suppose that $T^{-1}$ exists i.e., $T(\tilde{x}_1) = T(\tilde{x}_2)$ implies $\tilde{x}_1 = \tilde{x}_2$. Let $T(\tilde{x}) = \Theta$, then $T(\tilde{x}) = \Theta = T(\Theta)$ implying thereby $\tilde{x} = \Theta$. □

**Theorem 5.3.** *Let $T : SE(\check{X}) \to SE\left(\check{Y}\right)$ be a soft linear operator. If $T^{-1}$ exists, then $T^{-1}$ is soft linear.*

*Proof.* Let $\tilde{y}_1, \tilde{y}_2 \tilde{\in} R(T)$, then there exist $\tilde{x}_1, \tilde{x}_2 \tilde{\in} \check{X}$ such that $T(\tilde{x}_1) = \tilde{y}_1$, $T(\tilde{x}_2) = \tilde{y}_2$. So,

$$\begin{aligned} T^{-1}(\tilde{y}_1 + \tilde{y}_2) &= T^{-1}(T(\tilde{x}_1) + T(\tilde{x}_2)) \\ &= T^{-1}(T(\tilde{x}_1 + \tilde{x}_2)) \\ &= \tilde{x}_1 + \tilde{x}_2 = T^{-1}(\tilde{y}_1) + T^{-1}(\tilde{y}_2) \end{aligned}$$

and if $\tilde{c}$ be any soft scalar then

$$T^{-1}(\tilde{c}\tilde{y}_1) = T^{-1}(\tilde{c}T(\tilde{x}_1)) = T^{-1}(T(\tilde{c}\tilde{x}_1)) = \tilde{c}\tilde{x}_1 = \tilde{c}T^{-1}(\tilde{y}_1).$$

Hence $T^{-1}$ is soft linear. □

**Theorem 5.4.** *Let $T : SE(\check{X}) \to SE\left(\check{Y}\right)$ be a soft linear operator, where $\check{X}, \check{Y}$ are soft normed linear spaces with the same finite dimension $n$. If $R(T) = SE(\check{Y})$, then $T^{-1}$ exists.*

*Proof.* Let $\tilde{y}_1, \tilde{y}_2, \ldots.., \tilde{y}_n$ be a basis for $\check{Y}$. Since $R(T) = SE(\check{Y})$, there exists $\tilde{x}_1, \tilde{x}_2, \ldots.., \tilde{x}_n$ in $\check{X}$ such that $T(\tilde{x}_k) = \tilde{y}_k$, for $k = 1, 2, .., n$. Now $\tilde{c}_1.\ \tilde{x}_1 + \tilde{c}_2.\ \tilde{x}_2 + .. + \tilde{c}_n.\ \tilde{x}_n = \Theta$, implies $\Theta =$ T$(\tilde{c}_1.\ \tilde{x}_1 + \tilde{c}_2.\ \tilde{x}_2 + .. + \tilde{c}_n.\ \tilde{x}_n) = \tilde{c}_1.\ \tilde{y}_1 + \tilde{c}_2.\ \tilde{y}_2 + .. + \tilde{c}_n.\ \tilde{y}_n$, giving that $\tilde{c}_1 = \tilde{c}_2 = \cdots = \tilde{c}_n = \overline{0}$, because $\tilde{y}_1, \tilde{y}_2, \ldots.., \tilde{y}_n$ are linearly independent. This shows that $\tilde{x}_1, \tilde{x}_2, \ldots.., \tilde{x}_n$ are linearly independent and thus form a basis for $\check{X}$ since $\check{X}$ is of dimension $n$. Now, suppose that $T(\tilde{x}) = \Theta$, for some $\tilde{x} \tilde{\in} \check{X}$. Then $\tilde{x}$ can be expressed as $\tilde{x} = \tilde{c}_1.\ \tilde{x}_1 + \tilde{c}_2.\ \tilde{x}_2 + .. + \tilde{c}_n.\ \tilde{x}_n$. So, $\Theta = T(\tilde{x}) =$ T$(\tilde{c}_1.\ \tilde{x}_1 + \tilde{c}_2.\ \tilde{x}_2 + .. + \tilde{c}_n.\ \tilde{x}_n) = \tilde{c}_1.\ \tilde{y}_1 + \tilde{c}_2.\ \tilde{y}_2 + .. + \tilde{c}_n.\ \tilde{y}_n$. This implies that $\tilde{c}_1 = \tilde{c}_2 = \cdots = \tilde{c}_n = \overline{0}$ and so $\tilde{x} = \Theta$. Hence $T^{-1}$ exists by Theorem 5.2. □

**Definition 5.5.** An operator $T : SE(\check{X}) \to SE\left(\check{Y}\right)$ is called injective or one-to-one if $T(\tilde{x}_1) = T(\tilde{x}_2)$ implies $\tilde{x}_1 = \tilde{x}_2$. It is called surjective or onto if $R(T) = SE(\check{Y})$. The operator $T$ is bijective if $T$ is both injective and surjective.

**Theorem 5.6.** *Let $T : SE(\check{X}) \to SE\left(\check{Y}\right)$ be a soft linear operator, where $\check{X}, \check{Y}$ are soft normed linear spaces. Then $T^{-1}$ exists and is continuous on its domain of definition if there exists a soft real number $\tilde{m} \tilde{>} \overline{0}$ such that $\tilde{m}\,\|\tilde{x}\| \tilde{\leq} \|T(\tilde{x})\|$ for every $\tilde{x} \tilde{\in} \check{X}$.*

*Proof.* Suppose that $\tilde{m}\,\|\tilde{x}\| \tilde{\leq} \|T(\tilde{x})\|$ for every $\tilde{x} \tilde{\in} \check{X}$. Then $T(\tilde{x}) = \Theta$ implies $\tilde{x} = \Theta$ and so $T^{-1}$ exists by Theorem 5.2. Now, $T(\tilde{x}) = \tilde{y}$ is equivalent to $\tilde{x} = T^{-1}(\tilde{y})$ and so $\tilde{m}\,\|\tilde{x}\| = \tilde{m}\left\|T^{-1}(\tilde{y})\right\| \tilde{\leq} \|\tilde{y}\| = \|T(\tilde{x})\|$ or $\left\|T^{-1}(\tilde{y})\right\| \tilde{\leq} \frac{\|\tilde{y}\|}{\tilde{m}}$ for all $\tilde{y}$ in $R(T)$, which is the domain of $T^{-1}$. So, $T^{-1}$ is bounded and hence continuous. □

**Theorem 5.7.** *Let $\check{X}$ and $\check{Y}$ be soft normed linear spaces which satisfy (N5) and $T : SE(\check{X}) \to SE(\check{Y})$ be a soft linear operator. Let $T^{-1}$ be a continuous soft linear operator $T^{-1} : SE(\check{Y}) \to SE(\check{X})$ which satisfy (L3). Then there exists a soft real number $\tilde{m} \tilde{>} \overline{0}$ such that $\tilde{m}\,\|\tilde{x}\| \tilde{\leq} \|T(\tilde{x})\|$ for every $\tilde{x} \tilde{\in} \check{X}$.*

*Proof.* Suppose that $T^{-1}$ exists and is continuous. Since $\check{X}, \check{Y}$ satisfy (N5) and $T$ satisfy (L3), by Theorem 3.12, $T^{-1}$ is bounded. So there exists a positive soft real number $\tilde{M}\tilde{>}\bar{0}$ such that $\left\|T^{-1}(\tilde{y})\right\| \tilde{\leq} \tilde{M} \left\|\tilde{y}\right\|$ for all $\tilde{y}$ in $R(T)$. There exists $\tilde{x}\tilde{\in}\check{X}$ such that $T(\tilde{x}) = \tilde{y}$ i.e., $\tilde{x} = T^{-1}(\tilde{y})$. So, the above becomes $\|\tilde{x}\| \tilde{\leq} \tilde{M} \|T(\tilde{x})\|$ i.e., $\frac{1}{\tilde{M}} \|\tilde{x}\| \tilde{\leq} \|T(\tilde{x})\|$. So if $\tilde{m} = \frac{1}{\tilde{M}}$ the theorem is obtained. □

**Theorem 5.8.** *Let $\check{X}$ be a soft Banach space and $\check{Y}$ be a soft normed linear space both of which satisfy (N5) and $T : SE(\check{X}) \to SE(\check{Y})$ be a bijective continuous soft linear operator satisfying (L3). If $T^{-1}$ is continuous then $\check{Y}$ is a soft Banach space.*

*Proof.* We need only to show that $\check{Y}$ is complete. Let $\{y_n\}$ be a Cauchy sequence in $\check{Y}$. There exists $x_n\tilde{\in}\check{X}$ such that $T(x_n) = y_n$. So by Theorem 5.7,

$$\|y_n - y_m\| = \|T(x_n) - T(x_m)\| = \|T(x_n - x_m)\| \tilde{\geq} \tilde{m} \|x_n - x_m\|,$$

where $\tilde{m}\tilde{>}\bar{0}$. Thus $\{x_n\}$ is also a Cauchy sequence in $\check{X}$. Since $\check{X}$ is complete, $\lim_{n\to\infty} \tilde{x}_n = \tilde{x}_0$, for some $\tilde{x}_0\tilde{\in}\check{X}$. Let $T(x_0) = y_0$. Then

$$\lim_{n\to\infty} \tilde{y}_n = \lim_{n\to\infty} T(x_n) = T(x_0) = y_0,$$

proving the completeness of $\check{Y}$. So, $\check{Y}$ is a soft Banach space. □

## 6. Conclusions

In this paper we have introduced a concept of soft linear operator on a soft linear space. Some basic properties of such operators has been investigated with examples. Continuty and boundedness of such operators has been defined and studied some of their basic properties. Spaces of continuous soft linear operators and inverse of soft linear operators are studied . There is an ample scope for further research on soft normed linear spaces and soft linear operators.

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