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Medical diagnosis via interval valued intuitionistic fuzzy sets

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Abstract. We propose a new approach for medical diagnosis by interval valued intuitionistic fuzzy sets. In Sanchez's approach [5, 6] for medical diagnosis is studied and the concept is generalized by the application of IFS theory. Where fuzzy sets are $IFSs$, the converse is not always true as justified in [1, 8]. As a consequence, a study of Sanchez's approach for medical diagnosis has been made with a generalized notion (i.e., IFS theory). The nonmembership functions have more important roles here in comparison to the membership function corresponding to the complement of fuzzy sets because of the fact that in decision making problems, particularly in case of medical diagnosis, is a fair chance of the existence of a non-zero hesitation part at each moment of evaluation of any unknown object.

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1. Introduction

Intuitionistic fuzzy sets as a generalization of fuzzy sets can be useful in situations when description of a problem by a (fuzzy) linguistic variable, given in terms of a membership function only, seems too rough. For example, in decision making problems, particularly in the case of medial diagnosis, sales analysis, new product marketing, nancial services, etc. there is a fair chance of the existence of a non-null hesitation part at each moment of evaluation of an unknown object. To be more precise-intuitionistic fuzzy sets let us express, e.g., the fact that the temperature of a patient changes, and other symptoms are not quiteclear.In this article we will present intuitionistic fuzzy sets as a tool for reasoning in the presence of imperfect facts and imprecise knowledge. An example of medical diagnosis will be presented assuming there is a database, i.e. description of a set of symptoms S, and a set of diagnoses D. We will describe a state of a patient knowing results of his/her medical tests. Description of the problem uses the notion of an intuitionistic fuzzy set. The proposed method of diagnosis involves intuitionistic fuzzy distances as introduced in (Szmidtand Kacprzyk [7, 8]). Advantages of such an approach are pointed out in comparison with the method presented in (De, Biswas and Roy [4]) in which the max-min-max composition rule was applied.

2. Brief introduction to intuitionistic fuzzy sets

As opposed to fuzzy set (Zadeh [9]) in $X = x$, given by

$$A^{'} = \{\langle x, \mu_{A'}(x)\rangle\}$$

where $\mu_{A'} : X \to [0,1]$ is the membership function of the fuzzy set

$$A^{'} : \mu_{A'} \in [0,1],$$

is the membership of $x \in X$ in A, an intuitionistic fuzzy set (Atanassov [2]).

Definition 2.1. Let X and Y Be two sets. An intuitionistic fuzzy relation $(IFR)R$ from X to Y is an IFS of $X \times Y$ characterized by the membership function μ_R and non-membership function υ_R. An IFR R from X to Y will be denoted by $R\ (X \to Y)$.

Definition 2.2. If A is an IFS of X, then $max - min - max$ composition of the IFR $R(X \to Y)$ with A is an IFS B of Y denoted by $B = R \circ A$, and is defined by the membership function

$$\mu_{R\circ A}(y) = \bigvee_x [\mu_{A(x)} \wedge \mu_{R(x,y)}]$$

and the non-membership function

$$\nu_{R\circ A}(y) = \bigwedge_x [\nu_{A(x)} \wedge \nu_{R(x,y)}] \quad \forall y \in Y (\text{where } \bigvee = \max, \quad \bigwedge = \min\ ).$$

Definition 2.3. Let $Q(X \to Y)$ and $R(Y \to Z)$ be two $IFSs$. The max-min-max composition $R \circ Q$ is the intuitionistic fuzzy relation (IFR) from X to Z, defined by the membership function

$$\mu_{R\circ Q}(y) = \bigvee_y [\mu_{Q(x,y)} \wedge \mu_{R(y,z)}]$$

and the non-membership function

$$\nu_{R\circ Q}(y) = \bigwedge_y [\nu_{Q(x,y)} \wedge \nu_{R(y,z)}] \quad \forall (x,y) \in X \times Y \quad \text{and} \quad \forall y \in Y$$

In this section we present an application of intuitionistic fuzzy sets theory in Sanchez's approach [3, 4] for medical diagnosis. In given pathology, suppose $S =$ is a set of symptoms, D a set of diagnosis, and P a set patients.

Analogous to Sanchez's notion of Medical Knowledge we define intuitionistic Medical Knowledge as an intuitionistic fuzzy relation R from the set of symptoms S to the set of diagnosis D (i.e., on $S \times D$) which reveals the degree of association and the degree of non-association between symptoms and diagnosis.

Let A be an IFS of the set S, and R be an IFR from S to D. Then max-min-max composition [7] B of IFS A with the IFR $R(Srightarrow\ D)$ denoted by $B = A \circ R$ signifies the state of the patient in terms of diagnosis as an IFS B of D with the membership function given by

$$\mu_B(d) = \bigvee_{s \in S} [\mu_{A(s)} \wedge \mu_{R(s,d)}]$$

and the non-membership function given by

$$\mu_B(d) = \bigwedge_{s \in S} [\nu_{A(s)} \wedge \nu_{R(s,d)}] \quad \forall d \in D$$

. If the state of a give patient P is described in items of an IFS A of S, then P is assumed to be assigned diagnosis in terms of IFS B of D, through an IFR R of intuitionistic Medical Knowledge from S to D which is assumed to be given by a doctor who able to translate his owen precepation of the intuitionistic involved in degrees of association and non-association respectively between symptoms and diagnosis.

Now Let us extend this concept to a finite number of patient. Let there be n patients P_i, $i = 1, 2, 3, \ldots, n$ in a hospital. Thus $p_i \in P$. Let R be an IFR $(S \to D)$ and constract an IFR Q from the set of patients to the set of symptoms S. Clearly, the composition T of IFRs R and $Q(T = R \circ Q)$ describes the state of patients P_i in terms of the diagnosis as an IFR from P to D given by the membership function

$$\mu_T(p_i, d) = \bigvee_{s \in S} [(\bigvee_s \mu_{Q(p_i,s)}) \wedge (\bigvee_s \mu_{R(s,d)})]$$

and the non-membership function given by

$$\mu_T(p_i, d) = \bigwedge_{s \in S} [\bigwedge_s \nu_{Q(p_i,s)} \wedge \bigwedge_s \nu_{R(s,d)}] \quad \forall p_i \in P \quad \text{and} \quad d \in D.$$

For a given R and Q, the relation $T = R \circ Q$ can be computed. From the knowledge of Q and R, one may compute an improved version of the IFR R for which the following holds good:

(i) $S_R = \mu_R - \nu_R.\pi_R$ is greatest, and
(ii) the equality $T = R \circ Q$ is retained.

To see the application of the medical let us make a hypothetical case study:

Let there be four patients Case 1, Case 2, Case 3, Case 4 in a hospital. Their symptoms are temperature, headache, stomachpain, cough. Clearly,

$$P = \{Case\ 1,\ Case\ 2,\ Case\ 3,\ Case\ 4\}$$

and the set of symptoms

$$S = \{temperature,\ headache,\ stomachpain,\ cough\}.$$

The intuitionistic fuzzy relation $Q(P \to S)$ is given as in (hypothetical) Table 1.

Let the set of diagnosis be

$$D = \{Viral\ Fever,\ Malaria,\ Typhoid,\ Stomach\ problem\}.$$

TABLE 1.

Q	Temperature	Headache	Stomachpain	Cough
Case 1	$\langle[0.6, 0.8], [0.02, 0.1]\rangle$	$\langle[0.4, 0.6], [0.02, 0.1]\rangle$	$\langle[0.1, 0.2], [0.6, 0.8]\rangle$	$\langle[0.4, 0.6], [0.02, 0.1]\rangle$
Case 2	$\langle[0.0, 0.0], [0.6, 0.8]\rangle$	$\langle[0.2, 0.4], [0.2, 0.4]\rangle$	$\langle[0.4, 0.6], [0.02, 0.1]\rangle$	$\langle[0.02, 0.1], [0.5, 0.7]\rangle$
Case 3	$\langle[0.6, 0.8], [0.02, 0.1]\rangle$	$\langle[0.6, 0.8], [0.02, 0.1]\rangle$	$\langle[0.0, 0.0], [0.4, 0.6]\rangle$	$\langle[0.1, 0.2], [0.5, 0.7]\rangle$
Case 4	$\langle[0.4, 0.6], [0.02, 0.1]\rangle$	$\langle[0.3, 0.5], [0.2, 0.4]\rangle$	$\langle[0.1, 0.3], [0.2, 0.4]\rangle$	$\langle[0.5, 0.7], [0.1, 0.2]\rangle$

TABLE 2.

R	Viral fever	Malaria	Typhoid	Stomach problem
Temperature	$\langle[0.5, 0.4], [0.02, 0.1]\rangle$	$\langle[0.1, 0.7], [0.02, 0.1]\rangle$	$\langle[0.1, 0.3], [0.2, 0.3]\rangle$	$\langle[0.4, 0.1], [0.02, 0.7]\rangle$
Headache	$\langle[0.0, 0.3], [0.5, 0.5]\rangle$	$\langle[0.02, 0.2], [0.2, 0.6]\rangle$	$\langle[0.4, 0.6], [0.02, 0.1]\rangle$	$\langle[0.02, 0.2], [0.3, 0.4]\rangle$
Stomach pain	$\langle[0.06, 0.1], [0.02, 0.7]\rangle$	$\langle[0.0, 0.0], [0.02, 0.9]\rangle$	$\langle[0.0, 0.2], [0.3, 0.7]\rangle$	$\langle[0.1, 0.8], [0.0, 0.0]\rangle$
Cough	$\langle[0.2, 0.4], [0.02, 0.3]\rangle$	$\langle[0.3, 0.7], [0.0, 0.0]\rangle$	$\langle[0.1, 0.2], [0.2, 0.6]\rangle$	$\langle[0.1, 0.2], [0.1, 0.7]\rangle$

TABLE 3.

T	Viral fever	Malaria	Typhoid	Stomach problem
Case 1	$\langle[0.6, 0.4], [0.02, 0.1]\rangle$	$\langle[0.0, 0.7], [0.6, 0.1]\rangle$	$\langle[0.6, 0.6], [0.02, 0.1]\rangle$	$\langle[0.4, 0.2], [0.02, 0.4]\rangle$
Case 2	$\langle[0.4, 0.3], [0.02, 0.5]\rangle$	$\langle[0.2, 0.2], [0.2, 0.6]\rangle$	$\langle[0.6, 0.4], [0.02, 0.4]\rangle$	$\langle[0.3, 0.6], [0.2, 0.1]\rangle$
Case 3	$\langle[0.1, 0.4], [0.6, 0.1]\rangle$	$\langle[0.4, 0.7], [0.02, 0.1]\rangle$	$\langle[0.0, 0.6], [0.4, 0.1]\rangle$	$\langle[0.1, 0.2], [0.2, 0.4]\rangle$
Case 4	$\langle[0.4, 0.4], [0.02, 0.1]\rangle$	$\langle[0.02, 0.7], [0.5, 0.1]\rangle$	$\langle[0.1, 0.5], [0.5, 0.3]\rangle$	$\langle[0.5, 0.3], [0.1, 0.4]\rangle$

Table 4.

S_R	Viral fever	Malaria	Typhoid	Stomach problem
Case 1	0.35	0.68	0.57	0.04
Case 2	0.20	0.08	0.32	0.57
Case 3	0.35	0.68	0.57	0.04
Case 4	0.32	0.68	0.44	0.18

The intuitionistic fuzzy relation $R(S \to D)$ is given as in (hypothetical) Table 2. Therefore the composition $T = R \circ Q$ is as given in Table 3. We calculate S_R as given in Table 4.

From Table 4 it is obvious that, if the doctor agrees, then Case 1, Case 3, Case 4 suffer from Malaria whereas Case 2 faces Stomach problem.

3. Conclusions

Employing intuitionistic fuzzy sets in databases let us express hesitancy concerning examined objects. The method proposed in this article - making a diagnosis on the basis of calculating distances from a considered case to all considered illnesses, takes into account values of all symptoms. What is more, our approach let us introduce weights for all symptoms (for some illnesses some symptoms can be more important). Such an approach is impossible by the method described in (De, Biswas and Roy, 2001) - the max-min-max rule "neglects" in fact most values basing on extreme ones.

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