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Fuzzy P-closed spaces through fuzzy grill

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Abstract. In this article, an attempt has been made to study the existing concept of fuzzy P-closedness in terms of fuzzy grills.

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1. Introduction

It is well known that after the introduction of fuzzy sets by Zadeh [15] in 1965, Chang [4] first initiated the concept of fuzzy topology in 1968. These paved the way for the subsequent rapid development along different directions, encompassing various concepts and results of general topology for their generalizations in fuzzy settings. In 1995, Zahran [16] generalized the idea of P-closedness in fuzzy topological spaces, a further extension to L-fuzzy topological spaces was done by Aygun and Kudri [1]. In [9] we also undertook a study of fuzzy P-closedness. In the present article we continue the same but with fuzzy grill as the basic appliance.

It is a well accepted fact that certain special appliance like nets and filters are invoked and judiciously applied to draw interesting and useful conclusions while investigating different topological problems. Another such extremely useful tool, called grill, was first defined by Choquet [6], and it is seen from literature that investigations of many topological concepts like proximity, compactifications and problems of extension of spaces, can be nicely tackled by use of grills. The fuzzy version of grill was introduced by Azad [2] with some specific purpose. He showed the efficacy of employing fuzzy grill for the study of fuzzy proximity. Further applications of fuzzy grill are also found in the investigations of LO proximity by Srivastava and Gupta [14]. Day by day, the importance and applicational aspects of fuzzy grill are being noticed, as a pivotal supporting appliance. Out of the various rather recent investigations pursued in terms of fuzzy grills, one may refer for instance, to the works in [3], [5] and [14]. All these investigations amply point out how a fuzzy topological concept can be viewed and explored via the idea of fuzzy grills, and this has been our motivation to take up a study of fuzzy P-closedness- already studied by many topologists with considerable interest-from a new angle and with a new approach by means of fuzzy grills on a space endowed with fuzzy topology. This has enabled us to obtain a few new results on fuzzy P-closedness among certain other associated results.

Throughout the paper, by an fts X, we always mean a fuzzy topological space (X, τ). The fuzzy closure, fuzzy interior and fuzzy complement of a fuzzy set A in an fts X (i.e., $A \in I^X$ where $I = [0, 1]$) are denoted by cl$A$, int$A$ and $1_X - A$ respectively, while 0_X and 1_X will be used to stand for the fuzzy sets taking on respectively the values 0 and 1 at each point of X. A fuzzy singleton or fuzzy point [12] with support x and value α $(0 < \alpha < 1)$ is denoted by x_α. For $A, B \in I^X$, we write $A \leq B$ iff $A(x) \leq B(x)$, for all $x \in X$ and AqB (i.e., A is quasi-coincident with B) [12] iff $A(x)+B(x) > 1$ for some $x \in X$. The negations of these statements are denoted by $A \not\leq B$ and $A\overline{q}B$ respectively. We now recall a few definitions and results for their subsequent uses.

Definition 1.1 ([13])**.** A fuzzy set A in an fts X is said to be
(i) a fuzzy preopen set if $A \leq int(clA)$,
(ii) a fuzzy preclosed set if $A \geq cl(intA)$. Thus A is fuzzy preopen if and only if $1 - A$ is fuzzy preclosed.

Definition 1.2 ([11])**.** Let A be a fuzzy set in an fts X and x_α be a fuzzy point in X. Then A is called a fuzzy pre-q-nbd of x_α if there exists a fuzzy preopen set B in X such that x_αq$B \leq A$. In particular, if A is fuzzy preopen and x_αqA, then A will be called a preopen pre-q-nbd of x_α.

Definition 1.3 ([13])**.** Fuzzy preclosure and fuzzy preinterior of a fuzzy set A in an fts X, denoted by pcl$A$ and pint$A$ respectively, are defined as follows:
pcl$A = \bigwedge\{U : U$ is fuzzy preclosed and $A \leq U\}$
pint$A = \bigvee\{V : V$ is fuzzy preopen and $A \geq V\}$.

Theorem 1.4 ([13])**.** *For a fuzzy set A in an fts X,*
(i) *pint$(1 - A) = 1-$pcl$A$,*
(ii) *pcl$(1 - A) = 1-$pint$A$,*
(iii) *A is fuzzy preopen (fuzzy preclosed) iff $A =$pint$A$ (resp. $A =$pcl$A$),*
(iv) *pcl(pcl$A$) =pcl$A$ and hence also pint(pint$A$) =pint$A$, so that pcl$A$ (pint$A$) is fuzzy preclosed (resp. preopen) for any $A \in I^X$.*

Theorem 1.5 ([9])**.** *For every fuzzy set A in an fts X, $p(\theta)$-cl$A = \bigwedge\{$pcl$V$: $A \leq V$ and V is fuzzy preopen$\}$.*

Definition 1.6 ([8])**.** A collection $\mathcal{F}$ of fuzzy sets in an fts X is called a prefilterbase on X if
(i) $0_X \notin \mathcal{F}$, and
(ii) $A, B \in \mathcal{F} \Rightarrow \exists\, C \in \mathcal{F}$ such that $C \leq A \bigwedge B$.
If in addition,
(iii) $A \in \mathcal{F}$ and $A \leq B \in I^X \Rightarrow B \in \mathcal{F}$ holds, then $\mathcal{F}$ is called a prefilter on X.

Theorem 1.7 ([7]). *For a fuzzy ultrafilter $\mathcal{U}$ (i.e., a maximal, with respect to set inclusion, prefilter on X is called a fuzzy ultrafilter on X) on an fts X, the following holds:*
if $A \bigvee B \in \mathcal{U}$, then either $A \in \mathcal{U}$ or $B \in \mathcal{U}$.

Definition 1.8 ([10]). (i) Let A be any fuzzy set in an fts X and x_α be a fuzzy point in X. Then A is called a fuzzy pre-θ-nbd of x_α if there exists a preopen pre-q-nbd B of x_α such that pcl$B\bar{q}(1-A)$.

(ii) A fuzzy point x_α in an fts X is said to be a fuzzy pre-θ-adherent point of a fuzzy set A if for every preopen pre-q-nbd B of x_α, pclBqA. The union of all fuzzy pre-θ-adherent points of a fuzzy set A is called the fuzzy pre-θ-closure of A and is denoted by $p(\theta)$-cl$A$. A fuzzy set A is called fuzzy pre-θ-closed if $p(\theta)$-cl$A = A$ and the complements of fuzzy pre-θ-closed sets are called pre-θ-open.

Theorem 1.9 ([10]). *For a fuzzy preopen set A in an fts X, pcl$A = p(\theta)$-cl$A$.*

2. Fuzzy grill

This section is devoted to the introduction of $p(\theta)$-adherence and $p(\theta)$-convergence of fuzzy grills, and it is established that for a fuzzy grill $\mathcal{G}$, the concepts of $p(\theta)$-adherence at x_α and $p(\theta)$-convergence to x_α are independent of each other. Some necessary and sufficient conditions of $p(\theta)$-adherence and $p(\theta)$-convergence of a fuzzy grill $\mathcal{G}$ are also obtained. The definition of fuzzy grill, introduced by Azad [2] goes as follows:

Definition 2.1. A non-void collection $\mathcal{G}$ of fuzzy sets in an fts X is said to be a fuzzy grill on X if

(i) $0_X \notin \mathcal{G}$,
(ii) $A \in \mathcal{G}$, $B \in I^X$ and $A \leq B \Rightarrow B \in \mathcal{G}$, and
(iii) $A, B \in I^X$ and $A \bigvee B \in \mathcal{G} \Rightarrow A \in \mathcal{G}$ or $B \in \mathcal{G}$.

Remark 2.2. If $\mathcal{F}$ is a fuzzy ultrafilter on an fts X, then by Theorem 1.7, it follows that every fuzzy ultrafilter on an fts X is a fuzzy grill on X.

Definition 2.3. Let $\mathcal{G}$ be a fuzzy grill on an fts X and x_α be a fuzzy point on an fts X. Then $\mathcal{G}$ is said to

(i) $p(\theta)$-adhere at x_α if for each preopen pre-q-nbd U of x_α and each $G \in \mathcal{G}$, pclUqG.

(ii) $p(\theta)$-converge to x_α if for each preopen pre-q-nbd U of x_α, $G \leq$ pcl$U$ for some $G \in \mathcal{G}$.

Remark 2.4. (a) From Definition 2.3 (ii), it follows that a fuzzy grill on an fts X, $p(\theta)$-converges to some fuzzy point x_α iff { pcl$U$: U is a preopen pre-q-nbd of x_α} $\subseteq \mathcal{G}$.

(b) For a fuzzy grill $\mathcal{G}$, the concepts of $p(\theta)$-adherence at x_α and $p(\theta)$-convergence to x_α are independent of each other as is shown below.

Example 2.5. Let $X = \{a\}$ and $\tau = \{0_X, A, 1_X\}$ where $A(a) = 0.5$. Then (X, τ) is an fts. Let $\mathcal{G} = \{G \in I^X : 0.2 \leq G(a) \leq 1\}$. Then $\mathcal{G}$ is a fuzzy grill on the fts X. It is easy to see that in the fts (X, τ), every fuzzy set is fuzzy preopen as well as fuzzy preclosed. Now, for any preopen pre-q-nbd U of the fuzzy point $a_{0.6}$, we have

$(\text{pcl}U)(a) = U(a) > 0.4$, and then $\mathcal{G}$ clearly $p(\theta)$-converges to $a_{0.6}$. But the fuzzy set V with $V(a) = 0.5$, is a preopen pre-q-nbd of $a_{0.6}$ and G (with $G(a) = 0.2$) $\in \mathcal{G}$ such that $\text{pcl}V\overline{q}G$, so that $\mathcal{G}$ does not $p(\theta)$-adhere at $a_{0.6}$.

Example 2.6. Consider the fts (X, τ) of the above example. Let $\mathcal{G} = \{G \in I^X : 0.6 \leq G(a) \leq 1\}$. Then $\mathcal{G}$ is a fuzzy grill on the fts X. It is easy to check that $\mathcal{G}$ $p(\theta)$-adheres at $a_{0.6}$, but $\mathcal{G}$ does not $p(\theta)$-converge to $a_{0.6}$.

Definition 2.7 ([9])**.** A prefilter $\mathcal{F}$ on an fts X is said to

(i) $p(\theta)$-adhere at x_α if for each preopen pre-q-nbd U of x_α and each $F \in \mathcal{F}$, $\text{pcl}UqF$.

(ii) $p(\theta)$-converge to x_α if for each preopen pre-q-nbd U of x_α, $F \leq \text{pcl}U$ for some $F \in \mathcal{F}$.

Definition 2.8 ([3])**.** If $\mathcal{G}$ is a fuzzy grill (or a prefilter) on an fts X, then the section of $\mathcal{G}$, denoted by $\text{Sec}\mathcal{G}$, is given by $\text{Sec}\mathcal{G} = \{A \in I^X : AqG \text{ for each } G \in \mathcal{G}\}$.

Definition 2.9. Let X be an fts. Then for any fuzzy point x_α in an fts X, we adopt the following notations:

(a) $\mathcal{G}(p(\theta), x_\alpha) = \{A \in I^X : x_\alpha \leq p(\theta)\text{-cl}A\ \}$.

(b) $\text{Sec}\mathcal{G}(p(\theta), x_\alpha) = \{A \in I^X : AqG, \text{ for each } G \in \mathcal{G}(p(\theta), x_\alpha)\}$.

Theorem 2.10. *A fuzzy grill $\mathcal{G}$ on an fts X $p(\theta)$-adheres at a fuzzy point x_α in X if and only if $\mathcal{G} \subseteq \mathcal{G}(p(\theta), x_\alpha)$.*

Proof. Let $\mathcal{G}$ be a fuzzy grill on an fts X, which $p(\theta)$-adheres at a fuzzy point x_α. Then for each preopen pre-q-nbd U of x_α and each $G \in \mathcal{G}$, $\text{pcl}UqG$. Thus $x_\alpha \leq p(\theta)$-cl$G$, for each $G \in \mathcal{G} \Rightarrow G \in \mathcal{G}(p(\theta), x_\alpha)$, for each $G \in \mathcal{G} \Rightarrow \mathcal{G} \subseteq \mathcal{G}(p(\theta), x_\alpha)$.
Conversely, let $\mathcal{G} \subseteq \mathcal{G}(p(\theta), x_\alpha)$. Then for all $G \in \mathcal{G}$, $x_\alpha \leq p(\theta)\text{-cl}G \Rightarrow \text{pcl}UqG$ for all propen pre-q-nbd U of x_α and for all $G \in \mathcal{G} \Rightarrow \mathcal{G}$ $p(\theta)$-adheres at x_α. □

Similarly we can prove the following result:

Theorem 2.11. *A prefilter $\mathcal{G}$ on an fts X $p(\theta)$-adheres at a fuzzy point x_α in X if and only if $\mathcal{G} \subseteq \mathcal{G}(p(\theta), x_\alpha)$.*

Theorem 2.12. *A fuzzy grill $\mathcal{G}$ on an fts X is $p(\theta)$-convergent to a fuzzy point x_α in X if and only if $Sec\mathcal{G}(p(\theta), x_\alpha) \subseteq \mathcal{G}$.*

Proof. Let $\mathcal{G}$ be a fuzzy grill on an fts X, which is $p(\theta)$-convergent to a fuzzy point x_α in X. Then for each preopen pre-q-nbd U of x_α, $G \leq \text{pcl}U$ for some $G \in \mathcal{G} \Rightarrow$ $\text{pcl}U \in \mathcal{G}$ for all preopen pre-q-nbd U of x_α. ...(i).
Let $A \in \text{Sec}\mathcal{G}(p(\theta), x_\alpha)$. Then AqG, for each $G \in \mathcal{G}(p(\theta), x_\alpha)$. But $A\overline{q}(1-A)$ so that $1 - A \notin \mathcal{G}(p(\theta), x_\alpha)$. Thus $x_\alpha \not\leq p(\theta)\text{-cl}(1 - A) \Rightarrow$ there exists a preopen pre-q-nbd U of x_α such that $\text{pcl}U\overline{q}(1 - A) \Rightarrow \text{pcl}U \leq A \Rightarrow A \in \mathcal{G}$ (by using (i)).
Conversely, if possible let $\mathcal{G}$ be not $p(\theta)$-convergent to x_α. Then for some preopen pre-q-nbd U of x_α, $\text{pcl}U \notin \mathcal{G}$ and hence $\text{pcl}U \notin \text{Sec}\mathcal{G}(p(\theta), x_\alpha)$. Thus for some $G \in \mathcal{G}(p(\theta), x_\alpha)$, $\text{pcl}U\overline{q}G$ ....(ii). Now $G \in \mathcal{G}(p(\theta), x_\alpha) \Rightarrow x_\alpha \leq p(\theta)\text{-cl}G \Rightarrow \text{pcl}UqG$ contradicting (ii). □

Similarly we can prove the following result:

Theorem 2.13. *A prefilter $\mathcal{G}$ on an fts X is $p(\theta)$-convergent to a fuzzy point x_α in X if and only if $Sec\mathcal{G}(p(\theta), x_\alpha) \subseteq \mathcal{G}$.*

3. Fuzzy P-closedness

In this section, we investigate some characterizations and properties of fuzzy P-closedness of an fts with fuzzy grill as the basic appliance.

Definition 3.1 ([4])**.** A collection of fuzzy sets in an fts X is said to be a fuzzy cover of X if $\bigvee\{U : U \in \mathcal{U}\} = 1_X$. A fuzzy cover $\mathcal{U}$ of X is said to have finite subcover $\mathcal{U}_0$ if $\mathcal{U}_0$ is a finite subcollection of $\mathcal{U}$ and a fuzzy cover of X.

Definition 3.2 ([16])**.** An fts X is said to be fuzzy P-closed if for every fuzzy cover $\mathcal{U}$ of X by fuzzy preopen sets of X, there is a finite subfamily $\mathcal{U}_0$ of $\mathcal{U}$ such that $\bigvee\{\mathrm{pcl}U : U \in \mathcal{U}_0\} = 1_X$.

Theorem 3.3 ([9])**.** *For any fts X the following are equivalent:*
(i) *X is fuzzy P-closed.*
(ii) *Every prefilterbase in X $p(\theta)$-adheres at some fuzzy point in X.*
(iii) *For every family $\{V_\alpha : \alpha \in \Lambda\}$ of fuzzy preclosed sets in X with $\bigwedge\{V_\alpha : \alpha \in \Lambda\} = 0_X$, there is a finite subset Λ_0 of Λ such that $\bigwedge\{pintV_\alpha : \alpha \in \Lambda_0\}= 0_X$.*
(iv) *For every family $\mathcal{F}$ of non-null fuzzy sets in X with finite intersection property, $\bigwedge\{p(\theta)\text{-}clF : F \in \mathcal{F}\} \neq 0_X$.*

Theorem 3.4. *A fuzzy grill $\mathcal{G}$ on a fuzzy P-closed space X $p(\theta)$-adheres in X if for every finite subfamily $\{G_1, G_2, ..., G_n\}$ of $\mathcal{G}$, $pint[\bigwedge_{i=1}^{n} p(\theta)\text{-}clG_i] \neq 0_X$.*

Proof. Consider any fuzzy grill $\mathcal{G}$ with the given property, on a fuzzy P-closed space X. Now by Theorem 1.5, for any $G \in I^X$, $p(\theta)$-cl$G$ is fuzzy preclosed. Thus $\{p(\theta)\text{-cl}G : G \in \mathcal{G}\}$ is a collectoin of fuzzy preclosed sets in X such that $\mathrm{pint}[\bigwedge_{i=1}^{n} p(\theta)\text{-cl}G_i] \neq 0_X$ for any finite subcollection $\{G_1, G_2, ..., G_n\}$ of $\mathcal{G}$. Thus $\bigwedge_{i=1}^{n} \mathrm{pint}(p(\theta)\text{-cl}G_i) \geq \mathrm{pint}[\bigwedge_{i=1}^{n} p(\theta)\text{-cl}G_i] \neq 0_X$ and consequently by Theorem 3.3, we have $\bigwedge\{p(\theta)\text{-cl}G : G \in \mathcal{G}\} \neq 0_X$. Thus there is a fuzzy point x_α in X such that $x_\alpha \leq p(\theta)\text{-cl}G$, for each $G \in \mathcal{G}$. Hence $\mathcal{G} \subseteq \mathcal{G}(p(\theta), x_\alpha)$. So by Theorem 2.10, $\mathcal{G}$ $p(\theta)$-adheres at x_α in X. □

Definition 3.5 ([9])**.** A fuzzy set A in an fts X is called fuzzy preregular open if A=pintpcl$A$ and the complements of fuzzy preregular open sets are called fuzzy preregular closed.

Definition 3.6 ([9])**.** An fts X is called fuzzy pre-almost regular if for each fuzzy point x_α in X and each fuzzy preregular open set U with $x_\alpha q U$, there exists a fuzzy preregular open set V such that $x_\alpha q V$ and pcl$V \leq U$.

Theorem 3.7 ([9])**.** *An fts X is fuzzy pre-almost regular if and only if $p(\theta)$-$cl(p(\theta)$-$clA) = p(\theta)$-clA, for any $A \in I^X$.*

Theorem 3.8. *A fuzzy pre-almost regular space X is fuzzy P-closed if and only if every fuzzy grill $\mathcal{G}$ on X with the property that* $\bigwedge_{i=1}^{n} p(\theta)\text{-}clG_i \neq 0_X$ *for any finite subfamily* $\{G_1, G_2, ..., G_n\}$ *of $\mathcal{G}$, $p(\theta)$-adheres in X.*

Proof. Let X be a fuzzy pre-almost regular, fuzzy P-closed space and $\mathcal{G} = \{G_\alpha : \alpha \in \Lambda\}$ be a fuzzy grill on X with the property that $\bigwedge_{\alpha \in \Lambda_0} p(\theta)\text{-cl}G_\alpha \neq 0_X$ for any finite subfamily Λ_0 of Λ. We consider $\mathcal{F} = \{ \bigwedge_{\alpha \in \Lambda_0} p(\theta)\text{-cl}G_\alpha : \Lambda_0$ is a finite subfamily of $\Lambda\}$. Then $\mathcal{F}$ is a prefilterbase on X. Then by Theorem 3.3, $\mathcal{F}$ $p(\theta)$-adheres at some fuzzy point x_α in X and so that $x_\alpha \leq p(\theta)\text{-cl}(p(\theta)\text{-cl}G) = p(\theta)\text{-cl}G$ (by Theorem 3.7) for all $G \in \mathcal{G} \Rightarrow \mathcal{G} \subseteq \mathcal{G}(p(\theta), x_\alpha)$. Hence by Theorem 2.10, $\mathcal{G}$ $p(\theta)$-adheres at the fuzzy point x_α in X.
Conversely, let $\mathcal{F}$ be any prefilter on an fts X (not necessarily fuzzy pre-almost regular), in which the condition holds. Now $\mathcal{F}$ is contained in some fuzzy ultrafilter $\mathcal{U}$ on X. Then $\mathcal{U}$ is a fuzzy grill on X. Moreover, for any finite subcollection $\{U_1, U_2, ..., U_n\}$ of $\mathcal{U}$, $\bigwedge_{i=1}^{n} p(\theta)\text{-cl}U_i \geq \bigwedge_{i=1}^{n} U_i \neq 0_X$. Then by hypothesis, $\mathcal{U}$ $p(\theta)$-adheres in X and hence $\mathcal{F}$ $p(\theta)$-adheres in X. Thus by Theorem 3.3, X is fuzzy P-closed. □

Theorem 3.9. *Every fuzzy grill $\mathcal{G} = \{G_\alpha : \alpha \in \Lambda\}$ on an fts X with the property that $\bigwedge\{p(\theta)\text{-}clG : \alpha \in \Lambda_0\} \neq 0_X$ for every finite subset Λ_0 of Λ, $p(\theta)$-adheres in X iff for every family $\mathcal{U}$ of non-null fuzzy sets in X for which the family $\{p(\theta)\text{-}clU : U \in \mathcal{U}\}$ has the finite intersection property, we have $\bigwedge\{p(\theta)\text{-}clU : U \in \mathcal{U}\} \neq 0_X$.*

Proof. Let every fuzzy grill with the given property on an fts X, $p(\theta)$-adhere in X. Also, let $\mathcal{U}$ be a family of non-null fuzzy sets in X such that the family $\mathcal{U}^* = \{p(\theta)\text{-cl}U : U \in \mathcal{U}\}$ has the finite intersection property. Let Ω be the collection of all those families $\mathcal{G}$ of non-null fuzzy sets in X for which $\mathcal{G}^* = \{p(\theta)\text{-cl}G : G \in \mathcal{G}\}$ has the finite intersection property and $\mathcal{U} \subseteq \mathcal{G}$. Then $\mathcal{U} \in \Omega$ and Ω is partially ordered by set inclusion relation, in which every chain has an upper bound. Hence by Zorn's Lemma, $\mathcal{U}$ is then contained in a maximal family Ω^* in Ω. Then Ω^* is a fuzzy grill on X satisfying the given property. Thus $\bigwedge\{p(\theta)\text{-cl}U : U \in \mathcal{U}\} \geq \{p(\theta)\text{-cl}U : U \in \Omega^*\} \neq 0_X$.
Conversely, let $\mathcal{G}$ be any fuzzy grill on an fts X with the given condition. Then by hypothesis for every subfamily $\mathcal{G}_0$ of $\mathcal{G}$, $\bigwedge\{p(\theta)\text{-cl}G : G \in \mathcal{G}_0\} \neq 0_X$. Hence, by hypothesis $\bigwedge\{p(\theta)\text{-cl}G : G \in \mathcal{G}\} \neq 0_X$ and so that $\mathcal{G}$ $p(\theta)$-adheres in X. □

Corollary 3.10. *In a fuzzy pre-almost regular space, the following are equivalent:*

(a) *X is fuzzy P-closed.*

(b) *Every fuzzy grill on X with the property that* $\bigwedge_{i=1}^{n} p(\theta)\text{-}clG_i \neq 0_X$ *for every finite subfamily* $\{G_1, G_2, ..., G_n\}$ *of $\mathcal{G}$, $p(\theta)$-adheres at some fuzzy point in X.*

(c) *For every family $\mathcal{U}$ of non-null fuzzy sets in X for which the family $\{p(\theta)\text{-}clU : U \in \mathcal{U}\}$ has the finite intersection property, we have $\bigwedge\{p(\theta)\text{-}clU : U \in \mathcal{U}\} \neq 0_X$.*

Theorem 3.11. *For a fuzzy pre-almost regular space X, fuzzy P-closedness of X implies that every fuzzy grill $\mathcal{G}$ on X with the property that $p(\theta)$-$clAq$ $p(\theta)$-clB for any two members A, B of $\mathcal{G}$, $p(\theta)$-adheres in X.*

Proof. Let X be a fuzzy pre-almost regular, P-closed space and $\mathcal{G}$ be a fuzzy grill with the given property that does not $p(\theta)$-adhere at any fuzzy point of X. Now for each $x \in X$ and each $n \in \mathbb{N}$ ($\mathbb{N}$ = the set of all natural numbers), consider the fuzzy point $x_{1/n}$. Then there exists a $G_x^n \in \mathcal{G}$ such that $x_{1/n} \not\leq p(\theta)\text{-cl}G_x^n = p(\theta)\text{-cl}(p(\theta)\text{-cl}G_x^n)$ (by Theorem 3.8). So there exists a preopen pre-q-nbd U_x^n of $x_{1/n}$ such that $\text{pcl}U_x^n \overline{q} p(\theta)\text{-cl}G_x^n$ i.e., $p(\theta)\text{-cl}U_x^n \overline{q} p(\theta)\text{-cl}G_x^n$ (by Theorem 1.9) and hence $p(\theta)\text{-cl}(p(\theta)\text{-cl}U_x^n) \overline{q} p(\theta)\text{-cl}G_x^n$. Now $p(\theta)\text{-cl}G_x^n \in \mathcal{G}$ and so by the given condition $p(\theta)\text{-cl}U_x^n = \text{pcl}U_x^n \notin \mathcal{G}$. Now $\{U_x^n : x \in X, n \in \mathbb{N}\}$ is a fuzzy cover cover of X by fuzzy preopen sets of X, so by fuzzy P-closedness of X, there exist $x_1, x_2, \ldots, x_k \in X$ and $n_1, n_2, \ldots, n_k \in \mathbb{N}$ such that $\bigvee\{\text{pcl}U_{x_i}^{n_i} : i = 1, 2, ..., k\} = 1_X \in \mathcal{G} \Rightarrow \text{pcl}U_{x_i}^{n_i} \in \mathcal{G}$ for some i $(1 \leq i \leq k)$, a contradiction. □

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