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Softness of a soft set: Soft set entropy

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Abstract. Soft set is a new tool for modeling uncertainties that arises due to the parameterized classification of elements of a universe. Here the notion of softness of a soft set has been introduced and entropy has been used as a measure for this softness. In this paper we have proposed several measures of entropy for soft sets. Also the notion of soft entropy of a fuzzy set are discussed.

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1. Introduction

Uncertainty is a common phenomenon of our daily existence because our world is full of uncertainties. In our daily life we encounter many situations where we do take account of these uncertainties. Therefore it is natural for man to understand and try to model this uncertainty prevailing in physical world. From centuries, in almost all branches of Science or in Philosophy, attempts have been made to understand and represent the features of uncertainty. Perhaps that is the main reason behind the development of Probability theory and Stochastic techniques which started in early eighteenth century. Till mid-twentieth century, Probability theory was the only tool for handling certain type of uncertainty called "Randomness". But there are several other kinds of uncertainties; one such type is called "vagueness" or "imprecision" which is inherent in our natural languages. In 1965, L. A. Zadeh [13] coined his remarkable theory of Fuzzy sets that deals with such types of uncertainties and which is due to partial membership of an element in a set. Later this "Fuzziness" concept leads to the highly acclaimed theory of Fuzzy Logic. This theory has been applied with a good deal of success to engineering, with fuzzy control systems able to do things like cook rice, wash cloths or shift gears in a car with great efficiency.

After the invention of fuzzy sets many other hybrid concepts began to develop. In 1983, K. Atanasov [2] introduced the idea of Intuitionistic fuzzy sets, a set with each member having a degree of belongingness as well as a degree of non-belongingness. This is again a generalization of fuzzy set theory. Although Fuzzy set theory is very successful in handling uncertainties arising from vagueness or partial belongingness of an element in a set, it cannot model all sorts of uncertainties prevailing in different real physical problems. Thus search for new theories has been continued. In 1982, Z. Pawlak [10] came up with his Rough set theory. A Rough set is a set with imprecise boundaries. It is basically an approximate representation of a given crisp set in terms of two subsets of a crisp partition defined on the universal set involved. Thus it is a computational model of approximate reasoning. Except Rough sets there are also several other theories like multisets, vague sets, interval analysis etc. which have their own domain of applications in the field of uncertainty analysis. But there is one theory which is relatively new and having a lot of potential for being a major tool for modeling uncertainty, is the theory of Soft sets. Soft set was introduced by Molodtsov [9] in 1999 and is considered as the only theory having a "parameterization tool". Later Maji et al. [6, 7] defined operations on then and applied soft sets in decision making. A soft set is a classification of elements of the universe with respect to a given set of parameters. It has been shown that soft set is more general in nature and has more capabilities in handling uncertain information. Also a fuzzy set or a rough set can be considered as a special case of soft sets. Now different types of sets mentioned above, expresses different types of uncertainties and hence it is natural to measure the amount of uncertainty, whether randomness or fuzziness etc that is attached with it. But what type of uncertainty does a soft set represent? Soft sets actually deals with the uncertainty arising from the parameterized classification of elements of a universe. To illustrate the notion of soft set and the uncertainty associated with it, we here give an example:

Consider a collection $U = \{h_1, h_2, ..., h_5, h_6\}$ of six houses that is sorted from others by a buyer willing to buy a house. Some parameters of a good house are selected and expressed as the parameter set$E = \{e_1 = \mathit{cheap}, e_2 = \mathit{well\,constructed}, e_3 = \cos tly, e_4 = \mathit{in\,good\ neighbourhood}, e_5 = \mathit{good\ location}\}$. A soft set is a mapping $F : E \to P(U)$ which classify the elements of U according to the parameters given in E.For example here let us define the mapping $F : E \to P(U)$ as follows: Let $F(e_1) = \{h_1, h_4\}, F(e_2) = \{h_2, h_3, h_5\}, F(e_3) = \{h_2, h_3\}, F(e_4) = \{h_1, h_2, h_3\}, F(e_5) = \{h_4\}$.

Here apparently the situation may seem to be quite deterministic. But if we observe deeply we see the following facts: (i) This classification is dependent on E, (ii) Associated to each element of U, there are either no parameter, exactly one parameter or more than one parameter that is attached, e.g. consider the elements$h_6, h_5 \& h_2$. (iii) From a buyers prospective there is always an uncertainty to select one particular house, especially those one which have several positive and negative parameters attached to it, e.g. h_3. (iv) Buyer may also be worried about other unknown parameters which may later affect him. (v) The situations where one house does not have any given parameters are very much uncertain, e.g. h_6.

These are some of the points for which a soft set represents certain kind of uncertainty. Therefore it is significant to study the amount of uncertainty or 'softness' that is attached with a soft set. This paper aims to answer this question.

The rest of the paper is constructed as follows: In Section 2, we state some preliminary concepts, definitions and results that will be used in the rest of this paper. In section 3, measure of softness of a soft set has been discussed. An application of this softness measure in calculating the soft entropy of a fuzzy set has been discussed in section 4. Section 5 concludes the paper.

2. Preliminaries

In this section a few definitions and properties regarding soft sets are given.

Definition 2.1 ([7]). Let U be an initial universal set and Ebe a set of parameters. Let $P(U)$denote the power set of U. A pair(F, A) is called a soft set over U iff $F$is a mapping given by$F : A \to P(U)$, *where* $A \subset E$.

Example 2.2. As an illustration, consider the following example.

Suppose a soft set (F, A)describes attractiveness of the shirts which the authors are going to wear.

U = the set of all shirts under consideration = $\{x_1, x_2, x_3, x_4, x_5\}$

A = {colorful, bright, cheap, warm} =$\{e_1, e_2, e_3, e_4\}$.

Let $F(e_1) = \{x_1, x_2\}$,$F(e_2) = \{x_1, x_2, x_3\}$,$F(e_3) = \{x_4\}$,$F(e_4) = \{x_2, x_5\}$.

So, the soft set (F, A) is a subfamily $\{F(e_i), i = 1, 2, 3, 4\}ofP(U)$.

Here$F(e_i)$is called an e-approximation of(F, A).

Definition 2.3 ([7]). For two soft sets $(F, A)$and(G, B)over a common universe U,we say that $(F, A)$is a soft subset of (G, B)if $(i) A \subset B$,$(ii) \forall \varepsilon \in A, F(\varepsilon)$is a subset of$G(\varepsilon)$.

Definition 2.4 ([7]). (Equality of two soft sets) Two soft sets $(F, A)$and (G, B)over a common universe $U$are said to be soft equal if $(F, A)$is a soft subset of $(G, B)$and $(G, B)$is a soft subset of (F, A).

Definition 2.5 ([8]). The complement of a soft set $(F, A)$is denoted by $(F, A)^C$ and is defined by $(F, A)^C = (F^C, A)$,where $F^C : A \to P(U)$is a mapping given by $F^C(\alpha) = U - F(\alpha), \forall \alpha \in A$.

Definition 2.6 ([7]). (Null soft set) A soft set (F, A)over $U$is said to be null soft set denoted by$\tilde{\Phi}$, if $\forall \varepsilon \in A, F(\varepsilon)$ =null set ϕ .

Definition 2.7 ([7]). (Absolute soft set) A soft set (F, A)over $U$is said to be absolute soft set denoted by$\tilde{A}$, if $\forall \varepsilon \in A, F(\varepsilon) = U$.

Definition 2.8 ([7]). Union of two soft sets $(F, A)$and (G, B)over a common universe $U$is the soft set(H, C), where $C = A \bigcup B$,and $\forall e \in C$,

$$H(e) = F(e), e \in A - B, = G(e), e \in B - A, = F(e) \bigcup G(e), e \in A \bigcap B.$$

This is denoted by$(F, A)\tilde{\bigcup}(G, B)$.

Definition 2.9 ([7]). Intersection of two soft sets $(F, A)$and (G, B)over a common universe $U$is the soft set(H, C), where $C = A \bigcap B$,and $\forall e \in C$,$H(e) = F(e) \bigcap G(e)$. This is denoted by $(F, A)\tilde{\bigcap}(G, B)$.

Definition 2.10 ([8])**.** For two soft sets (F, A) and (G, A) the mean Hamming distance $d(F, G)$, between two soft sets is defined as:

$d(F,G) = \frac{1}{m}\{\sum_{i=1}^{m}\sum_{j=1}^{n}|F(e_i)(x_j) - G(e_i)(x_j)|\}$, where $F(e_i)(x_k) = 1,\ if\ x_k \in F(e_i) \& F(e_i)(x_k) = 0, if\ x_k \notin F(e_i)$.

3. Softness measure of a soft set:

In this section we introduce the notion of soft set entropy to measure the 'softness' of a soft set. Throughout this paper we assume the universe$U$ and parameter set$E$ to be finite and denote$|A|$ as the cardinality of the set$A$.

Entropy as a measure of fuzziness was first mentioned by Zadeh [14] in 1965. Later De Luca-Termini [3] axiomatized the non-probabilistic entropy. According to them the entropy E of a fuzzy set A should satisfy the following axioms:

(DT1) $E(A) = 0$ iff $A \in 2^X$

(DT2) $E(A) = 1$ iff $\mu_A(x) = 0.5, \forall x \in X$

(DT3) $E(A) \leq E(B)$ iff A is less fuzzy than B, i.e. if $\mu_A(x) \leq \mu_B(x) \leq 0.5 \forall x \in X$ or if $\mu_A(x) \geq \mu_B(x) \geq 0.5, \forall x \in X$.

(DT4) $E(A^c) = E(A)$.

Several other authors have investigated the notion of entropy. Kaufmann [4] proposed a distance based measure of soft entropy, Yager [12] gave another view of degree of fuzziness of any fuzzy set in terms of lack of distinction between the fuzzy set and its complement. Kosko [5] investigated the fuzzy entropy in relation to a measure of subsethood. Szmidt & Kacprzyk [11] studied the entropy of intuitionistic fuzzy sets etc. Following these concepts here we propose a set of axioms that should be satisfied by any measure of softness i.e. soft set entropy. For that purpose we give two new definitions, namely deterministic soft set and equivalent soft sets.

Definition 3.1. A soft set(F, A) is said to be a deterministic soft set over$U$if the following holds:

(i) $\bigcup_{e\in A} F(e) = U$

(ii) $F(e) \cap F(f) = \varphi$, where $e, f \in A$.

Definition 3.2. Let(F, A) be any soft set. Then another soft set $(F*, A)$ is said to be equivalent to(F, A)if there exists a bijective mapping $\sigma$from$A$to$A$defined as: $\sigma(F_x) = F^*_x$,where$F_x = \{e : x \in F(e)\} \& F^*_x = \{e' : x \in F^*(e)\}$.

Let $C(F)$denote the collection of all soft sets which are equivalent with(F, A).

Example 3.3. Let the universe and parameter set be $U = \{x_1, x_2, x_3, x_4\} \& A = \{e_1, e_2, e_3\}$. Then let us consider the following soft set(F, A)as follows:

$$F(e_1) = \{x_1, x_2\}, F(e_2) = \{x_2, x_3\}, F(e_3) = \{x_1, x_4\}.$$

Then the following soft set$(G, A)$is equivalent to the soft set$(F, A)$where

$$G(e_1) = \{x_1, x_4\}, G(e_2) = \{x_1, x_2\}, G(e_3) = \{x_2, x_3\}.$$

This is because there is a bijective mappingσ on A such that $\sigma(e_1) = e_2,\ \sigma(e_2) = e_3\ \&\ \sigma(e_3) = e_1$ and $\sigma(F_x) = G_x\ \forall x \in U$.

From intuition we understand that the 'softness', i.e. the associated uncertainty of a soft set is maximum if either the elements cannot be classified at all w.r.t. the parameters or every element of the universe belong to every parameter, i.e. to every e-approximation. Again the softness of a soft set is minimum if each element of the universe is associated with only one parameter. Now for a superset of a soft set the uncertainty ultimately increases in comparison with its subset, as new elements being introduced in the set(i.e. in e-approximations) which shares same parameters with other elements. The softness of a soft set and its equivalent soft sets are same because the amount of imperfectness or ambiguity of information is same in both cases. Based on the above discussion we have the following:

Definition 3.4. Let$\chi^*(U)$ be the collection of all soft sets over$U$. A mapping$S : \chi^*(U) \to [0,1]$ is said to be soft set entropy or softness measure if$S$satisfies the following properties:

(S1) $S(\tilde{\Phi}) = 1, S(\tilde{A}) = 1$
(S2) $S(F) = 0$, if F is det er min is tic soft set
(S3) $S(F) \leq S(G)$ if $F(\neq \tilde{\Phi}) \subseteq G$
(S4) $S(F^*) = S(F)$, where $F^* \in C(F)$

Remark 3.5. Note that according to this definition an ordinary set has softness zero, as an ordinary set can be thought of as a soft set with a single parameter and thus is a deterministic soft set.

Then we have the following theorem:

Theorem 3.6. *The function*$S : \chi^*(U) \to [0,1]$ *defined below is an entropy (or measure of softness) of a soft set:*

$$S(F) = 1 - \frac{|U|}{\sum_{x\in U}|\{e : x \in F(e)\}|},\ if\ F \neq \tilde{\Phi}\ or\ \tilde{A}$$
$$= 1,\ if\ F = \tilde{\Phi}\ or\ \tilde{A}$$

Proof. Here (S1) holds obviously from construction.

(S2) For a deterministic soft set$\sum_{x\in U} |\{e : x \in F(e)| = |U|$ because each element is attached with exactly one parameter$\Rightarrow |\{e : x \in F(e)| = 1\ \forall x \in U$.
Hence $S(F) = 0$.
Thus (S2) holds.

Next let$F \& G$ be two soft sets such that

$$F(\neq \tilde{\Phi}) \subseteq G \Rightarrow \forall e \in E, F(e) \subseteq G(e) \Rightarrow \{e : x \in F(e)\} \subseteq \{e : x \in G(e)\}$$
$$\therefore \sum_{x\in U} |\{e : x \in F(e)| \leq \sum_{x\in U} |\{e : x \in G(e)|$$
$$\Rightarrow \frac{|U|}{\sum_{x\in U} |\{e : x \in F(e)|} \geq \frac{|U|}{\sum_{x\in U} |\{e : x \in G(e)|}$$
$$\Rightarrow S(F) \leq S(G).$$

Thus (S3) also holds.

(S4) Let$F^* \in C(F)$
Then $\therefore \sum_{x\in U} |\{e : x \in F(e)| = \sum_{x\in U} |\{e : x \in F^*(e)|$

$$\therefore S(F) = S(G).$$

Hence the theorem. □

Example 3.7. Consider the example of house selection given in Section 1.

Consider a collection $U = \{h_1, h_2, ..., h_5, h_6\}$ of six houses that is sorted among others by a buyer willing to buy one house. Some parameters of a good house are collected and expressed as$E = \{e_1 = cheap, e_2 = well\, constructed, e_3 = \cos tly, e_4 = in\, good\ neighbourhood, e_5 = good\ location\}$. A soft set is a mapping $F : E \to P(U)$ which classify the elements of U according to the parameters given in E.Here we define the soft set $F : E \to P(U)$ as follows: Let $F(e_1) = \{h_1, h_4\}, F(e_2) = \{h_2, h_3, h_5\}, F(e_3) = \{h_2, h_3\}, F(e_4) = \{h_1, h_2, h_3\}, F(e_5) = \{h_4\}$. Here$S(F) = 1 - \frac{|U|}{\sum_{x \in U} |\{e : x \in F(e)\}|} = 1 - \frac{6}{11} \approx 0.45$.

The notion of mean Hamming distances between two soft sets was proposed by us in [8] in connection with the similarity of soft sets.

Again we can calculate entropy using the notion of distance between two soft sets. For this we give the following definitions:

Let(F, A) be a soft set over$U$. Then$F : A \to P(U)$& *let us denote* $R_T = Range\ of\ F$.

Definition 3.8. Let(F, A) be any soft set over the universe$U$. Then the nearest soft set of(F, A) is a soft set (F_{near}, A)which is a deterministic over soft set overR_T and is obtained from(F, A) by eliminating the least number of elements.

Definition 3.9. Let(F, A) be any soft set over the universe$U$. Then the farthest soft set of(F, A) is a soft set (F_{far}, A)which is obtained from(F, A) by including additional elements in (F, A)such that$F_{far}(e) = R_F\ \forall e \in A$.

Now let$d_1\ and\ d_2$ be the distances between$F\ and\ F_{near}$ and$F\ and\ F_{far}$ respectively.

Definition 3.10. A non null non absolute soft set (F, A) is said to be a soft set of TYPE- I, if$d_1 \le d_2$.

Definition 3.11. A non null non absolute soft set (F, A) is said to be a soft set of TYPE- II, if$d_1 > d_2$.

Example 3.12. Consider the following soft set (F, A)defined as follows:

Let $U = \{x_1, x_2, x_3, x_4, x_5\}$& $E = \{e_1, e_2, e_3\}$ be the universal set and the set of parameters. Let

$$F(e_1) = \{x_1, x_2, x_5\}, F(e_2) = \{x_2, x_3\}, F(e_3) = \{x_1, x_4, x_5\}.$$

Here $d_1 = \frac{3}{3}$ & $d_2 = \frac{7}{3}$.Hence(F, A) is TYPE-I soft set.

If we take another soft set(G, A) over same$U\ and\ E$ such that:

$$G(e_1) = \{x_1, x_2, x_3, x_4\}, G(e_2) = \{x_2, x_3, x_4, x_5\}, G(e_3) = \{x_1, x_4, x_5\}.$$

Then it is of TYPE-II because here $d_1 = \frac{6}{3} > d_2 = \frac{4}{3}$.

The distance based softness measure or soft entropy of(F, A) is defined as the ratio $E(F) = \left\{ \begin{array}{ll} \frac{d_1}{d_2}, & \text{if } d_1 \le d_2 \\ \frac{d_2}{d_1} & \text{if } d_2 \le d_1 \end{array} \right\}$ if $F \ne \Phi\,,\ \tilde{A}$ and $= 1$ if $F = \Phi\,,\ \tilde{A}$.

Theorem 3.13. *The softness measures defined in Definition 3.12 satisfy all the properties of Definition 3.4 for TYPE-I soft sets.*

Proof. Here (S1) is obvious.

Now if a soft set$F$ is deterministic soft set then$d_1 = d(F, F_{near}) = 0, \therefore E(F) = 0$ and hence (S2) holds.

Let $F(\neq \Phi)$ *and* Gbe two TYPE-I soft sets such that$F \subseteq G,\ i.e.\ F(e) \subseteq G(e) \forall e \in A$.

Now to findF_{near} from$F$ we have to delete some common elements from $F(e)'s$. Let the total number of such deletions be n_F. Now these elements are also common in$G$ as $F \subseteq G$. So to findG_{near} one have to delete at least n_F number of elements, $i.e.\ n_G \geq n_F \Rightarrow d_1^F \leq d_1^G$. Similarly number of additions required to findF_{far} will be more compared to $G_{far},\ i.e.\ d_2^F \geq d_2^G$.

Hence$E(F) = \frac{d_1^F}{d_2^F} \leq \frac{d_1^G}{d_2^G} = E(G)$.

Hence (S3) holds.

(S4) follows obviously for all soft sets $F \& F^* \in C(F)$from definition of equivalent soft sets.

This completes the proof. □

Example 3.14. Consider an example where the universe is$X = \{x_1, x_2, x_3, x_4\}$and the parameter set is

$A = \{e_1, e_2, e_3\}$.Let (F, A)be a soft set defined as follows:

$$F(e_1) = \{x_1, x_3\}, F(e_2) = \{x_2, x_3, x_4\}, F(e_3) = \{x_1, x_4\}.$$

Then $F_{near} = (G, A)$ is defined as follows: $F_{near}(e_1) = \{x_1\}, F_{near}(e_2) = \{x_2, x_3\}, F_{near}(e_3) = \{x_4\}$.

Again$F_{far} = (H, A)$ is defined as follows: $F_{far} = \tilde{A}$.

Now $d_1 = d(F, F_{near}) = 1,\ d_2 = d(F, F_{far}) = 5/3, \Rightarrow S(F) = \frac{d_1}{d_2} = 3/5$.

4. Soft entropy of a fuzzy set:

In this section we will show that a softness measure set of a soft can also be used as a measure the fuzziness of a fuzzy set.

In [1], Aktas & Cagman have made a comparative study between a soft set and a fuzzy set. According to them:

Proposition 4.1 ([1])**.** *Every fuzzy set may be considered as a soft set.*

They have shown the following example to explain the situation.

Example 4.2 ([1])**.** Let us consider the following fuzzy set:

$$F_{poor} = \{\frac{h_1}{0.9}, \frac{h_2}{0.3}, \frac{h_3}{1.0}, \frac{h_4}{1.0}, \frac{h_5}{0.2}\}$$

Let $X = \{h_1, h_2, h_3, h_4, h_5\}$ be a universe and the parameter set be $A = \{0.2, 0.3, 0.9, 1.0\} = \{\mu_{F_{poor}}(x) : x \in X\}$.

Then the fuzzy set F_{poor} can be represented by the soft set (F_S, A), where $F_S : A \to P(X)$ is defined as $F_S(\alpha) = \{x \in X : \mu_{F_{poor}}(x) \geq \alpha\}$.

$$\therefore F_S(0.2) = \{h_1, h_2, h_3, h_4, h_5\}, F_S(0.3) = \{h_1, h_2, h_3, h_4\},$$

$$F_S(0.9) = \{h_1, h_3, h_4\}, F_S(1.0) = \{h_3, h_4\}.$$

Definition 4.3. Let X be a universe and $I = [0, 1]$. A soft set $F : A \to P(X)$, with $A \subseteq I$ is called a softly fuzzy set. The collection of all such set are denoted by $SF(X)$.

The soft set (F_s, A) defined above is a softly fuzzy set.

Here we define two types of softly fuzzy sets corresponding to any fuzzy set.

Definition 4.4. Given a fuzzy set F, its corresponding upper (lower) soft set is a softly fuzzy set (F_s^u, A) (*or* (F_s^l, A)) defined as follows: $F_S^u(or\ F_S^l) : A \to P(X)$ such that

$$F_S^u(\alpha) = \{x \in X : \mu_F(x) > \alpha\}, where\, A = \{\alpha \in [0, 1] : \mu_F(x) = \alpha,\, x \in X\}.$$

$$(\text{or } F_S^l(\alpha) = \{x \in X : \mu_F(x) < \alpha\}, where\, A = \{\alpha \in [0, 1] : \mu_F(x) = \alpha,\, x \in X\}).$$

Definition 4.5. Given a fuzzy set F, its upper soft entropy is the soft entropy of the softly fuzzy set (F_s^u, A)& is denoted by $E_u(F)$.

Definition 4.6. Given a fuzzy set F, its lower soft entropy is the soft entropy of the softly fuzzy set (F_s^l, A)& is denoted by $E_l(F)$.

The upper soft entropy and lower soft entropy of a fuzzy set satisfies the following properties:

Theorem 4.7. *For a fuzzy set F the following holds:*
(A1) $E'(F) = 0$ *if* $F \in 2^X$.
(A2) $E'(F) = 1$ *if* $\mu_F(x) = 0.5 \forall x \in X$.
(A3) $E'(F) \leq E'(G)$ *if* $\mu_F(x) \leq \mu_G(x) \leq 0.5$.
(A4) $E_u(F) = E_l(F^c)$
Here $E' = E_u$ *or* E_l.

Proof. Proof for the first three results we prove for upper soft sets only. The case for lower soft set will be similar.

Let $F \in 2^X$, then its corresponding upper soft set (softly fuzzy set) (F_s^u, A)will be as follows:

$A = \{0, 1\}$ and $F_s^u : A \to P(X)$ such that $F_s^u(0) = X_1 \subseteq X \& F_s^u(1) = \varphi$.

Thus the softness measure S of (F_s^u, A) will be zero as(F_s^u, A) is also a deterministic soft set, i.e. $E_u(F) = S(F_s^u) = 0$.

Hence(A1) is satisfied.

Next let$F \in I^X s.t.\ \mu_F(x) = 0.5\ \forall x \in X$.

Then we take the parameter set as $A = \{0.5\}$ and $F_s^u : A \to P(X)$ such that $F_s^u(0.5) = \varphi$.

Therefore $(F_s^u, A) = \Phi$ and(F_s^u, A) becomes an null soft set and $E_u(F) = S(F_s^u) = 1$.

Hence (A2) is satisfied.

Now if$F, G \in I^X s.t.\ \mu_F(x) \leq \mu_G(x) \forall x \in X$.

Then let their corresponding soft sets be $(F_S^{u1}, A)$and(F_S^{u2}, A')

Where obviously$A'' = A \cup A'$. Now we change the parameter sets of the above two softly fuzzy sets to $A''$and define$F_S^{u1}(\beta) = \varphi \,\forall \beta \notin A \& F_S^{u2}(\beta) = \varphi\, \forall \beta \notin A'$.

Then

$$F_S^{u1}(\alpha) = \{x \in X : \mu_F(x) > \alpha\} \subseteq \{x \in X : \mu_G(x) > \alpha\} = F_S^{u2}(\alpha)\ \forall \alpha \in A''.$$

Because this will not change the soft entropy.

Then$S(F_S^{u1}) \leq S(F_S^{u2})$.

Therefore (A3) is also satisfied.

(A4): Let$F \in I^X$. Let the corresponding upper soft set be (F_s^u, A).

Then$A = \{\mu_F(x); x \in X\}$and

$$F_s^u(\alpha) = \{x : \mu_F(x) > \alpha\} = \{x : 1 - \mu_F(x) < 1 - \alpha\} = \{x : \mu_{F^c}(x) < 1 - \alpha\}$$
$$= (F_s^l)^c(1 - \alpha).$$

Now for $F^c \in I^X$, the corresponding lower soft set will be $((F_s^l)^c, A')$,

where$A' = \{\alpha : 1 - \alpha = \mu_F(x) \& x \in X\}$.

Thus $(F_s^u, A) = ((F_s^l)^c, A') \Rightarrow E_u(F) = E_l(F)$ and hence (A4) follows.

This completes the proof. □

Remark 4.8. Then upper (lower) soft entropy of a fuzzy set can be found from a measure of softness of a softly fuzzy set, i.e. soft set.

Remark 4.9. The softness measure defined in theorem 3.6 is also a measure of upper (lower) soft entropy of fuzzy sets.

Example 4.10. Let$F \in I^X$ be a fuzzy set over the universe$X = \{x_1, x_2, x_3, x_4, x_5\}$ where

$$F = \{\frac{x_1}{0.3}, \frac{x_2}{0.7}, \frac{x_3}{0.8}, \frac{x_4}{0.5}, \frac{x_5}{0.9}\}.$$

Let(F_s^u, A) be a the corresponding soft set, where $A = \{0.3, 0.5, 0.7, 0.8, 0.9\} \& F_s^u : A \to P(X)\, s.t.$

$F_s^u(0.3) = \{x_2, x_3, x_4, x_5\}$, $F_s^u(0.5) = \{x_2, x_3, x_5\}$, $F_s^u(0.7) = \{x_3, x_5\}$,

$F_s^u(0.8) = \{x_5\}$, $F_s^u(0.9) = \varphi$.

Then the softness measure = the lower soft entropy of the fuzzy set $F = S(F_s^u) = 1 - \frac{5}{10} = 0.5$.

5. Conclusions:

In this paper we have extended the De Luca and Termini's axioms to measure the uncertainty attached to soft sets and proposed two techniques to measures of softness of soft sets or soft set entropy for the first time. These measures also satisfy the softness measure axioms defined by us. We have also shown that this measure is consistent with similar considerations for fuzzy sets. And we have used soft entropy to measure the fuzziness of a fuzzy set. Measure of softness of a soft set is actually a measure of imperfectness of information represented by a soft set. Thus the study of soft entropy is important and the authors are sure that this notion will be useful in many domains of information theory and its applications.

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