Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 6, No. 1, (July 2013), pp. 33–45
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr

@FMI

Ordered (r,s) intuitionistic fuzzy quasi uniform regular G_δ extremally disconnected spaces

G. K. Revathi, E. Roja, M. K. Uma

Received 22 July 2012; Revised 8 November 2012; Accepted 19 November 2012

Abstract. In this paper,a new class of intuitionistic fuzzy quasi uniform topological space called ordered intuitionistic fuzzy quasi uniform topological space is introduced.Tietze extention theorem for ordered (r,s) intuitionistic fuzzy quasi uniform regular G_δ extremally disconnected space has been discussed besides providing several other propositions. .

2010 AMS Classification: 54A40, 03E72

Keywords: Ordered (r,s) intuitionistic fuzzy quasi uniform regular G_δ extremally disconnected space, ordered (r,s) intuitionistic fuzzy quasi uniform regular G_δ continuous mapping, lower (r,s) intuitionistic fuzzy quasi uniform regular G_δ continuous mapping and upper (r,s) intuitionistic fuzzy quasi uniform regular G_δ continuous mapping.

Corresponding Author: G. K. Revathi (gk_revathi@yahoo.co.in)

1. Introduction

The concept of fuzzy set was introduced by Zadeh [13].Since then the concept has invaded nearly all branches of Mathematics. Chang [2] introduced and developed the theory of fuzzy topological spaces and since then various notions in classical topology have been extended to fuzzy topological spaces. Fuzzy sets have applications in many fields such as information [11] and control [10]. Atanassov [1] generalised fuzzy sets to intuitionistic fuzzy sets. Coker [3, 4] introduced the notions of an intuitionistic fuzzy topological space, intuitionistic fuzzy continuous mapping, intuitionistic fuzzy compact space, intuitionistic fuzzy quasi coincident and some related concepts. Tomasz Kubiak [6, 7] studied L-Fuzzy normal spaces and Tietze extension Theorem and extending continuous L-Real mappings.M.K.Uma ,E.Roja and G.Balasubramanian [12] discussed Tietze extention theorem for ordered fuzzy pre-extremally disconnected spaces. G.K.Revathi, E.Roja and M.K.Uma [9] have introduced and studied (r,s) intuitionistic fuzzy quasi uniform regular G_δ set and (r,s) intuitionistic fuzzy quasi uniform regular G_δ compactness.In this paper,a new class of intuitionistic fuzzy quasi uniform topological spaces called ordered intuitionistic fuzzy quasi uniform topological spaces is introduced.Tietze extention theorem for ordered (r,s) intuitionistic fuzzy quasi uniform regular G_δ extremally disconnected spaces has been discussed besides providing several other propositions.

2. Preliminaries

Definition 2.1 ([1])**.** Let X be a non empty fixed set and I the closed interval [0,1].An intuitionistic fuzzy set (IFS) A is an object of the following form $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$ where the mapping $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non membership (namely $\gamma_A(x)$) for each element $x \in X$ to the set A respectively and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each x ∈ X.Obviously, every fuzzy set A on a nonempty set X is an IFS of the following form $A = \{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle : x \in X\}$. For the sake of simplicity, we shall use the symbol $A = \langle x, \mu_A(x), \gamma_A(x)\rangle$ for the intuitionistic fuzzy set $A = \{\langle x, \mu_A(x), \gamma_A(x) : x \in X\rangle\}$.
For a given non empty set X,denote the family of all intuitionistic fuzzy sets in X by the symbol ζ^X

Definition 2.2 ([1])**.** Let A and B be IFS's of the form $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$ and $B = \{\langle x, \mu_B(x), \gamma_B(x) :\rangle x \in X\}$. Then

(i) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$
(ii) $\overline{A} = \{\langle x, \gamma_A(x), \mu_A(x)\rangle : x \in X\}$
(iii) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \gamma_A(x) \vee \gamma_B(x)\rangle : x \in X\}$
(iv) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \gamma_A(x) \wedge \gamma_B(x)\rangle : x \in X\}$

Definition 2.3 ([1])**.** The IFSs $0_\sim$ and $1_\sim$ are defined by $0_\sim$=$\{\langle x, 0, 1\rangle : x \in X\}$ and $1_\sim$=$\{\langle x, 1, 0\rangle : x \in X\}$.

Definition 2.4 ([4])**.** An intuitionistic fuzzy topology (IFT) in Coker's sense on a non empty set X is a family τ of IFSs in X satisfying the following axioms.

(T_1) $0_\sim$,$1_\sim$ $\in \tau$
(T_2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$
(T_3) $\cup G_i \in \tau$ for arbitrary family $\{G_i / i \in I\} \subseteq \tau$

In this paper by (X , τ) or simply by X we will denote the Coker's intuitionistic fuzzy topological space (IFTS). Each IFSs in τ is called an intuitionistic fuzzy open set (IFOS) in X.The complement $\overline{A}$ of an IFOS A in X is called an intuitionistic fuzzy closed set (IFCS) in X.

Definition 2.5 ([8])**.** Let a and b be two real numbers in [0,1] satisfying the inequality $a + b \leq 1$.Then the pair $\langle a, b\rangle$ is called an intuitionistic fuzzy pair.Let $\langle a_1, b_1\rangle$,$\langle a_2, b_2\rangle$ be any two intuitionistic fuzzy pairs.Then define

(1) $\langle a_1, b_1\rangle \leq \langle a_2, b_2\rangle$ if and only if $a_1 \leq a_2$ and $b_1 \geq b_2$.
(2) $\langle a_1, b_1\rangle = \langle a_2, b_2\rangle$ if and only if $a_1 = a_2$ and $b_1 = b_2$.
(3) If $\{\langle a_i, b_i\rangle / i \in J\}$ is a family of intuitionistic fuzzy pairs,then $\vee\langle a_i, b_i\rangle = \langle \vee a_i, \wedge b_i\rangle$ and $\wedge\langle a_i, b_i\rangle = \langle \wedge a_i, \vee b_i\rangle$.
(4) The complement of an intuitionistic fuzzy pair $\langle a, b\rangle$ is the intuitionistic fuzzy pair defined by $\overline{\langle a, b\rangle} = \langle b, a\rangle$

(5) $1^{\sim} = \langle 1, 0\rangle$ and $0^{\sim} = \langle 0, 1\rangle$.

Definition 2.6 ([9]). Let Ω_X denotes the family of all intuitionistic fuzzy mappings $f : \zeta^X \to \zeta^X$ with the following properties.

(1) $f(0_\sim) = 0_\sim$
(2) $A \subseteq f(A)$ for every $A \in \zeta^X$
(3) $f(\cup A_i) = \cup f(A_i)$ for every $A_i \in \zeta^X, i \in J$

For $f \in \Omega_X$,the mapping $f^{-1} \in \Omega_X$ is defined by $f^{-1}(A) = \cap\{B/f(\overline{B}) \subseteq \overline{A}\}$
For $f, g \in \Omega_X$, we define, for all $A \in \zeta^X$,

$$f \sqcap g(A) = \cap\{f(A_1) \cup g(A_2)/A_1 \cup A_2 = A\},(f \circ g)(A) = f(g(A))$$

Definition 2.7 ([9]). Let $\mathcal{U} : \Omega_X \to I_0 \times I_1$ be an intuitionistic fuzzy mapping. Then $\mathcal{U}$ is called an intuitionistic fuzzy quasi uniformity on X if it satisfies the following conditions.

(1) $\mathcal{U}(f_1 \sqcap f_2) \supseteq \mathcal{U}(f_1) \cap \mathcal{U}(f_2)$ for $f_1, f_2 \in \Omega_X$
(2) For $f \in \Omega_X$ we have $\cup\{\mathcal{U}(f_1)/f_1 \circ f_1 \subseteq f\} \supseteq \mathcal{U}(f)$
(3) If $f_1 \supseteq f$ then $\mathcal{U}(f_1) \supseteq \mathcal{U}(f)$.
(4) There exists $f \in \Omega_X$ such that $\mathcal{U}(f) = \langle 1, 0\rangle$

Then the pair $(X,\mathcal{U})$ is said to be an intuitionistic fuzzy quasi uniform space where $I_0 = (0, 1]$ and $I_1 = [0, 1)$.

Definition 2.8 ([9]). Let $(X,\mathcal{U})$ be an intuitionistic fuzzy quasi uniform space.Define, for each $r \in (0, 1] = I_0$,$s \in [0, 1) = I_1$ with $r + s \leq 1$ and $A \in \zeta^X$

$(r, s)IFQI_{\mathcal{U}}(A) = \cup\{B/f(B) \subseteq A$ for some $f \in \Omega_X$ with $\mathcal{U}(f) > \langle r, s\rangle\}$

Definition 2.9 ([9]). Let $(X,\mathcal{U})$ be an intuitionistic fuzzy quasi uniform space. Then the mapping $T_{\mathcal{U}} : \zeta^X \to I_0 \times I_1$ is defined by $T_{\mathcal{U}}(A) = \cup\{\langle r, s\rangle/(r, s)IFQI_{\mathcal{U}}(A) = A$,$r \in I_0$, $s \in I_1$ with $r + s \leq 1\}$. The the pair $(X, T_{\mathcal{U}})$ is called an intuitionistic fuzzy quasi uniform topological space.The members of $(X, T_{\mathcal{U}})$ are called (r,s) intuitionistic fuzzy quasi uniform open sets.

Note 2.10 ([9]). The complement of (r,s) intuitionistic fuzzy quasi uniform open set is called (r,s) intuitionistic fuzzy quasi uniform closed set.

Definition 2.11 ([9]). Let $(X, T_{\mathcal{U}})$ be an intuitionistic fuzzy quasi uniform topological space and A be an intuitionistic fuzzy set.Then A is said to be a (r,s) intuitionistic fuzzy quasi uniform G_δ set if $A = \cap_{i=1}^{\infty} A_i$ where each A_i is a (r,s) intuitionistic fuzzy quasi uniform open set, where $r \in I_0$,$s \in I_1$ with $r + s \leq 1$. The complement of (r,s) intuitionistic fuzzy quasi uniform G_δ set is an (r,s) intuitionistic fuzzy quasi uniform F_σ set.

Definition 2.12 ([9]). Let $(X, T_{\mathcal{U}})$ be an intuitionistic fuzzy quasi uniform topological space and A be an intuitionistic fuzzy set.Then the intuitionistic fuzzy quasi uniform σ closure of A is denoted and defined by $(r, s)IFQ\sigma cl_{\mathcal{U}}(A) = \cap\{B/B \supseteq A$ and B is an (r,s) intuitionistic fuzzy quasi uniform F_δ set where $r \in I_0$,$s \in I_1$ with $r + s \leq 1$ $\}$.

Definition 2.13 ([9]). Let $(X, T_{\mathcal{U}})$ be an intuitionistic fuzzy quasi uniform topological space and A be an intuitionistic fuzzy set.Then A is said to be a (r,s) intuitionistic fuzzy quasi uniform regular open set if $A = (r, s)IFQint_{\mathcal{U}}((r, s)IFQcl_{\mathcal{U}}(A))$ where $r \in I_0$,$s \in I_1$ with $r + s \leq 1$.

The complement of (r,s) intuitionistic fuzzy quasi uniform regular open set is a (r,s) intuitionistic fuzzy quasi uniform regular closed set.

Definition 2.14 ([5]). An fuzzy topological space X is said to be fuzzy extremally disconnected if the closure of every fuzzy open set in X is fuzzy open in X.

3. Ordered intuitionistic fuzzy quasi uniform regular G_δ extremally disconnected spaces

Definition 3.1. Let X be a non empty set and $A \in \zeta^X$. Then A is said to be

(1) increasing intuitionistic fuzzy set if $x \leq y$ implies $A(x) \leq A(y)$. That is, $\mu_A(x) \leq \mu_A(y)$ and $\gamma_A(x) \geq \gamma_A(y)$.

(2) decreasing intuitionistic fuzzy set if $x \leq y$ implies $A(x) \geq A(y)$. That is, $\mu_A(x) \geq \mu_A(y)$ and $\gamma_A(x) \leq \gamma_A(y)$.

Definition 3.2. Let X be an ordered set. $T_\mathcal{U}$ is an intuitionistic fuzzy quasi uniform topology defined on X.Then $(X, T_\mathcal{U}, \leq)$ is said to be an ordered intuitionistic fuzzy quasi uniform topological space.

Definition 3.3. Let $(X, T_\mathcal{U})$ be an intuitionistic fuzzy quasi uniform topological space and A be (r,s) intuitionistic fuzzy quasi uniform regular open set. Then A is said to be

(1) (r, s) intuitionistic fuzzy quasi uniform increasing regular open set if $x \leq y$ implies $A(x) \leq A(y)$. That is, $\mu_A(x) \leq \mu_A(y)$ and $\gamma_A(x) \geq \gamma_A(y)$.

(2) (r, s) intuitionistic fuzzy quasi uniform decreasing regular open set if $x \leq y$ implies $A(x) \geq A(y)$. That is, $\mu_A(x) \geq \mu_A(y)$ and $\gamma_A(x) \leq \gamma_A(y)$.

Definition 3.4. Let $(X, T_\mathcal{U}, \leq)$ be an ordered intuitionistic fuzzy quasi uniform topological space and A be any intuitionistic fuzzy set in $(X, T_\mathcal{U}, \leq)$. Then A is said to be (r,s) intuitionistic fuzzy quasi uniform increasing/ decreasing regular G_δ set if there exists a (r,s) intuitionistic fuzzy quasi uniform increasing/decreasing regular open set U such that $U \subseteq A \subseteq (r, s)IFQ\sigma cl_\mathcal{U}(U)$.

The complement of (r,s) intuitionistic fuzzy quasi uniform increasing/decreasing regular G_δ set is said to be (r,s) intuitionistic fuzzy quasi uniform decreasing/increasing regular F_σ set.

Definition 3.5. Let $(X, T_\mathcal{U}, \leq)$ be an ordered intuitionistic fuzzy quasi uniform topological space and A be any intuitionistic fuzzy set in $(X, T_\mathcal{U}, \leq)$.Then we define

(1) $(r, s)IFQrG_\delta I_\mathcal{U}(A)$ = (r,s) Intuitionistic fuzzy quasi uniform increasing regular G_δ closure of A = The smallest (r,s)intuitionistic fuzzy quasi uniform increasing regular F_σ set containing A.

(2) $(r, s)IFQrG_\delta D_\mathcal{U}(A)$ =(r,s) Intuitionistic fuzzy quasi uniform decreasing regular G_δ closure of A = The smallest (r,s)intuitionistic fuzzy quasi uniform decreasing regular F_σ set containing A.

(3) $(r, s)IFQrG_\delta I^0_\mathcal{U}(A)$ = (r,s) Intuitionistic fuzzy quasi uniform increasing regular G_δ interior of A = The greatest (r,s) intuitionistic fuzzy quasi uniform increasing regular G_δ set contained in A.

(4) $(r, s)IFQrG_\delta D^0_\mathcal{U}(A)$ = (r,s) Intuitionistic fuzzy quasi uniform decreasing regular G_δ interior of A = The greatest (r, s) intuitionistic fuzzy quasi uniform decreasing regular G_δ set contained in A.

Definition 3.6. Let $(X, T_{\mathcal{U}}, \leq)$ be an ordered intuitionistic fuzzy quasi uniform topological space and A be any decreasing intuitionistic fuzzy set in $(X, T_{\mathcal{U}}, \leq)$.Then A is said to be (r,s) intuitionistic fuzzy quasi uniform decreasing regular G_δ F_σ set in $(X, T_{\mathcal{U}}, \leq)$ if A is both (r, s) intuitionistic fuzzy quasi uniform decreasing regular G_δ set and (r,s)intuitionistic fuzzy quasi uniform decreasing regular F_σ set.

Proposition 3.7. *Let $(X, T_{\mathcal{U}}, \leq)$ be an ordered intuitionistic fuzzy quasi uniform topological space.Then for any two intuitionistic fuzzy sets A and B in $(X, T_{\mathcal{U}}, \leq)$ the following are valid.*

(1) $\overline{(r,s)IFQrG_\delta I_{\mathcal{U}}(A)} = (r,s)IFQrG_\delta D^0_{\mathcal{U}}(\overline{A})$
(2) $\overline{(r,s)IFQrG_\delta D_{\mathcal{U}}(A)} = (r,s)IFQrG_\delta I^0_{\mathcal{U}}(\overline{A})$
(3) $\overline{(r,s)IFQrG_\delta I^0_{\mathcal{U}}(A)} = (r,s)IFQrG_\delta D_{\mathcal{U}}(\overline{A})$
(4) $\overline{(r,s)IFQrG_\delta D^0_{\mathcal{U}}(A)} = (r,s)IFQrG_\delta I_{\mathcal{U}}(\overline{A})$

Proof. (1) Since $(r,s)IFQrG_\delta I_{\mathcal{U}}(A)$ is a (r,s) intuitionistic fuzzy quasi uniform increasing regular F_σ set containing A, $\overline{(r,s)IFQrG_\delta I_{\mathcal{U}}(A)}$ is a (r,s) intuitionistic fuzzy quasi uniform decreasing regular G_δ set such that $\overline{(r,s)IFQrG_\delta I_{\mathcal{U}}(A)} \subseteq \overline{A}$. Let B be another (r,s) intuitionistic fuzzy quasi uniform decreasing regular G_δ set such that $B \subseteq \overline{A}$. Then $\overline{B}$ is a (r,s) intuitionistic fuzzy quasi uniform increasing regular F_σ set such that $\overline{B} \supseteq A$. It follows that $(r,s)IFQrG_\delta I_{\mathcal{U}}(A) \subseteq \overline{B}$. That is,

$$B \subseteq \overline{(r,s)IFQrG_\delta I_{\mathcal{U}}(A)}.$$

Thus, $\overline{(r,s)IFQrG_\delta I_{\mathcal{U}}(A)}$ is the largest (r,s) intuitionistic fuzzy quasi uniform decreasing regular G_δ set such that $\overline{(r,s)IFQrG_\delta I_{\mathcal{U}}(A)} \subseteq \overline{A}$. That is,

$$\overline{(r,s)IFQrG_\delta I_{\mathcal{U}}(A)} = (r,s)IFQrG_\delta D^0_{\mathcal{U}}(\overline{A}).$$

The proof of (2),(3) and (4) are similar to (1). □

Definition 3.8. Let $(X, T_{\mathcal{U}}, \leq)$ be an ordered intuitionistic fuzzy quasi uniform topological space.Let A be any (r,s) intuitionistic fuzzy quasi uniform increasing regular G_δ set in $(X, T_{\mathcal{U}}, \leq)$. If $(r,s)IFQrG_\delta I_{\mathcal{U}}(A)$ is (r,s) intuitionistic fuzzy quasi uniform increasing regular G_δ set in $(X, T_{\mathcal{U}}, \leq)$,then $(X, T_{\mathcal{U}}, \leq)$ is said to be upper (r,s) intuitionistic fuzzy quasi uniform regular G_δ extremally disconnected space.Similarly we can define lower (r,s) intuitionistic fuzzy quasi uniform regular G_δ extremally disconnected space.

Definition 3.9. An ordered intuitionistic fuzzy quasi uniform topological space $(X, T_{\mathcal{U}}, \leq)$ is said to be ordered (r,s) intuitionistic fuzzy quasi uniform regular G_δ extremally disconnected space if it is both upper (r,s) intuitionistic fuzzy quasi uniform regular G_δ extremally disconnected space and lower (r,s) intuitionistic fuzzy quasi uniform regular G_δ extremally disconnected space.

Proposition 3.10. *Let $(X, T_{\mathcal{U}}, \leq)$ be an ordered intuitionistic fuzzy quasi uniform topological space.Then the following statements are equivalent*

(1) *$(X, T_{\mathcal{U}}, \leq)$ is an upper (r,s) intuitionistic fuzzy quasi uniform regular G_δ extremally disconnected space .*

(2) *For each (r,s) intuitionistic fuzzy quasi uniform decreasing regular F_σ set A, $(r,s)IFQrG_\delta D^0_{\mathcal{U}}(A)$ is (r,s) intuitionistic fuzzy quasi uniform decreasing regular F_σ.*

(3) *For each (r,s) intuitionistic fuzzy quasi uniform increasing regular G_δ set A,we have* $(r,s)IFQrG_\delta D_{\mathcal{U}}(\overline{(r,s)IFQrG_\delta I_{\mathcal{U}}(A)}) = \overline{(r,s)IFQrG_\delta I_{\mathcal{U}}(A)}$

(4) *For each (r,s) intuitionistic fuzzy quasi uniform increasing regular G_δ set A and an (r,s) intuitionistic fuzzy quasi uniform decreasing regular G_δ set B in $(X, T_{\mathcal{U}}, \leq)$ with* $(r,s)IFQrG_\delta I_{\mathcal{U}}(A) = \overline{B}$, *we have ,*$(r,s)IFQrG_\delta D_{\mathcal{U}}(B) = \overline{(r,s)IFQrG_\delta I_{\mathcal{U}}(A)}$.

Proof. (1) ⇒(2) Let A be any (r,s) intuitionistic fuzzy quasi uniform decreasing regular F_σ set. Then $\overline{A}$ is an (r,s) intuitionistic fuzzy quasi uniform increasing regular G_δ set and so by assumption (1), $(r,s)IFQrG_\delta I_{\mathcal{U}}(\overline{A})$ is (r,s) intuitionistic fuzzy quasi uniform increasing regular G_δ set.That is,$(r,s)IFQrG_\delta D^0_{\mathcal{U}}(A)$ is (r,s) intuitionistic fuzzy quasi uniform decreasing regular F_σ set.

(2) ⇒(3) Let A be any (r,s) intuitionistic fuzzy quasi uniform increasing regular G_δ set.Then $\overline{A}$ is an (r,s) intuitionistic fuzzy quasi uniform decreasing regular F_σ set.Then by (2),$(r,s)IFQrG_\delta D^0_{\mathcal{U}}(\overline{A})$ is an (r,s) intuitionistic fuzzy quasi uniform decreasing regular F_σ set. Now,

$$\begin{aligned}(r,s)IFQrG_\delta D_{\mathcal{U}}((r,s)IFQrG_\delta D^0_{\mathcal{U}}(\overline{A})) &= (r,s)IFQrG_\delta D^0_{\mathcal{U}}(\overline{A}) \\ &= \overline{(r,s)IFQrG_\delta I_{\mathcal{U}}(A)}.\end{aligned}$$

(3) ⇒(4) Let A be a (r,s) intuitionistic fuzzy quasi uniform increasing regular G_δ set and B be a (r,s) intuitionistic fuzzy quasi uniform decreasing regular G_δ set such that $(r,s)IFQrG_\delta I_{\mathcal{U}}(A) = \overline{B}$. By (3),

$(r,s)IFQrG_\delta D_{\mathcal{U}}(\overline{(r,s)IFQrG_\delta I_{\mathcal{U}}(A)}) = \overline{(r,s)IFQrG_\delta I_{\mathcal{U}}(A)}$.

$(r,s)IFQrG_\delta D_{\mathcal{U}}(B) = \overline{(r,s)IFQrG_\delta I_{\mathcal{U}}(A)}$.

(4) ⇒(1) Let A be a (r,s) intuitionistic fuzzy quasi uniform increasing regular G_δ set.Put $B = \overline{(r,s)IFQrG_\delta I_{\mathcal{U}}(A)}$.Clearly,B is a (r,s) intuitionistic fuzzy quasi uniform decreasing regular G_δ set. By (4) it follows that

$$(r,s)IFQrG_\delta D_{\mathcal{U}}(B) = \overline{(r,s)IFQrG_\delta I_{\mathcal{U}}(A)}.$$

That is, $\overline{(r,s)IFQrG_\delta I_{\mathcal{U}}(A)}$ is a (r,s) intuitionistic fuzzy quasi uniform decreasing regular F_σ set.Hence $(X, T_{\mathcal{U}}, \leq)$ is an upper (r,s) intuitionistic fuzzy regular G_δ extremally disconnected space. □

Proposition 3.11. *Let $(X, T_{\mathcal{U}}, \leq)$ be an ordered intuitionistic fuzzy quasi uniform topological space. Then $(X, T_{\mathcal{U}}, \leq)$ is an upper (r,s) intuitionistic fuzzy quasi uniform regular G_δ extremally disconnected space if and only if for each (r,s) intuitionistic fuzzy quasi uniform decreasing regular G_δ set A and (r,s) intuitionistic fuzzy quasi uniform decreasing regular F_σ set B such that $A \subseteq B$ we have,* $(r,s)IFQrG_\delta D_{\mathcal{U}}(A) \subseteq (r,s)IFQrG_\delta D^0_{\mathcal{U}}(B)$.

Proof. Suppose $(X, T_{\mathcal{U}}, \leq)$ is an upper (r,s) intuitionistic fuzzy quasi uniform regular G_δ extremally disconnected space and let A be a (r,s) intuitionistic fuzzy quasi uniform decreasing regular G_δ set and B be a (r,s) intuitionistic fuzzy quasi uniform decreasing regular F_σ set such that $A \subseteq B$.Then by (2) of Proposition 3.10, $(r,s)IFQrG_\delta D^0_{\mathcal{U}}(B)$ is (r,s) intuitionistic fuzzy quasi uniform decreasing regular F_σ set. Also, since A is a (r,s) intuitionistic fuzzy quasi uniform decreasing regular G_δ set and $A \subseteq B$, it follows that $A \subseteq (r,s)IFQrG_\delta D^0_{\mathcal{U}}(B)$.Again,since

$(r,s)IFQrG_\delta D_{\mathcal{U}}(A)$ is a (r,s) intuitionistic fuzzy quasi uniform decreasing regular F_σ set,it follows that $(r,s)IFQrG_\delta D_{\mathcal{U}}(A) \subseteq (r,s)IFQrG_\delta D^0_{\mathcal{U}}(B)$.

Conversely, let B be any (r,s) intuitionistic fuzzy quasi uniform decreasing regular F_σ set. Then by Definition 3.5, $(r,s)IFQrG_\delta D^0_{\mathcal{U}}(B)$ is a (r,s) intuitionistic fuzzy quasi uniform decreasing regular G_δ set and it is also clear that $(r,s)IFQrG_\delta D^0_{\mathcal{U}}(B) \subseteq B$. Therefore by assumption, $(r,s)IFQrG_\delta D_{\mathcal{U}}((r,s)IFQrG_\delta D^0_{\mathcal{U}}(B)) \subseteq (r,s) IFQrG_\delta D^0_{\mathcal{U}}(B)$. This implies that $(r,s)IFQrG_\delta D^0_{\mathcal{U}}(B)$ is a (r,s) intuitionistic fuzzy quasi uniform decreasing regular F_σ set Hence by (2) of Proposition 3.10, it follows that $(X, T_{\mathcal{U}}, \leq)$ is an upper (r,s) intuitionistic fuzzy quasi uniform regular G_δ extremally disconnected space. □

Remark 3.12. Let $(X, T_{\mathcal{U}}, \leq)$ be an upper (r,s) intuitionistic fuzzy quasi uniform regular G_δ extremally disconnected space. Let $\{A_i, \overline{B_i}/i \in N\}$ be collection such that A_i's are (r,s) intuitionistic fuzzy quasi uniform decreasing regular G_δ sets and B_i are (r,s) intuitionistic fuzzy quasi uniform decreasing regular F_σ sets. Let A and $\overline{B}$ be (r,s) intuitionistic fuzzy quasi uniform decreasing regular G_δ set and (r,s) intuitionistic fuzzy quasi uniform increasing regular G_δ set respectively . If $A_i \subseteq A \subseteq B_j$ and $A_i \subseteq B \subseteq B_j$ for all $i, j \in N$, then there exists a (r,s) intuitionistic fuzzy quasi uniform decreasing regular $G_\delta F_\sigma$ set C such that $(r,s)IFQrG_\delta D_{\mathcal{U}}(A_i) \subseteq C \subseteq (r,s)IFQrG_\delta D^0_{\mathcal{U}}(B_j)$ for all $i, j \in N$.

Proof. By Proposition 3.11, $(r,s)IFQrG_\delta D_{\mathcal{U}}(A_i) \subseteq (r,s)IFQrG_\delta D_{\mathcal{U}}(A) \cap (r,s) IFQrG_\delta D^0_{\mathcal{U}}(B) \subseteq (r,s)IFQrG_\delta D^0_{\mathcal{U}}(B_j)$ for all $i, j \in N$. Letting $C = (r,s)IFQrG_\delta D_{\mathcal{U}}(A) \cap (r,s)IFQrG_\delta D^0_{\mathcal{U}}(B)$ in the above, we have C is a (r,s) intuitionistic fuzzy quasi uniform decreasing regular $G_\delta F_\sigma$ set satisfying the required conditions. □

Proposition 3.13. *Let $(X, T_{\mathcal{U}}, \leq)$ be an ordered (r,s) intuitionistic fuzzy quasi uniform regular G_δ extremally disconnected space. Let $\{A_q\}_{q\in Q}$ and $\{B_q\}_{q\in Q}$ be monotone increasing collections of (r,s) intuitionistic fuzzy quasi uniform decreasing regular G_δ sets and (r,s) intuitionistic fuzzy quasi uniform decreasing regular F_σ sets of $(X, T_{\mathcal{U}}, \leq)$ respectively. Suppose that $A_{q_1} \subseteq B_{q_2}$ whenever $q_1 < q_2$ (Q is the set of all rational numbers). Then there exists a monotone increasing collection $\{C_q\}_{q\in Q}$ of a (r,s) intuitionistic fuzzy quasi uniform decreasing regular $G_\delta F_\sigma$ sets of $(X, T_{\mathcal{U}}, \leq)$ such that $(r,s)IFQrG_\delta D_{\mathcal{U}}(A_{q_1}) \subseteq C_{q_2}$ and $C_{q_1} \subseteq (r,s)IFQrG_\delta D^0_{\mathcal{U}}(B_{q_2})$ whenever $q_1 < q_2$.*

Proof. Let us arrange all rational numbers into a sequence $\{q_n\}$ (without repetitions). For every $n \geq 2$, we shall define inductively a collection $\{C_{q_i}/1 \leq i < n\} \subset \zeta^X$ such that

$$(r,s)IFQrG_\delta D_{\mathcal{U}}(A_q) \subseteq C_{q_i} \text{if } q < q_i,$$
$$C_{q_i} \subseteq (r,s)IFQrG_\delta D^0_{\mathcal{U}}(B_q) \text{if } q_i < q, \text{for all } i < n \qquad (S_n)$$

By Proposition 3.11, the countable collections $\{(r,s)IFQrG_\delta D_{\mathcal{U}}(A_q)\}$ and $\{(r,s)IFQrG_\delta D^0_{\mathcal{U}}(B_q)\}$ satisfies $(r,s)IFQrG_\delta D_{\mathcal{U}}(A_{q_1}) \subseteq (r,s)IFQrG_\delta D^0_{\mathcal{U}}(B_{q_2})$ if $q_1 < q_2$. By Remark 3.10, there exists a (r,s) intuitionistic fuzzy quasi uniform decreasing regular $G_\delta F_\sigma$ set D_1 such that

$$(r,s)IFQrG_\delta D_{\mathcal{U}}(A_{q_1}) \subseteq D_1 \subseteq (r,s)IFQrG_\delta D^0_{\mathcal{U}}(B_{q_2})$$

Letting $C_{q_1} = D_1$, we get (S_2). Assume that intuitionistic fuzzy sets C_{q_i} are already defined for $i < n$ and satisfy (S_n). Define $E = \cup\{C_{q_i}/i < n, q_i < q_n\} \cup A_{q_n}$ and $F = \cap\{C_{q_j}/j < n, q_j > q_n\} \cap B_{q_n}$. Then

$$(r,s)IFQrG_\delta D_{\mathcal{U}}(C_{q_i}) \subseteq (r,s)IFQrG_\delta D_{\mathcal{U}}(E) \subseteq (r,s)IFQrG_\delta D^0_{\mathcal{U}}(C_{q_j})$$

and

$$(r,s)IFQrG_\delta D^0_{\mathcal{U}}(C_{q_i}) \subseteq (r,s)IFQrG_\delta D^0_{\mathcal{U}}(F) \subseteq (r,s)IFQrG_\delta D^0_{\mathcal{U}}(C_{q_j})$$

whenever $q_i < q_n < q_j (i, j < n)$, as well as $A_q \subseteq (r,s)IFQrG_\delta D_{\mathcal{U}}(E) \subseteq B_{q'}$ and $A_q \subseteq (r,s)IFQrG_\delta D^0_{\mathcal{U}}(F) \subseteq B_{q'}$ whenever $q < q_n < q^{'}$. This shows that the countable collection $\{C_{q_i}/i < n, q_i < q_n\} \cup \{A_q | q < q_n\}$ and $\{C_{q_j}/j < n, q_j > q_n\} \cup \{B_q | q > q_n\}$ together with E and F fulfil the conditions of Remark 3.12. Hence, there exists a (r,s) intuitionistic fuzzy quasi uniform decreasing regular $G_\delta F_\sigma$ set D_n such that $(r,s)IFQrG_\delta D_{\mathcal{U}}(D_n) \subseteq B_q$ if $q_n < q$, $A_q \subseteq (r,s)IFQrG_\delta D^0_{\mathcal{U}}(D_n)$ if $q < q_n$, $(r,s)IFQrG_\delta D_{\mathcal{U}}(C_{q_i}) \subseteq (r,s)IFQrG_\delta D^0_{\mathcal{U}}(D_n)$ if $q_i < q_n$ $(r,s).IFQrG_\delta D_{\mathcal{U}}(D_n) \subseteq (r,s)IFQrG_\delta D^0_{\mathcal{U}}(C_{q_i})$ if $q_n < q_j$ where $1 \leq i, j \leq n-1$. Letting $C_{q_n} = D_n$ we obtain an intuitionistic fuzzy sets $C_{q_1}, C_{q_2}, C_{q_3}, ..., C_{q_n}$ that satisfy $(\mathbf{S_{n+1}})$. Therefore, the collection $\{C_{q_i}/i = 1, 2, ...\}$ has the required property. □

Definition 3.14. Let $(X, T_{\mathcal{U}}, \leq)$ and $(Y, S_{\mathcal{V}}, \leq)$ be an ordered (r,s) intuitionistic fuzzy quasi uniform topological spaces and $f : (X, T, \leq) \to (Y, S, \leq)$ be an intuitionistic fuzzy mapping.Then f is said to be a (r,s) intuitionistic fuzzy quasi uniform increasing/decreasing regular G_δ continuous mapping if for any (r,s) intuitionistic fuzzy quasi uniform regular G_δ set A in $(Y, S_{\mathcal{V}}, \leq)$, $f^{-1}(A)$ is a (r,s) intuitionistic fuzzy quasi uniform increasing/decreasing regular G_δ set in $(X, T_{\mathcal{U}}, \leq)$.

If f is both (r,s) intuitionistic fuzzy quasi uniform increasing regular G_δ continuous mapping and (r,s) intuitionistic fuzzy quasi uniform decreasing regular G_δ continuous mapping then it is called ordered (r,s) intuitionistic fuzzy quasi uniform regular G_δ continuous mapping.

4. Tietze extention theorem for ordered (r,s) intuitionistic fuzzy quasi uniform regular G_δ extremally disconnected space

An intuitionistic fuzzy real line $\mathbb{R}_I(I)$ is the set of all monotone decreasing intuitionistic fuzzy set $A \in \zeta^{\mathbb{R}}$ satisfying

$$\cup\{A(t) : t \in \mathbb{R}\} = 1^\sim$$
$$\cap\{A(t) : t \in \mathbb{R}\} = 0^\sim$$

After the identification of intuitionistic fuzzy sets $A, B \in \mathbb{R}_I(I)$ if and only if $A(t-) = B(t-)$ and $A(t+) = B(t+)$ for all $t \in \mathbb{R}$ where

$$A(t-) = \cap\{A(s) : s < t\} \text{ and}$$
$$A(t+) = \cup\{A(s) : s > t\}$$

The natural intuitionistic fuzzy topology on $\mathbb{R}_I(I)$ is generated from the subbasis $\{L^I_t, R^I_t : t \in \mathbb{R}\}$ where L^I_t, R^I_t are mapping from $\mathbb{R}_I(I) \to \mathbb{I}_I(I)$ are given by $L^I_t[A] = \overline{A(t-)}$ and $R^I_t[A] = A(t+)$.

The intuitionistic fuzzy unit interval $\mathbb{I}_I(I)$ is a subset of $\mathbb{R}_I(I)$ such that $[A] \in \mathbb{I}_I(I)$ if the member and non member of A are defined by

$$\mu_A(t) = \begin{cases} 0 & \text{if } t > 1 \\ 1 & \text{if } t < 0 \end{cases}$$

and

$$\gamma_A(t) = \begin{cases} 1 & \text{if } t > 1 \\ 0 & \text{if } t < 0 \end{cases}$$

respectively.

Definition 4.1. Let $(X, T_{\mathcal{U}}, \leq)$ be an ordered intuitionistic fuzzy quasi uniform topological space and $f : X \to \mathbb{R}_I(I)$ be a mapping.Then f is said to be lower (r,s) intuitionistic fuzzy quasi uniform regular G_δ continuous mapping if $f^{-1}(R_t^I)$ is a (r,s) intuitionistic fuzzy quasi uniform increasing regular G_δ set or (r,s) intuitionistic fuzzy quasi uniform decreasing regular G_δ set , for $t \in \mathbb{R}$.

Definition 4.2. Let $(X, T_{\mathcal{U}}, \leq)$ be an ordered intuitionistic fuzzy quasi uniform topological space and $f : X \to \mathbb{R}_I(I)$ be a mapping.Then f is said to be upper (r,s) intuitionistic fuzzy quasi uniform regular G_δ continuous mapping if $f^{-1}(L_t^I)$ is a (r,s) intuitionistic fuzzy quasi uniform increasing regular G_δ set or (r,s) intuitionistic fuzzy quasi uniform decreasing regular G_δ set , for $t \in \mathbb{R}$.

Note 4.3. Let X be a non empty set and $A \in \zeta^X$.Then $A^\sim = \langle \mu_A(x), \gamma_A(x) \rangle$ for every $x \in X$.

Proposition 4.4. *Let $(X, T_{\mathcal{U}}, \leq)$ be an ordered intuitionistic fuzzy quasi uniform topological space, $A \in \zeta^X$ and $f : X \to \mathbb{R}_I(I)$ be such that*

$$f(x)(t) = \begin{cases} 1^\sim & \text{if } t < 0 \\ A^\sim & \text{if } 0 \leq t \leq 1 \\ 0^\sim & \text{if } t > 1 \end{cases}$$

and for all $x \in X$.Then f is lower/upper (r,s) intuitionistic fuzzy quasi uniform regular G_δ continuous mapping if and only if A is (r,s) intuitionistic fuzzy quasi uniform increasing or decreasing regular G_δ /regular F_σ set.

Proof. It suffices to observe that

$$f^{-1}(R_t^I) = \begin{cases} 1_\sim & \text{if } t < 0 \\ A & \text{if } 0 \leq t \leq 1 \\ 0_\sim & \text{if } t > 1 \end{cases}$$

implies f is lower (r,s) intuitionistic fuzzy quasi uniform regular G_δ continuous mapping if and only if A is (r,s) intuitionistic fuzzy quasi uniform increasing regular G_δ set and

$$f^{-1}(\overline{L_t^I}) = \begin{cases} 1_\sim & \text{if } t < 0 \\ A & \text{if } 0 \leq t \leq 1 \\ 0_\sim & \text{if } t > 1 \end{cases}$$

implies f is upper (r,s) intuitionistic fuzzy quasi uniform regular G_δ continuous mapping if and only if A is (r,s) intuitionistic fuzzy quasi uniform decreasing regular F_σ set.Thus proved. □

Definition 4.5. Let X be any non empty set.An intuitionistic fuzzy charectaristic mapping of an intuitionistic fuzzy set A in X is a map $\Psi_A : X \to \mathbb{I}_I(I)$ defined by $\Psi_A(x) = A^{\sim}$ for each $x \in X$.

Proposition 4.6. *Let $(X, T_{\mathcal{U}}, \leq)$ be an ordered intuitionistic fuzzy quasi uniform topological space, $A \in \zeta^X$ .Then Ψ_A is lower/upper (r,s) intuitionistic fuzzy quasi uniform regular G_δ continuous mapping if and only if A is (r,s) intuitionistic fuzzy quasi uniform increasing or decreasing regular G_δ / regular F_σ set.*

Proof. Proof is similar to Proposition 4.4. □

Proposition 4.7. *Let $(X, T_{\mathcal{U}}, \leq)$ be an ordered intuitionistic fuzzy quasi uniform topological space. Then the following are equivalent*

(1) *$(X, T_{\mathcal{U}}, \leq)$ is an upper (r,s) intuitionistic fuzzy quasi uniform regular G_δ extremally disconnected space*

(2) *If $g, h : X \to \mathbb{R}_I(I)$, g is lower (r,s) intuitionistic fuzzy quasi uniform regular G_δ continuous mapping, h is upper (r,s) intuitionistic fuzzy quasi uniform regular G_δ continuous mapping and $g \subseteq h$, then there exists an (r,s) intuitionistic fuzzy quasi uniform increasing regular G_δ continuous mapping $f : (X, T_{\mathcal{U}}, \leq) \to \mathbb{R}_I(I)$ such that $g \subseteq f \subseteq h$*

(3) *If $\overline{A}$ is a (r,s) intuitionistic fuzzy quasi uniform increasing regular G_δ set and B is a (r,s) intuitionistic fuzzy quasi uniform decreasing regular G_δ set such that $B \subseteq A$, then there exists (r,s) intuitionistic fuzzy quasi uniform increasing regular G_δ continuous mapping $f : (X, T, \leq) \to \mathbb{R}_I(I)$ such that $B \subseteq f^{-1}(\overline{L_1^I}) \subseteq f^{-1}(R_0^I) \subseteq A$.*

Proof. (1) ⇒ (2) Define $A_r = h^{-1}(L_r^I)$ and $B_r = g^{-1}(\overline{R_r^I})$, for all $r \in Q$ (Q is the set of all rationals). Clearly, $\{A_r\}_{r\in Q}$ and $\{B_r\}_{r\in Q}$ are monotone increasing families of a (r,s) intuitionistic fuzzy quasi uniform decreasing regular G_δ sets and (r,s) intuitionistic fuzzy quasi uniform decreasing regular F_σ sets of $(X, T_{\mathcal{U}}, \leq)$. Moreover $A_r \subseteq B_s$ if $r < s$.By Proposition 3.13,tere exists a monotone increasing family $\{C_r\}_{r\in Q}$ of a (r,s) intuitionistic fuzzy quasi uniform decreasing regular $G_\delta F_\sigma$ sets of $(X, T_{\mathcal{U}}, \leq)$ such that $(r,s)IFQrG_\delta D_{\mathcal{U}}(A_r) \subseteq C_s$ and $C_r \subseteq (r,s)IFQrG_\delta D^0_{\mathcal{U}}(B_s)$ whenever $r < s$ ($r, s \in Q$). Letting $V_t = \bigcap_{r<t} \overline{C_r}$ for $t \in \mathbb{R}$, we define a monotone decreasing family $\{V_t \mid t \in \mathbb{R}\} \subseteq \zeta^X$. Moreover we have $(r,s)IFQrG_\delta I_{\mathcal{U}}(V_t) \subseteq (r,s)IFQrG_\delta I^0_{\mathcal{U}}(V_s)$ whenever $s < t$. We have,

$$\bigcup_{t\in\mathbb{R}} V_t = \bigcup_{t\in\mathbb{R}} \bigcap_{r<t} \overline{C_r} \supseteq \bigcup_{t\in\mathbb{R}} \bigcap_{r<t} \overline{B_r} = \bigcup_{t\in\mathbb{R}} \bigcap_{r<t} g^{-1}(\overline{L_t^I})$$
$$= \bigcup_{t\in\mathbb{R}} g^{-1}(\overline{L_t^I}) = g^{-1}(\bigcup_{t\in\mathbb{R}} \overline{L_t^I}) = 1_\sim$$

Similarly, $\bigcap_{t\in\mathbb{R}} V_t = 0_\sim$. Now define a mapping $f : (X, T_{\mathcal{U}}, \leq) \to \mathbb{R}_I(I)$ possessing required conditions. Let $f(x)(t) = V_t(x)$, for all $x \in X$ and $t \in \mathbb{R}$. By the above discussion, it follows that f is well defined. To prove f is a (r,s) intuitionistic fuzzy quasi uniform increasing regular G_δ continuous mapping. Observe that

$$\bigcup_{s>t} V_s = \bigcup_{s>t} (r,s)IFQrG_\delta I^0_{\mathcal{U}}(V_s) \text{ and } \bigcap_{s<t} V_s = \bigcap_{s<t} (r,s)IFQrG_\delta I_{\mathcal{U}}(V_s).$$

Then $f^{-1}(R_t^I) = \bigcup_{s>t} V_s = \bigcup_{s>t}(r,s)IFQrG_\delta I^0_{\mathcal{U}}(V_s)$ is a (r,s) intuitionistic fuzzy quasi uniform increasing regular G_δ and

$$f^{-1}(\overline{L_t^I}) = \bigcap_{s<t} V_s = \bigcap_{s<t}(r,s)IFQrG_\delta I_{\mathcal{U}}(V_s)$$

is a (r,s) intuitionistic fuzzy quasi uniform increasing regular F_σ set. Therefore, f is (r,s) intuitionistic fuzzy quasi uniform increasing regular G_δ continuous mapping. To conclude the proof it remains to show that $g \subseteq f \subseteq h$. That is,

$$g^{-1}(\overline{L_t^I}) \subseteq f^{-1}(\overline{L_t^I}) \subseteq h^{-1}(\overline{L_t^I}) \text{ and } g^{-1}(R_t^I) \subseteq f^{-1}(R_t^I) \subseteq h^{-1}(R_t^I)$$

for each $t \in \mathbb{R}$. We have,

$$\begin{aligned} g^{-1}(\overline{L_t^I}) &= \bigcap_{s<t} g^{-1}(\overline{L_s^I}) = \bigcap_{s<t}\bigcap_{r<s} g^{-1}(R_r^I) = \bigcap_{s<t}\bigcap_{r<s} \overline{B_r} \\ &\subseteq \bigcap_{s<t}\bigcap_{r<s} \overline{C_r} = \bigcap_{s<t} V_s = f^{-1}(\overline{L_t^I}) \end{aligned}$$

and

$$\begin{aligned} f^{-1}(\overline{L_t^I}) &= \bigcap_{s<t} V_s = \bigcap_{s<t}\bigcap_{r<s} \overline{C_r} \subseteq \bigcap_{s<t}\bigcap_{r<s} \overline{A_r} \\ &= \bigcap_{s<t}\bigcap_{r<s} h^{-1}R_r^I = \bigcap_{s<t} h^{-1}(\overline{L_s^I}) = h^{-1}(\overline{L_t^I}) \end{aligned}$$

Similarly,

$$\begin{aligned} g^{-1}(R_t^I) &= \bigcup_{s>t} g^{-1}(R_s^I) = \bigcup_{s>t}\bigcup_{r>s} g^{-1}\overline{(L_r^I)} = \bigcup_{s>t}\bigcup_{r>s} \overline{B_r} \\ &\subseteq \bigcup_{s>t}\bigcap_{r<s} \overline{C_r} = \bigcup_{s>t} V_s = f^{-1}(R_t^I) \end{aligned}$$

and

$$\begin{aligned} f^{-1}(R_t^I) &= \bigcup_{s>t} V_s = \bigcup_{s>t}\bigcap_{r<s} \overline{C_r} \subseteq \bigcup_{s>t}\bigcup_{r>s} \overline{A_r} \\ &= \bigcup_{s>t}\bigcup_{r>s} h^{-1}(R_r^I) = \bigcup_{s>t} h^{-1}(R_s^I) = h^{-1}(R_t^I). \end{aligned}$$

Hence, the condition (ii) is proved.

(2) $\Rightarrow$ (3) $\overline{A}$ is a (r,s) intuitionistic fuzzy quasi uniform increasing regular G_δ set and B is a (r,s) intuitionistic fuzzy quasi uniform decreasing regular G_δ set such that $B \subseteq A$.Then, $\Psi_B \subseteq \Psi_A$, Ψ_B and Ψ_A lower and upper (r,s) intuitionistic fuzzy quasi uniform regular G_δ continuous mapping respectively.Hence by (2),there exists a (r,s) intuitionistic fuzzy quasi uniform increasing regular G_δ continuous mapping $f : (X, T_{\mathcal{U}}, \leq) \to \mathbb{I}_I(I)$ such that $\Psi_B \subseteq f \subseteq \Psi_A$. Clearly, $f(x) \in [0,1]$ for all $x \in X$ and $B = \Psi_B^{-1}(\overline{L_1^I}) \subseteq f^{-1}(\overline{L_1^I}) \subseteq f^{-1}(R_0^I) \subseteq \Psi_A^{-1}(R_0^I) = A$. Therefore, $B \subseteq f^{-1}(\overline{L_1^I}) \subseteq f^{-1}(R_0^I) \subseteq A$.

(3) $\Rightarrow$ (1) Since $f^{-1}(\overline{L_1^I})$ and $f^{-1}(R_0^I)$ are (r,s) intuitionistic fuzzy quasi uniform decreasing regular F_σ and (r,s) intuitionistic fuzzy quasi uniform decreasing regular G_δ sets by Proposition 3.11,$(X, T_{\mathcal{U}}, \leq)$ is a (r,s) intuitionistic fuzzy quasi uniform regular G_δ extremally disconnected space. □

Note 4.8. Let X be a non empty set and $A \subset X$. Then an intuitionistic fuzzy set χ_A^* is of the form $\langle x, \chi_A(x), 1-\chi_A(x)\rangle$ where

$$\chi_A(x) = \begin{cases} 1 & \text{if } x \in X \\ 0 & \text{if } x \notin X \end{cases}$$

Proposition 4.9. *Let $(X, T_{\mathcal{U}}, \leq)$ be an upper intuitionistic fuzzy quasi uniform regular G_δ extremally disconnected space. Let $A \subset X$ be such that χ_A^* is (r,s) intuitionistic fuzzy quasi uniform increasing regular G_δ in $(X, T_{\mathcal{U}}, \leq)$. Let $f : (A, T_{\mathcal{U}}/A) \to \mathbb{I}_I(I)$ be an (r,s) intuitionistic fuzzy quasi uniform increasing regular G_δ continuous mapping . Then f has an (r,s) intuitionistic fuzzy quasi uniform increasing regular G_δ continuous extension over $(X, T_{\mathcal{U}}, \leq)$.*

Proof. Let $g, h : X \to \mathbb{I}_I(I)$ be such that $g = f = h$ on A and $g(x) = \langle 0, 1\rangle = 0^\sim$, $h(x) = \langle 1, 0\rangle = 1^\sim$ if $x \notin A$. For every $t \in \mathbb{R}$, We have,

$$g^{-1}(R_t^I) = \begin{cases} B_t \cap \chi_A^* & \text{if } t \geq 0 \ , \\ 1_\sim & \text{if } t < 0, \end{cases}$$

where B_t is an (r,s) intuitionistic fuzzy quasi uniform increasing regular G_δ such that $B_t/A = f^{-1}(R_t^I)$ and

$$h^{-1}(L_t^I) = \begin{cases} D_t \cap \chi_A^* & \text{if } t \leq 1 \ , \\ 1_\sim & \text{if } t > 1, \end{cases}$$

where D_t is an (r,s) intuitionistic fuzzy quasi uniform increasing regular G_δ set such that $D_t/A = f^{-1}(L_t^I)$. Thus, g is lower (r,s) intuitionistic fuzzy quasi uniform regular G_δ continuous mapping and h is upper (r,s) intuitionistic fuzzy quasi uniform regular G_δ continuous mapping with $g \subseteq h$. By Proposition 4.7, there is an (r,s) intuitionistic fuzzy quasi uniform increasing regular G_δ continuous mapping F : $X \to \mathbb{I}_I(I)$ such that $g \subseteq F \subseteq h$. Hence $F \equiv f$ on A. □

Acknowledgements. The authors acknowledge the referees and the editors for their valuable suggestions resulting in improvement of the paper.

References

[1] K. T. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20 (1986) 87–96.
[2] C. L. Chang, Fuzzy topological spaces, J. Math. Annal. Appl. 24 (1968) 182–190.
[3] D. Coker and M. Demirci, On intuitionistic fuzzy points. Notes IFS 1(2) (1995) 79–84.
[4] D. Coker, An introduction to intuitionistic fuzzy topological spaces, Fuzzy Sets and Systems 88 (1997) 81–89.
[5] B. Ghosh, Fuzzy extremally disconnected space, Fuzzy Sets and Systems 46 (1992) 245–254.
[6] T. Kubiak, Extending continuous L-real functions, Math. Japonica 31(6) (1986) 875–887.
[7] T. Kubiak, L-fuzzy normal spaces and Tietze extension theorem, J. Math. Anal. App. 125(1) (1987) 141–152.
[8] A. A. Ramadan, S. E. Abbas and A. A. Abd El-Latif, Compactness in intuitionistic fuzzy topological spaces, Int. J. Math. Math. Sci. 2005, no. 1, 19–32.
[9] G. K. Revathi, E. Roja and M. K. Uma, A new approach to intuitionistic fuzzy quasi uniform regular G_δ compactness, Int. J. Math. Sci. Appl. 2(2) (2012) 601–610.
[10] P. Smets, The degree of belief in a fuzzy event, Inform. Sci. 25 (1981) 1–19.
[11] M. Sugeno, An introductory survey of fuzzy control, Inform. Sci. 36 (1985) 59–83.
[12] M. K. Uma, E. Roja and G. Balasubramanian, Tietze extension theorem for ordered fuzzy pre-extremally disconnected spaces, East Asian Math. J. 24 (2008) 1–9.

[13] L. A. Zadeh, Fuzzy sets, Information and Control 9 (1965) 338–353.

G. K. Revathi (gk_revathi@yahoo.co.in)
Department of Mathemathics, Sri Sarada College for Women, Salem - 636016, Tamil Nadu, India.

E. Roja (ar.udhay@yahoo.co.in)
Department of Mathemathics, Sri Sarada College for Women, Salem - 636016, Tamil Nadu, India.

M. K. Uma (ar.udhay@yahoo.co.in)
Department of Mathemathics, Sri Sarada College for Women, Salem - 636016, Tamil Nadu, India.