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On intuitionistic fuzzy Γ-ideals of Γ-LA-semigroups

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Abstract. We consider the intuitionistic fuzzification of the concept of $\Gamma-$ideals in a Γ-LA-semigroup S, and investigate some properties of them. We also prove in this paper the set of all intuitionistic fuzzy left(right) Γ-ideals of S is a LA-semigroup. We also prove that the intuitionistic fuzzy right Γ-ideals and intuitionistic fuzzy left Γ-ideals coincide in Γ-LA bands.

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1. Introduction

The concept of an LA-semigroup was first indroduced by Kazim and Naseerudin [13]. A non-empty set S with binary operation $*$ is said to be an LA-semigroup if the identity $(x * y) * z = (z * y) * x$ for all $x, y, z \in S$ holds. Later, Q. Mushtaq and others have been investigated the structure further and added many useful results to theory of LA-semigroups. T. Shah and I. Rehman have introduced the concept of a Γ-LA-semigroup [16]. Let S and Γ be two non-empty sets. If there exist a mapping $S \times \Gamma \times S \longrightarrow S$ written as (a, γ, b) by $a\gamma b$, then S is called a Γ-LA-semgroup if S satisfies the identity $(a\beta b)\gamma c = (c\beta b)\gamma a$ for all $a, b, c \in S$ and $\beta, \gamma \in \Gamma$. T. Shah and I. Rehman introduce the notion of Γ-ideals in Γ-LA-semigroups.

The concept of a fuzzy set was first initiated by Zadeh [18]. Fuzzy set theory has been shown to be a useful tool to describe situations in which the data are imprecise or vague. Fuzzy sets handle such situations by attributing a degree to which a certain object belongs to a set. The fuzzy algebraic structures play a prominent role in mathematics with wide applications in many other branches such as theoretical physics, computer sciences, control engineering, information sciences, coding theory, topological spaces, logic, set theory, group theory, real analysis, measure theory etc. Rosenfeld studied fuzzy subgroup of a group [15].

In 1986, Atanassov [11] initiated the concept of an intuitionistic fuzzy set (IFS). An Atanassov intuitionistic fuzzy set is considered as a generalization of fuzzy set [18] and has been found to be useful to deal with vagueness. In the sense of Atanassov an IFS is characterized by a pair of functions valued in $[0,1]$: the membership function and the non-membership function. The evaluation degrees of membership and non-membership are independent. Thus, an Atanassov intuitionistic fuzzy set is more material and concise to describe the essence of fuzziness, and Atanassov intuitionistic fuzzy set theory may be more suitable than fuzzy set theory for dealing with imperfect knowledge in many problems. The concept has been applied to various algebraic structures. In [14], Kim and Jun defined intuitionistic fuzzy ideals of semigroups. For further study see [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 17]

In this paper, we introduce the notion of intuitionistic fuzzy left (right) Γ-ideals of a Γ-LA-semigroup S, and also we introduce the notion of intuitionistic fuzzy Γ-ideals of a Γ-LA-semigroup S, then some related properties are investigated. Characterizations of intuitionistic fuzzy left (right) Γ-ideals are given. For a homomorphism f from a Γ-LA-semigroup S to a Γ-LA-semigroup T, if $B = (\mu_B, \gamma_B)$ is an intuitionistic fuzzy Γ-ideals of a Γ-LA-semigroup T, then the preimage $f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B))$ of B under f is an intuitionistic fuzzy Γ-ideals of Γ-LA-semigroup S.

2. Preliminaries

Definition 2.1 ([16]). Let $S = \{x, y, z, ...\}$ and $\Gamma = \{\alpha, \beta, \gamma, ...\}$ be two non-empty sets. Then S is called a Γ-LA-semigroup if it satisfies

1) $x\gamma y \in S$
2) $(x\beta y)\gamma z = (z\beta y)\gamma x$

for all $x, y, z \in S$ and $\beta, \gamma \in \Gamma$.

Definition 2.2 ([16]). A non-empty set U of a Γ-LA-semigroup S is said to be a Γ-sub LA-semigroup S if $U\Gamma U \subseteq U$.

Definition 2.3 ([16]). A left (right) Γ-ideal of a Γ-LA-semigroup S is a non-empty subset U of S such that $S\Gamma U \subseteq U$ $(U\Gamma S \subseteq U)$ if U is a left and a right Γ-ideal of a Γ-LA-semigroup S, then we say that U is Γ-ideal of S.

Definition 2.4 ([11]). Let X be a nonempty fixed set. An intuitionistic fuzzy set (briefly, IFS) A is an object having the form

$$A = \{(x, \mu_A(x), \gamma_A(x)) : x \in X\}$$

where the functions $\mu_A : X \longrightarrow [0,1]$ and $\gamma_A : X \longrightarrow [1,0]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\gamma_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for all $x \in S$ for the sake of simplicity, we use the symbol $A = (\mu_A, \gamma_A)$ for the IFS $A = \{(x, \mu_A(x), \gamma_A(x)) : x \in X\}$.

Definition 2.5 ([9]). A fuzzy set μ in a Γ-LA-semigroup S is called fuzzy Γ-subLA-semigroup of S, if $\mu_A(x\gamma y) \geq \mu_A(x) \wedge \mu_A(y)$ for all $x, y \in S$ and $\gamma \in \Gamma$.

Definition 2.6 ([9]). A fuzzy set μ in a Γ-LA-semigroup S is called a fuzzy left(resp, right) Γ-ideal of S, if $\mu_A(x\gamma y) \geq \mu_A(y)$ (resp, $\mu_A(x\gamma y) \geq \mu_A(x)$) for all $x, y \in S$ and

$\gamma \in S$. A fuzzy set μ in a Γ-LA-semigroup S is called fuzzy Γ-ideal of S, if fuzzy set μ is a fuzzy left Γ-ideal and a fuzzy right Γ-ideal of Γ-LA-semigroup S.

3. Major section

In what follows, S denote a Γ-LA-semigroup, unless otherwise specified.

Definition 3.1. An IFS $A = (\mu_A, \gamma_A)$ in S is called an intuitionistic fuzzy Γ-subLA-semigroup of S if the following conditions hold
(IF1) $\mu_A(x\gamma y) \geq \mu_A(x) \wedge \mu_A(y)$,
(IF2) $\gamma_A(x\gamma y) \leq \gamma_A(x) \vee \gamma_A(y)$,
for all $x, y \in S$.

Definition 3.2. An IFS $A = (\mu_A, \gamma_A)$ in S is called an intuitionistic fuzzy right Γ-idealof S if the following conditions hold
(IF3) $\mu_A(x\gamma y) \geq \mu_A(x)$,
(IF4) $\gamma_A(x\gamma y) \leq \gamma_A(x)$,
for all $x, y \in S$.

Definition 3.3. An IFS $A = (\mu_A, \gamma_A)$ in S is called an intuitionistic fuzzy left Γ-ideal of S if the following conditions hold
(IF5) $\mu_A(x\gamma y) \geq \mu_A(y)$,
(IF6) $\gamma_A(x\gamma y) \leq \gamma_A(y)$,
for all $x, y \in S$.

Example 3.4. Let $S = \{0, i, -i\}$ and $\Gamma = S$. Then S is Γ-LA-semigroup, by binary operation $S \times \Gamma \times S \longrightarrow S$ as $a\gamma b = a\gamma b$ for all $a, b \in S$ and $\gamma \in S$. Let $A = \langle \mu_A, \gamma_A \rangle$ be IFS on S and Defined $\mu_A : S \longrightarrow [1, 0]$ by $\mu_A(0) = 0.7, \mu_A(i) = \mu_A(-i) = 0.5$ and $\gamma_A : S \longrightarrow [0, 1]$ by $\gamma_A(0) = 0.2, \gamma_A(i) = \gamma_A(-i) = 0.4$ then clearly $A = \langle \mu_A, \gamma_A \rangle$ is an intuitionistic fuzzy left Γ- ideal of Γ-LA-semigroup S.

Example 3.5. Let $S = \{1, 2, 3, 4, 5\}$ and $\Gamma = \{\alpha, \beta, \gamma\}$ be two non-empty sets. Then, S is a Γ-LA-semigroup by the following Tables:

α	1	2	3	4	5
1	1	1	1	1	1
2	1	1	1	1	1
3	1	1	1	1	1
4	1	1	1	1	1
5	1	1	1	1	1

β	1	2	3	4	5
1	2	2	2	2	2
2	2	2	2	2	2
3	2	2	2	2	2
4	2	2	2	2	2
5	2	2	2	2	2

γ	1	2	3	4	5
1	1	1	1	1	1
2	1	1	1	1	1
3	1	1	1	1	1
4	1	1	1	1	1
5	1	1	1	3	3

Also S is non-associative because $(1\alpha 2)\beta 3 \neq 1\alpha\,(2\beta 3)$. Let $A = \langle \mu_A, \gamma_A \rangle$ be an IFS in S and defined as:

$$\begin{aligned} \mu_A\,(1) &= \mu_A\,(2) = 0.8,\ \mu_A\,(3) = 0.7,\ \mu_A\,(4) = 0.6,\ \mu_A\,(5) = 0.3, \\ \gamma_A\,(1) &= \gamma_A\,(2) = 0.1,\ \gamma_A\,(3) = 0.3,\ \gamma_A\,(4) = 0.4,\ \gamma_A\,(5) = 0.7. \end{aligned}$$

Thus, by routine calculation $A = \langle \mu_A, \gamma_A \rangle$ is an intuitionistic fuzzy Γ-ideal of S.

Theorem 3.6. *An intuitionistic fuzzy right Γ-ideal $A = \langle \mu_A, \gamma_A \rangle$ of a Γ-LA-semigroup S with left identity is an intuitionistic fuzzy left Γ- ideal of S.*

Proof. Let $A = \langle \mu_A, \gamma_A \rangle$ be an intuitionistic fuzzy right Γ- ideal of S and let $x, y \in S$ and $\alpha, \beta \in \Gamma$. Then, we have

$$\begin{aligned} \mu_A(x\alpha y) &= \mu_A((e\beta x)\alpha y) = \mu_A((y\beta x)\alpha e) \\ &\geq \mu_A(y\beta x) \geq \mu_A(y) \\ \mu_A(x\alpha y) &\geq \mu_A(y) \end{aligned}$$

and

$$\begin{aligned} \gamma_A(x\alpha y) &= \gamma_A((e\beta x)\alpha y) = \gamma_A((y\beta x)\alpha e) \\ &\leq \gamma_A(y\beta x) \leq \gamma_A(y) \\ \gamma_A(x\alpha y) &\leq \gamma_A(y). \end{aligned}$$

Hence, $A = \langle \mu_A, \gamma_A \rangle$ is an intuitionistic fuzzy left Γ- ideal of S. □

Corollary 3.7. *Let S be a Γ-LA-semigroup with left identity. Then, every intuitionistic fuzzy right Γ- ideal $A = \langle \mu_A, \gamma_A \rangle$ of S is an intuitionistic fuzzy Γ- ideal of S.*

Theorem 3.8. *Let $\{A_i\}_{i\in\Lambda}$ be a family of intuitionistic fuzzy Γ-ideals of a Γ-LA-semigroup S. Then, $\cap A_i$ is an intuitionistic fuzzy Γ-ideal of S, where*

$$\begin{aligned} \cap A_i &= \langle \wedge\mu_{A_i}, \vee\gamma_{A_i} \rangle \text{ and} \\ \wedge\mu_{A_i}(x) &= \inf\{\mu_{A_i}(x) \ / \ i \in \Lambda,\ x \in S\}, \\ \vee\gamma_{A_i}(x) &= \sup\{\gamma_{A_i}(x) \ / \ i \in \Lambda,\ x \in S\}. \end{aligned}$$

Proof. Let $\{A_i\}_{i\in\Lambda}$ be a family of intuitionistic fuzzy Γ-ideals of a Γ-LA-semigroup S and let for any $x, y \in S$ and $\gamma \in \Gamma$. Then, we have

$$\begin{aligned} \wedge\mu_{A_i}(x\gamma y) &\geq \wedge\mu_{A_i}(x) \\ \vee\gamma_{A_i}(x\gamma y) &\leq \vee\gamma_{A_i}(x) \end{aligned}$$

and

$$\begin{aligned} \wedge\mu_{A_i}(x\gamma y) &\geq \wedge\mu_{A_i}(y) \\ \vee\gamma_{A_i}(x\gamma y) &\leq \vee\gamma_{A_i}(y). \end{aligned}$$

Hence, $\cap A_i = \langle \wedge\mu_{A_i}, \vee\gamma_{A_i} \rangle$ is an intuitionistic fuzzy Γ-ideals of a Γ-LA-semigroup S, □

Theorem 3.9. *Let $A = \langle \mu_A, \gamma_A \rangle$ be an intuitionistic fuzzy left (resp. right) Γ- ideal of Γ-LA-semigroup S. Then, $\Box A = \langle \mu_A, \overline{\mu}_A \rangle$, $\overline{\mu}_A = 1 - \mu_A$ is an intuitionistic fuzzy left (resp. right) Γ-ideal of S.*

Proof. Let $A = \langle \mu_A, \gamma_A \rangle$ be an intuitionistic fuzzy left Γ-ideal of a Γ-LA-semigroup S and let for any $x, y \in S$ and $\gamma \in \Gamma$. Then,

$$\begin{aligned} \mu_A(x\gamma y) &\geq \mu_A(y) \\ -\mu_A(x\gamma y) &\leq -\mu_A(y) \\ 1 - \mu_A(x\gamma y) &\leq 1 - \mu_A(y) \\ \bar{\mu_A}(x\gamma y) &\leq \bar{\mu_A}(y). \end{aligned}$$

Hence, $\Box A = \langle \mu_A, \bar{\mu_A} \rangle$ is an intuitionistic fuzzy left Γ- ideal of a Γ-LA-semigroup S $\Box$

Definition 3.10. Let $A = \langle \mu_A, \gamma_A \rangle$ be an IFS in S and $\alpha \in [0, 1]$. Then the sets

$$\mu^{\geq}_{A,\alpha} := \{x \in S \ / \ \mu_A(x) \geq \alpha\},\ \gamma^{\leq}_{A,\alpha} := \{x \in S \ / \ \gamma_A(x) \leq \alpha\}$$

are called a μ-level set and γ-level *set* of A, respectively.

Theorem 3.11. *Let $A = \langle \mu_A, \gamma_A \rangle$ be an IFS in a Γ-LA-semigroup S. Then, $A = \langle \mu_A, \gamma_A \rangle$ is an intuitionistic fuzzy left (resp. right) Γ- ideal of a Γ-LA-semigroup S if and only if μ-level set and γ-level set of A are left (resp. right) Γ- ideals of a Γ-LA-semigroup S.*

Proof. Suppose $\mu^{\geq}_{A,\alpha}(\neq \Phi)$, and $\gamma^{\leq}_{A,\alpha}(\neq \Phi)$ are left Γ- ideals of a Γ-LA-semigroup S for $\alpha \in (0, 1]$. We have to show that $A = \langle \mu_A, \gamma_A \rangle$ is an intuitionistic fuzzy left Γ- ideal of S. Suppose $A = \langle \mu_A, \gamma_A \rangle$ is not an intuitionistic fuzzy left Γ- ideal of S, then there exit $x_\circ, y_\circ$ in S and $\gamma \in \Gamma$ such that

$$\mu_A(x_\circ \gamma y_\circ) < \mu_A(y_\circ)$$

taking

$$\alpha_\circ = \frac{1}{2}\{\mu_A(x_\circ \gamma y_\circ) + \mu_A(y_\circ)\}$$

we have $\mu_A(x_\circ \gamma y_\circ) < \alpha_\circ < \mu_A(y_\circ)$. It follows that $y_\circ \in \mu^{\geq}_{A,\alpha}$ but $x_\circ \gamma y_\circ \notin \mu^{\geq}_{A,\alpha}$ for $\gamma \in \Gamma$. This is a contradiction. Thus

$$\mu_A(x\gamma y) \geq \mu_A(y).$$

Similarly,

$$\gamma_A(x_\circ \gamma y_\circ) \leq \gamma_A(y_\circ).$$

Hence, $A = \langle \mu_A, \gamma_A \rangle$ is an intuitionistic fuzzy left Γ- ideal of a Γ-LA-semigroup S.

Conversely, suppose that $A = \langle \mu_A, \gamma_A \rangle$ is an intuitionistic fuzzy left Γ-ideal of a Γ-LA-semigroup S. Let $\alpha \in [0, 1]$ and for any $x \in S$, $\gamma \in \Gamma$ and $y \in \mu^{\geq}_{A,\alpha}$. Then,

$$\begin{aligned} \mu_A(x\gamma y) &\geq \mu_A(y) \geq \alpha \\ \mu_A(x\gamma y) &\geq \alpha \end{aligned}$$

$x\gamma y \in \mu^{\geq}_{A,\alpha}$ for all $x \in S$, $\gamma \in \Gamma$ and $y \in S$. Hence, $\mu^{\geq}_{A,\alpha}$ is a left Γ- ideal of a Γ-LA-semigroup. Now $x \in S$, $\gamma \in \Gamma$ and $y \in \gamma^{\geq}_{A,\alpha}$. Then,

$$\gamma_A(x\gamma y) \leq \gamma_A(y) \leq \alpha$$

$x\gamma y \in \gamma^{\geq}_{A,\alpha}$ for all $x \in S$, $\gamma \in \Gamma$ and $y \in S$. Hence, $\gamma^{\geq}_{A,\alpha}$ is a left Γ- ideal of a Γ-LA-semigroup S. $\Box$

Example 3.12. Let $S = \{0, i, -i\}$ and $\Gamma = S$ be two non-empty sets. Then S is a Γ-LA-semigroup by binary operation $S \times \Gamma \times S \longrightarrow S$ as $(a, \gamma, b) = a\gamma b$ for all $a, b \in S$ and $\gamma \in \Gamma$. Let $A = \langle \mu_A, \gamma_A \rangle$ be an IFS on S and defined by $\mu_A : S \longrightarrow [1, 0]$ $\mu_A(0) = 0.7, \mu_A(i) = \mu_A(-i) = 0.5$ and $\gamma_A(0) = 0.2, \gamma_A(i) = \gamma_A(-i) = 0.4$. Then, clearly $A = \langle \mu_A, \gamma_A \rangle$ is an intuitionistic fuzzy left Γ-ideal of a Γ-LA-semigroup S.

Theorem 3.13. *Every intuitionistic fuzzy left (right) Γ-ideal of a Γ-LA-semigroup S is an intuitionistic fuzzy bi-Γ-ideal of a Γ-LA-semigroup S.*

Proof. Let $A = \langle \mu_{A,}\gamma_A \rangle$ be an intuitionistic fuzzy left Γ-ideal of a Γ-LA-semigroup S. Let $w, x, y \in S$ and $\alpha, \gamma \in \Gamma$. Then,

$$\begin{aligned} \mu_A((x\alpha w)\gamma y) &\geq \mu_A(y) \\ \mu_A((x\alpha w)\gamma y) &= \mu_A((y\alpha w)\gamma x) \geq \mu_A(y) \\ \mu_A((x\alpha w)\gamma y) &\geq \min\{\mu_A(z), \mu_A(y) \end{aligned}$$

and

$$\begin{aligned} \gamma_A((x\alpha w)\gamma y) &\leq \gamma_A(y) \\ \gamma_A((x\alpha w)\gamma y) &= \gamma_A((y\alpha w)\gamma x) \leq \gamma_A(x) \\ \gamma_A((x\alpha w)\gamma y) &\leq \max\{\gamma_A(x), \gamma_A(y). \end{aligned}$$

Hence, $A = \langle \mu_{A,}\gamma_A \rangle$ is an intuitionistic fuzzy bi-Γ-ideal of a Γ-LA-semigroup S. □

Theorem 3.14. *Let IF(S) denotes the set of all intuitionistic fuzzy left (right)* Γ-*ideal of a* Γ-*LA-semigroup S. Then,* $(IF(S), \subseteq, U, \cap)$ *is a lattice.*

Proof. Let for any $A, B, C \in IF(S)$. Then, we have to show that the following properties hold:

1) Reflexive: Since

$$\mu_A(x) \leq \mu_A(x) \text{ and } \gamma_A(x) \geq \gamma_A(x)$$

so $A \subseteq A$

2) Antisymmetric: For all $A, B \in IF(S)$, we have $A \subseteq B$ and $B \subseteq A$. Then

$$\mu_A(x) \leq \mu_B(x), \gamma_A(x) \geq \gamma_B(x)$$

and

$$\mu_B(x) \leq \mu_A(x), \gamma_B(x) \geq \gamma A(x).$$

Thus, $A = B$

3) Transitive For all $A, B, C \in IF(S)$ such that

$$A \subseteq B \text{ and } B \subseteq C.$$

We have:

$$\mu_A(x) \leq \mu_B(x), \gamma_A(x) \geq \gamma_B(x)$$
$$\mu_B(x) \leq \mu_C(x), \gamma_B(x) \geq \gamma_C(x)$$

and it follows that

$$\mu_A(x) \leq \mu_C(x), \gamma_A(x) \geq \gamma_C(x).$$

Thus $A \subseteq C$. Hence, $(IF(S), \subseteq)$ is a Poset.

Inf: For any two $A, B \in (IF(S)$ $\text{Inf}\{A, B\} = A \cap B$

$$A \cap B = \{\mu_A \wedge \mu_B, \gamma_A \vee \gamma_B$$

Now we show that $A \cap B$ is an intuitionistic fuzzy right Γ-ideal of a Γ-LA-semigroup S. For any $x, y \in S$ and $\alpha \in \Gamma$

$$\begin{aligned} (\mu_A \wedge \mu_B)(x\alpha y) &= \mu_A(x\alpha y) \wedge \mu_A(x\alpha y) \\ &\geq \mu_A(x) \wedge \mu_A(x) = (\mu_A \wedge \mu_B)(x) \\ (\mu_A \wedge \mu_B)(x\alpha y) &\geq (\mu_A \wedge \mu_B)(x) \end{aligned}$$

and

$$\begin{aligned} (\gamma_A \vee \gamma_B)(x\alpha y) &= \gamma_A(x\alpha y) \vee \gamma_A(x\alpha y) \\ &\leq \gamma_A(x) \vee \gamma_A(x) = (\gamma_A \vee \gamma_B)(x) \\ (\gamma_A \vee \gamma_B)(x\alpha y) &\leq (\gamma_A \vee \gamma_B)(x) \end{aligned}$$

$A\cap B$ is an intuitionistic fuzzy right Γ-ideal of a Γ-LA-semigroup S. This means $A\cap B\in IF(S)$, $\inf\{A,B\}$ exists in $IF(S)$.

For any two $A,B\in(IF(S)$ $\sup\{A,B\}=A\cup B$

$$\begin{aligned}
A\cup B &= \{\mu_A\vee\mu_B,\gamma_A\wedge\gamma_B\}\\
(\mu_A\vee\mu_B)(x\alpha y) &= \mu_A(x\alpha y)\vee\mu_A(x\alpha y)\\
&\geq \mu_A(x)\vee\mu_A(x)=(\mu_A\vee\mu_B)(x)\\
(\mu_A\vee\mu_B)(x\alpha y) &\geq (\mu_A\vee\mu_B)(x)
\end{aligned}$$

and

$$\begin{aligned}
(\gamma_A\wedge\gamma_B)(x\alpha y) &= \gamma_A(x\alpha y)\wedge\gamma_A(x\alpha y)\\
&\leq \gamma_A(x)\wedge\gamma_A(x)=(\gamma_A\wedge\gamma_B)(x)\\
(\gamma_A\wedge\gamma_B)(x\alpha y) &\leq (\gamma_A\wedge\gamma_B)(x).
\end{aligned}$$

Thus, $A\cup B$ is an intuitionistic fuzzy right Γ-ideal of a Γ-LA-semigroup S. $\sup\{A,B\}$ exists in $IF(S)$. Hence $(IF(S),\subseteq,U,\cap)$ is a lattice. □

Definition 3.15. Let f be a mapping from a set X to Y and μ be fuzzy set in Y, then the pre-image of μ under f is denoted by $f^{-1}(\mu)$ and is defined by

$$f^{-1}(\mu(x))=\mu(f(x)) \text{ for all } x\in S$$

Definition 3.16. Let $f:S\longrightarrow S_1$ be a homomorphism from a Γ-LA-semigroup S to a Γ-LA-semigroup S_1. If $A=\langle\mu_A,\gamma_A\rangle$ is an intuitionistic fuzzy set in S_1, then the preimage of $A=\langle\mu_A,\gamma_A\rangle$ is denoted by $f^{-1}(A)=\langle f^{-1}(\mu_A),f^{-1}(\gamma_A)\rangle$ and is defined by $f^{-1}(\mu_A(x))=(\mu_A(f(x)))$ and $f^{-1}(\gamma_A(x))=(\gamma_A(f(x)))$

Theorem 3.17. *Let the pair of mappings $f:S\longrightarrow S_1$ and $h:\Gamma\longrightarrow\Gamma_1$ be a homomorphism from a Γ-LA-semigroup S to a Γ-LA-semigroup S_1 and let $A=\langle\mu_A,\gamma_A\rangle$ be an intuitionistic fuzzy left (resp. right) Γ-ideal of a Γ-LA-semigroup S_1. Then, $f^{-1}(A)=\langle f^{-1}(\mu_A),f^{-1}(\gamma_A)\rangle$ is an intuitionistic fuzzy left (resp. right) Γ-ideal of a Γ-LA-semigroup S.*

Proof. Let $A=\langle\mu_A,\gamma_A\rangle$ be an intuitionistic fuzzy left Γ-ideal of a Γ-LA-semigroup S_1. Let $x,y\in S$ and $\alpha\in\Gamma$. Then,

$$\begin{aligned}
f^{-1}(\mu_A(x\alpha y) &= (\mu_A(f(x\alpha y)))=(\mu_A(f(x)h(\alpha)f(y)))\\
f^{-1}(\mu_A(x\alpha y) &\geq \mu_A(f(y))=f^{-1}\mu_A(y)
\end{aligned}$$

and

$$\begin{aligned}
f^{-1}(\gamma_A(x\alpha y)) &= (\gamma_A(f(x\alpha y)))=(\gamma_A(f(x)h(\alpha)f(y)))\\
f^{-1}(\gamma_A(x\alpha y)) &\leq \gamma_A(f(y))=f^{-1}(\gamma_A(y)).
\end{aligned}$$

Hence, $f^{-1}(A)=\langle f^{-1}(\mu_A),f^{-1}(\gamma_A)\rangle$ is an intuitionistic fuzzy left Γ-ideal of Γ-LA-semigroup S. Similarly we will proof for an intuitionistic fuzzy right Γ-ideal of Γ-LA-semigroup S. □

Definition 3.18. Let $f:[1,0]\longrightarrow[1,0]$ be an increasing function and $A=\langle\mu_A,\gamma_A\rangle$ an IFS of a Γ-LA-semigroup S. Then, $A_f=(\mu_{A_f},\gamma_{A_f})$ is an IFS of a Γ-LA-semigroup S, and is defined by $\mu_{A_f}(x)=f(\mu_A(x))$ and $\gamma_{A_f}(x)=f(\gamma_A(x))$ for all $x\in S$.

Proposition 3.19. *Let S be a Γ-LA-semigroup. If $A = \langle \mu_{A,} \gamma_A \rangle$ is an intuitionistic fuzzy left (resp. right) Γ-ideal of S, then $A_f = (\mu_{A_f}, \gamma_{A_f})$ is an intuitionistic fuzzy left (resp. right) Γ-ideal of S.*

Proof. Let $A = \langle \mu_{A,} \gamma_A \rangle$ be an intuitionistic fuzzy left Γ-ideal of S. Let $x, y \in S$ and $\alpha \in \Gamma$ and $A_f = (\mu_{A_f}, \gamma_{A_f})$ an IFS of S. Then,

$$\mu_{A_f}(x\alpha y) = f(\mu_A(x\alpha y)) \geq f(\mu_A(y))$$

and

$$\gamma_{A_f}(x\alpha y) = f(\gamma_A(x\alpha y)) \leq f(\gamma_A(y))$$
$$\mu_{A_f}(x\alpha y) \geq f(\mu_A(y)) \text{ and } \gamma_{A_f}(x\alpha y) \leq f(\gamma_A(y)).$$

Hence, $A_f = (\mu_{A_f}, \gamma_{A_f})$ is an intuitionistic fuzzy left Γ-ideal of S. □

Proposition 3.20. *Let I be a left (resp. right) Γ-ideal of a Γ-LA-semigroup S. Then, $A = (x_{I,}, \overline{x_{I,}})$ is an intuitionistic fuzzy left (resp. right) Γ-ideal of a Γ-LA-semigroup S.*

Proof. Let $y, z \in S$ and $\alpha \in \Gamma$ and $A = (x_{I,} \bar{x_I})$ be IFS of S. Since I is a left Γ-ideal of a Γ-LA-semigroup S, so we have two case's i) if $y \in I$ and ii) $y \notin I$

case i) if $y \in I$, then $y\alpha z \in I$

$$x_I(y) = 1 \text{ and } x_I(y\alpha z) = 1$$

and also

$$x_I(y\alpha z) = 1 = x_I(y)$$

ii) if $y \notin I$, then

$$x_I(y) = 0 \text{ and } x_I(y\alpha z) \geq 0$$
$$x_I(y\alpha z) \geq 0 = x_I(y) \Longrightarrow x_I(y\alpha z) \geq x_I(y)$$

if $y \in I$

$$1 - x_I(y) = 1 - 1 = 0 \text{ and } 1 - x_I(y\alpha z) = 1 - 1 = 0$$
$$\bar{x_I}(y\alpha z) = \bar{x_I}(y)$$

if $y \notin I$, then

$$\bar{x_I}(x) = 1 - x_I(y) = 1 - 0 = 1$$
$$\bar{x_I}(y\alpha z) \leq \bar{x_I}(x)$$

Hence $A = (x_{I,} \bar{x_I})$ is an intuitionistic fuzzy left Γ-ideal of Γ-LA-semigroup S. □

Definition 3.21. Let $A = \langle \mu_{A,} \gamma_A \rangle$ and $B = (\mu_{B,} \gamma_B)$ be two intuitionistic fuzzy sets of a Γ-LA-semigroup S. Then, the product of $A = \langle \mu_{A,} \gamma_A \rangle$ and $B = (\mu_{B,} \gamma_B)$ is denoted by $A \circ_\Gamma B$, and is defined by

$$\mu_{A\Gamma B}(x) = \bigvee_{x = y\alpha z} \{\mu_A(y) \wedge \mu_B(x)\},$$
$$\gamma_{A\Gamma B}(x) = \bigwedge_{x = y\alpha z} \{\gamma_A(y) \vee \gamma_A(z)\}.$$

Lemma 3.22. *Let $A = \langle \mu_{A,} \gamma_A \rangle$ and $B = (\mu_{B,} \gamma_B)$ be any two intuitionistic fuzzy right (left) Γ-ideals of a Γ-LA-semigroup S with left identity. Then, $A\Gamma B$ is an intuitionistic fuzzy right (left) ideal of S*

Theorem 3.23. *Let $IF(S)$ denotes the set of all intuitionistic fuzzy left (right) Γ-ideals of a Γ-LA-semigroup S with left identity. Then, $(IF(S), \Gamma)$ is an LA-semigroup.*

Proof. Let $IF(S)$ denotes the set of all intuitionistic fuzzy left (right) ideals of S. Then, clearly $(IF(S), \Gamma)$ is closed by Lemma 3.22. Let $A = \langle \mu_{A,} \gamma_A \rangle$, $B = \langle \mu_{B,} \gamma_B \rangle$

and $C = \langle \mu_{C,} \gamma_C \rangle \in IF(S)$. Then, we have

$$\begin{aligned}
\mu_{(A\Gamma B)\Gamma C}(x) &= \vee_{x=y\alpha z}\{\mu_{A\Gamma B}(y) \wedge \mu_C(z)\} \\
&= \vee_{x=y\alpha z}\{\vee_{y=p\beta q}\{\mu_A(p) \wedge \mu_B(q)\} \wedge \mu_C(z)\} \\
&= \vee_{x=(p\beta q)\alpha z}\{\mu_A(p) \wedge \mu_B(q) \wedge \mu_C(z)\} \\
&= \vee_{x=(z\beta q)\alpha p}\{\mu_C(z) \wedge \mu_B(q) \wedge \mu_A(p)\} \\
&\leq \vee_{x=w\alpha p}\{\vee_{w=z\beta q}\{\mu_C(z) \wedge \mu_B(q)\} \wedge \mu_C(p)\} \\
&= \vee_{x=w\alpha p}\{\mu_{C\Gamma B}(w) \wedge \mu_A(p)\} = \mu_{(C\Gamma B)\Gamma A}(x) \\
\mu_{(A\Gamma B)\Gamma C}(x) &\leq \mu_{(C\Gamma B)\Gamma A}(x) \\
\text{Similarly } \mu_{(C\Gamma B)\Gamma A}(x) &\leq \mu_{(A\Gamma B)\Gamma C}(x) \text{ and thus } \mu_{(A\Gamma B)\Gamma C}(x) = \mu_{(C\Gamma B)\Gamma A}(x)
\end{aligned}$$

and

$$\begin{aligned}
\gamma_{(A\Gamma B)\Gamma C}(x) &= \wedge_{x=y\alpha z}\{\gamma_{A\Gamma B}(y) \vee \gamma_C(z)\} \\
&= \wedge_{x=y\alpha z}\{\wedge_{y=m\beta n}\{\gamma_A(m) \vee \gamma_B(n)\} \vee \gamma_C(z)\} \\
&= \wedge_{x=(m\beta n)\alpha z}\{\gamma_A(m) \vee \gamma_B(n) \vee \gamma_C(z)\} \\
&= \wedge_{x=(z\beta n)\alpha m}\{\gamma_C(z) \vee \gamma_B(n) \vee \gamma_A(m)\} \\
&\geq \wedge_{x=l\alpha m}\{\wedge_{x=z\beta n}\{\gamma_C(z) \vee \gamma_B(n)\} \vee \gamma_A(m)\} \\
&= \wedge_{x=l\alpha m}\{\gamma_{A\Gamma B}(l) \vee \gamma_C(m)\} = \gamma_{(C\Gamma B)\Gamma A}(x) \\
\gamma_{(A\Gamma B)\Gamma C}(x) &\geq \gamma_{(C\Gamma B)\Gamma A}(x) \\
\text{Similarly } \gamma_{(C\Gamma B)\Gamma A}(x) &\geq \gamma_{(A\Gamma B)\Gamma C}(x) \text{ and thus } \gamma_{(A\Gamma B)\Gamma C}(x) = \gamma_{(C\Gamma B)\Gamma A}(x)
\end{aligned}$$

Hence

$$(A\Gamma B)\Gamma C = (C\Gamma B)\Gamma A$$

Thus $(IF(S), \Gamma)$ is a Γ-LA-semigroup S. ◻

Proposition 3.24. *Let $A = \langle \mu_A, \gamma_A \rangle$ be an intuitionistic fuzzy right Γ-ideal of a Γ -LA-semigroup S with left identity. Then, $A\Gamma A$ is an intuitionistic fuzzy Γ-ideal of S.*

Proof. Since $A = \langle \mu_A, \gamma_A \rangle$ is an intuitionistic fuzzy right Γ-ideal of S, so $A = \langle \mu_A, \gamma_A \rangle$ is an intuitionistic fuzzy left Γ-ideal of S. Let $a, b \in S$ and $\alpha, \gamma \in \Gamma$ if $a \neq x\gamma y$. Then,

$$\mu_{A\Gamma A}(a) = 0 \text{ and } \mu_{A\Gamma A}(a\alpha b) \geq \mu_{A\Gamma A}(a)$$

and

$$\gamma_{A\Gamma A}(a) = 0 \text{ and } \gamma_{A\Gamma A}(a\alpha b) \leq \gamma_{A\Gamma A}(a)$$

otherwise

$$\begin{aligned}
\mu_{A\Gamma A}(a) &= \bigvee_{a=x\gamma y} \{\mu_A(x) \wedge \mu_A(y)\} \\
\text{if } a &= x\gamma y, \text{ then } a\alpha b = (x\gamma y)\alpha b = (b\gamma y)\alpha x \text{ by left invertible law.} \\
\mu_{A\Gamma A}(a) &= \bigvee_{a=x\gamma y} \{\mu_A(y) \wedge \mu_A(x)\} \\
\mu_{A\Gamma A}(a) &\leq \bigvee_{a=x\gamma y} \{\mu_A(b\gamma y) \wedge \mu_A(x)\} \text{ since } A \text{ is IF left } \Gamma\text{-ideal} \\
&\leq \bigvee_{a\alpha b=(b\gamma y)\alpha x} \{\mu_A(b\gamma y) \wedge \mu_A(x)\} = \mu_{A\Gamma A}(a\alpha b) \\
\mu_{A\Gamma A}(a\alpha b) &\geq \mu_{A\Gamma A}(a)
\end{aligned}$$

and

$$\begin{aligned}
\gamma_{A\Gamma A}(a) &= \bigwedge_{a=x\gamma y} \{\gamma_A(x) \vee \gamma_A(y)\} \\
\gamma_{A\Gamma A}(a) &= \bigwedge_{a=x\gamma y} \{\gamma_A(y) \vee \gamma_A(x)\} \\
\gamma_{A\Gamma A}(a) &\geq \bigwedge_{a=x\gamma y} \{\gamma_A(b\gamma y) \vee \gamma_A(x)\} \text{ since } A \text{ is IF left } \Gamma - \text{ideal} \\
&\geq \bigwedge_{a\alpha b=(b\gamma y)\alpha x} \{\gamma_A(b\gamma y) \vee \gamma_A(x)\} = \gamma_{A\Gamma A}(a\alpha b) \\
\gamma_{A\Gamma A}(a\alpha b) &\leq \gamma_{A\Gamma A}(a)
\end{aligned}$$

Hence $A\Gamma A = \langle \mu_{A\Gamma A}, \gamma_{A\Gamma A} \rangle$ is an intuitionistic fuzzy right Γ-ideal of S, and by Theorem 3.6 $A\Gamma A = \langle \mu_{A\Gamma A}, \gamma_{A\Gamma A} \rangle$ is an intuitionistic fuzzy left Γ-ideal of S. □

Theorem 3.25. *Let S be a Γ-LA-semigroup with left identity. Then for any A, B, C IFS of S. $A\Gamma(B\Gamma C) = B\Gamma(A\Gamma C)$.*

Proof. Let $x \in S$ and $A = \langle \mu_A, \gamma_A \rangle, B = \langle \mu_B, \gamma_B \rangle, C = \langle \mu_C, \gamma_C \rangle$ be any IFS of S. Then

$$\begin{aligned}
\mu_{A\Gamma(B\Gamma C)}(x) &= \bigvee_{x=y\alpha z} \{\mu_A(y) \wedge \mu_{B\Gamma C}(z)\} \\
&= \bigvee_{x=y\alpha z} \{\mu_A(y) \wedge [\bigvee_{z=s\beta t} \{\mu_B(s) \wedge \mu_C(t)\}]\} \\
&= \bigvee_{x=y\alpha(s\beta t)} \{\mu_A(y) \wedge \mu_B(s) \wedge \mu_C(t)\} \\
&= \bigvee_{x=s\alpha(y\beta t)} \{\mu_B(s) \wedge \mu_A(y) \wedge \mu_C(t)\}
\end{aligned}$$

since

$$\begin{aligned}
\mu_A(y)\wedge\mu_C(t) &\leq \bigvee_{y\alpha t=a\gamma b}\{\mu_A(a)\wedge\mu_C(b)\}\\
\text{so} \qquad &\leq \bigvee_{x=s\alpha(y\beta t)}\{\mu_B(s)\wedge[\bigvee_{y\beta t=a\gamma b}\{\mu_A(a)\wedge\mu_C(b)\}]\}\\
&= \bigvee_{x=s\alpha(y\beta t)}\{\mu_B(s)\wedge\mu_{A\Gamma C}(y\beta t)\}\\
&\leq \bigvee_{x=p\alpha q}\{\mu_B(p)\wedge\mu_{A\Gamma C}(q)\}=\mu_{B\Gamma(A\Gamma C)}(x)\\
\mu_{A\Gamma(B\Gamma C)}(x) &\leq \mu_{B\Gamma(A\Gamma C)}(x)\Longrightarrow\mu_{A\Gamma(B\Gamma C)}\leq\mu_{B\Gamma(A\Gamma C)}\\
\text{Similarly } \gamma_{A\Gamma(B\Gamma C)}(x) &\geq \gamma_{B\Gamma(A\Gamma C)}(x)\Longrightarrow\gamma_{A\Gamma(B\Gamma C)}\geq\gamma_{B\Gamma(A\Gamma C)}
\end{aligned}$$

and

$$\begin{aligned}
\gamma_{A\Gamma(B\Gamma C)}(x) &= \bigwedge_{x=y\alpha z}\{\gamma_A(y)\vee\gamma_{B\Gamma C}(z)\}\\
&= \bigwedge_{x=y\alpha z}\{\gamma_A(y)\vee[\bigwedge_{z=s\beta t}\{\gamma_B(s)\vee\gamma_C(t)\}]\}\\
&= \bigwedge_{x=y\alpha(s\beta t)}\{\gamma_A(y)\vee\gamma_B(s)\vee\gamma_C(t)\}\\
&= \bigwedge_{x=s\alpha(y\beta t)}\{\gamma_B(s)\vee\gamma_A(y)\vee\gamma_C(t)\}\\
\text{since } \gamma_A(y)\wedge\gamma_C(t) &\geq \bigwedge_{y\alpha t=a\gamma b}\{\gamma_A(a)\wedge\gamma_C(b)\}\\
&\geq \bigwedge_{x=s\alpha(y\beta t)}\{\gamma_B(s)\vee[\bigwedge_{y\beta t=a\gamma b}\{\gamma_A(a)\vee\gamma_C(b)\}]\}\\
&= \bigwedge_{x=s\alpha(y\beta t)}\{\gamma_B(s)\vee\gamma_{A\Gamma C}(y\beta t)\}\\
&\geq \bigwedge_{x=p\alpha q}\{\gamma_B(p)\vee\gamma_{A\Gamma C}(q)\}=\gamma_{B\Gamma(A\Gamma C)}(x)
\end{aligned}$$

Thus $A\Gamma(B\Gamma C)\leq B\Gamma(A\Gamma C)$ and similarly $A\Gamma(B\Gamma C)\geq B\Gamma(A\Gamma C)$. Hence $A\Gamma(B\Gamma C)$ $= B\Gamma(A\Gamma C)$. □

Lemma 3.26. *Let S be a Γ-LA-semigroup. Let $A=\langle\mu_A,\gamma_A\rangle$ be an intuitionistic fuzzy right Γ-ideal of S and $B=\langle\mu_B,\gamma_B\rangle$ an intuitionistic fuzzy left Γ-ideal of S. Then $A\Gamma B\subseteq A\cap B$*

Proof. Let for any $x\in S$ and $\alpha\in\Gamma$. If $x\neq y\alpha z$ for any $y,z\in S$, then

$$\mu_{A\Gamma B}(x)=0\leq\mu_{A\cap B}(x)=\mu_A\wedge\mu_B(x)$$

otherwise

$$\begin{aligned} \mu_{A\Gamma B}(x) &= \bigvee_{x=y\alpha z} \{\mu_A(y) \wedge \mu_B(z)\} \\ &\leq \bigvee_{x=y\alpha z} \{\mu_A(y\alpha z) \wedge \mu_B(y\alpha z)\} \\ &= \bigvee_{x=y\alpha z} \{\mu_A(x) \wedge \mu_B(x)\} \\ \mu_{A\Gamma B}(x) &\leq (\mu_A \wedge \mu_B)(x) \Longrightarrow \mu_{A\Gamma B} \leq (\mu_A \wedge \mu_B). \end{aligned}$$

If $x \neq y\alpha z$ for any $y, z \in S$, then

$$\gamma_{A\Gamma B}(x) = 0 \geq \gamma_{A\cap B}(x) = \gamma_A \vee \gamma_B(x)$$

otherwise

$$\begin{aligned} \mu_{A\Gamma B}(x) &= \bigwedge_{x=y\alpha z} \{\gamma_A(y) \vee \gamma_B(z)\} \\ &\leq \bigwedge_{x=y\alpha z} \{\gamma_A(y\alpha z) \vee \gamma_B(y\alpha z)\} \\ &= \bigwedge_{x=y\alpha z} \{\gamma_A(x) \vee \gamma_B(x)\} \\ \gamma_{A\Gamma B}(x) &\leq (\gamma_A \vee \gamma_B)(x) \Longrightarrow \gamma_{A\Gamma B} \leq (\gamma_A \vee \gamma_B) \end{aligned}$$

Hence $A\Gamma B = \langle \mu_{A\Gamma B}, \gamma_{A\Gamma B}\rangle \subseteq \langle \mu_A \wedge \mu_B, \gamma_A \vee \gamma_B\rangle = A \cap B$. □

Corollary 3.27. *Let S be a Γ-LA-semigroup and $A = \langle \mu_A, \gamma_A\rangle, B = \langle \mu_B, \gamma_B\rangle$ any intuitionistic fuzzy Γ-ideal of S. Then, $A\Gamma B \subseteq A \cap B$*

Remark 3.28. If S is a Γ-LA-semigroup with left identity e and $A = \langle \mu_A, \gamma_A\rangle$ and $B = \langle \mu_B, \gamma_B\rangle$ are intuitionistic fuzzy right Γ-ideals of S. Then $A\Gamma B \subseteq A \cap B$

Remark 3.29. If S is a Γ-LA-semigroup and $A = \langle \mu_A, \gamma_A\rangle$ an intuitionistic fuzzy Γ-ideal of S. Then $A\Gamma A \subseteq A$

Definition 3.30. A Γ-LA-semigroup S is called regular if for every $a \in S$, there exists x in S and $\alpha, \beta \in \Gamma$ such that $a = (a\alpha x)\beta a$, or equivalently, $a \in (a\Gamma S)\Gamma a$.

For regular Γ-LA-semigroup it is easy to see that $S\Gamma S = S$

Proposition 3.31. *Every intuitionistic fuzzy right Γ-ideal of a regular Γ-LA-semigroup S is an intuitionistic fuzzy left Γ-ideal of S.*

Proof. Let $A = \langle \mu_A, \gamma_A\rangle$ be an intuitionistic fuzzy right Γ-ideal of S and $a, b \in S$ and $\gamma \in \Gamma$. Since S is regular, so there exist $x \in S$, and $\alpha, \beta \in \Gamma$ such that $a = (a\alpha x)\beta a$. We have

$$\begin{aligned} \mu_A(a\gamma b) &= \mu_A(((a\alpha x)\beta a)\gamma b) \\ &= \mu_A((b\beta a)\gamma(a\alpha x)) \geq \mu_A(b\beta a) \\ \mu_A(a\gamma b) &\geq \mu_A(b) \end{aligned}$$

and

$$\begin{aligned} \gamma_A(a\gamma b) &= \gamma_A(((a\alpha x)\beta a)\gamma b) \\ &= \gamma_A((b\beta a)\gamma(a\alpha x)) \geq \gamma_A(b\beta a) \\ \gamma_A(a\gamma b) &\geq \gamma_A(b). \end{aligned}$$

Hence, $A = \langle \mu_A, \gamma_A \rangle$ is an intuitionistic fuzzy left Γ-ideal of S. □

Proposition 3.32. *Let $A = \langle \mu_A, \gamma_A \rangle$ and $B = \langle \mu_B, \gamma_B \rangle$ be any intuitionistic fuzzy right Γ- ideal of regular Γ - LA - semigroup S. Then, $A\Gamma B = A \cap B$*

Proof. Since S regular, so by Proposition 3.31 and Lemma 3.26, we have $A\Gamma B \subseteq A \cap B$.

On the other hand, let $a \in S$. Then, there exist $x \in S$ and $\alpha, \beta \in \Gamma$ such that $a = (a\alpha x)\beta a$. Thus

$$\begin{aligned} (\mu_A \wedge \mu_B)(a) &= \mu_A(a) \wedge \mu_B(a) \\ &\leq \mu_A(a\alpha x) \wedge \mu_B(a) \\ &\leq \bigvee_{a=(a\alpha x)\beta a} \mu_A(a\alpha x) \wedge \mu_B(a) \\ (\mu_A \wedge \mu_B)(a) &\leq \mu_{A\Gamma B}(a) \Longrightarrow \mu_A \wedge \mu_B \leq \mu_{A\Gamma B} \end{aligned}$$

and

$$\begin{aligned} (\gamma_A \vee \gamma_B)(a) &= \gamma_A(a) \vee \gamma_B(a) \\ &\geq \gamma_A(a\alpha x) \vee \gamma_B(a) \\ &\geq \bigwedge_{a=(a\alpha x)\beta a} \gamma_A(a\alpha x) \vee \gamma_B(a) \\ (\gamma_A \vee \gamma_B)(a) &\geq \gamma_{A\Gamma B}(a) \Longrightarrow \gamma_A \vee \gamma_B \geq \gamma_{A\Gamma B} \end{aligned}$$

Thus $A \cap B \subseteq A\Gamma B$, therefore

$$A\Gamma B \subseteq A \cap B \text{ and } A \cap B \subseteq A\Gamma B \Longrightarrow A \cap B = A\Gamma B.$$

□

Definition 3.33. A Γ-LA-semigroup S is called Γ-LA band if all of its elements are idempotent *i.e* for all $x \in S$, there exist $\alpha \in \Gamma$, such that $x\alpha x = x$.

Theorem 3.34. *The concept of intuitionistic fuzzy right and left Γ-ideal in a Γ-LA band coincides.*

Proof. Let $A = \langle \mu_A, \gamma_A \rangle$ be an intuitionistic fuzzy right Γ-ideal in a Γ-LA band S and $x, y \in S$ and $\alpha, \beta, \gamma \in \Gamma$. Then

$$\begin{aligned} \mu_A(x\alpha y) &= \mu_A((x\beta x)\alpha y) \\ &= \mu_A((y\beta x)\alpha x) \text{ by left invertible law} \\ &\geq \mu_A(y\beta x) \geq \mu_A(y) \\ \mu_A(x\alpha y) &\geq \mu_A(y) \end{aligned}$$

and

$$\begin{aligned}\gamma_A(x\alpha y) &= \gamma_A((x\beta x)\alpha y)\\ &= \gamma_A((y\beta x)\alpha x) \text{ by left invertible law}\\ &\leq \gamma_A(y\beta x) \leq \gamma_A(y)\\ \mu_A(x\alpha y) &\leq \mu_A(y)\end{aligned}$$

Therefore $A = \langle \mu_A, \gamma_A \rangle$ is an intuitionistic fuzzy left Γ-ideal in a Γ-LA band S

Conversely suppose that $A = \langle \mu_A, \gamma_A \rangle$ is an intuitionistic fuzzy left Γ-ideal in a Γ-LA band S and $x, y \in S$ and $\alpha, \beta, \gamma \in \Gamma$. Then

$$\begin{aligned}\mu_A(x\alpha y) &= \mu_A((x\beta x)\alpha y)\\ &= \mu_A((y\beta x)\alpha y)) \geq \mu_A(y\beta x)\\ &\Longrightarrow \mu_A(x\alpha y) \geq \mu_A(x)\end{aligned}$$

and

$$\begin{aligned}\gamma_A(x\alpha y) &= \gamma_A((x\beta x)\alpha y)\\ &= \gamma_A((y\beta x)\alpha y) \geq \gamma_A(y\beta x)\\ &\Longrightarrow \gamma_A(x\alpha y) \geq \gamma_A(x)\end{aligned}$$

Thererfore $A = \langle \mu_A, \gamma_A \rangle$ is an intuitionistic fuzzy right Γ-ideal in a Γ-LA band. S □

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