Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 6, No. 1, (July 2013), pp. 115–126
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr

Intuitionistic fuzzy rough relations

Anjan Mukherjee, Ajoy Kanti Das

Received 18 July 2012; Revised 19 September 2012; Accepted 2 October 2012

Abstract. In this paper, we introduce intuitionistic fuzzy rough relation on a set and then it is to be proved that the collection of such relations is closed under different binary compositions such as, algebraic sum, algebraic product etc. Also the definitions of reflexive, symmetric, anti symmetric, transitive and anti transitive intuitionistic fuzzy rough relations on a set are to be defined and a few properties of them are to be investigated. Lastly, we define an operation '©' which is a composition of two intuitionistic fuzzy rough relations, with the help of '$\circ$'(maxmin relation) and '$\diamond$'(minmax relation). Thereafter it is shown that the collection of such relations is closed under the operation '©'.

2010 AMS Classification: 03E72, 03E02.

Keywords: Fuzzy set, Intuitionistic fuzzy set, Rough set, Rough relations, Fuzzy rough relations, Intuitionistic fuzzy rough relations.

Corresponding Author: Anjan Mukherjee (anjan2002_m@yahoo.co.in)

1. Introduction

Theory of fuzzy sets and rough sets are powerful mathematical tools for modeling various types of uncertainty. In 1965, Prof. L. A. Zadeh [11] initiated the concept of fuzzy set theory, thereafter in 1982, the concept of rough set theory was first given by Pawlak [8]. The concept of fuzzy relation on a set was defined by Prof. L. A. Zadeh [12] and several authors have considered it further. In fact, all these concepts have good applications in other disciplines and real life problems.

Nanda and Majumdar (1993) [6] introduced the notion of fuzzy rough sets. In 1998, Chakrabarty et al.'s [2] approached intuitionistic fuzzy rough sets(IF rough set), they constructed an IF rough set (A,B) of the rough set (P,Q), where A and B are both IF sets in X such that A $\subseteq$ B i.e. $\mu_A \leq \mu_B$ and $\nu_A \geq \nu_B$.

From this point of view the lower approximation A and the upper approximation B are both IF sets. Jena and Ghosh (2002)[4] reintroduced the same notion.

Samanta and Mondal (2001)[10] also introduced this notion but they called it a rough IF set. They also defined the concept of IF rough set according to them an IF rough set is a couple (A,B) such that A and B are both fuzzy rough sets (in the sense of Nanda and Majumdar[6]) and A is included in the complement of B according to Samanta and Mondal (2001)[10] an intuitionistic fuzzy rough set (A,B) is a generalization of an IF set in which membership and non-membership functions are no longer fuzzy sets but fuzzy rough sets A and B. On the other hand, for Chakrabarty et al.[2], an intuitionistic fuzzy rough set [13] (A,B) is a generalization of a fuzzy rough set in which upper and lower approximation are no longer fuzzy sets but IF sets A and B. Rizvi et al.(2002)[9] described their proposal as "Rough intuitionistic fuzzy set" in which the lower and upper approximations themselves are not Intuitionistic fuzzy sets in X but intuitionistic fuzzy sets in the class of equivalence classes. Recently Gangwal and Bhaumik (2012)[3] defined Intuitionistic fuzzy rough relation and applied it to some medical applications. But the theories of Intuitionistic fuzzy rough relations have not been developed there.

In this paper, we introduce the intuitionistic fuzzy rough relation on a set in a different approach. It is to be proved that the collection of such relations is closed under different binary compositions such as, algebraic sum, algebraic product etc. We also define reflexive, symmetric, anti symmetric, transitive and anti transitive intuitionistic fuzzy rough relations on a set and investigate a few properties on them.

We now give some ready references for further discussion. After the introduction of the concept of fuzzy set by Zadeh [8], several researchers were conducted on the generalization of the notion of a fuzzy set. The idea of "Intuitionistic fuzzy set" was first published by Atanassov [1].

Definition 1.1 ([11])**.** Let U be a non empty set. Then a fuzzy set A on U is a set having the form A=$\{$(x, μ_A(x)): x $\in$ U$\}$ where the function μ_A:U$\to$[0,1] is called the membership function and μ_A(x) represents the degree of membership of each element x $\in$ U.

Definition 1.2 ([1])**.** Let U be a non empty set. Then an intuitionistic fuzzy set (IFS for short) A on U is a set having the form A=$\{\langle$x, μ_A(x), ν_A(x)$\rangle$: x $\in$ U$\}$ where the functions μ_A:U$\to$[0,1] and ν_A:U$\to$[0,1] represents the degree of membership and the degree of non-membership respectively of each element x$\in$U and 0$\leq\mu_A$(x)+ ν_A(x)$\leq$1 for each x$\in$U.

Let U be a nonempty set of universe and R be a equivalence relation on U, then (U, R) is called approximation space. The product space is also an approximation space, which is (U$\times$U,S) , where the indiscernibility relation S$\subseteq$U$\times$U is defined by ((x_1,y_1),(x_2,y_2))$\in$S if and only if (x_1,y_1)$\in$R and (x_2,y_2)$\in$R, for each x_1,y_1,x_2,$y_2$$\in$U. It can be easily verified that S is an equivalence relation on U. The elements (x_1,y_1) and (x_2,y_2) are indiscernible in S if and only if the elements x_1 and x_2 are indiscernible in R and so y_1, y_2 are in R. This implies that the equivalence class containing the element (x, y) with respect to S denoted by $[x,y]_s$ and should be equal to the Cartesian product of $[x]_R$ and $[y]_R$.

The concepts of rough set can be easily extended to a relation, mainly due to the fact that a relation is also a set, i.e. a subset of a Cartesian product. So, let (U,R) be an approximation space. Let X$\subseteq$U. A relation T on X is said to be a rough relation [5, 7] on X if $\underline{T}\neq\overline{T}$, where $\underline{T}$ and $\overline{T}$ are lower and upper approximation of T, respectively defined by $\overline{T} = \{(x,y) \in U \times U : [x,y]_S \bigcap X \times X \neq \varnothing\}$ and $\underline{T} = \{(x,y) \in U \times U : [x,y]_S \subseteq X \times X\}$.

2. Intuitionistic fuzzy rough relations

In this section we introduce **Intuitionistic Fuzzy Rough Relation** on a set and study their basic properties. Before we can prepare a hybrid theory, it is necessary to check the origin of all ingredients, for they can have an important influence on the flavor of the resulting product.

Let A=(U,R) be an approximation space and X⊆U be a rough set in A. Let Y be a fuzzy set in U with membership grade μ_Y: X→[0,1]. Then Y is said to be fuzzy rough set in (U,R) if the following condition holds:

(i) μ_Y(x)=1, ∀x∈$\underline{X}$

(ii) μ_Y(x)=0, ∀x∈(U\\$\overline{X}$)

(iii) 0<μ_Y(x)<1, ∀x∈($\overline{X}$)\\$\underline{X}$).

Let A=(U,R) be approximation space and X⊆U be be a rough set in A. Let Y be a fuzzy set in A characterized by the membership function μ_Y:U→[0,1]. Then the product Y × Y is defined by the membership function $\mu_{Y\times Y}$(x,y)=min{μ_Y(x),μ_Y(y)}, ∀(x,y)∈U×U.

A Fuzzy Rough Relation R_1 on A is a fuzzy subset of Y×Y

i.e. ∀(x,y)∈U×U, μ_{R_1}(x,y)≤$\mu_{Y\times Y}$(x,y), satisfying the following:

(i) μ_{R_1}(x,y) =1, ∀(x,y)∈$\underline{X \times X}$

(ii) μ_{R_1}(x,y)=0, ∀(x,y)∈[U×U\\$\overline{X \times X}$]

(iii) 0<μ_{R_1}(x,y)<1, ∀(x,y)∈[$\overline{X \times X}$\\$\underline{X \times X}$]

where $\underline{X \times X}$={(x,y)∈U×U: $[x,y]_S$ ⊆ X×X } and

$\overline{X \times X}$={(x,y)∈U×U: $[x,y]_S$ ⋂ X×X ≠ ∅}.

Definition 2.1. Let A=(U,R) be approximation space and X⊆U be be a rough set in A. Let Y be a IF set in A characterized by the membership function μ_Y:U→[0,1] and non-membership functionν_Y: U→[0,1], Then the product Y × Y is defined by the membership function $\mu_{Y\times Y}$(x,y)=min{μ_Y(x),μ_Y(y)} and the non membership function $\nu_{Y\times Y}$(x,y)=max{ν_Y(x),ν_Y(y)}, ∀(x,y)∈U×U.

An Intuitionistic Fuzzy Rough Relation(in short IFR relation)R_1 on Y is an IF rough subset of Y×Y

i.e. ∀(x,y)∈U×U, μ_{R_1}(x,y) ≤ $\mu_{Y\times Y}$(x,y) and ν_{R_1}(x,y) ≥ $\nu_{Y\times Y}$(x,y), satisfying the following:

(i) μ_{R_1}(x,y)=1 and ν_{R_1}(x,y)=0, ∀(x,y)∈$\underline{X \times X}$

(ii) μ_{R_1}(x,y)=0 and ν_{R_1}(x,y)=1, ∀(x,y)∈[U×U\\$\overline{X \times X}$]

(iii) 0<μ_{R_1}(x,y), ν_{R_1}(x,y)<1, ∀(x,y)∈[$\overline{X \times X}$\\$\underline{X \times X}$].

where $\underline{X \times X}$={(x,y)∈U×U: $[x,y]_S$ ⊆ X×X }

$\overline{X \times X}$={(x,y)∈U×U: $[x,y]_S$⋂ X×X ≠ ∅} .

The pair [μ_{R_1},ν_{R_1}] is called the IFR relation on X⊆U w. r. t. (U×U,S).

Definition 2.2. If [μ_{R_1},ν_{R_1}] and [μ_{R_2},ν_{R_2}] are two IFR relations on X⊆U, then for every (x,y)∈U×U, we define

[i] [μ_{R_1},ν_{R_1}]⊆ [μ_{R_2},ν_{R_2}] ⇔ μ_{R_1}(x,y)≤μ_{R_2}(x,y) and ν_{R_1}(x,y)≥ν_{R_2}(x,y).

[ii] Union: [μ_{R_1},ν_{R_1}]⋁[μ_{R_2},ν_{R_2}]=[μ_{R_1} ⋁μ_{R_2},ν_{R_1}⋀ν_{R_2}]

where (μ_{R_1}⋁μ_{R_2})(x,y)=μ_{R_1}(x,y)⋁μ_{R_2}(x,y)= max{μ_{R_1}(x,y), μ_{R_2}(x,y)},

$(\nu_{R_1}\bigwedge\nu_{R_2})$(x,y)=$\nu_{R_1}(x,y)\bigwedge\nu_{R_2}$(x,y)= min{$\nu_{R_1}$(x,y), ν_{R_2}(x,y)}.
[iii] intersection: $[\mu_{R_1},\nu_{R_1}]\bigwedge[\mu_{R_2},\nu_{R_2}]=[\mu_{R_1}\bigwedge\mu_{R_2},\ \nu_{R_1}\bigvee\nu_{R_2}]$
where $(\mu_{R_1}\bigwedge\mu_{R_2})$(x,y)=$\mu_{R_1}(x,y)\bigwedge\mu_{R_2}$(x,y)= min{$\mu_{R_1}$(x,y), μ_{R_2}(x,y)},
$(\nu_{R_1}\bigvee\nu_{R_2})$(x,y)=$\nu_{R_1}(x,y)\bigvee\nu_{R_2}$(x,y)= max{$\nu_{R_1}$(x,y), ν_{R_2}(x,y)}.
[iv] Algebraic Product: $[\mu_{R_1},\nu_{R_1}]\cdot[\mu_{R_2},\nu_{R_2}]=[\mu_{R_1}\cdot\mu_{R_2},\nu_{R_1}\cdot\nu_{R_2}]$
where $(\mu_{R_1}\cdot\mu_{R_2})$(x,y)=$\mu_{R_1}(x,y)\cdot\mu_{R_2}$(x,y),
$(\nu_{R_1}\cdot\nu_{R_2})$(x,y)=$\nu_{R_1}(x,y)\cdot\nu_{R_2}$(x,y).
[v] Algebraic Sum: $[\mu_{R_1},\nu_{R_1}]\oplus[\mu_{R_2},\nu_{R_2}]=[\mu_{R_1}\oplus\mu_{R_2},\nu_{R_1}.\nu_{R_2}]$
where $(\mu_{R_1}\oplus\mu_{R_2})$(x,y)=$\mu_{R_1}$(x,y)+$\mu_{R_2}$(x,y) - μ_{R_1}(x,y)$\cdot\mu_{R_2}$(x,y),
$(\nu_{R_1}.\nu_{R_2})$(x,y)=ν_{R_1}(x,y)$\cdot\nu_{R_2}$(x,y).
[vi] Arithmetic Mean: $[\mu_{R_1},\nu_{R_1}]@[\mu_{R_2},\nu_{R_2}]=[\mu_{R_1}@\mu_{R_2},\nu_{R_1}@\nu_{R_2}]$
where $(\mu_{R_1}@\mu_{R_2})$(x,y)=(μ_{R_1}(x,y)+μ_{R_2}(x,y))/2,
$(\nu_{R_1}@\nu_{R_2})$(x,y)=(ν_{R_1}(x,y)+ν_{R_2}(x,y))/2.
[vi] Geometric Mean: $[\mu_{R_1},\nu_{R_1}]\sharp[\mu_{R_2},\nu_{R_2}]=[\mu_{R_1}\sharp\mu_{R_2},\nu_{R_1}\sharp\nu_{R_2}]$
where $(\mu_{R_1}\sharp\mu_{R_2})$(x,y)=$\sqrt{\mu_{R_1}(x,y)\cdot\mu_{R_2}(x,y)}$ and
$(\nu_{R_1}\sharp\nu_{R_2})$(x,y)=$\sqrt{\nu_{R_1}(x,y)\cdot\nu_{R_2}(x,y)}$.

Proposition 2.3. *Union and intersection of two IFR relations on X⊆U are also an IFR relations on $X\subseteq U$.*

Proof. Let $[\mu_{R_1},\nu_{R_1}]$ and $[\mu_{R_2},\nu_{R_2}]$ are two IFR relations on X⊆U and let

$$\mu^{/}=\mu_{R_1}\bigvee\mu_{R_2},\ \nu^{/}=\nu_{R_1}\bigwedge\nu_{R_2}.$$

Then $\mu^{/}$(x,y)=$(\mu_{R_1}\bigvee\mu_{R_2})$(x,y)= max{$\mu_{R_1}$(x,y), μ_{R_2}(x,y)}. Since

$$[\mu_{R_1},\nu_{R_1}] \text{ and } [\mu_{R_2},\nu_{R_2}]$$

are two IFR relations, therefore
μ_{R_1}(x,y)=1=μ_{R_2}(x,y), ∀(x,y)∈ $\underline{X\times X}$ ⇒ max{μ_{R_1}(x,y), μ_{R_2}(x,y)}=1
⇒ $\mu^{/}$(x,y)=1, ∀(x,y)∈ $\underline{X\times X}$.
Also, since $[\mu_{R_1},\nu_{R_1}]$ and $[\mu_{R_2},\nu_{R_2}]$ are two IFR relations,
⇒ μ_{R_1}(x,y)= 0 =μ_{R_2}(x,y), ∀(x,y)∈[U×U\\$\overline{X\times X}$]
therefore max{μ_{R_1}(x,y), μ_{R_2}(x,y)}=0
⇒ $\mu^{/}$(x,y)=0, ∀(x,y)∈[U×U\\$\overline{X\times X}$].
Again since 0<μ_{R_1}(x,y),μ_{R_2}(x,y)<1, ∀(x,y)∈[$\overline{X\times X}$\\$\underline{X\times X}$]
⇒ 0<max{μ_{R_1}(x,y),μ_{R_2}(x,y)}<1.
Thus 0<$\mu^{/}$(x,y)<1, ∀(x,y)∈[U×U\\$\overline{X\times X}$].

and $\nu^{/}$(x,y)=$(\nu_{R_1}\bigwedge\nu_{R_2})$(x,y)= min{$\nu_{R_1}$(x,y), ν_{R_2}(x,y)}.
Now since $[\mu_{R_1},\nu_{R_1}]$ and $[\mu_{R_2},\nu_{R_2}]$ are two IFR relations,
therefore ν_{R_1}(x,y)=0=ν_{R_2}(x,y), ∀(x,y)∈$\underline{X\times X}$
⇒ min{ν_{R_1}(x,y), ν_{R_2}(x,y)}=0.
Thus $\nu^{/}$(x,y)=0, ∀(x,y)∈$\underline{X\times X}$.
Also since $[\mu_{R_1},\nu_{R_1}]$ and $[\mu_{R_2},\nu_{R_2}]$ are two IFR relations,
therefore ν_{R_1}(x,y)=1=ν_{R_2}(x,y), ∀(x,y)∈[U×U\\$\overline{X\times X}$]
⇒ min{ν_{R_1}(x,y), ν_{R_2}(x,y)}=1.
Thus $\nu^{/}$(x,y)=1, ∀(x,y)∈[U×U\\$\overline{X\times X}$].
Again since 0<ν_{R_1}(x,y), ν_{R_2}(x,y)<1, ∀(x,y)∈[$\overline{X\times X}$\\$\underline{X\times X}$]

$\Rightarrow$ 0<min{ν_{R_1}(x,y), ν_{R_2}(x,y)}<1.
$\Rightarrow$ 0<$\mu^/$(x,y)<1, $\forall$(x,y)$\in$[U$\times$U$\setminus\overline{X \times X}$].

Thus [$\mu^/$,$\nu^/$] i.e. [μ_{R_1},ν_{R_1}]$\bigvee$[μ_{R_2},ν_{R_2}] is an IFR relation on X$\subseteq$U.

Similarly we can prove that intersection of two IFR relations on X$\subseteq$U is also an IFR relation on X$\subseteq$U. □

Proposition 2.4. *Algebraic product of two IFR relations on X$\subseteq$U is also an IFR relation on X$\subseteq$U..*

Proof. Let [μ_{R_1},ν_{R_1}] and [μ_{R_2},ν_{R_2}] are two IFR relations on X$\subseteq$U and
let $\mu^/$=$\mu_{R_1}\cdot\mu_{R_2}$, $\nu^/$= $\nu_{R_1}\cdot\nu_{R_2}$. Then $\mu^/$(x,y)=μ_{R_1}(x,y)$\cdot\mu_{R_2}$(x,y).
Since [μ_{R_1},ν_{R_1}] and [μ_{R_2},ν_{R_2}] are two IFR relations,
$\Rightarrow$ μ_{R_1}(x,y)= 1 = μ_{R_2}(x,y), $\forall$(x,y)$\in\underline{X \times X}$
therefore μ_{R_1}(x,y)$\cdot\mu_{R_2}$(x,y)=1
$\Rightarrow$ $\mu^/$(x,y)=1, $\forall$(x,y)$\in\underline{X \times X}$.
Also, since [μ_{R_1},ν_{R_1}] and [μ_{R_2},ν_{R_2}] are two IFR relations,
$\Rightarrow$ μ_{R_1}(x,y)= 0 = μ_{R_2}(x,y), $\forall$(x,y)$\in$[U$\times$U$\setminus\overline{X \times X}$]
therefore μ_{R_1}(x,y)$\cdot\mu_{R_2}$(x,y)=0
$\Rightarrow$ $\mu^/$(x,y)=0, $\forall$(x,y)$\in$[U$\times$U$\setminus\overline{X \times X}$].
Again, since 0<μ_{R_1}(x,y), μ_{R_2}(x,y)<1, $\forall$(x,y)$\in$[$\overline{X \times X}\setminus\underline{X \times X}$]
$\Rightarrow$ 0 < μ_{R_1}(x,y)$\cdot\mu_{R_2}$(x,y) < 1
$\Rightarrow$ 0 < $\mu^/$(x,y)< 1, $\forall$(x,y)$\in$[U$\times$U$\setminus\overline{X \times X}$]
and $\nu^/$(x,y)=ν_{R_1}(x,y)$\cdot\nu_{R_2}$(x,y)
Now since [μ_{R_1},ν_{R_1}] and [μ_{R_2},ν_{R_2}] are two IFR relations,
$\Rightarrow$ ν_{R_1}(x,y)= 0 = ν_{R_2}(x,y), $\forall$(x,y)$\in\underline{X \times X}$
therefore ν_{R_1}(x,y)$\cdot\nu_{R_2}$(x,y)=0.
$\Rightarrow$ $\nu^/$(x,y)=0, $\forall$(x,y)$\in\underline{X \times X}$.
Also since [μ_{R_1},ν_{R_1}] and [μ_{R_2},ν_{R_2}] are two IFR relations,
$\Rightarrow$ ν_{R_1}(x,y)= 1 = ν_{R_2}(x,y), $\forall$(x,y)$\in$[U$\times$U$\setminus\overline{X \times X}$]
therefore ν_{R_1}(x,y)$\cdot\nu_{R_2}$(x,y)=1.
$\Rightarrow$ $\nu^/$(x,y)=1, $\forall$(x,y)$\in$ [U$\times$U$\setminus\overline{X \times X}$].
Again, since 0 < ν_{R_1}(x,y), ν_{R_2}(x,y)< 1, $\forall$(x,y)$\in$[$\overline{X \times X}\setminus\underline{X \times X}$]
$\Rightarrow$ 0 < ν_{R_1}(x,y)$\cdot\nu_{R_2}$(x,y) < 1
$\Rightarrow$ 0 < $\nu^/$(x,y) < 1, $\forall$(x,y)$\in$[U$\times$U$\setminus\overline{X \times X}$].
Thus [$\mu^/$,$\nu^/$] i.e. [μ_{R_1},ν_{R_1}]$\cdot$[μ_{R_2},ν_{R_2}] is an IFR relation on X$\subseteq$U. □

Proposition 2.5. *Algebraic sum of two IFR relations on X$\subseteq$U is also an IFR relation on X$\subseteq$U.*

Proof. Same as above. □

Proposition 2.6. *The arithmetic mean and geometric mean of two IFR relations on X$\subseteq$U are also IFR relations on X$\subseteq$U.*

Proof. Obvious. □

Definition 2.7. An IFR relation $[\mu_{R_1},\nu_{R_1}]$ on X⊆U is said to be reflexive IFR relation if μ_{R_1}(x,x)=1 and ν_{R_1}(x,x)=0, ∀x∈U.

Definition 2.8. An IFR relation $[\mu_{R_1},\nu_{R_1}]$ on X⊆U is said to be reflexive IFR relation of order α>0, if μ_{R_1}(x,x)≥α and ν_{R_1}(x,x)≤α, ∀x∈U.

Definition 2.9. An IFR relation $[\mu_{R_1},\nu_{R_1}]$ on X⊆U is said to be weakly reflexive IFR relation if μ_{R_1}(x,x)≥μ_{R_1}(x,y) and ν_{R_1}(x,x)≤ν_{R_1}(x,y) , ∀x,y∈U.

Definition 2.10. An IFR relation $[\mu_{R_1},\nu_{R_1}]$ on X⊆U is said to be w-reflexive IFR relation if μ_{R_1}(x,x) ≥ μ_Y(x) and ν_{R_1}(x,x) ≤ ν_Y(x) , ∀ x∈U.

Theorem 2.11. *Intersection and union of two reflexive IFR relations on X⊆U is also reflexive IFR relations on X⊆U.*

Proof. Let $[\mu_{R_1},\nu_{R_1}]$ and $[\mu_{R_2},\nu_{R_2}]$ be two reflexive IFR relations on X⊆U and let
$\mu^/=\mu_{R_1}\bigwedge\mu_{R_2}$, $\nu^/=\nu_{R_1}\bigvee\nu_{R_2}$
Then $\mu^/$(x,y)=$(\mu_{R_1}\bigwedge\mu_{R_2})$(x,y)= min{$\mu_{R_1}$(x,y), μ_{R_2}(x,y)}
and $\nu^/$(x,y)=$(\nu_{R_1}\bigvee\nu_{R_2})$(x,y)= max{$\nu_{R_1}$(x,y), ν_{R_2}(x,y)}.

Since $[\mu_{R_1},\nu_{R_1}]$ and $[\mu_{R_2},\nu_{R_2}]$ be two reflexive IFR relations,
therefore μ_{R_1}(x,x)= 1 = μ_{R_2}(x,x) and ν_{R_1}(x,x)= 0 = ν_{R_2}(x,x) ∀ x ∈ U
⇒ min{μ_{R_1}(x,x), μ_{R_2}(x,x)}=1 and max{ν_{R_1}(x,x), ν_{R_2}(x,x)}=0.
Thus $\mu^/$(x,x)=1 and $\nu^/$(x,x)=0, ∀ x ∈ U.
Hence $[\mu_{R_1},\nu_{R_1}]\bigwedge[\mu_{R_2},\nu_{R_2}]$ is reflexive IFR relation on X⊆U. Similarly we can prove that union of two reflexive IFR relations on X⊆U is also reflexive IFR relation on X⊆U. □

Proposition 2.12. *Algebraic product of two reflexive IFR relations on X⊆U is also reflexive IFR relation on X⊆U.*

Proof. Obvious. □

Theorem 2.13. *The arithmetic mean and geometric mean of two reflexive IFR relations on X⊆U are also reflexive IFR relations on X⊆U.*

Proof. Let $[\mu_{R_1},\nu_{R_1}]$ and $[\mu_{R_2},\nu_{R_2}]$ are two reflexive IFR relations on X⊆U and let
$\mu^/=\mu_{R_1}@\mu_{R_2}$ and $\nu^/=\nu_{R_1}@\nu_{R_2}$
Then ∀x∈U,
$\mu^/$(x,x)=$(\mu_{R_1}@\mu_{R_2})$(x,x) =(μ_{R_1}(x,x)+μ_{R_2}(x,x))/2=1,
[since $[\mu_{R_1},\nu_{R_1}]$ and $[\mu_{R_2},\nu_{R_2}]$ are two reflexive IFR relations
⇒ μ_{R_1}(x,x)=1=μ_{R_2}(x,x), ∀x∈U]
and $\nu^/$(x,x)=$(\nu_{R_1}@\nu_{R_2})$(x,x)=(ν_{R_1}(x,x)+ν_{R_2}(x,x))/2=0,
[since $[\mu_{R_1}, \nu_{R_1}]$ and $[\mu_{R_2}, \nu_{R_2}]$ are two reflexive IFR relations
⇒ ν_{R_1}(x,x)=0=ν_{R_2}(x,x), ∀x∈U].
Thus arithmetic mean of two reflexive IFR relations is also reflexive IFR relation. Similarly we can prove that geometric mean of two reflexive IFR relations is also reflexive IFR relation. □

3. Composition of two IFR relations

Definition 3.1. Let $[\mu_{R_1},\nu_{R_1}]$ and $[\mu_{R_2},\nu_{R_2}]$ are two IFR relations on $X\subseteq U$. Then composition of two IFR relation denoted by $[\mu_{R_1},\nu_{R_1}]©[\mu_{R_2},\nu_{R_2}]$ is defined by

$([\mu_{R_1},\nu_{R_1}]©[\mu_{R_2},\nu_{R_2}])(x,y) = [(\mu_{R_1}\circ\mu_{R_2})(x,y),(\nu_{R_1}\diamondsuit\nu_{R_2})(x,y)]$

where $(\mu_{R_1}\circ\mu_{R_2})(x,y)=\max_{u\in U}\min\{\mu_{R_1}(x,u),\ \mu_{R_2}(u,y)\}$

$(\nu_{R_1}\diamondsuit\nu_{R_2})(x,y)=\min_{u\in U}\max\{\nu_{R_1}(x,u),\ \nu_{R_2}(u,y)\}$, $\forall x,y\in U$.

Property:

(i) Commutative: $[\mu_{R_1},\nu_{R_1}]©[\mu_{R_2},\nu_{R_2}] = [\mu_{R_2},\nu_{R_2}]©[\mu_{R_1},\nu_{R_1}]$

(ii) Associative:

$[\mu_{R_1},\nu_{R_1}]©([\mu_{R_2},\nu_{R_2}]©[\mu_{R_3},\nu_{R_3}])=([\mu_{R_1},\nu_{R_1}]©[\mu_{R_2},\nu_{R_2}])©[\mu_{R_3},\nu_{R_3}]$.

Theorem 3.2. *Let $[\mu_{R_1},\nu_{R_1}]$ and $[\mu_{R_2},\nu_{R_2}]$ are two IFR relations on $X\subseteq U$. Then $[\mu_{R_1},\nu_{R_1}]©[\mu_{R_2},\nu_{R_2}]$ is also an IFR relation on $X\subseteq U$.*

Proof. Let $\mu^{/}=\mu_{R_1}\circ\mu_{R_2}$ and $\nu^{/}= \nu_{R_1}\diamondsuit\nu_{R_2}$.

Then $\mu^{/}(x,y)=(\mu_{R_1}\circ\mu_{R_2})(x,y)=\max_{u\in U}\min\{\mu_{R_1}(x,u),\ \mu_{R_2}(u,y)\}$.
Since $[\mu_{R_1},\nu_{R_1}]$ and $[\mu_{R_2},\nu_{R_2}]$ are two IFR relations,
implies that $\mu_{R_1}(x,y)= 1 = \mu_{R_2}(x,y)$, $\forall(x,y)\in\underline{X\times X}$.
Therefore $\max_{u\in U}\min\{\mu_{R_1}(x,u),\ \mu_{R_2}(u,y)\}=1$,
implies $\mu^{/}(x,y)=1$, $\forall(x,y)\in\underline{X\times X}$.
Also, since $[\mu_{R_1},\nu_{R_1}]$ and $[\mu_{R_2},\nu_{R_2}]$ are two IFR relations,
implies $\mu_{R_1}(x,y)= 0 = \mu_{R_2}(x,y)$, $\forall(x,y)\in[U\times U\backslash\overline{X\times X}]$.
Therefore $\max_{u\in U}\min\{\mu_{R_1}(x,u),\ \mu_{R_2}(u,y)\}=0$,
implies $\mu^{/}(x,y)=0$, $\forall(x,y)\in[U\times U\backslash\overline{X\times X}]$.
Again since $0<\mu_{R_1}(x,y),\ \mu_{R_2}(x,y)<1$, $\forall(x,y)\in[\overline{X\times X}\backslash\underline{X\times X}]$
implies $0<\max_{u\in U}\min\{\mu_{R_1}(x,u),\ \mu_{R_2}(u,y)\}<1$, $\forall(x,y)\in[\overline{X\times X}\backslash\underline{X\times X}]$
implies $0<\mu^{/}(x,y)<1$, $\forall(x,y)\in[\overline{X\times X}\backslash\underline{X\times X}]$
and $\nu^{/}(x,y)=(\nu_{R_1}\diamondsuit\nu_{R_2})(x,y)=\min_{u\in U}\max\{\nu_{R_1}(x,u),\ \nu_{R_2}(u,y)\}$.
Since $[\mu_{R_1},\nu_{R_1}]$ and $[\mu_{R_2},\nu_{R_2}]$ are two IFR relations,
implies that $\nu_{R_1}(x,y)= 0 = \nu_{R_2}(x,y)$, $\forall(x,y)\in\underline{X\times X}$.
Therefore $\min_{u\in U}\max\{\nu_{R_1}(x,u),\ \nu_{R_2}(u,y)\}=0$,
implies $\mu^{/}(x,y)=0$, $\forall(x,y)\in\underline{X\times X}$.
Also, since $[\mu_{R_1},\nu_{R_1}]$ and $[\mu_{R_2},\nu_{R_2}]$ are two IFR relations,
implies $\nu_{R_1}(x,y)= 1 = \nu_{R_2}(x,y)$, $\forall(x,y)\in[U\times U\backslash\overline{X\times X}]$.
Therefore $\min_{u\in U}\max\{\nu_{R_1}(x,u),\ \nu_{R_2}(u,y)\}=1$,
implies $\nu^{/}(x,y)=1$, $\forall(x,y)\in[U\times U\backslash\overline{X\times X}]$.
Again since $0<\nu_{R_1}(x,y),\ \nu_{R_2}(x,y)<1$, $\forall(x,y)\in[\overline{X\times X}\backslash\underline{X\times X}]$
implies $0<\min_{u\in U}\max\{\nu_{R_1}(x,u),\ \nu_{R_2}(u,y)\}<1$, $\forall(x,y)\in[\overline{X\times X}\backslash\underline{X\times X}]$
implies $0< \nu^{/}(x,y)<1$, $\forall(x,y)\in[\overline{X\times X}\backslash\underline{X\times X}]$.
Thus $[\mu_{R_1},\nu_{R_1}]©[\mu_{R_2},\nu_{R_2}]$ is IFR relation on $X\subseteq U$. □

Theorem 3.3. *Let $[\mu_{R_1},\nu_{R_1}]$ and $[\mu_{R_2},\nu_{R_2}]$ be two reflexive IFR relations on $X\subseteq U$. Then $[\mu_{R_1},\nu_{R_1}]©[\mu_{R_2},\nu_{R_2}]$ is also reflexive IFR relation on $X\subseteq U$.*

Proof. Let $\mu^{/}=\mu_{R_1}\circ\mu_{R_2}$ and $\nu^{/}= \nu_{R_1}\diamondsuit\nu_{R_2}$.

Then $\forall x\in U$

$\mu^{/}(x,x)$

$=(\mu_{R_1} \circ \mu_{R_2})$(x,x)
$=\max_{u \in U} \min\{\mu_{R_1}$(x,u), μ_{R_2}(u,x)$\}$
$=\max_u\{\max_{u=x} \min\{\mu_{R_1}$(x,u), μ_{R_2}(u,x)$\}$, $\max_{u \neq x} \min\{\mu_{R_1}$(x,u), μ_{R_2}(u,x)$\}\}$
$=\max_u\{\min\{\mu_{R_1}$(x,x), μ_{R_2}(x,x)$\}$, $\max_{u \neq x} \min\{\mu_{R_1}$(x,u), μ_{R_2}(u,x)$\}\}=1$,
[Since $[\mu_{R_1},\nu_{R_1}]$ and $[\mu_{R_2},\nu_{R_2}]$ be two reflexive IFR relations,
$\Rightarrow \mu_{R_1}$(x,x)$= 1 = \mu_{R_2}$(x,x), $\forall$x$\in$U]
and $\nu^{/}$(x,x)
$=(\nu_{R_1} \diamondsuit \nu_{R_2})$(x,x)
$=\min_{u \in U} \max\{\nu_{R_1}$(x,u), ν_{R_2}(u,x)$\}$
$=\min_u\{\min_{u=x} \max\{\nu_{R_1}$(x,u), ν_{R_2}(u,x)$\}$, $\min_{u \neq x} \max\{\nu_{R_1}$(x,u), ν_{R_2}(u,x)$\}\}$
$= \min_u\{\max\{\nu_{R_1}$(x,x), ν_{R_2}(x,x)$\}$, $\min_{u \neq x} \max\{\nu_{R_1}$(x,u), μ_{R_2}(u,x)$\}\}=0$,
[Since $[\mu_{R_1},\nu_{R_1}]$ and $[\mu_{R_2},\nu_{R_2}]$ be two reflexive IFR relations,
$\Rightarrow \mu_{R_1}$(x,x)$= 0 = \mu_{R_2}$(x,x), $\forall$x$\in$U].
Thus $[\mu_{R_1},\nu_{R_1}]$©$[\mu_{R_2},\nu_{R_2}]$ is reflexive IFR relation on X$\subseteq$U. □

Definition 3.4. An IFR relation $[\mu_{R_1},\nu_{R_1}]$ on X$\subseteq$U is said to be Symmetric IFR relation if μ_{R_1}(x,y)$=\mu_{R_1}$(y,x) and ν_{R_1}(x,y)$=\nu_{R_1}$(y,x), $\forall$x,y$\in$U.

Definition 3.5. An IFR relation $[\mu_{R_1},\nu_{R_1}]$ on X$\subseteq$U is said to be Anti-Symmetric IFR relation if μ_{R_1}(x,y)$\neq\mu_{R_1}$(y,x) or μ_{R_1}(x,y)$=\mu_{R_1}$(y,x)$=0$ and ν_{R_1}(x,y)$\neq\nu_{R_1}$(y,x) or ν_{R_1}(x,y)$=\nu_{R_1}$(y,x)$=1$, $\forall$x,y$\in$U.

Definition 3.6. An IFR relation $[\mu_{R_1},\nu_{R_1}]$ on X$\subseteq$U is said to be Transitive if $[\mu_{R_1},\nu_{R_1}]$©$[\mu_{R_1},\nu_{R_1}]\subseteq[\mu_{R_1},\nu_{R_1}]$.

Definition 3.7. An IFR relation $[\mu_{R_1},\nu_{R_1}]$ on X$\subseteq$U is said to be Equivalence IFR relation if it is reflexive, symmetric and transitive IFR relation.

Definition 3.8. An IFR relation $[\mu_{R_1},\nu_{R_1}]$ on X$\subseteq$U is said to be Equivalence IFR relation of order α if it is reflexive of order α, symmetric and transitive IFR relation.

Definition 3.9. An IFR relation $[\mu_{R_1},\nu_{R_1}]$ on X$\subseteq$U is said to be IFR order relation if it is reflexive, anti-symmetric and transitive IFR relation.

Definition 3.10. An IFR relation $[\mu_{R_1},\nu_{R_1}]$ on X$\subseteq$U is said to be IFR order relation of order α if it is reflexive of order α, anti-symmetric and transitive IFR relation.

Theorem 3.11. *If $[\mu_{R_1},\nu_{R_1}]$ be a symmetric and transitive IFR relation on X$\subseteq$U. Then $[\mu_{R_1},\nu_{R_1}]$ is weakly reflexive IFR relation on X$\subseteq$U.*

Proof. Since $[\mu_{R_1},\nu_{R_1}]$ be a symmetric IFR relation,
implies that μ_{R_1}(x,y)$=\mu_{R_1}$(y,x), $\forall$x, y$\in$U.
Again since $[\mu_{R_1},\nu_{R_1}]$ be a transitive IFR relation, $\forall$x,y$\in$U, we have
μ_{R_1}(x,y)$\geq\max_{u \in U} \min\{\mu_{R_1}$(x,u), μ_{R_2}(u,y)$\}$
ν_{R_1}(x,y)$\leq\min_{u \in U} \max\{\nu_{R_1}$(x,u), ν_{R_2}(u,y)$\}$.
Taking y=x, we get
μ_{R_1}(x,x)
$\geq\max_{u \in U} \min\{\mu_{R_1}$(x,u), μ_{R_2}(u,x)$\}$
$=\max_{u \in U} \min\{\mu_{R_1}$(x,u), μ_{R_2}(x,u)$\}$
$=\max_{u \in U}\{\mu_{R_1}(x,u)\}$

$\geq\mu_{R_1}$(x,y), ∀x,y∈U
and ν_{R_1}(x,x)
$\leq\min_{u\in U}\max\{\nu_{R_1}$(x,u), ν_{R_2}(u,x)}
$=\min_{u\in U}\max\{\nu_{R_1}$(x,u), ν_{R_2}(x,u)}
$=\min_{u\in U}\{\nu_{R_1}$(x,u)}
$\leq\nu_{R_1}$(x,y), ∀x,y∈U.
Thus $[\mu_{R_1},\nu_{R_1}]$ is weakly reflexive IFR relation on X⊆U. □

Proposition 3.12. *If $[\mu_{R_1},\nu_{R_1}]$ and $[\mu_{R_2},\nu_{R_2}]$ are two w-reflexive IFR relations on X⊆U. Then $[\mu_{R_1},\nu_{R_1}]$ $\bigvee$ $[\mu_{R_2},\nu_{R_2}]$ ⊆ $[\mu_{R_1},\nu_{R_1}]$©$[\mu_{R_2},\nu_{R_2}]$.*

Proof. We have ∀x,y∈U
$(\mu_{R_1}\circ\mu_{R_2})$(x,y)
$=\max_{u\in U}\min\{\mu_{R_1}$(x,u), μ_{R_2}(u,y)}
$\geq\min\{\mu_{R_1}$(x,x), μ_{R_2}(x,y)}
$\geq\min\{\mu_Y$(x), μ_{R_2}(x,y)}, [as $[\mu_{R_1},\nu_{R_1}]$ is w-reflexive].
Again, μ_{R_2}(x,y)$\leq\mu_{Y\times Y}$(x,y)=min{μ_Y(x), μ_Y(y)}$\leq\mu_Y$(x),
so $(\mu_{R_1}\circ\mu_{R_2})$(x,y)≥ μ_{R_2}(x,y), ∀x,y∈U.
Therefore, $\mu_{R_2}\leq\mu_{R_1}\circ\mu_{R_2}$. Similarly, we can show that $\mu_{R_1}\leq\mu_{R_1}\circ\mu_{R_2}$.
Hence $\mu_{R_1}\bigvee\mu_{R_2}\leq\mu_{R_1}\circ\mu_{R_2}$.
Also $(\nu_{R_1}\diamondsuit\nu_{R_2})$(x,y)
$=\min_{u\in U}\max\{\nu_{R_1}$(x,u), ν_{R_2}(u,y)}
≤ max{ν_{R_1}(x,x), ν_{R_2}(x,y)}
≤ max{ν_Y(x), ν_{R_2}(x,y)}, [as $[\mu_{R_1},\nu_{R_1}]$ is w-reflexive]
Again, ν_{R_2}(x,y)$\geq\nu_{Y\times Y}$(x,y)=min{ν_Y(x), ν_Y(y)} ≥ ν_Y(x),
so $(\nu_{R_1}\diamondsuit\nu_{R_2})(x,y)\geq\nu_{R_2}$(x,y), ∀x,y∈U.
Therefore, $\nu_{R_2}\geq\nu_{R_1}\diamondsuit\nu_{R_2}$. Similarly, we can show that $\nu_{R_1}\geq\nu_{R_1}\diamondsuit\nu_{R_2}$.
Hence $\nu_{R_1}\bigwedge\nu_{R_2}\geq\nu_{R_1}\diamondsuit\nu_{R_2}$.
Thus $[\mu_{R_1},\nu_{R_1}]\bigvee[\mu_{R_2},\nu_{R_2}]$ ⊆$[\mu_{R_1},\nu_{R_1}]$©$[\mu_{R_2},\nu_{R_2}]$. □

Proposition 3.13. *Let X is an intuitionistic fuzzy rough set [[13]] of universe U, where μ is the membership function and ν be a non-membership function on U. Also let $[\mu_{R_1},\nu_{R_1}]$ be an equivalence IFR relation of order α. Then for each x∈U, there exist an intuitionistic fuzzy rough subset $\langle\mu_{R_x},\nu_{R_x}\rangle$ of X determined by membership function μ_{R_x} and non-membership function ν_{R_x}, satisfying the following:*
(i) $\mu_{R_x}(x)\geq\alpha$ and $\nu_{R_x}(x)\leq\alpha$,
(ii) $\mu_{R_x}(y)=\mu_{R_y}(x)$ and $\nu_{R_x}(y)=\nu_{R_y}(x)$
(iii) $\mu_{R_x}(y)>\beta$, $\mu_{R_y}(z)>\beta\Rightarrow\mu_{R_x}(z)>\beta$, $0\leq\beta<1$
and $\nu_{R_x}(y)<\lambda$, $\nu_{R_y}(z)<\lambda\Rightarrow\nu_{R_x}(z)<\lambda$, $0<\lambda\leq1$
(iv) $\mu_{R_x}(y)=0$ and $\nu_{R_x}(y)=1\Rightarrow\langle\mu_{R_x},\nu_{R_x}\rangle\bigwedge\langle\mu_{R_y},\nu_{R_y}\rangle=\varnothing$

Proof. For each x∈U, we define μ_{R_x}(y)=μ_{R_1}(x,y) and ν_{R_x}(y)= ν_{R_1}(x,y), ∀y∈U.
Since μ_{R_1}(x,y) ≤ min{μ(x),μ(y)} and ν_{R_1}(x,y) ≥ max{ν(x),ν(y)}, ∀x,y∈U.
We note that
μ_{R_x}(y)=μ_{R_1}(x,y)≤ min{μ(x),μ(y)}≤ μ(y)
and ν_{R_x}(y)=ν_{R_1}(x,y)≥ max{ν(x),ν(y)}≥ν(y), ∀y∈U.
Therefore the intuitionistic fuzzy rough set determined by $\langle\mu_{R_x},\nu_{R_x}\rangle$ is an intuitionistic fuzzy rough subset of X.

(i) μ_{R_x}(x)=μ_{R_1}(x,x)$\geq\alpha$
and ν_{R_x}(x)=ν_{R_1}(x,x)$\leq\alpha$, [since [μ_{R_1},ν_{R_1}] is reflxive IFR relation of order α].
(ii) μ_{R_x}(y)=μ_{R_1}(x,y)=μ_{R_1}(y,x)=μ_{R_y}(x)
and ν_{R_x}(y)=ν_{R_1}(x,y)=ν_{R_1}(y,x)=ν_{R_y}(x), [as [μ_{R_1},ν_{R_1}] is symmetric IFR relation].
(iii) Let μ_{R_x}(y)>β and μ_{R_y}(z)>β
$\Rightarrow$ μ_{R_1}(x,y), μ_{R_1}(y,z)>β
$\Rightarrow$ min{μ_{R_1}(x,y), μ_{R_1}(y,z)}>β ....(1)
Since [μ_{R_1},ν_{R_1}] is transitive,
μ_{R_1}(x,z)
$\geq$max$_{u\in U}$min{μ_{R_1}(x,u), μ_{R_2}(u,z)}
$\geq$min{μ_{R_1}(x,y), μ_{R_2}(y,z)}>β, [by (1)]
$\Rightarrow$ μ_{R_x}(z)=μ_{R_1}(x,z)>β.
Again, we suppose ν_{R_x}(y)<λ and ν_{R_y}(z)<λ
$\Rightarrow$ ν_{R_1}(x,y), ν_{R_1}(y,z)<λ
$\Rightarrow$ max{ν_{R_1}(x,y), ν_{R_1}(y,z)}<λ ....(2)
Also by transitive of [μ_{R_1},ν_{R_1}],
ν_{R_1}(x,z)
$\leq$min$_{u\in U}$max{ν_{R_1}(x,u), ν_{R_2}(u,z)}
$\leq$ max{ν_{R_1}(x,y), ν_{R_2}(y,z)}<λ, [by (2)]
$\Rightarrow$ ν_{R_x}(z)=ν_{R_1}(x,z)< λ.

(iv) Let μ_{R_x}(y)=0 and ν_{R_x}(y)=1.
Now we have to show that $\langle\mu_{R_x},\nu_{R_x}\rangle\bigwedge\langle\mu_{R_y},\nu_{R_y}\rangle=\varnothing$
i.e. (μ_{R_x} $\bigwedge$ μ_{R_y})(u)=0 and (ν_{R_x} $\bigvee$ ν_{R_y})(u)=1, $\forall$u$\in$U.
If possible let $\exists$ z$\in$U, such that
($\mu_{R_x}\bigwedge\mu_{R_y}$)(z)>0
$\Rightarrow$ min{μ_{R_x}(z), μ_{R_y}(z)}>0
$\Rightarrow$ μ_{R_x}(z), μ_{R_y}(z)>0
$\Rightarrow$ μ_{R_x}(z), μ_{R_z}(y)>0, [using (ii)]
$\Rightarrow$ μ_{R_x}(y) > 0, [using (iii), puting β=0]
This contradicts μ_{R_x}(y)=0. Thus ($\mu_{R_x}\bigwedge\mu_{R_y}$)(u)=0, $\forall$u$\in$U.
Also if possible let $\exists$ t$\in$U, such that
($\nu_{R_x}\bigvee\nu_{R_y}$)(t)<1
$\Rightarrow$ max{ν_{R_x}(t), ν_{R_y}(t)}<1
$\Rightarrow$ ν_{R_x}(t), ν_{R_y}(t)<1
$\Rightarrow$ ν_{R_x}(t), ν_{R_t}(y)<1, [using (ii)]
$\Rightarrow$ ν_{R_x}(y)<1, [using (iii), puting λ=1]
This contradicts ν_{R_x}(y)=1. Thus ($\nu_{R_x}\bigvee$ ν_{R_y})(u)=1, $\forall$u$\in$U.
Hence $\langle\mu_{R_x},\nu_{R_x}\rangle\bigwedge\langle\mu_{R_y},\nu_{R_y}\rangle=\varnothing$. □

Definition 3.14. An IFR relation [μ_{R_1},ν_{R_1}] on X$\subseteq$U is said to be Anti-Transitive if [μ_{R_1},ν_{R_1}]$\subseteq$[μ_{R_1}, ν_{R_1}]©[μ_{R_1},ν_{R_1}].

Theorem 3.15. *If [μ_{R_1},ν_{R_1}] be a w-reflexive IFR relation on X$\subseteq$U. Then [μ_{R_1},ν_{R_1}] is an Anti-Transitive IFR relation on X$\subseteq$U.*

Proof. $\forall$x,y$\in$U, we have
($\mu_{R_1}\circ\mu_{R_1}$)(x,y)

$=\max_{u\in U}\min\{\mu_{R_1}$(x,u), μ_{R_1}(u,y)$\}$
$\geq\min\{\mu_{R_1}$(x,x), μ_{R_1}(x,y)$\}$
$\geq\min\{\mu_Y$(x), μ_{R_1}(x,y)$\}$, [as $[\mu_{R_1},\nu_{R_1}]$ is w-reflexive]
Again, μ_{R_1}(x,y)$\leq\mu_{Y\times Y}$(x,y) $=\min\{\mu_Y$(x), μ_Y(y)$\}\leq\mu_Y$(x),
so μ_{R_1}(x,y)$\leq(\mu_{R_1}\circ\mu_{R_1})$(x,y), $\forall$x,y$\in$U.
Therefore, $\mu_{R_1}\leq\mu_{R_1}\circ\mu_{R_1}$.
Also, $(\nu_{R_1}\Diamond\nu_{R_1})$(x,y)
$=\min_{u\in U}\max\{\nu_{R_1}$(x,u), ν_{R_1}(u,y)$\}$
$\leq\max\{\nu_{R_1}$(x,x), ν_{R_2}(x,y)$\}$
$\leq\max\{\nu_Y$(x), ν_{R_1}(x,y)$\}$, [as $[\mu_{R_1},\nu_{R_1}]$ is w-reflexive]
Again, ν_{R_1}(x,y) $\geq\nu_{Y\times Y}$(x,y) $=\max\{\nu_Y$(x), ν_Y(y)$\}\geq\nu_Y$(x),
so $(\nu_{R_1}\Diamond\nu_{R_1})(x,y)\leq\nu_{R_1}$(x,y), $\forall$x,y$\in$U.
Therefore, $\nu_{R_1}\geq\nu_{R_1}\Diamond\nu_{R_1}$.
Thus $[\mu_{R_1},\nu_{R_1}]\subseteq[\mu_{R_1},\nu_{R_1}]$©$[\mu_{R_2},\nu_{R_1}]$. □

4. Conclusions

Rough sets and IF sets both capture particular facets of the same notion-imprecision. In this paper, it was shown how they can be usefully combined into a single framework. Here we introduced intuitionistic fuzzy rough relations in a different approach and established some important properties. In future the generalization of the above concepts will be done.

References

[1] K. Atanassov, Intutionistic fuzzy sets, Fuzzy Sets and Systems 64 (1986) 87–96.
[2] K. Chakrabarty, T. Gedeon and I. Koczy, Intuitionistic fuzzy rough sets, in Proceeding of the Fourth Joint Conference of Information Sciences, Durham, NC, JCTS, (1998) 211–214.
[3] C. Gangwal and R. N. Bhaumik, Intuitionistic fuzzy rough relationin some medical applications, International Journal of Advance Research in Computer Engineering and Technology 1 (2012) 28–32.
[4] S. P. Jena and S. K. Ghosh, Intuitionistic fuzzy rough sets, Notes on Intuitionistic Fuzzy Sets. 8 (2002) 1–18.
[5] Maria Do Nicoletti, Joaquim Quinteiro Uchôa, Margarete T. Z. Baptistini, Rough relation properties, Int. J. Appl. Math. Comput. Sci. 11(3) (2001) 621–635.
[6] S. Nanda and S. Majumdar, Fuzzy rough sets, Fuzzy Sets and Systems 45 (1992) 157–160.
[7] Z. Pawlak, Rough Relations, Techn. Rep. No. 435, Institute of Computer Science, Polish Academy of Science, warsaw 1981.
[8] Z. Pawlak, Rough sets, Int. J. Comput. Inform. Sci. 11 (1982) 341–356.
[9] S. Rizvi, H. J. Nagvi and D. Nadeem, Rough intuitionistic fuzzy sets, in Proceeding of the 6th Joint Conference on information Sciences, Durham, NC, JCTS, (2002) 101–104.
[10] S. K. Samanta and T. K. Mondal, Intuitioistic fuzzy rough sets and rough intuitionistic fuzzy sets, J. Fuzzy Math. 9 (2001) 561–582.
[11] L. A. Zadeh, Fuzzy sets, Information and control 8 (1965) 338–353.
[12] L. A. Zadeh, Similarity relations and fuzzy orderings, Inform. Sci. 3 (1971) 177–200.
[13] Lei Zhou, Wei-Zhi Wu and Wen-Xiu Zhang, On intuitionistic fuzzy rough sets and their topological structures, International Journal of General Systems 38 (2009) 589–616.

Anjan Mukherjee (anjan2002_m@yahoo.co.in)
Department of Mathematics, Tripura University, Suryamaninagar, Agartala-799022, Tripura, India

Ajoy Kanti Das (ajoykantidas@gmail.com)
Department of Mathematics, Tripura University, Suryamaninagar, Agartala-799022, Tripura, India