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Weakly generalized homeomorphism in intuitionistic fuzzy topological space

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Abstract. The purpose of this paper is to introduce and study the concepts of intuitionistic fuzzy weakly generalized homeomorphism and intuitionistic fuzzy weakly generalized * homeomorphism in intuitionistic fuzzy topological space. Some of their properties are explored.

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Keywords: Intuitionistic fuzzy topology, Intuitionistic fuzzy weakly generalized closed set, Intuitionistic fuzzy weakly generalized continuous mapping, Intuitionistic fuzzy weakly generalized closed mapping, Intuitionistic fuzzy weakly generalized homeomorphism, Intuitionistic fuzzy weakly generalized * homeomorphism.

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1. Introduction

Fuzzy set (FS) as proposed by Zadeh ([15]) in 1965, is a framework to encounter uncertainty, vagueness and partial truth and it represents a degree of membership for each member of the universe of discourse to a subset of it. After the introduction of fuzzy topology by Chang ([2]) in 1968, there have been several generalizations of notions of fuzzy sets and fuzzy topology. By adding the degree of non-membership to FS, Atanassov ([1]) proposed intuitionistic fuzzy set (IFS) in 1986 which appeals more accurate to uncertainty quantification and provides the opportunity to precisely model the problem, based on the existing knowledge and observations. In 1997, Coker ([3]) introduced the concept of intuitionistic fuzzy topological space. In this paper, we introduce the notion of intuitionistic fuzzy weakly generalized homeomorphism and intuitionistic fuzzy weakly generalized * homeomorphism in intuitionistic fuzzy topological space and study some of their properties. We provide some characterizations of intuitionistic fuzzy weakly generalized homeomorphism and establish the relationships with other classes of early defined forms of intuitionistic fuzzy homeomorphisms.

2. Preliminaries

Definition 2.1 ([1]). Let X be a non empty fixed set. An *intuitionistic fuzzy set* (IFS in short) A in X is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$ where the functions $\mu_A(x) : X \to [0, 1]$ and $\nu_A(x) : X \to [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element x $\in$ X to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

Definition 2.2 ([1]). Let A and B be IFSs of the forms

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\} \text{ and } B = \{\langle x, \mu_B(x), \nu_B(x)\rangle / x \in X\}.$$

Then

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$,
(b) A = B if and only if A $\subseteq B$ and B $\subseteq$ A,
(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle / x \in X\}$,
(d) A $\cap$ B = $\{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle / x \in X\}$,
(e) A $\cup$ B = $\{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle / x \in X\}$.

For the sake of simplicity, the notation A = $\langle x, \mu_A, \nu_A\rangle$ shall be used instead of A = $\{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$. Also for the sake of simplicity, we shall use the notation A= $\langle x, (\mu_A, \mu_B), (\nu_A, \nu_B)\rangle$ instead of A= $\langle x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B)\rangle$

The intuitionistic fuzzy sets $0_\sim = \{\langle x, 0, 1\rangle / x \in X\}$ and $1_\sim = \{\langle x, 1, 0\rangle / x \in X\}$ are the *empty set* and the *whole set* of X, respectively.

Definition 2.3 ([3]). An *intuitionistic fuzzy topology* (IFT in short) on a non empty set X is a family τ of IFSs in X satisfying the following axioms:

(a) $0_\sim, 1_\sim \in \tau$,
(b) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,
(c) $\cup G_i \in \tau$ for any arbitrary family $\{G_i / i \in J\} \subseteq \tau$.

In this case, the pair (X, τ) is called an *intuitionistic fuzzy topological space* (IFTS in short) and any IFS in τ is known as an *intuitionistic fuzzy open set* (IFOS in short) in X.

The complement A^c of an IFOS A in an IFTS (X, τ) is called an *intuitionistic fuzzy closed set* (IFCS in short) in X.

Definition 2.4 ([3]). Let (X, τ) be an IFTS and A = $\langle x, \mu_A, \nu_A\rangle$ be an IFS in X. Then the *intuitionistic fuzzy interior* and an *intuitionistic fuzzy closure* are defined by

int(A)= $\cup\{G/G\, is\, an\, IFOS\, in\, X\, and\, G \subseteq A\}$,
cl(A)= $\cap\{K/K\, is\, an\, IFCS\, in\, X\, and\, A \subseteq K\}$.

Note that for any IFS A in (X, τ), we have $cl(A^c) = (int(A))^c$ and $int(A^c) = (cl(A))^c$.

Definition 2.5. An IFS A = $\{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$ in an IFTS (X, τ) is said to be

(a)([6]) *intuitionistic fuzzy semi closed set* (IFSCS in short) if $int(cl(A)) \subseteq A$,
(b)([6]) *intuitionistic fuzzy α-closed set* (IFαCS in short) if $cl(int(cl(A))) \subseteq A$,
(c)([6]) *intuitionistic fuzzy pre-closed set* (IFPCS in short) if $cl(int(A)) \subseteq A$,
(d)([6]) *intuitionistic fuzzy regular closed set* (IFRCS in short) if $cl(int(A)) = A$,

(e)([14]) *intuitionistic fuzzy generalized closed set* (IFGCS in short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS,

(f)([13]) *intuitionistic fuzzy generalized semi closed set* (IFGSCS in short) if $scl(A) \subseteq U$, whenever $A \subseteq U$ and U is an IFOS,

(g)([12]) *intuitionistic fuzzy α generalized closed set* (IFαGCS in short) if $\alpha cl(A) \subseteq U$, whenever $A \subseteq U$ and U is an IFOS,

(h)([5]) *intuitionistic fuzzy γ closed set* (IF γCS in short) if $int(cl(A)) \cap cl(int(A)) \subseteq A$.

An IFS A is called *intuitionistic fuzzy semi open set, intuitionistic fuzzy α-open set, intuitionistic fuzzy pre-open set, intuitionistic fuzzy regular open set, intuitionistic fuzzy generalized open set, intuitionistic fuzzy generalized semi open set, intuitionistic fuzzy α generalized open set and intuitionistic fuzzy γ open set* (IFSOS, IFαOS, IFPOS, IFROS, IFGOS, IFGSOS, IFαGOS and IFγOS) if the complement A^c is an IFSCS, IFαCS, IFPCS, IFRCS, IFGCS, IFGSCS, IFαGCS and IFγCS respectively.

Definition 2.6 ([7])**.** An IFS A $= \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$ in an IFTS (X, τ) is said to be an *intuitionistic fuzzy weakly generalized closed set* (IFWGCS in short) if $cl(int(A)) \subseteq U$, whenever $A \subseteq U$ and U is an IFOS in X.

The family of all IFWGCSs of an IFTS (X, τ) is denoted by IFWGC(X).

Definition 2.7 ([7])**.** An IFS A $= \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$ is said to be an *intuitionistic fuzzy weakly generalized open set* (IFWGOS in short) in an IFTS (X, τ) if the complement A^c is an IFWGCS in X.

The family of all IFWGOSs of an IFTS (X, τ) is denoted by IFWGO(X).

Remark 2.8 ([7])**.** Every IFCS, IFαCS, IFGCS, IFRCS, IFPCS, IFαGCS is an IFWGCS but the converses need not be true in general.

Definition 2.9 ([8])**.** Let (X, τ) be an IFTS and A $= \langle x, \mu_A, \nu_A\rangle$ be an IFS in X. Then the *intuitionistic fuzzy weakly generalized interior* and an *intuitionistic fuzzy weakly generalized closure* are defined by

wgint(A)$= \cup\{G/G\, is\, an\, IFWGOS\, in\, X\, and\, G \subseteq A\}$,
wgcl(A)$= \cap\{K/K\, is\, an\, IFWGCS\, in\, X\, and\, A \subseteq K\}$.

Definition 2.10 ([3])**.** Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). If B $= \{\langle y, \mu_B(y), \nu_B(y)\rangle / y \in Y\}$ is an IFS in Y, then the *pre-image* of B under f is the IFS in X defined by $f^{-1}(B) = \left\{\left\langle x, f^{-1}(\mu_B(x)), f^{-1}(\nu_B(x))\right\rangle / x \in X\right\}$, where $f^{-1}(\mu_B(x)) = \mu_B(f(x))$.

If A $= \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$ is an IFS in X, then the *image* of A under f denoted by f(A), is the IFS in Y defined by f(A) $=\{\langle y, f(\mu_A(y)), f_-(\nu_A(y))\rangle / y \in Y\}$ where $f_-(\nu_A) = 1 - f(1 - \nu_A)$.

Definition 2.11. Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be

(a)([4]) *intuitionistic fuzzy continuous* (IF continuous in short) if $f^{-1}(B)$ is an IFOS in X for every IFOS B in Y,

(b)([6]) *intuitionistic fuzzy α continuous* (IFα continuous in short) if $f^{-1}(B)$ is an IFαOS in X for every IFOS B in Y, (c)([14]) *intuitionistic fuzzy generalized continuous* (IFG continuous in short) if $f^{-1}(B)$ is an IFGOS in X for every IFOS B in Y,

(d)([12]) *intuitionistic fuzzy α generalized continuous* (IFαG continuous in short) if $f^{-1}(B)$ is an IFαGOS in X for every IFOS B in Y,

(e)([9]) *intuitionistic fuzzy weakly generalized continuous* (IFWG continuous in short) if $f^{-1}(B)$ is an IFWGOS in X for every IFOS B in Y,

(f)([10]) *intuitionistic fuzzy weakly generalized closed mapping* (IFWGCM in short) if $f(B)$ is an IFWGCS in Y for every IFCS B in X,

(g)([10]) *intuitionistic fuzzy weakly generalized open mapping* (IFWGOM in short) if $f(B)$ is an IFWGOS in Y for every IFOS B in X,

(h)([10]) *intuitionistic fuzzy weakly generalized *closed mapping* (IFWG*CM in short) if $f(B)$ is an IFWGCS in Y for every IFWGCS B in X,

(i)([10]) *intuitionistic fuzzy weakly generalized *open mapping* (IFWG*OM in short) if $f(B)$ is an IFWGOS in Y for every IFWGOS B in X,

(j)([11]) *intuitionistic fuzzy homeomorphism* (IF homeomorphism in short) if f and f^{-1} are IF continuous mappings,

(k)([11]) *intuitionistic fuzzy alpha homeomorphism* (IFα homeomorphism in short) if f and f^{-1} are IFα continuous mappings,

(l)([11]) *intuitionistic fuzzy generalized homeomorphism* (IFG homeomorphism in short) if f and f^{-1} are IFG continuous mappings,

(m)([11]) *intuitionistic fuzzy alpha generalized homeomorphism* (IFαG homeomorphism in short) if f and f^{-1} are IFαG continuous mappings.

Definition 2.12 ([7])**.** An IFTS (X, τ) is said to be an *intuitionistic fuzzy $_wT_{1/2}$ space* (IF$_wT_{1/2}$ space) if every IFWGCS in X is an IFCS in X.

Definition 2.13 ([7])**.** An IFTS (X, τ) is said to be an *intuitionistic fuzzy $_{wg}T_q$ space* (IF$_{wg}T_q$ space) if every IFWGCS in X is an IFPCS in X.

3. Intuitionistic fuzzy weakly generalized homeomorphism

In this section, we introduce intuitionistic fuzzy weakly generalized homeomorphism and study some of their properties.

Definition 3.1. A bijective mapping $f : (X, \tau) \to (Y, \sigma)$ is called an *intuitionistic fuzzy weakly generalized homeomorphism* (IFWG homeomorphism in short) if f and f^{-1} are IFWG continuous mappings.

Example 3.2. Let X = $\{a, b\}$, Y = $\{u, v\}$ and $T_1 = \langle x, (0.2, 0.3), (0.8, 0.7)\rangle$, $T_2 = \langle y, (0.8, 0.7), (0.2, 0.2)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Consider a bijective mapping $f : (X, \tau) \to (Y, \sigma)$ defined as f(a) = u and f(b) = v. Then f and f^{-1} are IFWG continuous mappings. Hence f is an IFWG homeomorphism.

Theorem 3.3. *Every IF homeomorphism is an IFWG homeomorphism but not conversely.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be an IF homeomorphism. Then f and f^{-1} are IF continuous mappings. This implies f and f^{-1} are IFWG continuous mappings. Hence f is an IFWG homeomorphism. □

Example 3.4. Let X = $\{a, b\}$, Y = $\{u, v\}$ and $T_1 = \langle x, (0.4, 0.5), (0.6, 0.5)\rangle$, $T_2 = \langle y, (0.8, 0.7), (0.2, 0.3)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Consider a bijective mapping $f : (X, \tau) \to (Y, \sigma)$ defined as f(a) = u and f(b) = v. Then f is an IFWG homeomorphism but not an IF homeomorphism, since f and f^{-1} are not IF continuous mappings.

Theorem 3.5. *Let $f : (X, \tau) \to (Y, \sigma)$ be an IFWG homeomorphism from an IFTS (X, τ) into an IFTS (Y, σ). Then f is an IF homeomorphism if (X, τ) and (Y, σ) are $IF_wT_{1/2}$ spaces.*

Proof. Let B be an IFCS in Y. By hypothesis, $f^{-1}(B)$ is an IFWGCS in X. Since (X, τ) is an $IF_wT_{1/2}$ space, $f^{-1}(B)$ is an IFCS in X. Hence f is an IF continuous mapping. Also by hypothesis, $f^{-1} : (Y, \sigma) \to (X, \tau)$ is an IFWG continuous mapping. Let A be an IFCS in X. Then $(f^{-1})^{-1}(A) = f(A)$ is an IFWGCS in Y, by hypothesis. Since (Y, σ) is an $IF_wT_{1/2}$ space, $f(A)$ is an IFCS in Y. Hence f^{-1} is an IF continuous mapping. Thus f is an IF homeomorphism. □

Theorem 3.6. *Every IFα homeomorphism is an IFWG homeomorphism but not conversely.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be an IFα homeomorphism. Then f and f^{-1} are IFα continuous mappings. This implies f and f^{-1} are IFWG continuous mappings. Hence f is an IFWG homeomorphism. □

Example 3.7. Let X = $\{a, b\}$, Y = $\{u, v\}$ and $T_1 = \langle x, (0.5, 0.3), (0.5, 0.7)\rangle$, $T_2 = \langle y, (0.6, 0.8), (0.4, 0.2)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Consider a bijective mapping $f : (X, \tau) \to (Y, \sigma)$ defined as f(a) = u and f(b) = v. Then f is an IFWG homeomorphism but not an IFα homeomorphism, since f and f^{-1} are not IFα continuous mappings.

Theorem 3.8. *Every IFG homeomorphism is an IFWG homeomorphism but not conversely.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be an IFG homeomorphism. Then f and f^{-1} are IFG continuous mappings. This implies f and f^{-1} are IFWG continuous mappings. Hence f is an IFWG homeomorphism. □

Example 3.9. Let X = $\{a, b\}$, Y = $\{u, v\}$ and $T_1 = \langle x, (0.3, 0.4), (0.7, 0.6)\rangle$, $T_2 = \langle y, (0.8, 0.6), (0.2, 0.4)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Consider a bijective mapping $f : (X, \tau) \to (Y, \sigma)$ defined as f(a) = u and f(b) = v. Then f is an IFWG homeomorphism but not an IFG homeomorphism, since f and f^{-1} are not IFG continuous mappings.

Theorem 3.10. *Every IFαG homeomorphism is an IFWG homeomorphism but not conversely.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be an IFαG homeomorphism. Then f and f^{-1} are IFαG continuous mappings. This implies f and f^{-1} are IFWG continuous mappings. Hence f is an IFWG homeomorphism. □

Example 3.11. Let X = $\{a, b\}$, Y = $\{u, v\}$ and $T_1 = \langle x, (0.5, 0.6), (0.5, 0.4)\rangle$, $T_2 = \langle y, (0.6, 0.5), (0.3, 0.5)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Consider a bijective mapping $f : (X, \tau) \to (Y, \sigma)$ defined as f(a) = u and f(b) = v. Then f is an IFWG homeomorphism but not an IFαG homeomorphism, since f and f^{-1} are not IFαG continuous mappings.

Theorem 3.12. *Let $f : (X, \tau) \to (Y, \sigma)$ be a bijective mapping from an IFTS (X, τ) into an IFTS (Y, σ), then the following statements are equivalent.*
(a) *f is an IFWGOM,*
(b) *f is an IFWGCM,*
(c) *$f^{-1} : (Y, \sigma) \to (X, \tau)$ is an IFWG continuous mapping.*

Proof. (a) $\Rightarrow$ (b): Let A be an IFCS in X, then A^c is an IFOS in X. By hypothesis, $f(A^c) = (f(A))^c$ is an IFWGOS in Y. Therefore f(A) is an IFWGCS in Y. Hence f is an IFWGCM.

(b) $\Rightarrow$ (c): Let B be an IFCS in X. Since f is an IFWGCM, $f(A) = (f^{-1})^{-1}(A)$ is an IFWGCS in Y. Hence f^{-1} is an IFWG continuous mapping.

(c) $\Rightarrow$ (a): Let A be an IFOS in X. By hypothesis, $(f^{-1})^{-1}(A) = f(A)$ is an IFWGOS in Y. Hence f is an IFWGOM. □

Corollary 3.13. *Let $f : (X, \tau) \to (Y, \sigma)$ be a bijective mapping from an IFTS (X, τ) into an IFTS (Y, σ). If f is an IFWG continuous mapping, then the following statements are equivalent.*
(a) *f is an IFWGCM,*
(b) *f is an IFWGOM,*
(c) *f is an IFWG homeomorphism.*

Proof. Obvious. □

Theorem 3.14. *The composition of two IFWG homeomorphism need not be an IFWG homeomorphism in general.*

Proof. Let $X = \{a, b\}, Y = \{c, d\}$ and $Z = \{u, v\}$ and $T_1 = \langle x, (0.8, 0.6), (0.2, 0.4)\rangle$, $T_2 = \langle y, (0.5, 0.6), (0.5, 0.4)\rangle$, $T_3 = \langle z, (0.6, 0.5), (0.3, 0.5)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$, $\sigma = \{0_\sim, T_2, 1_\sim\}$ and $\delta = \{0_\sim, T_3, 1_\sim\}$ are IFTs on X , Y and Z respectively. Consider a bijective mapping $f : (X, \tau) \to (Y, \sigma)$ defined as f(a) = c , f(b) = d and $g : (Y, \sigma) \to (Z, \delta)$ by f(c) = u , f(d) = v. Then f and f^{-1} are IFWG continuous mappings. Also g and g^{-1} are IFWG continuous mappings. Hence f and g are IFWG homeomorphism. But the composition $gof : X \to Z$ is not an IFWG homeomorphism, since gof is not an IFWG continuous mapping. □

Theorem 3.15. *Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \delta)$ be two IFWG homeomorphisms and (Y, σ) an $IF_wT_{1/2}$ space. Then gof is an IFWG homeomorphism.*

Proof. Let A be an IFCS in Z. Since $g : (Y, \sigma) \to (Z, \delta)$ is an IFWG continuous mapping, $g^{-1}(A)$ is an IFWGCS in Y. Then $g^{-1}(A)$ is an IFCS in Y as (Y, σ) is an $IF_wT_{1/2}$ space. Also since $f : (X, \tau) \to (Y, \sigma)$ is an IFWG continuous mapping, $f^{-1}(g^{-1}(A)) = (gof)^{-1}(A)$ is an IFWGCS in X. Hence gof is an IFWG continuous mapping.

Let A be an IFCS in X. Since $f^{-1} : (Y, \sigma) \to (X, \tau)$ is an IFWG continuous mapping, $(f^{-1})^{-1}(A) = f(A)$ is an IFWGCS in Y. Then f(A) is an IFCS in Y as (Y, σ) is an $IF_wT_{1/2}$ space. Also since $g^{-1} : (Z, \delta) \to (Y, \sigma)$ is an IFWG continuous mapping, $(g^{-1})^{-1}(f(A)) = g(f(A)) = (gof)(A)$ is an IFWGCS in Z. Therefore $((gof)^{-1})^{-1}(A) = (gof)(A)$ is an IFWGCS in Z. Hence $(gof)^{-1}$ is an IFWG continuous mapping. Thus gof is an IFWG homeomorphism. □

4. Intuitionistic fuzzy weakly generalized* homeomorphism

In this section, we introduce intuitionistic fuzzy weakly generalized * homeomorphism and study some of their properties.

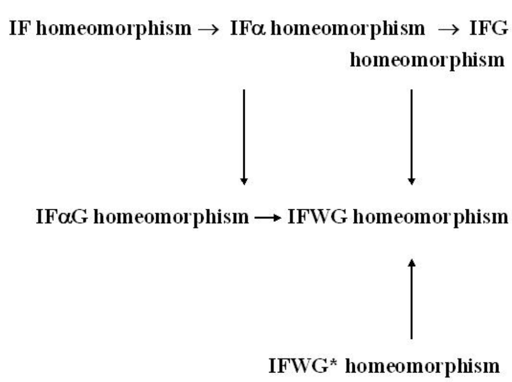
Definition 4.1. A bijective mapping $f : (X, \tau) \to (Y, \sigma)$ is called an *intuitionistic fuzzy weakly generalized * homeomorphism* (IFWG* homeomorphism in short) if f and f^{-1} are IFWG irresolute mappings.

Theorem 4.2. *Every IFWG* homeomorphism is an IFWG homeomorphism but not conversely.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be an IFWG* homeomorphism. Let B be an IFCS in Y. This implies B is an IFWGCS in Y. By hypothesis, $f^{-1}(B)$ is an IFWGCS in X. Hence f is an IFWG continuous mapping. Similarly we can prove f^{-1} is an IFWG continuous mapping. Hence f and f^{-1} are IFWG continuous mapping. Thus f is an IFWG homeomorphism. □

Example 4.3. Let X = $\{a, b\}$, Y = $\{u, v\}$ and $T_1 = \langle x, (0.3, 0.5), (0.7, 0.5)\rangle$, $T_2 = \langle y, (0.8, 0.7), (0.2, 0.2)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Consider a bijective mapping $f : (X, \tau) \to (Y, \sigma)$ defined as f(a) = u and f(b) = v. Then f is an IFWG homeomorphism. Let A = $\langle y, (0.3, 0.5), (0.7, 0.5)\rangle$ be an IFS in Y. Clearly A is an IFWGCS in Y. But $f^{-1}(A)$ is not an IFWGCS in X. This implies f is not an IFWG irresolute mapping. Hence f is not an IFWG* homeomorphism.

The relation among various types of intuitionistic fuzzy homeomorphisms are given in the following diagram.



The reverse implications are not true in general in the above diagram. In this diagram by "$A \to B$"we mean A implies B but not conversely.

Theorem 4.4. *Let $f : (X, \tau) \to (Y, \sigma)$ be an IFWG* homeomorphism from an IFTS (X, τ) into an IFTS (Y, σ). Then $wgcl(f^{-1}(B)) = f^{-1}(wgcl(B))$ for every IFS B in Y.*

Proof. Since f is an IFWG* homeomorphism, f is an IFWG irresolute mapping. Consider an IFS B in Y. Clearly wgcl(B) is an IFWGCS in Y. By hypothesis, $f^{-1}(wgcl(B))$ is an IFWGCS in X. Since $f^{-1}(B) \subseteq f^{-1}(wgcl(B)), wgcl(f^{-1}(B)) \subseteq wgcl(f^{-1}(wgcl(B))) = f^{-1}(wgcl(B))$. This implies $wgcl(f^{-1}(B)) \subseteq f^{-1}(wgcl(B))$ for every IFS B in Y.

Also by hypothesis, $f^{-1} : (Y, \sigma) \to (X, \tau)$ is an IFWG irresolute mapping. Consider an IFS $f^{-1}(B)$ in X. Clearly $wgcl(f^{-1}(B))$ is an IFWGCS in X. This implies $(f^{-1})^{-1}(wgcl(f^{-1}(B))) = f(wgcl(f^{-1}(B)))$ is an IFWGCS in Y. Clearly $B = (f^{-1})^{-1}(f^{-1}(B)) \subseteq (f^{-1})^{-1}(wgcl(f^{-1}(B))) = f(wgcl(f^{-1}(B)))$. Therefore $wgcl(B) \subseteq wgcl(f(wgcl(f^{-1}(B)))) = f(wgcl(f^{-1}(B)))$, since f^{-1} is an IFWG irresolute mapping. Hence $f^{-1}(wgcl(B)) \subseteq f^{-1}(f(wgcl(f^{-1}(B)))) = wgcl(f^{-1}(B))$. That is $f^{-1}(wgcl(B)) \subseteq wgcl(f^{-1}(B))$. Hence $wgcl(f^{-1}(B)) = f^{-1}(wgcl(B))$ for every IFS B in Y. □

Theorem 4.5. *Let $f : (X, \tau) \to (Y, \sigma)$ be a bijective mapping from an IFTS (X, τ) into an IFTS (Y, σ), then the following statements are equivalent.*

(a) *f is an IFWG* homeomorphism,*

(b) *f is an IFWG irresolute and IFWG*OM,*

(c) *f is an IFWG irresolute and IFWG*CM.*

Proof. (a) ⇒ (b): Let f be an IFWG* homeomorphism. Then f and f^{-1} are IFWG irresolute mappings. To prove that f is an IFWG*OM. Let A be an IFWGOS in X. Since $f^{-1} : (Y, \sigma) \to (X, \tau)$ is an IFWG irresolute mapping, $(f^{-1})^{-1}(A) = f(A)$ is an IFWGOS in Y. Hence f is an IFWG*OM.

(b) ⇒ (a): Let f be an IFWG irresolute and IFWG*OM. To prove that $f^{-1} : (Y, \sigma) \to (X, \tau)$ is an IFWG irresolute mapping. Let A be an IFWGOS in X. Since f is an IFWG*OM, f(A) is an IFWGOS in Y. Now $(f^{-1})^{-1}(A) = f(A)$ is an IFWGOS in Y. Therefore $f^{-1} : (Y, \sigma) \to (X, \tau)$ is an IFWG irresolute mapping. Hence f is an IFWG* homeomorphism.

(b) ⇒ (c): Let f be an IFWG irresolute and IFWG*OM. To prove that f is an IFWG*CM. Let B be an IFWGCS in X. Then B^c is an IFWGOS in X. Since f is an IFWG*OM, $f(B^c) = (f(B))^c$ is an IFWGOS in Y. Therefore f(B) is an IFWGCS in Y. Hence f is an IFWG*CM.

(c) ⇒ (b): Let f be an IFWG irresolute and IFWG*CM. To prove that f is an IFWG*OM. Let A be an IFWGOS in X. Then A^c is an IFWGCS in X. Since f is an IFWG*CM, $f(A^c) = (f(A))^c$ is an IFWGCS in Y. Therefore f(A) is an IFWGOS in Y. Hence f is an IFWG*OM. □

Theorem 4.6. *The composition of two IFWG* homeomorphism is an IFWG* homeomorphism in general.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \delta)$ be any two IFWG* homeomorphisms. Let A be an IFWGCS in Z. Since $g : (Y, \sigma) \to (Z, \delta)$ is an IFWG irresolute mapping, $g^{-1}(A)$ is an IFWGCS in Y. Also since $f : (X, \tau) \to (Y, \sigma)$ is an IFWG irresolute mapping, $f^{-1}(g^{-1}(A)) = (gof)^{-1}(A)$ is an IFWGCS in X. Hence gof is an IFWG irresolute mapping.

Again, let A be an IFWGCS in X. Since $f^{-1} : (Y, \sigma) \to (X, \tau)$ is an IFWG irresolute mapping, $(f^{-1})^{-1}(A) = f(A)$ is an IFWGCS in Y. Also since $g^{-1} : (Z, \delta) \to (Y, \sigma)$ is an IFWG irresolute mapping, $(g^{-1})^{-1}(f(A)) = g(f(A)) = (gof)(A)$ is an IFWGCS in Z. Therefore $((gof)^{-1})^{-1}(A) = (gof)(A)$ is an IFWGCS in Z. Hence $(gof)^{-1}$ is an IFWG irresolute mapping. Thus gof is an IFWG* homeomorphism. $\square$

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