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A new view on intuitionistic fuzzy $\mathscr{C}$ structure compactification

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Abstract. In this paper, the notions of an intuitionistic fuzzy T^c prefilter, intuitionistic fuzzy T^c ultrafilter, intuitionistic fuzzy normal family, intuitionistic fuzzy F^* space, intuitionistic fuzzy $\mathscr{C}$ structure and intuitionistic fuzzy $\mathscr{C}$ space are introduced. The concept of $\mathscr{C}$ structure compactification in an intuitionistic fuzzy topological space is introduced. In this connection, some interesting propositions are established.

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Keywords: Intuitionistic fuzzy T^c prefilter, Intuitionistic fuzzy T^c ultrafilter, Intuitionistic fuzzy normal family, Intuitionistic fuzzy F^* space, Intuitionistic fuzzy $\mathscr{C}$ structure, Intuitionistic fuzzy $\mathscr{C}$ space, Intuitionistic fuzzy $\mathscr{C}st$ prefilter.

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1. Introduction

The concept of fuzzy sets was introduced by Zadeh [6] in 1965. The theory of fuzzy topological spaces was introduced and developed by Chang [3] in 1968. Atanassov [1] introduced and studied intuitionistic fuzzy sets. On the otherhand, Coker [4] introduced the notions of an intuitionistic fuzzy topological space and some other concepts. Later N.Blasco Mardones, M.Macho Stadler and M. A. de Prada Vincente [2] were introduced the concepts of fuzzy compactifications. In this paper, the concepts of intuitionistic fuzzy T^c prefilter, intuitionistic fuzzy T^c ultrafilter, intuitionistic fuzzy prime T^c prefilter, intuitionistic fuzzy normal family, intuitionistic fuzzy F^* space, intuitionistic fuzzy $\mathscr{C}$ structure, intuitionistic fuzzy $\mathscr{C}$ space and intuitionistic fuzzy $\mathscr{C}st$ prefilter are discused. Also a new process of $\mathscr{C}$ structure compactification for an intuitionistic fuzzy topological space is established. In this connection, some interesting propositions are discussed.

2. Preliminaries

Definition 2.1 ([1]). Let X be a nonempty fixed set and I be the closed interval [0,1]. An intuitionistic fuzzy set(IFS) A is an object of the following form $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$, where the mappings $\mu_A : X \longrightarrow I$ and $\gamma_A : X \longrightarrow I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\gamma_A(x)$) for each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each $x \in X$. Obviously,every fuzzy set A on a nonempty set X is an IFS of the following form, $A = \{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle : x \in X\}$. For the sake of simplicity, we shall use the symbol $A = \langle x, \mu_A, \gamma_A\rangle$ for the intuitionistic fuzzy set $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$.

Definition 2.2 ([1]). Let X be a nonempty set and the IFSs A and B in the form $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$, $B = \{\langle x, \mu_B(x), \gamma_B(x)\rangle : x \in X\}$. Then

(i) $A \subseteq B$ iff $\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for all $x \in X$;
(ii) $\overline{A} = \{\langle x, \gamma_A(x), \mu_A(x)\rangle : x \in X\}$;
(iii) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \gamma_A(x) \vee \gamma_B(x)\rangle : x \in X\}$;
(iv) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \gamma_A(x) \wedge \gamma_B(x)\rangle : x \in X\}$.

Definition 2.3 ([1]). The IFSs $0_\sim$ and $1_\sim$ are defined by $0_\sim = \{\langle x, 0, 1\rangle : x \in X\}$ and $1_\sim = \{\langle x, 1, 0\rangle : x \in X\}$.

Definition 2.4 ([4]). An intuitionistic fuzzy topology (IFT) in Coker's sense on a nonempty set X is a family τ of IFSs in X satisfying the following axioms:

(i) $0_\sim, 1_\sim \in \tau$;
(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$;
(iii) $\cup G_i \in \tau$ for arbitrary family $\{G_i \mid i \in I\} \subseteq \tau$.

In this paper by (X, τ) or simply by X we will denote the Coker's intuitionistic fuzzy topological space (IFTS). Each IFS in τ is called intuitionistic fuzzy open set (IFOS) in X. The complement $\overline{A}$ of an IFOS A in X is called an intuitionistic fuzzy closed set (IFCS) in X.

Definition 2.5 ([4]). Let A be an IFS in IFTS X. Then

$int(A) = \bigcup\{G \mid G$ is an IFOS in X and $G \subseteq A\}$ is called an intuitionistic fuzzy interior of A;

$clA = \bigcap\{G \mid G$ is an IFCS in X and $G \supseteq A\}$ is called an intuitionistic fuzzy closure of A.

Definition 2.6 ([4]). Let (X, τ) and (Y, σ) be two IFTSs and let $f : X \to Y$ be a function. Then f is said to be intuitionistic fuzzy continuous function iff the preimage of each IFS in σ is an IFS in τ.

Definition 2.7 ([4]). $f : X \to Y$ is intuitionistic fuzzy continuous function iff the preimage of each IFCS in σ is an IFCS in τ.

Definition 2.8 ([4]). Let (X, τ) and (Y, σ) be two IFTSs and let $f : X \to Y$ be a function. Then f is said to be intuitionistic fuzzy closed function iff the image of each IFCS in τ is an IFCS in σ.

Definition 2.9 ([5]). Let X be a nonempty set and $x \in X$ a fixed element in X. If $r \in I_0$, $s \in I_1$ are fixed real numbers such that $r + s \leq 1$, then the IFS

$x_{r,s} = \langle x, x_r, 1 - x_{1-s}\rangle$ is called an intuitionistic fuzzy point(IFP) in X, where r denotes the degree of membership of $x_{r,s}$, s denotes the degree of nonmembership of $x_{r,s}$ and $x \in X$ the support of $x_{r,s}$. The IFP $x_{r,s}$ is contained in the IFS $A(x_{r,s} \in A)$ if and only if $r < \mu_A(x), s > \gamma_A(x)$.

Definition 2.10 ([4])**.** Let (X, τ) be an IFTS. If a family $\{\langle x, \mu_{G_i}, \gamma_{G_i}\rangle : i \in J\}$ of IFOSs in X satisfies the condition $\bigcup\{\langle x, \mu_{G_i}, \gamma_{G_i}\rangle : i \in J\} = 1_\sim$ then it is called a intuitionistic fuzzy open cover of X.

Definition 2.11 ([4])**.** Let (X, τ) be an IFTS. If a family $\{\langle x, \mu_{K_i}, \gamma_{K_i}\rangle : i \in J\}$ of IFCSs in X satisfies the finite intersection property(FIP for short) iff every finite subfamily $\{\langle x, \mu_{K_i}, \gamma_{K_i}\rangle : i = 1, 2, ..., n\}$ of the family satisfies the condition $\bigcap_{i=1}^{n}\{\langle x, \mu_{K_i}, \gamma_{K_i}\rangle\} \neq 0_\sim$.

Definition 2.12 ([4])**.** An IFTS (X, τ) is called intuitionistic fuzzy compact iff every intuitionistic fuzzy open cover of X has a finite subcover.

Corollary 2.13 ([4])**.** *An IFTS (X, τ) is intuitionistic fuzzy compact iff every family $\{\langle x, \mu_{K_i}, \gamma_{K_i}\rangle : i \in J\}$ of IFCSs in X having the FIP has a nonempty intersection.*

3. T^c prefilters in an intuitionistic fuzzy topological space

In the following sections, the collection of all intuitiionistic fuzzy closed sets in an intuitionistic fuzzy topological space(IFTS) is denoted by T^c.

Definition 3.1. Let (X, T) be an intuitionistic fuzzy topological space. Let $\mathscr{F} \subset T^c$ satisfying the following conditions.

(i) $\mathscr{F}$ is a nonempty family and $0_\sim \notin \mathscr{F}$
(ii) If $A_1, A_2 \in \mathscr{F}$ the $A_1 \cap A_2 \in \mathscr{F}$
(iii) If $A \in \mathscr{F}$ and $B \in T^c$ with $A \subseteq B$, then $B \in \mathscr{F}$.

$\mathscr{F}$ is called an intuitionistic fuzzy T^c prefilter on X.

Notation 3.1.

(i) If $\mathscr{F}$ and $\mathscr{G}$ are two intuitionistic fuzzy T^c prefilters on X. Then we say they are intuitionistic fuzzy compatible (in short, $\mathscr{F} \sim \mathscr{G}$)if every element of $\mathscr{F}$ meets every element of $\mathscr{G}$. In other words $\mathscr{F} \sim \mathscr{G}$ if and only if for every $A \in \mathscr{F}$, for every $B \in \mathscr{G}$ such that $A \cap B \neq 0_\sim$. If $\mathscr{F} \sim \mathscr{G}$ we can construct an intuitionistic fuzzy T^c prefilters which contains them both.
(ii) If $\mathscr{F}$ and $\mathscr{G}$ are two intuitionistic fuzzy T^c prefilters on X.Then we say that $\mathscr{G}$ is finer than $\mathscr{F}$ or $\mathscr{F}$ is coarser than $\mathscr{G}$ if $\mathscr{F} \subseteq \mathscr{G}$.
(iii) The collection of all intuitionistic fuzzy T^c prefilters on X is denoted by $\mathscr{P}^{T^c}$

Example 3.2. Let $X = \{a, b\}$ be a nonempty set. Let $A = \langle x, (\frac{a}{0.5}, \frac{b}{0.4}), (\frac{a}{0.5}, \frac{b}{0.6})\rangle$, $B = \langle x, (\frac{a}{0.2}, \frac{b}{0.4}), (\frac{a}{0.8}, \frac{b}{0.6})\rangle$,$C = \langle x, (\frac{a}{0.5}, \frac{b}{0.3}), (\frac{a}{0.5}, \frac{b}{0.7})\rangle$,$D = \langle x, (\frac{a}{0.2}, \frac{b}{0.3}), (\frac{a}{0.8}, \frac{b}{0.7})\rangle$ and $F = \langle x, (\frac{a}{1}, \frac{b}{0.4}), (\frac{a}{0}, \frac{b}{0.6})\rangle$ be intuitionistic fuzzy sets of X. Then the family $T = \{0_\sim, A, B, C, D, F, 1_\sim\}$ is an intuitionistic fuzzy topology on X. Consider $\mathscr{G}_1 = \{\overline{C}, \overline{D}, 1_\sim\}$ and $\mathscr{G}_2 = \{\overline{A}, \overline{C}, \overline{D}, 1_\sim\}$ are intuitionistic fuzzy T^c prefilters on X. Therefore, $\mathscr{G}_1 \sim \mathscr{G}_2$.

Definition 3.3. Let (X, T) be an intuitionistic fuzzy topological space. Let $\mathscr{F}$ be an intuitionistic fuzzy T^c prefilter and let $\mathscr{B} \subset \mathscr{F}$. $\mathscr{B}$ is called an intuitionistic fuzzy base for $\mathscr{F}$ if for each intuitionistic fuzzy set $A \in \mathscr{F}$, there is an intuitionistic fuzzy set $B \in \mathscr{B}$ such that $B \subseteq A$.

Definition 3.4. Let (X, T) be an intuitionistic fuzzy topological space.Let $\mathscr{H} \subset T^c$.$\mathscr{H}$ is an intuitionistic fuzzy subbase for some intuitionistic fuzzy T^c prefilter if the collection $\{\cap_{i=1}^{n} A_i : A_i \in \mathscr{H}\}$ is an intuitionistic fuzzy base for some intuitionistic fuzzy T^c prefilter.

Proposition 3.5. *Let (X, T) be an intuitionistic fuzzy topological space. Let $\mathscr{B} \subset T^c$.Then the following statements are equivalent.*

(i) *There is a unique intuitionistic fuzzy T^c prefilter $\mathscr{F}$ such that $\mathscr{B}$ is an intuitionistic fuzzy base for it.*

(ii) *(a) $\mathscr{B}$ is a nonempty family and $0_\sim \notin \mathscr{B}$.*
(b) If $B_1, B_2 \in \mathscr{B}$, there is an intuitionistic fuzzy set $B_3 \in \mathscr{B}$ with $B_3 \subseteq B_1 \cap B_2$.

Proof. Follows from the Definitions 3.1, 3.3 and 3.4. □

Definition 3.6. Let (X, T) be an intuitionistic fuzzy topological space. Let $\mathscr{B}$ be an intuitionistic fuzzy base satisfying the above conditions (a) and (b). Then the generated intuitionistic fuzzy T^c prefilter $\mathscr{F}$ is defined by $\mathscr{F}$=$\{A \in T^c : \exists B \in \mathscr{B}$ with $B \subseteq A\}$.

Definition 3.7. Let (X, T) be an intuitionistic fuzzy topological space. Let $\mathscr{G} \subset T^c$ with the property that the intersection of any finite subcollection from $\mathscr{G}$ is nonempty. There exists a unique intuitionistic fuzzy T^c prefilter containing $\mathscr{G}$, whose intuitionistic fuzzy base is the set of all finite intersections of elements in $\mathscr{G}$. Such an intuitionistic fuzzy T^c prefilter is called an intuitionistic fuzzy T^c prefilter generated by $\mathscr{G}$.

Proposition 3.8. *Let (X, T) be an intuitionistic fuzzy topological space. Let $\mathscr{F}$ be an intuitionistic fuzzy T^c prefilter and $A \in T^c$. Then the following statements are equivalent.*

(i) *There is an intuitionistic fuzzy T^c prefilter $\mathscr{F}_1$ which is finer than $\mathscr{F}$ such that $A \in \mathscr{F}_1$.*

(ii) *For each $B \in \mathscr{F}$, we have $A \cap B \neq 0_\sim$.*

Proof. (i) ⇒ (ii) Let $A \in T^c$ and $\mathscr{F}_1$ be an intuitionistic fuzzy T^c prefilter such that $\mathscr{F}_1$ which is finer than $\mathscr{F}$ such that $A \in \mathscr{F}_1$. Suppose that $A \cap B = 0_\sim$, for each $B \in \mathscr{F}$. This implies that $A \cap B = 0_\sim \in \mathscr{F}_1$. which is contradiction. Thus (ii) is proved.

(ii) ⇒ (i) Let $A \cap B \neq 0_\sim$,for each $B \in \mathscr{F}$. Suppose that there is an intuitionistic fuzzy T^c prefilter $\mathscr{F}_1$ which is not finer than $\mathscr{F}$ such that $A \in \mathscr{F}_1$. This implies that $A \in \mathscr{F}$. But by assumption $A \notin \mathscr{F}$, which is contradiction. Thus (i) is proved. □

Definition 3.9. Let (X, T) be an intuitionistic fuzzy topological space. Let $\mathscr{F}$ be an intuitionistic fuzzy T^c prefilter. $\mathscr{F}$ is an intuitionistic fuzzy T^c ultrafilter if $\mathscr{F}$ is a maximal element in the set of an intuitionistic fuzzy T^c prefilters ordered by the inclusion relation.

Proposition 3.10. *Every intuitionistic fuzzy T^c prefilter is contained in some intuitionistic fuzzy T^c ultrafilter.*

Proof. Let $\mathscr{F}$ be any intuitionistic fuzzy T^c prefilter on a set X. Define $\Phi = \{\mathscr{G}/\mathscr{G} \in \mathscr{P}^{T^c}$ and $\mathscr{F} \subseteq \mathscr{G}\}$. Inclusion is a partial ordering on Φ. Let $\mathscr{G}_i$ be a chain in Φ. Then it is easy to check that $\mathscr{E} = \bigcup \mathscr{G}_i$ is an intuitionistic fuzzy T^c prefilter on X. Then by applying Zorn's Lemma to deduce that Φ has a maximal element $\mathscr{G}$. Then $\mathscr{F} \subseteq \mathscr{G}$ and $\mathscr{G}$ is maximal. □

Proposition 3.11. *Let (X, T) be an intuitionistic fuzzy topological space. Let $\mathscr{F}$ be an intuitionistic fuzzy T^c prefilter on X. Then the following statements are equivalent*

(i) *$\mathscr{F}$ is an intuitionistic fuzzy T^c ultrafilter.*
(ii) *If A is an element of intuitionistic fuzzy T^c prefilter such that $A \cap B \neq 0_\sim$ for each $B \in \mathscr{F}$, then $A \in \mathscr{F}$.*
(iii) *If $A \in T^c$ and $A \notin \mathscr{F}$, then there is $B \in \mathscr{F}$ such that $A \cap B = 0_\sim$.*

Proof. (i) $\Rightarrow$ (ii) Suppose $A \in T^c$ and $A \cap B \neq 0_\sim$ for each $B \in \mathscr{F}$. By Proposition 3.8, there is an intuitionistic fuzzy T^c prefilter $\mathscr{F}^*$ generated by $\mathscr{F} \cup \{A\}$. Then $\mathscr{F} \subset \mathscr{F}^*$. Since $\mathscr{F}$ is an intuitionistic fuzzy T^c ultrafilter, it must be $\mathscr{F} = \mathscr{F}^*$.Therefore $A \in \mathscr{F}$.

(ii)$\Rightarrow$ (iii) Let $A \in T^c$. Suppose $A \notin \mathscr{F}$. By assumption, there exists $B \in \mathscr{F}$ such that $A \cap B = 0_\sim$.

(iii) $\Rightarrow$ (i) Let $\mathscr{G}$ be an intuitionistic fuzzy T^c prefilter with $\mathscr{F} \subset \mathscr{G}$ and $\mathscr{F} \neq \mathscr{G}$. Let $A \in \mathscr{G}$ such that $A \notin \mathscr{F}$. By assumption, there exists $B \in \mathscr{F}$ such that $A \cap B = 0_\sim$. Since $A, B \in \mathscr{G}$ then $A \cap B \in \mathscr{G}$ implies that $0_\sim \in \mathscr{G}$, which is a contradiction. Therefore $\mathscr{F} = \mathscr{G}$. Hence $\mathscr{F}$ is an intuitionistic fuzzy T^c ultrafilter. □

Proposition 3.12. *Let (X, T) be an intuitionistic fuzzy topological space. Let $\mathscr{U}_1$ and $\mathscr{U}_2$ be a pair of different intuitionistic fuzzy T^c ultrafilters on X. Then*

$$\bigcap_{A_i \in \mathscr{U}_1} A_i \cap \bigcap_{A_j \in \mathscr{U}_2} A_j = 0_\sim.$$

Proof. Suppose $\bigcap_{A_i \in \mathscr{U}_1} A_i \cap \bigcap_{A_j \in \mathscr{U}_2} A_j \neq 0_\sim$. Then for some $x \in X$,

$$\mu_{(\bigcap_i A_i \cap \bigcap_j A_j)}(x) > 0$$

and

$$\gamma_{(\bigcap_i A_i \cap \bigcap_j A_j)}(x) < 1,$$

for all i, j. This implies that $\mu_{A_i}(x) \wedge \mu_{A_j}(x) > 0$ and $\gamma_{A_i}(x) \vee \gamma_{A_j}(x) < 1$, for all i, j. This implies that $A_i \cap A_j \neq 0_\sim$. By Proposition 3.11, $A_i \in \mathscr{U}_2$ and $A_j \in \mathscr{U}_1$, for all i, j. Thus $\mathscr{U}_1 = \mathscr{U}_2$, which is contradiction. Hence, $\bigcap_{A_i \in \mathscr{U}_1} A_i \cap \bigcap_{A_j \in \mathscr{U}_2} A_2 = 0_\sim$. □

Definition 3.13. Let (X, T) be an intuitionistic fuzzy topological space. Let $\mathscr{F}$ be an intuitionistic fuzzy T^c prefilter on X. $\mathscr{F}$ is called an intuitionistic fuzzy prime T^c prefilter if for each $A, B \in T^c$ such that $A \cup B \in \mathscr{F}$ then $A \in \mathscr{F}$ (or) $B \in \mathscr{F}$.

Proposition 3.14. *Every intuitionistic fuzzy T^c ultrafilter $\mathscr{U}$ on X is an intuitionistic fuzzy T^c prefilter.*

Proof. Let $A, B \in T^c$ such that $A \cup B \in \mathscr{U}$. Suppose $A, B \notin \mathscr{U}$. Then there exist $A^*, B^* \in \mathscr{U}$ with $A^* \cap A = 0_\sim = B^* \cap B$. Since $A \cup B, A^*$ and $B^* \in \mathscr{U}$, we have $(A \cup B) \cap A^* \cap B^* \in \mathscr{U}$. Then

$$\begin{aligned}(A \cup B) \cap A^* \cap B^* &= (A \cup B) \cap A^* \cap B^* \\ &= ((A \cap A^*) \cup (B \cap A^*)) \cap B^* \\ &= (A \cap A^* \cap B^*) \cup (B \cap A^* \cap B^*) \\ &= 0_\sim\end{aligned}$$

which is a contradiction. Hence $\mathscr{U}$ is an intuitionistic fuzzy prime T^c prefilter. □

Proposition 3.15. *Let (X, T) be an intuitionistic fuzzy topological space. Let $\mathscr{F}$ be an intuitionistic fuzzy T^c prefilter. Let $\mathscr{P}(\mathscr{F})$ be the family of all intuitionistic fuzzy prime T^c prefilters which contains $\mathscr{F}$. Then*

$$\mathscr{F} = \bigcap_{\mathscr{G} \in \mathscr{P}(\mathscr{F})} \mathscr{G}$$

Proof. It is clear that

$$\mathscr{F} \subset \bigcap_{\mathscr{G} \in \mathscr{P}(\mathscr{F})} \mathscr{G} \tag{3.1}$$

Let $A \in T^c$ such that $A \notin \mathscr{F}$. Consider the family $\mathscr{L} = \{\mathscr{G} \in \mathscr{P}^{T^c} : \mathscr{F} \subset \mathscr{G}$ and $A \notin \mathscr{G}\}$. Now, $\mathscr{L}$ is an inductive set and there exist maximal elements. Let $\mathscr{U}$ be a maximal element in $\mathscr{L}$. Let $C_1, C_2 \in T^c$ with $C_1 \cup C_2 \in \mathscr{U}$ such that $C_1, C_2 \notin \mathscr{U}$. Let the family $\mathscr{I} = \{B \in T^c : B \cup C_2 \in \mathscr{U}\}$.

(i) Since $C_1 \in T^c$ then $C_1 \cup A_2 \in \mathscr{U}$ implies that $C_1 \in \mathscr{I}$. Hence $\mathscr{I}$ is an nonempty family. Suppose $0_\sim \in \mathscr{I}$. By Definition of $\mathscr{I}$, $C_2 \in \mathscr{I}$ which is a contradiction. Hence $0_\sim \notin \mathscr{I}$.

(ii) If $B_1, B_2 \in \mathscr{I}$, then $B_1 \cup C_2 \in \mathscr{U}$ and $B_2 \cup C_2 \in \mathscr{U}$. Since $\mathscr{U}$ is an intuitionistic fuzzy T^c prefilter, $(B_1 \cup C_2) \cap (B_2 \cup C_2) \in \mathscr{U}$ that is, $((B_1 \cap B_2) \cup C_2) \in \mathscr{U}$. Therefore $(B_1 \cap B_2) \in \mathscr{I}$.

(iii) If $B \in \mathscr{I}$ and $C \in T^c$ such that $B \subseteq C$ then $B \cup C_2 \subseteq C \cup C_2$. Since $\mathscr{U}$ is an intuitionistic fuzzy T^c prefilter and $C \cup C_2 \in \mathscr{U}$ implies that $C \in \mathscr{I}$. Hence $\mathscr{I}$ is an intuitionistic fuzzy T^c prefilter.

If $B \in \mathscr{U}$ then $B \cup C_2 \in \mathscr{U}$ implies that $B \in \mathscr{I}$. Hence $\mathscr{U} \subset \mathscr{I}$. Since $A_1 \in \mathscr{I}$ and $A_1 \notin \mathscr{U}$. Therefore $\mathscr{U} \neq \mathscr{I}$.
Let $\mathscr{K} = \{C \in T^c : A \cup C_2 \in \mathscr{U}\}$.

(i) Suppose $0_\sim \in \mathscr{K}$. By Definition of $\mathscr{K}$, $C_2 \in \mathscr{U}$ which is a contradiction to our assumption $\mathscr{U} \in \mathscr{L}$ and $C_2 \notin \mathscr{U}$. Hence $0_\sim \notin \mathscr{K}$. Since $1_\sim \in \mathscr{U}$ implies that $1_\sim \in \mathscr{K}$. Thus $\mathscr{K}$ is an nonempty family and $0_\sim \notin \mathscr{K}$.

(ii) If $C^*, C^{**} \in \mathscr{K}$, then $B \cup C^* \in \mathscr{U}$ and $B \cup C^{**} \in \mathscr{U}$. Since $\mathscr{U}$ is an intuitionistic fuzzy T^c prefilter, $(B \cup C^*) \cap (B \cup C^{**}) \in \mathscr{U}$ that is, $B \cup (C^* \cap C^{**}) \in \mathscr{U}$. Therefore $(C^* \cap C^{**}) \in \mathscr{K}$.

(iii) If $C \in \mathscr{K}$ and $C^* \in T^c$ such that $C^* \supseteq C$ then $C^* \in \mathscr{K}$.

Hence $\mathscr{K}$ is an intuitionistic fuzzy T^c prefilter. Now,

(i) $\mathscr{F} \subset \mathscr{K}$ follows from $\mathscr{F} \subset \mathscr{U}$ and $\mathscr{U} \subset \mathscr{K}$

(ii) $A \notin \mathscr{K}$, for $A \notin \mathscr{U}$.

Thus $\mathscr{K} \in \mathscr{L}$ and $\mathscr{U} \subset \mathscr{K}$. Maximality of $\mathscr{U}$ implies that $\mathscr{U} = \mathscr{K}$. Suppose $A \in \mathscr{I}$. Then $A \cup C_2 \in \mathscr{U}$ implies that $C_2 \in \mathscr{K} = \mathscr{U}$, which is contradiction to $C_2 \notin \mathscr{U}$. Therefore $A \notin \mathscr{I}$. Since $\mathscr{F} \subset \mathscr{I}$ and $A \notin \mathscr{I}$, $\mathscr{I} \in \mathscr{L}$. Since $\mathscr{U} \subset \mathscr{I}$ and $A \neq \mathscr{I}$, which is contradiction to the maximality of $\mathscr{U}$. Thus, $C_1, C_2 \in \mathscr{U}$. Therefore $\mathscr{U}$ is an intuitionistic fuzzy prime T^c prefilter and $A \notin \mathscr{U}$. This proves

$$\bigcap_{\mathscr{G} \in \mathscr{P}(\mathscr{F})} \mathscr{G} \subset \mathscr{F} \tag{3.2}$$

From (3.1) and (3.2), it follows $\bigcap_{\mathscr{G} \in \mathscr{P}(\mathscr{F})} \mathscr{G} = \mathscr{F}$ □

Proposition 3.16. *Let (X, T) be an intuitionistic fuzzy topological space. Then the following statements are equivalent:*

(i) *(X, T) is an intuitionistic fuzzy compact space.*
(ii) *every intuitionistic fuzzy T^c prefilter $\mathscr{F}$ satisfies $\bigcap_{A \in \mathscr{F}} A \neq 0_\sim$.*
(iii) *every intuitionistic fuzzy prime T^c prefilters $\mathscr{F}$ satisfies $\bigcap_{A \in \mathscr{F}} A \neq 0_\sim$.*
(iv) *every intuitionistic fuzzy T^c ultrafilter $\mathscr{U}$ satisfies $\bigcap_{A \in \mathscr{U}} A \neq 0_\sim$.*

Proof. (i) ⇒ (ii) Suppose $\bigcap_{A \in \mathscr{F}} A = 0_\sim$. Then $\bigcup_{A \in \mathscr{F}} \overline{A} = 1_\sim$. Since $\overline{A} \in T$ and (X, T) is an intuitionistic fuzzy compact space, there must exist a finite subcollection $\{\overline{A_1}, \overline{A_2}, ..., \overline{A_n}\}$such that $1_\sim = \bigcup_{i=1}^{n} \overline{A_i}$. That is $\bigcap_{i=1}^{n} \overline{A_i} = 0_\sim$ which is contradiction to the Definition 3.1. Therefore $\bigcap_{A \in \mathscr{F}} A \neq 0_\sim$.

(ii) ⇒ (iii) Suppose that every intuitionistic fuzzy prime T^c prefilter satisfies $\bigcap_{A \in \mathscr{F}} A = 0_\sim$. Since every intuitionistic fuzzy prime T^c prefilter is an intuitionistic fuzzy T^c prefilter, which is a contradiction. Hence every intuitionistic fuzzy prime T^c prefilters $\mathscr{F}$ satisfies $\bigcap_{A \in \mathscr{F}} A \neq 0_\sim$.

(iii) ⇒ (iv) Suppose that every intuitionistic fuzzy T^c ultrafilter $\mathscr{U}$ satisfies $\bigcap_{A \in \mathscr{U}} A = 0_\sim$. Since every intuitionistic fuzzy T^c ultrafilter is an intuitionistic fuzzy prime T^c prefilter, which is a contradiction. Hence every intuitionistic fuzzy T^c ultrafilters $\mathscr{U}$ satisfies $\bigcap_{A \in \mathscr{U}} A \neq 0_\sim$.

(iv) ⇒ (i) Suppose $\mathscr{H}$ is a family of intuitionistic fuzzy closed sets on X with the finite intersection property. For each $B \in \mathscr{H}$, we consider the family $\mathscr{G}_B = \{A \in T^c : A \supseteq B\}$. Then, $B \in \mathscr{G}_B$. Let $\mathscr{G} = \bigcup_{B \in \mathscr{H}} \mathscr{G}_B$. Since $\mathscr{H}$ has the finite intersection property,$\mathscr{G}$ has as well. Thus $\mathscr{H}$ and $\mathscr{G}$ aremintuitionistic fuzzy T^c prefilters. Therefore, there exists an intuitionistic fuzzy T^c ultrafilter $\mathscr{U}$ such that $\mathscr{H} \subseteq \mathscr{G} \subseteq \mathscr{U}$. Therefore, $\bigcap_{A \in \mathscr{U}} A \subseteq \bigcap_{A \in \mathscr{G}} A \subseteq \bigcap_{A \in \mathscr{H}} A$. By (iv), $\bigcap_{A \in \mathscr{U}} A \neq 0_\sim$. Therefore $\bigcap_{A \in \mathscr{H}} A \neq 0_\sim$. Hence (X, T) is an intuitionistic fuzzy compact space. □

Definition 3.17. Let (X, T) be an intuitionistic fuzzy topological space. Let $x_{r,s}$ be any intuitionistic fuzzy point. The nonempty collection $\mathscr{F}_{x_{r,s}} = \{A \in T^c : x_{r,s} \in A\}$ is an intuitionistic fuzzy prime T^c prefilter on X.Then $\mathscr{F}_{x_{r,s}}$ is called an intuitionistic fuzzy T^c prefilter generated by $x_{r,s}$.

Definition 3.18. Let (X, T) be an intuitionistic fuzzy topological space. The collection T^c is said to be an intuitionistic fuzzy normal if given $A_1, A_2 \in T^c$ such that $A_1 \cap A_2 = 0_\sim$, there exist $B_1, B_2 \in T^c$ with $B_1 \cup B_2 = 1_\sim$, $A_1 \cap B_1 = 0_\sim$ and $A_2 \cap B_2 = 0_\sim$.

Proposition 3.19. *Let (X,T) be an intuitionistic fuzzy topological space and T^c be an intuitionistic fuzzy normal family. Every intuitionistic fuzzy prime T^c prefilter $\mathscr{F}$ is contained in an unique intuitionistic fuzzy T^c ultrafilter.*

Proof. Let $\mathscr{U}_1$ and $\mathscr{U}_2$ be an intuitionistic fuzzy T^c ultrafilters such that $\mathscr{F} \subset \mathscr{U}_1$ and $\mathscr{F} \subset \mathscr{U}_2$. Suppose $\mathscr{U}_1 \neq \mathscr{U}_2$. Then, there exist $A_1 \in \mathscr{U}_1$and $A_2 \in \mathscr{U}_2$ with $A_1 \cap A_2 = 0_\sim$. Thus, there exist $B_1, B_2 \in T^c$ with $B_1 \cup B_2 = 1_\sim$, $A_1 \cap B_1 = 0_\sim$ and $A_2 \cap B_2$. Since $B_1 \cup B_2 = 1_\sim$ and $\mathscr{F}$ is an intuitionistic fuzzy prime T^c prefilter, $B_1 \in \mathscr{F}$ (or) $B_2 \in \mathscr{F}$. Suppose $B_1 \in \mathscr{F}$. Then $B_1 \in \mathscr{U}_1$ and $B_1 \in \mathscr{U}_2$. Thus $A_1 \cap B_1 = 0_\sim$ with $A_1, B_1 \in \mathscr{U}_2$, which is a contradiction. Similarly, $B_2 \in \mathscr{F}$, $A_2, B_2 \in \mathscr{U}_2$ with $A_2 \cap B_2 = 0_\sim$, which is a contradiction. Hence $\mathscr{U}_1 = \mathscr{U}_2$. Thus, every intuitionistic fuzzy prime T^c prefilter $\mathscr{F}$ is contained in an unique intuitionistic fuzzy T^c ultrafilter. □

Corollary 3.20. *Let (X,T) be an intuitionistic fuzzy topological space. Let T^c be an intuitionistic fuzzy normal family. Then for each $x \in X$, an intuitionistic fuzzy point $x_{r,s}$, there exists an unique intuitionistic fuzzy T^c ultrafilter $\mathscr{U}_{x_{r,s}}$ which contains $\mathscr{F}_{x_{r,s}}$.*

Proof. Let $x_{r,s}$ be any intuitionistic fuzzy point. Then $\mathscr{F}_{x_{r,s}}$ is an intuitionistic fuzzy T^c prefilter generated by $x_{r,s}$. By Definition 3.17 and Proposition 3.19, $\mathscr{F}_{x_{r,s}}$ is an intuitionistic fuzzy prime T^c prefilter contained in an unique intuitionistic fuzzy T^c ultrafilter $\mathscr{U}_{x_{r,s}}$. □

Corollary 3.21. *Let (X,T) be an intuitionistic fuzzy topological space. Let T^c be an intuitionistic fuzzy normal family. Suppose $x_{r,s}$ and $y_{m,n}$ are the intuitionistic fuzzy points of (X,T) with $x = y$, then $\mathscr{U}_{x_{r,s}} = \mathscr{U}_{y_{m,n}}$.*

Proof. Suppose $x_{r,s}$ and $y_{m,n}$ are intuitionistic fuzzy points of (X,T) with $x = y$. Let $\mathscr{U}_{x_{r,s}} \neq \mathscr{U}_{y_{m,n}}$. By Proposition 3.5, $\bigcap_i A_i \cap \bigcap_j A_j = 0_\sim$, for all $A_i \in \mathscr{U}_{x_{r,s}}$ and $A_j \in \mathscr{U}_{y_{m,n}}$. Thus $x_{r,s} \in \mathscr{U}_{x_{r,s}}$ and $y_{m,n} \in \mathscr{U}_{y_{m,n}}$. Since $x = y$, $x_{r,s} \cap y_{m,n} \neq 0_\sim$ which is a cotradiction. Hence $\mathscr{U}_{x_{r,s}} = \mathscr{U}_{y_{m,n}}$. □

Definition 3.22. Let (X,T) be an intuitionistic fuzzy topological space. For each $x \in X$, the collection of intuitionistic fuzzy points $\mathscr{P}_x = \{x_{r,s} : r \in (0,1]$ and $s \in [0,1)\}$. For each $x_{r,s} \in \mathscr{P}_x$, the only intuitionistic fuzzy T^c ultrafilter which contains $\mathscr{F}_{x_{r,s}}$ is denoted by $\mathscr{U}^x$

Definition 3.23. An intuitionistic fuzzy topological space (X,T) is said to be an intuitionistic fuzzy F^* space, if for each $x \in X$, there exists a minimum value $r \in (0,1]$ and a maximum value $s \in [0,1)$ such that intuitionistic fuzzy point $x_{r,s}$ belongs to T^c.

Proposition 3.24. *Let (X,T) be an intuitionistic fuzzy F^* space and T^c be an intuitionistic fuzzy normal family. For each $x \in X$ and an intuitionistic fuzzy T^c ultrafilter $\mathscr{U}^x$, $\bigcap_{A\in\mathscr{U}^x} A$ is an atmost intuitionistic fuzzy point $x_{r,s}$.*

Proof. Let $x \in X$. Since (X,T) is an intuitionistic fuzzy F^* space, there exists a minimum value $r \in (0,1]$ and a maximum value $s \in [0,1)$ such that $x_{r,s} \in T^c$. By Definition 3.10, $x_{r,s} \in \mathscr{U}^x$. Hence $\bigcap_{A\in\mathscr{U}^x} A = x_{r,s}$. □

4. $\mathscr{C}$ Structure Compactification in an Intuitionistic Fuzzy Topological Space

Let (X, T) be an intuitionistic fuzzy noncompact space. Let $\gamma(X)$ be the collection of all intuitionistic fuzzy T^c ultrafilters on X. Suppose (X, T) is an intuitionistic fuzzy F^* space and T^c is an intuitionistic fuzzy normal family. Associated with each $A \in T^c$, we define an intuitionistic fuzzy set $A^* = \langle \mathscr{U}, \mu_{A^*}, \gamma_{A^*} \rangle$. For each $\mathscr{U} \in \gamma(X)$, the membership function $\mu_{A^*} : \gamma(X) \to I$ is defined by

$$\mu_{A^*}(\mathscr{U}) = \begin{cases} 0 & \text{if } \forall\, x \in X \text{ and } \mu_A \notin \mathscr{U},\ \mathscr{U} \neq \mathscr{U}^x, \\ 1 & \text{if } \forall\, x \in X \text{ and } \mu_A \in \mathscr{U},\ \mathscr{U} \neq \mathscr{U}^x, \\ \mu_A(x) & \text{if } \exists\, x \in X \text{ with } \mathscr{U} = \mathscr{U}^x. \end{cases}$$

and the nonmembership function $\gamma_{A^*} : \gamma(X) \to I$ is defined by

$$\gamma_{A^*}(\mathscr{U}) = \begin{cases} 1 & \text{if } \forall\, x \in X \text{ and } \gamma_A \notin \mathscr{U},\ \mathscr{U} \neq \mathscr{U}^x, \\ 0 & \text{if } \forall\, x \in X \text{ and } \gamma_A \in \mathscr{U},\ \mathscr{U} \neq \mathscr{U}^x, \\ \gamma_A(x) & \text{if } \exists\, x \in X \text{ with } \mathscr{U} = \mathscr{U}^x. \end{cases}$$

Notation 4.1. For each $\mathscr{U}^x \in \gamma(X)$, the intuitionistic fuzzy point(IFP) of $\gamma(X)$ is denoted by $\mathscr{U}^x_{r,s}$

Proposition 4.1. *Under the previous conditions, the following identities holds:*

(i) $0^*_{\sim X} = 0_{\sim\gamma(X)}$
(ii) $1^*_{\sim X} = 1_{\sim\gamma(X)}$
(iii) $(x_{r,s})^* = \mathscr{U}^x_{r,s}$

Proof. Obvious. □

Definition 4.2. An intuitionistic fuzzy $\mathscr{C}$ structure(in short, $IF\mathscr{C}st$) consists of all intuitionisitic fuzzy sets of the form A^*. An intuitionistic fuzzy $\mathscr{C}$ space $(\gamma(X), IF\mathscr{C}st)$ is a space which admits intuitionistic fuzzy $\mathscr{C}$ structure.

Definition 4.3. An intuitionistic fuzzy $\mathscr{C}$ closure of an intuitionisttic fuzzy set A in an intuitionistic fuzzy $\mathscr{C}$ space is defined by $IFcl_{\mathscr{C}st}(A^*) = \bigcap\{B^* \in IF\mathscr{C}st / A^* \subseteq B^*\}$.

Proposition 4.4. *Let $e : X \to \gamma(X)$ is defined by $e(x) = \mathscr{U}^x$, for each $x \in X$. Then $e(1_{\sim X})$ is an intuitionistic fuzzy $\mathscr{C}$ dense in an intuitionistic fuzzy $\mathscr{C}$ space $(\gamma(X), IF\mathscr{C}st)$ that is, $IFcl_{\mathscr{C}st}(e(1_{\sim X})) = 1_{\sim\gamma(X)}$.*

Proof. Let A be an intuitionistic fuzzy set with $\mu, \gamma \in I^X$. Now,$e(A)$ is an intuitionistic fuzzy set in an intuitionistic fuzzy $\mathscr{C}$ space with $e(A) \in I^{\gamma(X)}$ and it is defined for each $\mathscr{U} \in \gamma(X)$,

$$e(\mu_A(\mathscr{U})) = \begin{cases} \mu_A(x) & \text{if } \exists\, x \in X \text{ such that } \mathscr{U} = \mathscr{U}^x, \\ 0 & \text{if } \forall\, x \in X,\ \mathscr{U} \neq \mathscr{U}^x. \end{cases}$$

and

$$e(\gamma_A(\mathscr{U})) = \begin{cases} \gamma_A(x) & \text{if } \exists\, x \in X \text{ such that } \mathscr{U} = \mathscr{U}^x, \\ 1 & \text{if } \forall\, x \in X,\ \mathscr{U} \neq \mathscr{U}^x. \end{cases}$$

Let $C = IFcl_{\mathscr{C}st}(e(1_{\sim X}))$. We know that $e(1_{\sim X}) \subseteq C$, then for each $x \in X$ and $\mathscr{U}^x \in \gamma(X)$, $1_{\sim\gamma(X)} \subseteq C$. Therefore $C = 1_{\sim\gamma(X)}$. Hence $IFcl_{\mathscr{C}st}(e(1_{\sim X})) = 1_{\sim\gamma(X)}$. Thus $e(1_{\sim X})$ is an intuitionistic fuzzy $\mathscr{C}$ dense in an intuitionistic fuzzy $\mathscr{C}$ space. □

Definition 4.5. Let (X, T) be an intuitionistic fuzzy topological space and $(\gamma(X), IF\mathscr{C}st)$ be an intuitionistic fuzzy $\mathscr{C}$ space. Then $f : (X, T) \to (\gamma(X), IF\mathscr{C}st)$ is said to be intuitionistic fuzzy $\mathscr{C}st$ continuous* function, if the inverse image of every intuitionistic fuzzy set in $(\gamma(X), IF\mathscr{C}st)$ is an intuitionistic fuzzy closed set in (X, T).

Definition 4.6. Let (X, T) be an intuitionistic fuzzy topological space and $(\gamma(X), IF\mathscr{C}st)$ be an intuitionistic fuzzy $\mathscr{C}$ space. Then $f : (X, T) \to (\gamma(X), IF\mathscr{C}st)$ is said to be
intuitionistic fuzzy $\mathscr{C}st$ closed* function, if the image of every intuitionistic fuzzy closed set in (X, T) is an intuitionistic fuzzy set in $(\gamma(X), IF\mathscr{C}st)$.

Notation 4.2. For an intuitionistic fuzzy point $x_{r,s}$,
(i) the degree of membership is denoted by r_x.
(ii) the degree of nonmembership is denoted by s_x.

Proposition 4.7. *The function e is an intuitionistic fuzzy embedding* of X into $\gamma(X)$.*

Proof. (i) Let $x, y \in X$ and $x \neq y$, then $\mathscr{U}^x \neq \mathscr{U}^y$. Let $x_{r,s}$ and $y_{m,n}$ be any two distinct intuitionistic fuzzy points.

(a) If $x \neq y$, for each $\mathscr{U} \in \gamma(X)$,

$$e(r_x)(\mathscr{U}) = \begin{cases} r_x & \text{if } \mathscr{U} = \mathscr{U}^x, \\ 0 & \text{if } \mathscr{U} \neq \mathscr{U}^x. \end{cases}$$

and

$$e(s_x)(\mathscr{U}) = \begin{cases} s_x & \text{if } \mathscr{U} = \mathscr{U}^x, \\ 1 & \text{if } \mathscr{U} \neq \mathscr{U}^x. \end{cases}$$

Similarly,

$$e(m_y)(\mathscr{U}) = \begin{cases} m_y & \text{if } \mathscr{U} = \mathscr{U}^y, \\ 0 & \text{if } \mathscr{U} \neq \mathscr{U}^y. \end{cases}$$

and

$$e(n_y)(\mathscr{U}) = \begin{cases} n_y & \text{if } \mathscr{U} = \mathscr{U}^y, \\ 1 & \text{if } \mathscr{U} \neq \mathscr{U}^y. \end{cases}$$

Therefore $e(x_{r,s}) \neq e(y_{m,n})$.

(b) If $x = y$ then $(r, s) \neq (m, n)$. Thus $e(x_{r,s}) = \mathscr{U}^x_{r,s} = \mathscr{U}^y_{r,s} \neq \mathscr{U}^y_{m,n}$ Therefore $e(x_{r,s}) \neq e(y_{m,n})$. Hence e is an intuitionistic fuzzy $\mathscr{C}st$ 1-1 function.

(ii) For each $A^* \in IF\mathscr{C}st$, $\mathscr{U}^x \in \gamma(X)$ and $x \in X$, $e^{-1}(\mu_{A^*})(x) = \mu_{A^*}(e(x)) = \mu_{A^*}(\mathscr{U}^x) = \mu_A(x)$ and $e^{-1}(\gamma_{A^*})(x) = \gamma_{A^*}(e(x)) = \gamma_{A^*}(\mathscr{U}^x) = \gamma_A(x)$. Then, for each $A \in T^c$, $e^{-1}(A^*) = A$. Thus the inverse image of every intuitionistic fuzzy set A^* in $(\gamma(X), IF\mathscr{C}st)$ is an intuitionistic fuzzy closed set A in (X, T). Hence e is an intuitionistic fuzzy $\mathscr{C}st$ continuous* function.

(iii) For each $\mathscr{U} \in \gamma(X)$ and $A \in T^c$,

$$e(\mu_A(\mathscr{U})) = \begin{cases} 0 & \text{if } \forall\, x \in X, \mathscr{U} \neq \mathscr{U}^x, \\ \mu_A(x) & \text{if } \exists\, x \in X \text{ such that } \mathscr{U} = \mathscr{U}^x. \end{cases}$$

and

$$e(\gamma_A(\mathscr{U})) = \begin{cases} 1 & \text{if } \forall\, x \in X, \mathscr{U} \neq \mathscr{U}^x, \\ \gamma_A(x) & \text{if } \exists\, x \in X \text{ such that } \mathscr{U} = \mathscr{U}^x. \end{cases}$$

$e(A)$ is an intuitionistic fuzzy set in $(\gamma(X), IF\mathscr{C}st)$. Hence e is an intuitionistic fuzzy $\mathscr{C}st$ closed* function. Hence the function e is an intuitionistic fuzzy embedding* of X into $\gamma(X)$. □

Definition 4.8. Let $(\gamma(X), IF\mathscr{C}st)$ be an intuitionistic fuzzy $\mathscr{C}$ space.
Let $\mathscr{F} \subset IF\mathscr{C}st$ satisfying the following conditions.

(i) $\mathscr{F}$ is a nonempty family and $0_\sim \notin \mathscr{F}$
(ii) If $A_1^*, A_2^* \in \mathscr{F}$ the $A_1^* \cap A_2^* \in \mathscr{F}$
(iii) If $A^* \in \mathscr{F}$ and $B^* \in IF\mathscr{C}st$ with $A^* \subseteq B^*$ then $B^* \in \mathscr{F}$.

$\mathscr{F}$ is called an intuitionistic fuzzy $\mathscr{C}st$ prefilter(in short, $(T_{\gamma(X)})^c$) on $\gamma(X)$.

Definition 4.9. An intuitionistic fuzzy $\mathscr{C}$ space $(\gamma(X), IF\mathscr{C}st)$ is said to be an intuitionistic fuzzy $\mathscr{C}$ compact* space if whenever $\bigcup_{i\in I} A_i^* = 1_{\sim\gamma}(X), A_i^* \in IF\mathscr{C}st$, $i \in I$, there is a finite subfamily J of I with $\bigcup_{j\in J} {A_j}^* = 1_{\sim\gamma}(X)$.

Definition 4.10. An intuitionistic fuzzy $\mathscr{C}$ space $(\gamma(X), IF\mathscr{C}st)$ is an intuitionistic fuzzy $\mathscr{C}$ compact* space if and only if for any family of intuitionistic fuzzy sets $\{A_i^* : i \in I\}$ in an intuitionistic fuzzy $\mathscr{C}$ structure with the property that $\bigcap_{j\in J} A_j^* \neq 0_\sim$, for any finite subfamily J of I, we have $\bigcap_{i\in I} A_i^* \neq 0_\sim$.

Proposition 4.11. *An intuitionistic fuzzy $\mathscr{C}$ space $(\gamma(X), IF\mathscr{C}st)$ is an intuitionistic fuzzy $\mathscr{C}st$ compact* space.*

Proof. Let $\mathscr{F}$ be an intuitionistic fuzzy $\mathscr{C}st$ prefilter on $\gamma(X)$. Let $B \in \mathscr{F}$. Since $B \in IF\mathscr{C}st$, there is an index family I_B such that $B = \bigcap_{i\in I_B} A_i^*$ for $A_i \in T^c$. Since $B \subseteq A_i^*$ for each $i \in I_B$. Therefore for each $B \in \mathscr{F}$ and $i \in I_B$, ${A_i}^* \in \mathscr{F}$. Consider the family of intuitionistic fuzzy closed sets in (X, T), $\mathcal{C} = \{A \in T^c : A^* \in \mathscr{F}\}$. Since $1^*_{\sim X} = 1_{\sim\gamma}(X) \in \mathscr{F}$ implies $1_{\sim X} \in \mathcal{C}$. Thus $\mathcal{C}$ is nonempty family.

(i) $0^*_{\sim X} \notin \mathscr{F}$ implies that $0_\sim \notin \mathcal{C}$

(ii) If $A_1, A_2 \in \mathcal{C}$, then $A_1^*, A_2^* \in \mathscr{F}$. By Definition 4.6, $A_1^* \cap A_2^* = (A_1 \cap A_2)^* \in \mathscr{F}$ implies that $A_1 \cap A_2 \in \mathcal{C}$. Hence $\mathcal{C}$ is an intuitionistic fuzzy base for an intuitionistic fuzzy T^c prefilter on X.

Let $\mathscr{U}_0$ be an intuitionistic fuzzy T^c ultrafilter contains $\mathcal{C}$. Then, for each $A \in \mathcal{C}$,

$$\mu_{A^*}(\mathscr{U}_0) = \begin{cases} 0 & \text{if } \forall\, x \in X \text{ and } \mu_A \notin \mathscr{U}_0,\, \mathscr{U}_0 \neq \mathscr{U}^x, \\ 1 & \text{if } \forall\, x \in X \text{ and } \mu_A \in \mathscr{U}_0,\, \mathscr{U}_0 \neq \mathscr{U}^x, \\ \mu_A(x) & \text{if } \exists\, x \in X \text{ with } \mathscr{U}_0 = \mathscr{U}^x. \end{cases}$$

$$= \begin{cases} 1 & \text{if } \forall\, x \in X,\, \mathscr{U}_0 \neq \mathscr{U}^x, \\ \mu_A(x) & \text{if } \exists\, x \in X,\, \mathscr{U}_0 = \mathscr{U}^x. \end{cases}$$

and

$$\gamma_{A^*}(\mathscr{U}_0) = \begin{cases} 1 & \text{if } \forall\, x \in X \text{ and } \gamma_A \notin \mathscr{U}_0,\, \mathscr{U}_0 \neq \mathscr{U}^x, \\ 0 & \text{if } \forall\, x \in X \text{ and } \gamma_A \in \mathscr{U}_0,\, \mathscr{U}_0 \neq \mathscr{U}^x, \\ \gamma_A(x) & \text{if } \exists\, x \in X \text{ with } \mathscr{U}_0 = \mathscr{U}^x. \end{cases}$$

$$= \begin{cases} 0 & \text{if } \forall\, x \in X,\, \mathscr{U}_0 \neq \mathscr{U}^x, \\ \gamma_A(x) & \text{if } \exists\, x \in X,\, \mathscr{U}_0 = \mathscr{U}^x. \end{cases}$$

This implies that

$$\bigwedge_{A\in\mathcal{C}} \mu_{A^*}(\mathscr{U}_0) = \bigwedge_{A^*\in\mathscr{F}} \mu_{A^*}(\mathscr{U}_0) = \begin{cases} 1 & \text{if } \forall\, x \in X,\, \mathscr{U}_0 \neq \mathscr{U}^x, \\ \bigwedge_{A\in\mathcal{C}} \mu_A(x), & \text{if } \exists\, x \in X,\, \mathscr{U}_0 = \mathscr{U}^x. \end{cases}$$

and

$$\bigvee_{A\in\mathcal{C}} \gamma_{A^*}(\mathscr{U}_0) = \bigwedge_{A^*\in\mathscr{F}} \gamma_{A^*}(\mathscr{U}_0) = \begin{cases} 0 & \text{if } \forall\, x \in X,\ \mathscr{U}_0 \neq \mathscr{U}^x, \\ \bigvee_{A\in\mathcal{C}} \gamma_A(x), & \text{if } \exists\, x \in X,\ \mathscr{U}_0 = \mathscr{U}^x. \end{cases}$$

Hence $\bigcap_{A\in\mathcal{C}} A^* \neq 0_\sim$. Now, $\bigcap_{B\in\mathscr{F}} B = \bigcap_{B\in\mathscr{F}}(\bigcap_{i\in I_B} A^*) = \bigcap_{A\in\mathcal{C}} A^* \neq 0_\sim$ implies that $\bigcap_{B\in\mathscr{F}} B \neq 0_\sim$. Therefore $(\gamma(X), IF\mathscr{C}st)$ is an intuitionistic fuzzy $\mathscr{C}st$ compact* space. □

Proposition 4.12. *Let (X,T) be an intuitionistic fuzzy F^* space. Suppose T^c is an intuitionistic fuzzy normal family. Under these conditions, the intuitionistic fuzzy $\mathscr{C}$ space $(\gamma(X), IF\mathscr{C}st)$ is an intuitionistic fuzzy $\mathscr{C}st$ compactification of an intuitionistic fuzzy topological space (X,T).*

Proof. Follows from the Propositions 4.1, 4.4, 4.7 and 4.11. □

References

[1] K. T. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20 (1986) 87–96.

[2] N. Blasco Mardones, M. Macho Stadler and M. A. de Prada, On fuzzy compactifications, Fuzzy Sets and Systems 43 (1991) 189–197.

[3] C. L. Chang, Fuzzy topological spaces, J. Math. Anal. Appl. 24 (1968) 182–190.

[4] D. Coker, An introduction to intuitionistic fuzzy topological spaces, Fuzzy Sets and Systems 88(1) (1997) 81–89.

[5] D. Coker and M. Demirci, On intuitionistic fuzzy points, Notes IFS 1(2) (1995) 79–84.

[6] L. A. Zadeh, Fuzzy sets, Information and Control 9 (1965) 338–353.

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