Annals of Fuzzy Mathematics and Informatics
Volume 5, No. 3, (May 2013), pp. 533–547
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr

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Interval-valued intuitionistic fuzzy soft rough sets

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Received 2 May 2012; Accepted 2 August 2012

Abstract. In this paper the concept of interval valued intuitionistic fuzzy soft rough sets is introduced. Also interval valued intuitionistic fuzzy soft rough set based multi criteria group decision making scheme is presented, which refines the primary evaluation of the whole expert group and enables us to select the optimal object in a most reliable manner. The proposed scheme is illustrated by an example regarding the candidate selection problem.

2010 AMS Classification: 54A40, 03E72, 03E02, 06D72

Keywords: Soft set, Soft rough set, Interval valued intuitionistic fuzzy set, Intuitionistic fuzzy soft rough set, Interval valued intuitionistic fuzzy soft rough set, Decision making.

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1. Introduction

There are many complicated problems in economics, engineering, environmental science and social science which can not be solved by the well known methods of classical mathematics as various types of uncertainties are presented in these problems. To overcome these uncertainties, some kind of theories were given like theory of fuzzy sets [13], rough sets [12], soft sets [9] etc. a mathematical tools for dealing with uncertainties. In 1999, Molodtsov [9] introduced soft set theory which is a new mathematical tool for dealing with uncertainties and is free from the difficulties affecting the existing methods. Research works on soft set theory are progressing rapidly. Combining soft sets with fuzzy sets and intuitionistic fuzzy sets, Maji et al. [7, 8] defined fuzzy soft sets and intuitionistic fuzzy soft sets which are rich potentials for solving decision making problems. It has been found that soft set, fuzzy set and rough set are closely related concepts. Based on the equivalence relation on the universe of discourse, Dubois and Prade (1990)[3] introduced the lower and upper approximation of fuzzy sets in a Pawlak's approximation space [12] and obtained a new notion called rough fuzzy sets. Feng et al. (2010)[4] introduced lower and upper soft rough approximations of fuzzy sets in a soft approximation space and obtained a new hybrid model called soft rough fuzzy sets which is the extension of Dubois and Prade's rough fuzzy sets [3]. Considering lower and upper intuitionistic fuzzy soft approximation space(IF soft approximation space),A.Mukherjee [11],obtained a new hybrid model called intuitionistic fuzzy soft rough set which can be seen as extension of both the previous work by Dubois , Prade [3] and Feng et al. [4,5]. The notion of the interval-valued intuitionistic fuzzy set was first introduced by Atanassov and Gargov [2]. It is characterized by an interval-valued membership degree and an interval-valued non-membership degree. In 2010, Y. Jiang et al. [6] introduced the concept of interval-valued intuitionistic fuzzy soft sets. The aim of this paper is to introduce a new concept- interval valued intuitionistic fuzzy soft rough sets. Also interval valued intuitionistic fuzzy soft rough set based multi criteria group decision making scheme is to be presented.

2. Preliminaries

This section presents a review of some fundamental notions of fuzzy sets, rough sets and soft sets. We refer to [9, 12, 13] for details. The theory of fuzzy sets initiated by Zadeh provides a framework for representing and processing vague concepts by allowing partial membership. A fuzzy set α in U (U be a nonempty set, called universe) is a membership function α: U $\rightarrow$[0, 1]. For x$\in$ U, the membership value α(x) specifies the degree to which x belongs to the fuzzy set α.

Definition 2.1 ([9])**.** Let U be an initial universe and E be a set of parameters. Let P(U) denotes the power set of U and A$\subseteq$E. Then the pair (F, A) is called a soft set over U, where F is a mapping given by F: A$\rightarrow$ P(U).

Definition 2.2 ([7])**.** Let U be an initial universe and E be a set of parameters. Let I^U be the set of all fuzzy subsets of U and A$\subseteq$E. Then the pair (F, A) is called a fuzzy soft set over U, where F is a mapping given by F: A $\rightarrow I^U$.

Definition 2.3 ([1])**.** Let X be a non empty set. Then an intuitionistic fuzzy set (IFS for short) A is a set having the form A=$\{\langle$x, μ_A(x), γ_A(x)$\rangle$: x $\in$ X$\}$ where the functions μ_A: X$\rightarrow$[0, 1] and γ_A: X$\rightarrow$[0, 1] represents the degree of membership and the degree of non-membership respectively of each element x$\in$X and 0$\leq\mu_A$(x)+ γ_A(x)$\leq$1 for each x$\in$X.

Definition 2.4 ([8])**.** Let U be an initial universe and E be a set of parameters. Let IF^U be the set of all intuitionistic fuzzy subsets of U and A$\subseteq$E. Then the pair (F, A) is called an intuitionistic fuzzy soft set over U, where F is a mapping given by F: A$\rightarrow IF^U$.

Definition 2.5 ([2])**.** An interval valued intuitionistic fuzzy set A over a universe set U is defined as the object of the form A=$\{\langle$x, μ_A(x), γ_A(x)$\rangle$: x$\in$U$\}$, where μ_A: U$\rightarrow$Int([0, 1]) and γ_A: U $\rightarrow$Int([0, 1]) are functions such that the condition: x$\in$U, $\sup\mu_A$(x) +$\sup\gamma_A$(x)$\leq$ 1 is satisfied (where Int[0, 1]is the set of all closed intervals of [0, 1]).
We denote the class of all interval valued intuitionistic fuzzy sets on U by $IVIFS^U$.

Definition 2.6 ([4]). Let $\odot$=(f, A) be a soft set over U. The pair S=(U, $\odot$) is called a soft approximation space. Based on S, the operators $\overline{apr}_S$ and $\underline{apr}_S$ are defined as:

$\underline{apr}_S$(X)={u∈U: ∃ a∈A (u∈f(a)⊆X)},

$\overline{apr}_S(X) = \{u \in U : \exists a \in A(u \in f(a), f(a) \bigcap X) \neq \phi\}$

for every $X \subseteq U$. The two sets $\overline{apr}_S$(X) and $\underline{apr}_S$(X) are called the upper and lower soft rough approximations of X in S respectively. If $\overline{apr}_S$(X)=$\underline{apr}_S$(X), then X is said to be soft definable; otherwise X is called a soft rough set.

Definition 2.7 ([10]). Let $\odot$=(f, A) be a full soft set over U i.e; $\bigcup_{a\in A} f(a) = U$ and the pair S=(U, $\odot$) is the soft approximation space. Then for a fuzzy set λ∈ I^U , the lower and upper soft rough approximations of λ with respect to S are denoted by $\underline{sap}_S(\lambda)$ and $\overline{sap}_S(\lambda)$ respectively, which are fuzzy sets in U given by:

$\underline{sap}_S(\lambda)$={(x, $\underline{sap}_S(\lambda)$(x)): x∈U}, $\overline{sap}_S(\lambda)$={(x, $\overline{sap}_S(\lambda)$(x)): x∈U},

where

$$\underline{sap}_S(\lambda)(x) = \bigwedge\{\mu_\lambda(y) : \exists a \in A(\{x, y\} \subseteq f(a))\}$$

and

$$\overline{sap}_S(\lambda)(x) = \bigvee\{\mu_\lambda(y) : \exists a \in A(\{x, y\} \subseteq f(a))\}$$

for every $x \in U$. The operators $\underline{sap}_S$ and $\overline{sap}_S$ are called the lower and upper soft rough approximation operators on fuzzy sets. If $\underline{sap}_S(\lambda)$=$\overline{sap}_S(\lambda)$, then λ is said to be fuzzy soft definable; otherwise is called a soft rough fuzzy set.

Definition 2.8 ([10]). Let $\odot$=(f, A) be a fuzzy soft set over U. Then the pair SF=(U, $\odot$) is called a soft fuzzy approximation space. Then for a fuzzy set λ∈ I^U, the lower and upper soft fuzzy rough approximations of λ with respect to SF are denoted by $\underline{Apr}_{SF}(\lambda)$and $\overline{Apr}_{SF}(\lambda)$ respectively, which are fuzzy sets in U given by:

$\underline{Apr}_{SF}(\lambda)$={(x, $\underline{Apr}_{SF}(\lambda)$(x)): x∈U}, $\overline{Apr}_{SF}(\lambda)$={(x, $\overline{Apr}_{SF}(\lambda)$(x)): x∈U}

where

$$\underline{Apr}_{SF}(\lambda)(x) = \bigwedge_{a\in A}((1 - f(a)(x)) \bigvee(\bigwedge_{y\in U}((1 - f(a)(y)) \bigvee \mu_\lambda(y))))$$

and

$$\overline{Apr}_{SF}(\lambda)(x) = \bigvee_{a\in A}(f(a)(x) \bigwedge(\bigvee_{y\in U}(f(a)(y) \bigwedge \mu_\lambda(y))))$$

for every $x \in U$ and μ_λ is the degree of membership of $y \in U$. The operators $\underline{Apr}_{SF}$ and $\overline{Apr}_{SF}$ are called the lower and upper soft fuzzy rough approximation operators on fuzzy sets. If $\underline{Apr}_{SF}(\lambda) = \overline{Apr}_{SF}(\lambda)$, then λ is said to be soft fuzzy definable; otherwise λ is called a soft fuzzy rough set.

3. Interval Valued Intuitionistic Fuzzy Soft Rough Sets

Definition 3.1. Let $\odot$=(f, A) be a full soft set over U and S=(U, $\odot$) be the soft approximation space. Then for $\tau \in IVIFS^U$, the the lower and upper soft rough approximations of τ with respect to S are denoted by $\downarrow sap_S(\tau)$ and $\uparrow sap_S(\tau)$ respectively, which are interval valued intuitionistic fuzzy sets in U given by:

$$\begin{aligned}\downarrow sap_S(\tau) = \{\langle x, [&\bigwedge\{\inf \mu_\tau(y) : \exists a \in A(\{x,y\} \subseteq f(a))\},\\ &\qquad\bigwedge\{\sup \mu_\tau(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}],\\ [&\bigvee\{\inf \gamma_\tau(y) : \exists a \in A(\{x,y\} \subseteq f(a))\},\\ &\qquad\bigvee\{\sup \gamma_\tau(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}]\rangle : x \in U\},\end{aligned}$$

$$\begin{aligned}\uparrow sap_S(\tau) = \{\langle x, [&\bigvee\{\inf \mu_\tau(y) : \exists a \in A(\{x,y\} \subseteq f(a))\},\\ &\qquad\bigvee\{\sup \mu_\tau(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}],\\ [&\bigwedge\{\inf \gamma_\tau(y) : \exists a \in A(\{x,y\} \subseteq f(a))\},\\ &\qquad\bigwedge\{\sup \gamma_\tau(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}]\rangle : x \in U\}.\end{aligned}$$

The operators $\downarrow sap_S$ and $\uparrow sap_S$ are called the lower and upper soft rough approximation operators on interval valued intuitionistic fuzzy sets. If $\downarrow sap_S(\tau) = \uparrow sap_S(\tau)$, then τ is said to be soft interval valued intuitionistic fuzzy definable; otherwise τ is called an interval valued intuitionistic fuzzy soft rough set.

Example 3.2. Let U={x, y, z} and A={a, b, c}. Let f: A→P(U) be defined by f(a)={x, y, z}, f(b)={x, y}, f(c)={x, z}. Let

$$\tau = \{\langle x, [0.3, 0.4], [0.1, 0.2]\rangle, \langle y, [0.6, 0.7], [0.1, 0.2]\rangle, \langle z, [0.5, 0.6], [0.2, 0.3]\rangle\}.$$

Then $\tau \in IVIFS^U$. So, we have,

$$\downarrow sap_S(\tau) = \{\langle x, [0.5, 0.6], [0.1, 0.2]\rangle, \langle y, [0.3, 0.4], [0.1, 0.2]\rangle, \langle z, [0.3, 0.4], [0.2, 0.3]\rangle\}$$

and

$$\uparrow sap_S(\tau) = \{\langle x, [0.6, 0.7], [0.2, 0.3]\rangle, \langle y, [0.5, 0.6], [0.2, 0.3]\rangle, \langle z, [0.6, 0.7], [0.1, 0.2]\rangle\}.$$

Since $\downarrow sap_S(\tau) \neq\uparrow sap_S(\tau)$, τ is an interval valued intuitionistic fuzzy soft rough set.

Theorem 3.3. *Let $\odot$=(f, A) be a full soft set over U and S=(U, $\odot$) be the soft approximation space. Then for $\tau \in IVIFS^U$ we have*
(i) $\downarrow sap_S(\tau) = \{\langle x, [\bigwedge_{x\in f(a)} \bigwedge_{y\in f(a)} \inf \mu_\tau(y), \bigwedge_{x\in f(a)} \bigwedge_{y\in f(a)} \sup \mu_\tau(y)],$
$[\bigvee_{x\in f(a)} \bigvee_{y\in f(a)} \inf \gamma_\tau(y), \bigvee_{x\in f(a)} \bigvee_{y\in f(a)} \sup \gamma_\tau(y)]\rangle : x \in U\},$
(ii) $\uparrow sap_S(\tau) = \{\langle x, [\bigvee_{x\in f(a)} \bigvee_{y\in f(a)} \inf \mu_\tau(y), \bigvee_{x\in f(a)} \bigvee_{y\in f(a)} \sup \mu_\tau(y)],$
$[\bigwedge_{x\in f(a)} \bigwedge_{y\in f(a)} \inf \gamma_\tau(y), \bigwedge_{x\in f(a)} \bigwedge_{y\in f(a)} \sup \gamma_\tau(y)]\rangle : x \in U\}.$

Proof. (i) Let a∈ A and x∈ f(a). Then for y∈ f(a), we have $\{x, y\} \subseteq f(a)$ and hence $\inf \mu_\tau(y) \geq \bigwedge\{\inf \mu_\tau(z) : \exists a \in A(\{x, z\} \subseteq f(a))\}$. Consequently,

$$\bigwedge_{y\in f(a)} \inf \mu_\tau(y) \geq \bigwedge\{\inf \mu_\tau(z) : \exists a \in A(\{x, z\} \subseteq f(a))\}$$

and so

$$\bigwedge_{x\in f(a)}\bigwedge_{y\in f(a)} \inf \mu_\tau(y) \geq \bigwedge\{\inf \mu_\tau(z) : \exists a \in A(\{x,z\} \subseteq f(a))\}.$$

Similarly, it can be shown that

$$\bigwedge_{x\in f(a)}\bigwedge_{y\in f(a)} \sup \mu_\tau(y) \geq \bigwedge\{\sup \mu_\tau(z) : \exists a \in A(\{x,z\} \subseteq f(a))\}.$$

Thus, we get

$$\begin{aligned}&\Big[\bigwedge\{\inf \mu_\tau(z) : \exists a \in A(\{x,z\} \subseteq f(a))\}, \bigwedge\{\sup \mu_\tau(z) : \exists a \in A(\{x,z\} \subseteq f(a))\}\Big]\\ &\subseteq \left[\bigwedge_{x\in f(a)}\bigwedge_{y\in f(a)} \inf \mu_\tau(y), \bigwedge_{x\in f(a)}\bigwedge_{y\in f(a)} \sup \mu_\tau(y)\right]. \qquad (3.3.1)\end{aligned}$$

In a similar manner it can be shown that

$$\begin{aligned}&\Big[\bigvee\{\inf \gamma_\tau(z) : \exists a \in A(\{x,z\} \subseteq f(a))\}, \bigvee\{\sup \gamma_\tau(z) : \exists a \in A(\{x,z\} \subseteq f(a))\}\Big]\\ &\subseteq \left[\bigvee_{x\in f(a)}\bigvee_{y\in f(a)} \inf \gamma_\tau(y), \bigvee_{x\in f(a)}\bigvee_{y\in f(a)} \sup \gamma_\tau(y)\right]. \qquad (3.3.2)\end{aligned}$$

From (3.3.1) and (3.3.2) we see that

$$\begin{aligned}\downarrow sap_S(\tau) \subseteq \Bigg\{\Bigg\langle x, &\left[\bigwedge_{x\in f(a)}\bigwedge_{y\in f(a)} \inf \mu_\tau(y), \bigwedge_{x\in f(a)}\bigwedge_{y\in f(a)} \sup \mu_\tau(y)\right],\\ &\left[\bigvee_{x\in f(a)}\bigvee_{y\in f(a)} \inf \gamma_\tau(y), \bigvee_{x\in f(a)}\bigvee_{y\in f(a)} \sup \gamma_\tau(y)\right]\Bigg\rangle : x \in U\Bigg\}. \qquad (3.3.3)\end{aligned}$$

Now to prove that

$$\begin{aligned}&\Bigg\{\Bigg\langle x, \left[\bigwedge_{x\in f(a)}\bigwedge_{y\in f(a)} \inf \mu_\tau(y), \bigwedge_{x\in f(a)}\bigwedge_{y\in f(a)} \sup \mu_\tau(y)\right],\\ &\qquad \left[\bigvee_{x\in f(a)}\bigvee_{y\in f(a)} \inf \gamma_\tau(y), \bigvee_{x\in f(a)}\bigvee_{y\in f(a)} \sup \gamma_\tau(y)\right]\Bigg\rangle : x \in U\Bigg\}\\ &\subseteq \downarrow sap_S(\tau),\end{aligned}$$

let us suppose that a∈ A such that {x, z}⊆ f(a). Then x∈ f(a), z∈ f(a) and hence

$$\inf \mu_\tau(z) \geq \bigwedge_{x\in f(a)}\bigwedge_{y\in f(a)} \inf \mu_\tau(y).$$

Consequently,

$$\bigwedge\{\inf \mu_\tau(z) : \exists a \in A(\{x,z\} \subseteq f(a))\} \geq \bigwedge_{x\in f(a)}\bigwedge_{y\in f(a)} \inf \mu_\tau(y).$$

Similarly it can be shown that

$$\bigwedge\{\sup \mu_\tau(z) : \exists a \in A(\{x, z\} \subseteq f(a))\} \geq \bigwedge_{x\in f(a)} \bigwedge_{y\in f(a)} \sup \mu_\tau(y).$$

Thus we get

$$\begin{aligned}&\left[\bigwedge_{x\in f(a)} \bigwedge_{y\in f(a)} \inf \mu_\tau(y), \bigwedge_{x\in f(a)} \bigwedge_{y\in f(a)} \sup \mu_\tau(y)\right]\\ &\subseteq \left[\bigwedge\{\inf \mu_\tau(z) : \exists a \in A(\{x, z\} \subseteq f(a))\},\right.\\ &\qquad\left.\bigwedge\{\sup \mu_\tau(z) : \exists a \in A(\{x, z\} \subseteq f(a))\}\right]. \qquad (3.3.4)\end{aligned}$$

In a similar manner it can be shown that

$$\begin{aligned}&\left[\bigvee_{x\in f(a)} \bigvee_{y\in f(a)} \inf \gamma_\tau(y), \bigvee_{x\in f(a)} \bigvee_{y\in f(a)} \sup \gamma_\tau(y)\right]\\ &\subseteq \left[\bigvee\{\inf \gamma_\tau(z) : \exists a \in A(\{x, z\} \subseteq f(a))\},\right.\\ &\qquad\left.\bigvee\{\sup \gamma_\tau(z) : \exists a \in A(\{x, z\} \subseteq f(a))\}\right]. \qquad (3.3.5)\end{aligned}$$

From (3.3.4) and (3.3.5) we see that

$$\begin{aligned}&\left\{\left\langle x, \left[\bigwedge_{x\in f(a)} \bigwedge_{y\in f(a)} \inf \mu_\tau(y), \bigwedge_{x\in f(a)} \bigwedge_{y\in f(a)} \sup \mu_\tau(y)\right],\right.\right.\\ &\qquad\left.\left.\left[\bigvee_{x\in f(a)} \bigvee_{y\in f(a)} \inf \gamma_\tau(y), \bigvee_{x\in f(a)} \bigvee_{y\in f(a)} \sup \gamma_\tau(y)\right]\right\rangle : x \in U\right\}\\ &\subseteq \downarrow sap_S(\tau). \qquad (3.3.6)\end{aligned}$$

From (3.3.3) and (3.3.6), we have

$$\begin{aligned}\downarrow sap_S(\tau) = &\left\{\left\langle x, \left[\bigwedge_{x\in f(a)} \bigwedge_{y\in f(a)} \inf \mu_\tau(y), \bigwedge_{x\in f(a)} \bigwedge_{y\in f(a)} \sup \mu_\tau(y)\right],\right.\right.\\ &\qquad\left.\left.\left[\bigvee_{x\in f(a)} \bigvee_{y\in f(a)} \inf \gamma_\tau(y), \bigvee_{x\in f(a)} \bigvee_{y\in f(a)} \sup \gamma_\tau(y)\right]\right\rangle : x \in U\right\}.\end{aligned}$$

(ii) Proof is similar as in(i). □

Theorem 3.4. *Let* $\odot$*=(f, A) be a full soft set over U and S=(U,* $\odot$*) be the soft approximation space and* $\tau, \delta \in IVIFS^U$*. Then*

(1) $\downarrow sap_S(\phi) = \phi = \uparrow sap_S(\phi)$
(2) $\downarrow sap_S(U) = U = \uparrow sap_S(U)$
(3)$\tau \subseteq \delta \Rightarrow \downarrow sap_S(\tau) \subseteq \downarrow sap_S(\delta)$

(4)$\tau \subseteq \delta \Rightarrow \uparrow sap_S(\tau) \subseteq \uparrow sap_S(\delta)$
(5) $\downarrow sap_S(\tau \bigcap \delta) \subseteq \downarrow sap_S(\tau) \bigcap \downarrow sap_S(\delta)$
(6) $\uparrow sap_S(\tau \bigcap \delta) \subseteq \uparrow sap_S(\tau) \bigcap \uparrow sap_S(\delta)$
(7) $\downarrow sap_S(\tau) \bigcup \downarrow sap_S(\delta) \subseteq \downarrow sap_S(\tau \bigcup \delta)$
(8) $\uparrow sap_S(\tau) \bigcup \uparrow sap_S(\delta) \subseteq \uparrow sap_S(\tau \bigcup \delta)$

Proof. (1)-(4) are straight forward.
(5) We have

$$\tau = \{\langle x, [\inf \mu_\tau(x), \sup \mu_\tau(x)], [\inf \gamma_\tau(x), \sup \gamma_\tau(x)]\rangle : x \in U\},$$

$$\delta = \{\langle x, [\inf \mu_\delta(x), \sup \mu_\delta(x)], [\inf \gamma_\delta(x), \sup \gamma_\delta(x)]\rangle : x \in U\}$$

and

$$\tau \bigcap \delta = \{\langle x, [\inf \mu_{\tau \bigcap \delta}(x), \sup \mu_{\tau \bigcap \delta}(x)], [\inf \gamma_{\tau \bigcap \delta}(x), \sup \gamma_{\tau \bigcap \delta}(x)]\rangle : x \in U\}.$$

Now,

$$\begin{aligned}
&\downarrow sap_S(\tau \bigcap \delta) \\
&= \Bigg\{\Bigg\langle x, \Bigg[\bigwedge\{\inf \mu_{\tau \bigcap \delta}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}, \\
&\qquad \bigwedge\{\sup \mu_{\tau \bigcap \delta}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}\Bigg], \\
&\quad \Bigg[\bigvee\{\inf \gamma_{\tau \bigcap \delta}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}, \\
&\qquad \bigvee\{\sup \gamma_{\tau \bigcap \delta}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}\Bigg]\Bigg\rangle : x \in U\Bigg\} \\
&= \Bigg\{\Bigg\langle x, \Bigg[\bigwedge\{\min(\inf \mu_\tau(y), \inf \mu_\delta(y)) : \exists a \in A(\{x,y\} \subseteq f(a))\}, \\
&\qquad \bigwedge\{\min(\sup \mu_\tau(y), \sup \mu_\delta(y)) : \exists a \in A(\{x,y\} \subseteq f(a))\}\Bigg], \\
&\quad \Bigg[\bigvee\{\max(\inf \gamma_\tau(y), \inf \gamma_\delta(y)) : \exists a \in A(\{x,y\} \subseteq f(a))\}, \\
&\quad \bigvee\{\max(\sup \gamma_\tau(y), \sup \gamma_\delta(y)) : \exists a \in A(\{x,y\} \subseteq f(a))\}\Bigg]\Bigg\rangle : x \in U\Bigg\}.(3.4.1)
\end{aligned}$$

Since

$$\min(\inf \mu_\tau(y), \inf \mu_\delta(y)) \leq \inf \mu_\tau(y)$$

and

$$\min(\inf \mu_\tau(y), \inf \mu_\delta(y)) \leq \inf \mu_\delta(y),$$

we have

$$\begin{aligned}
&\bigwedge\{\min(\inf \mu_\tau(y), \inf \mu_\delta(y)) : \exists a \in A(\{x,y\} \subseteq f(a))\} \\
&\leq \bigwedge\{\inf \mu_\tau(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}
\end{aligned}$$

and

$$\bigwedge\{\min(\inf \mu_\tau(y), \inf \mu_\delta(y)) : \exists a \in A(\{x, y\} \subseteq f(a))\}$$
$$\leq \bigwedge\{\inf \mu_\delta(y) : \exists a \in A(\{x, y\} \subseteq f(a))\}.$$

Consequently,

$$\begin{aligned}&\bigwedge\{\min(\inf \mu_\tau(y), \inf \mu_\delta(y)) : \exists a \in A(\{x, y\} \subseteq f(a))\}\\ &\leq \min(\bigwedge\{\inf \mu_\tau(y) : \exists a \in A(\{x, y\} \subseteq f(a))\},\\ &\qquad \bigwedge\{\inf \mu_\delta(y) : \exists a \in A(\{x, y\} \subseteq f(a))\}). \qquad (3.4.2)\end{aligned}$$

Similarly we can get

$$\begin{aligned}&\bigwedge\{\min(\sup \mu_\tau(y), \sup \mu_\delta(y)) : \exists a \in A(\{x, y\} \subseteq f(a))\}\\ &\leq \min(\bigwedge\{\sup \mu_\tau(y) : \exists a \in A(\{x, y\} \subseteq f(a))\},\\ &\qquad \bigwedge\{\sup \mu_\delta(y) : \exists a \in A(\{x, y\} \subseteq f(a))\}). \qquad (3.4.3)\end{aligned}$$

Again since

$$\max(\inf \gamma_\tau(y), \inf \gamma_\delta(y)) \geq \inf \gamma_\tau(y)$$

and

$$\max(\inf \gamma_\tau(y), \inf \gamma_\delta(y)) \geq \inf \gamma_\delta(y),$$

we have

$$\bigvee\{\max(\inf \gamma_\tau(y), \inf \gamma_\delta(y)) : \exists a \in A(\{x, y\} \subseteq f(a))\}$$
$$\geq \bigvee\{\inf \gamma_\tau(y) : \exists a \in A(\{x, y\} \subseteq f(a))\}$$

and

$$\bigvee\{\max(\inf \gamma_\tau(y), \inf \gamma_\delta(y)) : \exists a \in A(\{x, y\} \subseteq f(a))\}$$
$$\geq \bigvee\{\inf \gamma_\delta(y) : \exists a \in A(\{x, y\} \subseteq f(a))\}.$$

Consequently,

$$\begin{aligned}&\bigvee\{\max(\inf \gamma_\tau(y), \inf \gamma_\delta(y)) : \exists a \in A(\{x, y\} \subseteq f(a))\}\\ &\geq \max(\bigvee\{\inf \gamma_\tau(y) : \exists a \in A(\{x, y\} \subseteq f(a))\},\\ &\qquad \bigvee\{\inf \gamma_\delta(y) : \exists a \in A(\{x, y\} \subseteq f(a))\}). \qquad (3.4.4)\end{aligned}$$

Similarly we can get

$$\begin{aligned}&\bigvee\{\max(\sup \gamma_\tau(y), \sup \gamma_\delta(y)) : \exists a \in A(\{x, y\} \subseteq f(a))\}\\ &\geq \max(\bigvee\{\sup \gamma_\tau(y) : \exists a \in A(\{x, y\} \subseteq f(a))\},\\ &\qquad \bigvee\{\sup \gamma_\delta(y) : \exists a \in A(\{x, y\} \subseteq f(a))\}). \qquad (3.4.5)\end{aligned}$$

Using (3.4.2)-(3.4.5), we get from (3.4.1),

$$\downarrow sap_S(\tau \bigcap \delta) \subseteq \downarrow sap_S(\tau) \bigcap \downarrow sap_S(\delta).$$

(6) Proof is similar to (5).

(7) We have

$$\tau = \{\langle x, [\inf \mu_\tau(x), \sup \mu_\tau(x)], [\inf \gamma_\tau(x), \sup \gamma_\tau(x)]\rangle : x \in U\},$$
$$\delta = \{\langle x, [\inf \mu_\delta(x), \sup \mu_\delta(x)], [\inf \gamma_\delta(x), \sup \gamma_\delta(x)]\rangle : x \in U\}$$

and

$$\tau \bigcup \delta = \{\langle x, [\inf \mu_{\tau \bigcup \delta}(x), \sup \mu_{\tau \bigcup \delta}(x)], [\inf \gamma_{\tau \bigcup \delta}(x), \sup \gamma_{\tau \bigcup \delta}(x)]\rangle : x \in U\}.$$

Now,

$$\begin{aligned}
&\downarrow sap_S(\tau \textstyle\bigcup \delta) \\
&= \Bigg\{\Bigg\langle x, \Bigg[\bigwedge\{\inf \mu_{\tau \bigcup \delta}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}, \\
&\qquad \bigwedge\{\sup \mu_{\tau \bigcup \delta}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}\Bigg], \\
&\quad \Bigg[\bigvee\{\inf \gamma_{\tau \bigcup \delta}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}, \\
&\qquad \bigvee\{\sup \gamma_{\tau \bigcup \delta}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}\Bigg]\Bigg\rangle : x \in U\Bigg\} \\
&= \Bigg\{\Bigg\langle x, \Bigg[\bigwedge\{\max(\inf \mu_\tau(y), \inf \mu_\delta(y)) : \exists a \in A(\{x,y\} \subseteq f(a))\}, \\
&\qquad \bigwedge\{\max(\sup \mu_\tau(y), \sup \mu_\delta(y)) : \exists a \in A(\{x,y\} \subseteq f(a))\}\Bigg], \\
&\quad \Bigg[\bigvee\{\min(\inf \gamma_\tau(y), \inf \gamma_\delta(y)) : \exists a \in A(\{x,y\} \subseteq f(a))\}, \\
&\quad \bigvee\{\min(\sup \gamma_\tau(y), \sup \gamma_\delta(y)) : \exists a \in A(\{x,y\} \subseteq f(a))\}\Bigg]\Bigg\rangle : x \in U\Bigg\}. \quad (3.4.6)
\end{aligned}$$

Since

$$\max(\inf \mu_\tau(y), \inf \mu_\delta(y)) \geq \inf \mu_\tau(y)$$

and

$$\max(\inf \mu_\tau(y), \inf \mu_\delta(y)) \geq \inf \mu_\delta(y),$$

we have

$$\begin{aligned}
&\bigwedge\{\max(\inf \mu_\tau(y), \inf \mu_\delta(y)) : \exists a \in A(\{x,y\} \subseteq f(a))\} \\
&\geq \bigwedge\{\inf \mu_\tau(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}
\end{aligned}$$

and

$$\begin{aligned}
&\bigwedge\{\max(\inf \mu_\tau(y), \inf \mu_\delta(y)) : \exists a \in A(\{x,y\} \subseteq f(a))\} \\
&\geq \bigwedge\{\inf \mu_\delta(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}.
\end{aligned}$$

Consequently,

$$\begin{aligned}&\bigwedge\{\max(\inf \mu_\tau(y), \inf \mu_\delta(y)) : \exists a \in A(\{x, y\} \subseteq f(a))\}\\ &\geq \max(\bigwedge\{\inf \mu_\tau(y) : \exists a \in A(\{x, y\} \subseteq f(a))\},\\ &\qquad \bigwedge\{\inf \mu_\delta(y) : \exists a \in A(\{x, y\} \subseteq f(a))\}). \end{aligned} \tag{3.4.7}$$

Similarly we can get

$$\begin{aligned}&\bigwedge\{\max(\sup \mu_\tau(y), \sup \mu_\delta(y)) : \exists a \in A(\{x, y\} \subseteq f(a))\}\\ &\geq \max(\bigwedge\{\sup \mu_\tau(y) : \exists a \in A(\{x, y\} \subseteq f(a))\},\\ &\qquad \bigwedge\{\sup \mu_\delta(y) : \exists a \in A(\{x, y\} \subseteq f(a))\}). \end{aligned} \tag{3.4.8}$$

Again since

$$\min(\inf \gamma_\tau(y), \inf \gamma_\delta(y)) \leq \inf \gamma_\tau(y)$$

and

$$\min(\inf \gamma_\tau(y), \inf \gamma_\delta(y)) \leq \inf \gamma_\delta(y),$$

we have

$$\begin{aligned}&\bigvee\{\min(\inf \gamma_\tau(y), \inf \gamma_\delta(y)) : \exists a \in A(\{x, y\} \subseteq f(a))\}\\ &\leq \bigvee\{\inf \gamma_\tau(y) : \exists a \in A(\{x, y\} \subseteq f(a))\}\end{aligned}$$

and

$$\begin{aligned}&\bigvee\{\min(\inf \gamma_\tau(y), \inf \gamma_\delta(y)) : \exists a \in A(\{x, y\} \subseteq f(a))\}\\ &\leq \bigvee\{\inf \gamma_\delta(y) : \exists a \in A(\{x, y\} \subseteq f(a))\}.\end{aligned}$$

Consequently,

$$\begin{aligned}&\bigvee\{\min(\inf \gamma_\tau(y), \inf \gamma_\delta(y)) : \exists a \in A(\{x, y\} \subseteq f(a))\}\\ &\leq \min(\bigvee\{\inf \gamma_\tau(y) : \exists a \in A(\{x, y\} \subseteq f(a))\},\\ &\qquad \bigvee\{\inf \gamma_\delta(y) : \exists a \in A(\{x, y\} \subseteq f(a))\}). \end{aligned} \tag{3.4.9}$$

Similarly we can get

$$\begin{aligned}&\bigvee\{\min(\sup \gamma_\tau(y), \sup \gamma_\delta(y)) : \exists a \in A(\{x, y\} \subseteq f(a))\}\\ &\leq \min(\bigvee\{\sup \gamma_\tau(y) : \exists a \in A(\{x, y\} \subseteq f(a))\},\\ &\qquad \bigvee\{\sup \gamma_\delta(y) : \exists a \in A(\{x, y\} \subseteq f(a))\}). \end{aligned} \tag{3.4.10}$$

Using (3.4.7)-(3.4.10), we get from (3.4.6),

$$\downarrow sap_S(\tau) \bigcup \downarrow sap_S(\delta) \subseteq \downarrow sap_S(\tau \bigcup \delta).$$

(8) Proof is similar to (7). □

4. A multicriteria group decision making problem

Soft sets and fuzzy soft sets, intuitionistic fuzzy soft sets have been applied by many authors in solving decision making problems. In this section ,we illustrate the use of Soft sets and fuzzy soft sets, intuitionistic fuzzy soft sets, interval -valued intuitionistic fuzzy soft sets , rough sets, interval-valued intuitionistic fuzzy soft rough sets and related notions in object evaluation and group decision making.
Let U=$\{o_1, o_2, o_3,...,o_l\}$ be a set of objects and E be a set of parameters and A=$\{e_1, e_2, e_3,....,e_m\}$⊆E and S=(F, A) be a full soft set over U. Let us assume that we have an expert group G=$\{T_1, T_2,.....,T_n\}$ consisting of n specialists to evaluate the objects in U. Each specialist will examine all the objects in U and will point out his/her evaluation result. Let X_i denote the primary evaluation result of the specialist T_i. It is easy to see that the primary evaluation result of the whole expert group G can be represented as an interval valued intuitionistic fuzzy evaluation soft set $S^* = (F^*, G)$ over U, where $F^* : G \to IVIFS^U$ is given by $F^*(T_i) = X_i$, for i=1, 2,......, n. Now we consider the soft rough interval valued intuitionistic fuzzy approximations of the specialist $T_i's$ primary evaluation result X_i w.r.t the soft approximation space P=(U, S). Then we obtain two other interval valued intuitionistic fuzzy soft sets $\downarrow S^* = (\downarrow F^*, G)$ and $\uparrow S^* = (\uparrow F^*, G)$ over U, where $\downarrow F^*$: G→ IVIFS^U is given by $\downarrow F^*(T_i) =\downarrow apr_P(X_i)$ and $\uparrow F^* : G \to IVIFS^U$ is given by $\uparrow F^*(T_i) =\uparrow apr_P(X_i)$, for i=1, 2,...., n. Here $\downarrow S^*$ can be considered as the evaluation result for the whole expert group G with 'low confidence' , $\uparrow S^*$ can be considered as the evaluation result for the whole expert group G with 'high confidence' and S^* can be considered as the evaluation result for the whole expert group G with 'middle confidence' Let us define two interval valued intuitionistic fuzzy sets IVIFSet$_{\downarrow S^*}$ and IVIFSet$_{\uparrow S^*}$ by

$$IVIFSet_{\downarrow S^*} = \left\{ \left\langle o_k, \left[\frac{1}{n}\sum_{j=1}^{n} \inf \mu_{\downarrow F^*(T_j)}(o_k), \frac{1}{n}\sum_{j=1}^{n} \sup \mu_{\downarrow F^*(T_j)}(o_k) \right], \left[\frac{1}{n}\sum_{j=1}^{n} \inf \gamma_{\downarrow F^*(T_j)}(o_k), \frac{1}{n}\sum_{j=1}^{n} \sup \gamma_{\downarrow F^*(T_j)}(o_k) \right] \right\rangle : k = 1, 2, ..., l \right\}$$

and

$$IVIFSet_{\uparrow S^*} = \left\{ \left\langle o_k, \left[\frac{1}{n}\sum_{j=1}^{n} \inf \mu_{\uparrow F^*(T_j)}(o_k), \frac{1}{n}\sum_{j=1}^{n} \sup \mu_{\uparrow F^*(T_j)}(o_k) \right], \left[\frac{1}{n}\sum_{j=1}^{n} \inf \gamma_{\uparrow F^*(T_j)}(o_k), \frac{1}{n}\sum_{j=1}^{n} \sup \gamma_{\uparrow F^*(T_j)}(o_k) \right] \right\rangle : k = 1, 2, ..., l \right\}.$$

Now we define another interval valued intuitionistic fuzzy set $IVIFSet_{S*}$ by

$$IVIFSet_{S^*} = \Bigg\{ \Bigg\langle o_k, \left[\frac{1}{n}\sum_{j=1}^{n} \inf \mu_{F^*(T_j)}(o_k), \frac{1}{n}\sum_{j=1}^{n} \sup \mu_{F^*(T_j)}(o_k) \right],$$
$$\left[\frac{1}{n}\sum_{j=1}^{n} \inf \gamma_{F^*(T_j)}(o_k), \frac{1}{n}\sum_{j=1}^{n} \sup \gamma_{F^*(T_j)}(o_k) \right] \Bigg\rangle : k = 1, 2, ..., l \Bigg\}.$$

Then clearly,

$$IVIFSet_{\downarrow S^*} \subseteq IVIFSet_{S^*} \subseteq IVIFSet_{\uparrow S^*}.$$

Let C={L(low confidence), M(middle confidence), H(high confidence)} be a set of parameters. Let us consider the interval valued intuitionistic fuzzy soft set S^{**}=(f, C) over U, where f: C→ IVIFS^U is given by $f(L) = IVIFSet_{\downarrow S^*}, f(M) = IVIFSet_{S^*}, f(H) = IVIFSet_{\uparrow S^*}$. Now given a weighting vector $W = (w_L, w_M, w_H)$ such that $w_L, w_M, w_H \in Int([0,1])$, we define $\alpha : U \to P(U)$ by $\alpha(o_k) = \sup w_L \diamond \sup \mu_{f(L)}(o_k) + \sup w_M \diamond \sup \mu_{f(M)}(o_k) + \sup w_H \diamond \sup \mu_{f(H)}(o_k), o_k \in U$ ($\diamond$ represents ordinary multiplication). Here $\alpha(o_k)$ is called the weighted evaluation value of the alternative o_k ∈U. Finally, we can select the object o_p such that $\alpha(o_p)$=max{$\alpha(o_k)$: k=1, 2,....., l} as the most preferred alternative.

✠ Algorithm:
(1) Input the original description soft set (F, A).
(2) Construct the interval valued intuitionistic fuzzy evaluation soft set $S^* = (F^*, G)$.
(3) Compute the soft rough interval valued intuitionistic fuzzy approximations and then construct the interval valued intuitionistic fuzzy soft sets $\downarrow S^*$ and $\uparrow S^*$.
(4) Construct the interval valued intuitionistic fuzzy sets $IVIFSet_{\downarrow S^*}, IVIFSet_{S^*}$, $IVIFSet_{\uparrow S^*}$.
(5) Construct the interval valued intuitionistic fuzzy soft set S^{**}.
(6) Input the weighting vector W and compute the weighted evaluation values $\alpha(o_k)$ of each alternative $o_k \in U$.
(7) Select the object o_p such that $\alpha(o_p)$= max{$\alpha(o_k)$: k=1, 2,...., r} as the most preferred alternative.

5. An Illustrative Example

Let us consider a staff selection problem to fill a position in a private company. Let $U = \{c_1, c_2, c_3, c_4, c_5\}$ is the universe set consisting of five candidates. Let us consider the soft set S=(F, A), which describes the "quality of the candidates", where A={e_1(experience), e_2(computer knowledge), e_3(young and efficient), e_4(good communication skill)}. Let the tabular representation of the soft set (F, A) be:

	c_1	c_2	c_3	c_4	c_5
e_1	1	0	1	1	0
e_2	1	1	0	1	0
e_3	0	1	1	1	1
e_4	1	1	0	0	1

Let $G = \{T_1, T_2, T_3, T_4\}$ be the set of interviewers to judge the quality of the candidate in U. Now if X_i denote the primary evaluation result of the interviewer T_i (for i=1, 2, 3, 4), then the primary evaluation result of the whole expert group G can be represented as an interval valued intuitionistic fuzzy evaluation soft set $S^* = (F^*, G)$ over U, where $F^* : G \to IVIFS^U$ is given by $F^*(T_i) = X_i$ for i=1, 2, 3, 4.
Let the tabular representation of S^* be given as;

	c_1	c_2	c_3	c_4	c_5
T_1	([.2,.4],[.4,.5])	([.6,.7],[.1,.2])	([.3,.4],[.3,.5])	([.2,.4],[.4,.6])	([.3,.6],[.2,.3])
T_2	([.1,.3],[.6,.7])	([.3,.4],[.4,.5])	([.5,.7],[.1,.2])	([.7,.8],[.1,.2])	([.1,.3],[.1,.5])
T_3	([.4,.6],[.2,.3])	([.1,.4],[.2,.4])	([.2,.5],[.2,.4])	([.3,.5],[.2,.4])	([.4,.5],[.2,.5])
T_4	([.3,.5],[.3,.4])	([.5,.6],[.2,.3])	([.4,.5],[.2,.5])	([.4,.7],[.1,.2])	([.6,.8],[.1,.2])

Let us choose P=(U, S) as the soft interval valued intuitionistic fuzzy approximation space. Let us consider the interval valued intuitionistic fuzzy evaluation soft sets $\downarrow S^* = (\downarrow F^*, G)$ and $\uparrow S^* = (\uparrow F^*, G)$ over U.
Then the tabular representation of these sets are:
$\downarrow S^* = (\downarrow F^*, G)$:

	c_1	c_2	c_3	c_4	c_5
T_1	([.2,.4],[.4,.6])	([.2,.3],[.4,.6])	([.2,.3],[.4,.6])	([.2,.3],[.4,.6])	([.2,.3],[.4,.6])
T_2	([.1,.2],[.6,.8])	([.1,.3],[.6,.7])	([.1,.3],[.6,.7])	([.1,.3],[.6,.7])	([.1,.3],[.6,.7])
T_3	([.1,.4],[.2,.5])	([.1,.2],[.2,.5])	([.1,.4],[.2,.5])	([.1,.4],[.2,.5])	([.1,.4],[.2,.6])
T_4	([.2,.4],[.4,.5])	([.3,.5],[.3,.5])	([.3,.5],[.3,.5])	([.3,.5],[.3,.5])	([.3,.5],[.3,.5])

$\uparrow S^* = (\uparrow F^*, G)$:

	c_1	c_2	c_3	c_4	c_5
T_1	([.6,.7],[.1,.2])	([.6,.8],[.1,.2])	([.6,.7],[.1,.2])	([.6,.7],[.1,.2])	([.6,.7],[.1,.2])
T_2	([.7,.8],[.1,.2])	([.7,.8],[.1,.2])	([.7,.8],[.1,.2])	([.5,.7],[.1,.2])	([.7,.8],[.1,.2])
T_3	([.4,.7],[.2,.3])	([.4,.6],[.2,.3])	([.4,.6],[.2,.3])	([.4,.6],[.2,.3])	([.4,.6],[.2,.3])
T_4	([.6,.8],[.1,.2])	([.6,.8],[.1,.2])	([.6,.8],[.1,.2])	([.6,.8],[.1,.2])	([.5,.7],[.1,.2])

Here, $\downarrow S^* \subseteq S^* \subseteq \uparrow S^*$. Then we have,

$$\begin{aligned} IVIFSet_{\downarrow S^*} = \{ & \langle c_1, [0.15, 0.35], [0.4, 0.625]\rangle, \langle c_2, [0.175, 0.325], [0.375, 0.575]\rangle, \\ & \langle c_3, [0.175, 0.375], [0.375, 0.575]\rangle, \langle c_4, [0.175, 0.375], [0.375, 0.575]\rangle, \\ & \langle c_5, [0.175, 0.375], [0.375, 0.6]\rangle\}, \end{aligned}$$

$$\begin{aligned} IVIFSet_{\uparrow S^*} = \{ & \langle c_1, [0.575, 0.75], [0.125, 0.225]\rangle, \langle c_2, [0.575, 0.75], [0.125, 0.225]\rangle, \\ & \langle c_3, [0.575, 0.725], [0.125, 0.225]\rangle, \langle c_4, [0.525, 0.700], [0.125, 0.225]\rangle, \\ & \langle c_5, [0.55, 0.700], [0.125, 0.225]\rangle\}, \end{aligned}$$

$$\begin{aligned} IVIFSet_{S^*} = \{ & \langle c_1, [0.25, 0.45], [0.375, 0.475]\rangle, \langle c_2, [0.375, 0.525], [0.225, 0.35]\rangle, \\ & \langle c_3, [0.350, 0.525], [0.2, 0.4]\rangle, \langle c_4, [0.4, 0.6], [0.20, 0.35]\rangle, \\ & \langle c_5, [0.35, 0.55], [0.15, 0.375]\rangle\}. \end{aligned}$$

Here $IVIFSet_{\downarrow S^*} \subseteq IVIFSet_{S^*} \subseteq IVIFSet_{\uparrow S^*}$. Let

C={L(low confidence), M(middle confidence), H(high confidence)} be a set of parameters. Let us consider the interval valued intuitionistic fuzzy soft set S^{**}=(f, C) over U, where $f : C \to IVIFS^U$ is given by f(L)=$IVIFSet_{\downarrow S^*}$, f(M)=$IVIFSet_{S^*}$, f(H)= $IVIFSet_{\uparrow S^*}$. Now assuming the weighting vector $W = (w_L, w_M, w_H)$ such that w_L=[0.5, 0.7], w_M=[0.4, 0.6], w_H=[0.4, 0.8], we have,

$\alpha(c_1) = 0.7 \diamond 0.35 + 0.6 \diamond 0.45 + 0.8 \diamond 0.75 = 1.115,$
$\alpha(c_2) = 0.7 \diamond 0.325 + 0.6 \diamond 0.525 + 0.8 \diamond 0.75 = 1.1425,$
$\alpha(c_3) = 0.7 \diamond 0.375 + 0.6 \diamond 0.525 + 0.8 \diamond 0.725 = 1.1575,$
$\alpha(c_4) = 0.7 \diamond 0.375 + 0.6 \diamond 0.6 + 0.8 \diamond 0.7 = 1.1825,$
$\alpha(c_5) = 0.7 \diamond 0.375 + 0.6 \diamond 0.55 + 0.8 \diamond 0.7 = 1.1525.$

Since $\max\{\alpha(c_1, \alpha(c_2), \alpha(c_3), \alpha(c_4), \alpha(c_5)\}$=1.1825, so the candidate c_4 will be selected as the most preferred alternative.

6. Conclusions

In this paper we first defined Interval-valued intuitionistic fuzzy soft rough sets (IVIFSsets). Finally we provided an example that demonstrated that this method can be successfully worked. It can be applied to problems of many fields that contain uncertainty. However the approach should be more comprehensive in the future to solve the related problems. It is clear that IVIF soft rough sets are IF soft rough sets due to A.Mukherjee. Also IFsoft rough sets are soft rough Fuzzy sets due to Feng et al. Further Feng et al. showed that soft rough fuzzy sets are the extension of rough fuzzy sets due to Dubois and Prade. Thus our work is the extension of the previous works of Mukherjee, Dubois, prade and Feng et.al. This work is supported by the UGC, New Delhi, INDIA under the UGC Major Research Project No.F.No.37-388/2009(SR).

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