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Generalized rough sets via ideals

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Abstract. The aim of this paper to construct a new rough set structure for a given ideal and to study many of their properties.

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1. Introduction

A classic paper of Z. pawlak [9] of Rough Sets, published in 1982, which declared the birth of the rough set theory. A lot of mathematicians, logicians, and researchers of computers have become interested in the theory and have done a lot of research work of rough set theory [5, 7] and applications. Its applications are shown in wide fields such as machine learning [4], data mining [3], decision- making support and analysis [8, 10, 11] and expert system [12]. In this paper, a new definitions of lower and upper approximations via ideal have been introduced. These new definitions are compared with Pawlak's, Yao's and Allam's definitions. It's therefore shown that the current definitions are more generally. It's shown that the present method decreases the boundary region and we get a topology finer than Allam's one which is a generalization of that obtained by Yao's method. In addition, T_1 topological spaces are obtained by relations and ideals which are not discrete.

2. Preliminaries

2.1. Pawlak approximation space.

Definition 2.1 ([9])**.** Let R be an equivalence relation on a universe X, $[x]_R$ be the equivalence class containing x. For any set $A \subseteq X$, the lower approximation $\underline{R}(A)$ and the upper approximation $\overline{R}(A)$ are defined by:

$$\underline{R}(A) = \{x \in X : [x]_R \subseteq A\} \tag{2.1}$$

(2.2) $$\overline{R}(A) = \{x \in X : [x]_R \cap A \neq \phi\}$$

Theorem 2.2 ([14])**.** *The upper approximation, defined by (2.2), have the following properties: for subsets $A, B \subseteq X$,*

(i) $\overline{R}(\phi) = \phi$,
(ii) $A \subseteq \overline{R}(A)$,
(iii) $\overline{R}(A \cup B) = \overline{R}(A) \cup \overline{R}(B)$,
(iv) $\overline{R}(\overline{R}(A)) = \overline{R}(A)$,
(v) $\overline{R}(A) = (\underline{R}(A'))'$.

where A' denotes the complement of A.

Corollary 2.3. *Let R be an equivalence relation on X. Then the operator $\overline{R}$ on $P(X)$ defined by (2.2) satisfies the Kuratowski's axioms and induces a topology on X called τ_R given by*

(2.3) $$\tau_R = \{A \subseteq X : \overline{R}(A') = A'\}$$

2.2. **Yao approximation space.**

Definition 2.4 ([13])**.** Let R be a binary relation on X. For any set $A \subseteq X$, a pair of lower and upper approximations, $\underline{R}(A)$ and $\overline{R}(A)$, are defined by:

(2.4) $$\underline{R}(A) = \{x \in X : xR \subseteq A\}$$

(2.5) $$\overline{R}(A) = \{x \in X : xR \cap A \neq \phi\}$$

where xR, which is called the after set of x, is

(2.6) $$xR = \{y \in X : xRy\}$$

Theorem 2.5. *If R is a Preorder relation on X (a reflexive and a transitive relation on X), then the upper approximation, defined by (2.5), satisfies the properties in Theorem 2.2.*

2.3. **Allam approximation space.**

Definition 2.6 ([1])**.** Let R be a reflexive binary relation on X. For any set $A \subseteq X$, a pair of lower and upper approximations, $\underline{R}(A)$ and $\overline{R}(A)$, are defined by:

(2.7) $$\underline{R}(A) = \{x \in X : \langle x\rangle_R \subseteq A\}$$

(2.8) $$\overline{R}(A) = \{x \in X : \langle x\rangle_R \cap A \neq \phi\}$$

where,

(2.9) $$\langle x\rangle_R = \cap\{pR : x \in pR\}$$

Proposition 2.7 ([1])**.** *Let R be a binary relation on X and $y \in \langle x\rangle_R$. Then*

(2.10) $$\langle y\rangle_R \subseteq \langle x\rangle_R$$

Theorem 2.8 ([2])**.** *Let R be a reflexive relation on X. Then the upper approximation, defined by (2.8), satisfies the properties in Theorem 2.2.*

Theorem 2.9 ([2])**.** *Let R be a binary relation on X. Then the operator*

$$cl_R : P(X) \to P(X)$$

given by

$$cl_R(A) = A \cup \{x \in X : \langle x \rangle_R \cap A \neq \phi\} \tag{2.11}$$

satisfies Kuratowski's axioms.

Definition 2.10 ([6])**.** A non empty collection $\mathcal{I}$ of subsets of a set X is said to be an ideal on X, if it satisfies the following conditions

(i) $A \in \mathcal{I}$ and $B \in \mathcal{I} \Rightarrow A \cup B \in \mathcal{I}$,
(ii) $A \in \mathcal{I}$ and $B \subseteq A \Rightarrow B \in \mathcal{I}$,

i.e., $\mathcal{I}$ is closed under finite unions and under subsets.

Example 2.11 ([6])**.** Let X be a non empty set. Then the following families are ideals on X

(i) $\mathcal{I} = \{\phi\}$
(ii) $\mathcal{I} = P(X) = \{A : A \subseteq X\}$
(iii) $\mathcal{I}_f = \{A \subseteq X : A \; is \; finite\}$, called ideal of finite subsets of X
(iv) $\mathcal{I}_c = \{A \subseteq X : A \; is \; countable\}$, called ideal of countable subsets of X
(v) $\mathcal{I}_A = \{B \subseteq X : B \subseteq A\}$

3. Rough sets via ideal

Definition 3.1. Let R be a reflexive relation on X, $A \subseteq X$ and $\mathcal{I}$ be an ideal on X, The R^*- upper and R_*- lower approximations of A are defined respectively by:

$$R^*(A) := \{x \in X : \langle x \rangle_R \cap A \notin \mathcal{I}\} \tag{3.1}$$

$$R_*(A) := \{x \in X : \langle x \rangle_R \cap A' \in \mathcal{I}\} \tag{3.2}$$

Theorem 3.2. *Let $\mathcal{I} = \{\phi\}$ in Definition 3.1.*

(i) *If R is an equivalence relation, then we get Pawlak's Definition 2.1*
(ii) *If R is a preorder relation, then we get Yao's Definition 2.4*
(iii) *If R is reflexive, then we get Allam's Definition 2.6.*

Proof. Straightforward. □

Theorem 3.3. *Let R be a reflexive relation on X and $\mathcal{I}$ and $\mathcal{J}$ be ideals on X. Then the R^*- upper approximation, defined in (3.1), satisfies the following properties:*

(i) $R^*(\phi) = \phi$,
(ii) $A \subseteq B \Rightarrow R^*(A) \subseteq R^*(B)$,
(iii) $R^*(A \cup B) = R^*(A) \cup R^*(B)$,
(iv) $R^*(R^*(A)) \subseteq R^*(A)$,
(v) $\mathcal{I} \subseteq \mathcal{J} \Rightarrow R^*_{\mathcal{J}}(A) \subseteq R^*_{\mathcal{I}}(A)$,
(vi) $R_*(A) = (R^*(A'))'$,
(vii) $A \nsubseteq R^*(A)$, *in general.*

Proof. (i) Straightforward.

(ii) Let $x \in R^*(A)$. Then $\langle x\rangle_R \cap A \notin \mathcal{I}$. Since $\langle x\rangle_R \cap A \subseteq \langle x\rangle_R \cap B$, it follows that $\langle x\rangle_R \cap B \notin \mathcal{I}$, and hence $x \in R^*(B)$. Then the result.

(iii) We want to show that $R^*(A \cup B) \subseteq R^*(A) \cup R^*(B)$ and the other inclusion follows from part (ii).
Let $x \in R^*(A \cup B)$. Then $\langle x\rangle_R \cap (A \cup B) \notin \mathcal{I}$. It follows that
$\langle x\rangle_R \cap (A) \notin \mathcal{I}$ or $\langle x\rangle_R \cap (B) \notin \mathcal{I}$, and hence $x \in R^*(A)$ or $x \in R^*(B)$, i.e.
$x \in R^*(A) \cup R^*(B)$.

(iv) Let $x \in R^*(R^*(A))$. Then $\langle x\rangle_R \cap R^*(A) \notin \mathcal{I}$, and hence $\langle x\rangle_R \cap R^*(A) \neq \phi$. Hence, there exists $y \in \langle x\rangle_R \cap R^*(A)$. It follows that $\langle y\rangle_R \subseteq \langle x\rangle_R$ by Proposition 2.7, and $\langle y\rangle_R \cap A \notin \mathcal{I}$. Since $\langle y\rangle_R \cap A \subseteq \langle x\rangle_R \cap A$. Hence $\langle x\rangle_R \cap A \notin \mathcal{I}$, i.e. $x \in R^*(A)$.

(v) Let $x \in R^*_{\mathcal{J}}(A)$. Then $\langle x\rangle_R \cap A \notin \mathcal{J}$, since $\mathcal{I} \subseteq \mathcal{J}$. It follows that $\langle x\rangle_R \cap A \notin \mathcal{I}$, i.e. $x \in R^*_{\mathcal{I}}(A)$.

(vi) $(R^*(A'))' = \{x \in X : \langle x\rangle_R \cap A' \notin \mathcal{I}\}' = \{x \in X : \langle x\rangle_R \cap A' \in \mathcal{I}\} = R_*(A)$.

(vii) We give an example. Let $X = \{a, b, c, d\}$. Then $\mathcal{I} = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$ is ideal on X, and let $R = \triangle \cup \{(a, b), (a, c), (b, c), (b, d), (c, d), (c, a)\}$, where $\triangle$ is the identity relation on X. Then $R^*(\{a\}) = \phi$. □

Definition 3.4. Let R be a reflexive relation on X, $A \subseteq X$ and $\mathcal{I}$ be an ideal on X. The upper approximation of A is defined by

$$\overline{R}(A) := A \cup R^*(A) \tag{3.3}$$

and the lower approximation of A is defined by:

$$\underline{R}(A) = \{x \in A : \langle x\rangle_R \cap A' \in \mathcal{I}\} \tag{3.4}$$

With respect to any subset $A \subseteq X$, the universe can be divided into three disjoint regions using the lower and upper approximations:

$$BND(A) = \overline{R}(A) \backslash \underline{R}(A) \tag{3.5}$$

$$POS(A) = \underline{R}(A) \tag{3.6}$$

$$NEG(A) = X \backslash \overline{R}(A). \tag{3.7}$$

Theorem 3.5. *Let R be a reflexive relation on X. Then the upper approximation defined by (3.3) satisfies the following properties:*

(i) $\overline{R}(\phi) = \phi$
(ii) $A \subseteq \overline{R}(A)$
(iii) $A \subseteq B \Rightarrow \overline{R}(A) \subseteq \overline{R}(B)$
(iv) $\overline{R}(A \cup B) = \overline{R}(A) \cup \overline{R}(B)$
(v) $\overline{R}(\overline{R}(A)) = \overline{R}(A)$
(vi) $\overline{R}(A) = (\underline{R}(A'))'$

Proof. The result follows immediately from Theorem 3.3. □

Corollary 3.6. *Let R be a reflexive relation on X. Then the lower approximation defined by (3.4) satisfies Kuratowski's axioms and induces a topology on X called τ_R^* given by*

$$\tau_R^* = \{A \subseteq X : \underline{R}(A) = A\} \tag{3.8}$$

In such case interior of A, $int_R^*(A)$, is identical with $\underline{R}(A)$ defined in (3.4) and closure of A, $cl_R^*(A)$, is identical with $\overline{R}(A)$ defined in (3.3).

Proof. The result follows immediately from Theorem 3.5. □

Theorem 3.7. *Let (X, τ_R^*) be a topological space defined in (3.8). Then*

(i) $cl_R^*(A) \subseteq cl_R(A)$, *(for $cl_R(A)$, see (2.11))*
(ii) $R^*(A)$ *is closed, i.e.* $cl_R^*(R^*(A)) = R^*(A)$, *(for $R^*(A)$, see Definition 3.1)*

Proof. (i) Let $x \in cl_R^*(A)$. Hence $x \in A$ or $\langle x\rangle_R \cap A \notin \mathcal{I}$. It follows that $x \in A$ or $\langle x\rangle_R \cap A \neq \phi$, and hence $x \in cl_R(A)$.
(ii) We want to prove that $cl_R^*(R^*(A)) \subseteq R^*(A)$. Let $x \in cl_R^*(R^*(A))$. It implies that $x \in R^*(A)$ or $x \in R^*(R^*(A))$, and hence $x \in R^*(A)$ by Theorem 3.3. □

In the following corollary, we compare between τ_R and τ_R^*, where τ_R is the topology generated by closure operator defined in (2.11) and τ_R^* is that one defined in (3.8).

Corollary 3.8. *Let R be a reflexive relation on X. Then $\tau_R \subseteq \tau_R^*$, i.e. τ_R^* is finer than τ_R, where τ_R is the topology generated by closure operator defined in (2.11) and τ_R^* is that defined in (3.8).*

Proof. By Theorem 3.7 (i). □

The following theorem shows that the boundary of a subset decreases as the ideal on X increases.

Theorem 3.9. *Let R be a reflexive relation on X and $\mathcal{I}$ and $\mathcal{J}$ be two ideals on X. If $\mathcal{I} \subseteq \mathcal{J}$, then $BND_{\mathcal{J}}(A) \subseteq BND_{\mathcal{I}}(A)$.*

Proof. Let $x \in BND_{\mathcal{J}}(A)$. Then $x \in \overline{R}_{\mathcal{J}}(A)$ and $x \in (\underline{R}_{\mathcal{J}}(A))'$, by Theorem 3.3 . It follows that $x \in \overline{R}_{\mathcal{I}}(A)$ and $x \in (\underline{R}_{\mathcal{I}}(A))'$. Hence $x \in BND_{\mathcal{I}}(A)$. □

In the following example, we see that the current method in Definition 3.4 reduce the boundary in comparison of Allam's method [2].

Example 3.10. Let $X = \{a, b, c, d\}$, $R = \{(a, a), (b, b), (c, c), (d, d), (a, b), (a, d), (b, c), (c, b)\}$ and $\mathcal{I} = \{\phi, \{a\}, \{b\}, \{a, b\}\}$ be an ideal on X (See Table 1)

Theorem 3.11. *Let R be a reflexive binary relation on X and $\mathcal{I}$ be an ideal on X. Then*

$$\beta = \{\langle x\rangle_R - I : \ x \in X, \ I \in \mathcal{I}\} \tag{3.9}$$

is a basis for τ_R^*.

Table 1. Comparison between Allam's method and our method

A	Allam method $\underline{R}(A)$	present method $\underline{R}_{\mathcal{I}}(A)$	Allam method $\overline{R}(A)$	present method $\overline{R}_{\mathcal{I}}(A)$	Allam method $BND(A)$	present method $BND_{\mathcal{I}}$
ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ
X	X	X	X	X	ϕ	ϕ
$\{a\}$	ϕ	ϕ	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$
$\{b\}$	$\{b\}$	$\{b\}$	$\{a,b,c\}$	$\{b\}$	$\{a,c\}$	ϕ
$\{c\}$	ϕ	$\{c\}$	$\{c\}$	$\{c\}$	$\{c\}$	ϕ
$\{d\}$	$\{d\}$	$\{d\}$	$\{a,d\}$	$\{a,d\}$	$\{a\}$	$\{a\}$
$\{a,b\}$	$\{b\}$	$\{b\}$	$\{a,b,c\}$	$\{a,b\}$	$\{a,c\}$	$\{a\}$
$\{a,c\}$	ϕ	$\{c\}$	$\{a,c\}$	$\{a,c\}$	$\{a,c\}$	$\{a\}$
$\{a,d\}$	$\{d\}$	$\{a,d\}$	$\{a,d\}$	$\{a,d\}$	$\{a\}$	ϕ
$\{b,c\}$	$\{b,c\}$	$\{b,c\}$	$\{a,b,c\}$	$\{b,c\}$	$\{a\}$	ϕ
$\{b,d\}$	$\{b,d\}$	$\{b,d\}$	X	$\{a,b,d\}$	$\{a,c\}$	$\{a\}$
$\{c,d\}$	$\{d\}$	$\{c,d\}$	$\{c,d\}$	$\{a,c,d\}$	$\{a,c\}$	$\{a\}$
$\{a,b,c\}$	$\{b,c\}$	$\{b,c\}$	$\{a,b,c\}$	$\{a,b,c\}$	$\{a\}$	$\{a\}$
$\{a,b,d\}$	$\{a,b,d\}$	$\{a,b,d\}$	X	$\{a,b,d\}$	$\{c\}$	ϕ
$\{a,c,d\}$	$\{d\}$	$\{a,c,d\}$	$\{a,c,d\}$	$\{a,c,d\}$	$\{a,c\}$	ϕ
$\{b,c,d\}$	$\{b,c,d\}$	$\{b,c,d\}$	X	X	$\{a\}$	$\{a\}$

Proof. We want to prove that every element of β belongs to τ_R^*.
i.e, $\underline{R}(\langle x\rangle_R - I) = \langle x\rangle_R - I$. Let $y \in \langle x\rangle_R - I$. Then $\langle y\rangle_R \subseteq \langle x\rangle_R$ by Proposition 2.7.
We want to prove that $\langle y\rangle_R \cap (\langle x\rangle_R - I)' \in \mathcal{I}$, Now

$$\begin{aligned}\langle y\rangle_R \cap (\langle x\rangle_R - I)' &= \langle y\rangle_R \cap ((\langle x\rangle_R)' \cup I)\\ &= \langle y\rangle_R \cap I \subseteq I.\end{aligned}$$

It follows that $\langle y\rangle_R \cap (\langle x\rangle_R - I)' \in \mathcal{I}$ by Definition 2.10

Now, we prove that β is a basis for τ_R^*,

(i) Let $\langle x\rangle_R - I_1$, $\langle y\rangle_R - I_2 \in \beta$ such that $z \in (\langle x\rangle_R - I_1) \cap (\langle y\rangle_R - I_2)$. It follows that $\langle z\rangle_R \subseteq \langle x\rangle_R$ and $\langle z\rangle_R \subseteq \langle y\rangle_R$ by Proposition 2.7, and hence $(\langle z\rangle_R - (I_1 \cup I_2)) \subseteq \langle x\rangle_R - I_1$ and $(\langle z\rangle_R - (I_1 \cup I_2)) \subseteq \langle y\rangle_R - I_1$, and hence $\exists\ (\langle z\rangle_R - (I_1 \cup I_2)) \in \beta$ such that $z \in (\langle z\rangle_R - (I_1 \cup I_2)) \subseteq (\langle x\rangle_R - I_1) \cap (\langle y\rangle_R - I_2)$.

(ii) $\cup\{\langle x\rangle_R - I : \ x \in X,\ I \in \mathcal{I}\} = X$.

□

Example 3.12. Let $X = \{a,b,c,d\}$, $R = \Delta \cup \{(a,b),(a,c),(c,d),(b,d)\}$ and $\mathcal{I} = \{\phi,\{a\},\{b\},\{d\},\{a,b\},\{a,d\},\{b,d\},\{a,b,d\}\}$. Then $\langle a\rangle_R = \{a,b,c\}$, $\langle b\rangle_R = \{b\}$, $\langle c\rangle_R = \{c\}$, $\langle d\rangle_R = \{d\}$ and the basis of τ_R^* is $\beta = \{\phi,\{a,b,c\},\{b\},\{c\},\{d\},\{a,c\},\{b,c\}\}$. To form τ_R^* $\underline{R}(X) = X$, $\underline{R}(\phi) = \phi$, $\underline{R}(\{a\}) = \phi$, $\underline{R}(\{b\}) = \{b\}$, $\underline{R}(\{c\}) = \{c\}$, $\underline{R}(\{d\}) = \{d\}$, $\underline{R}(\{a,b\}) = \{b\}$, $\underline{R}(\{a,c\}) = \{a,c\}$, $\underline{R}(\{a,d\}) = \{d\}$, $\underline{R}(\{b,c\}) = \{b,c\}$, $\underline{R}(\{b,d\}) = \{b,d\}$, $\underline{R}(\{c,d\}) = \{c,d\}$, $\underline{R}(\{a,b,c\}) = \{a,b,c\}$, $\underline{R}(\{a,b,d\}) = \{b,d\}$, $\underline{R}(\{a,c,d\}) = \{a,c,d\}$, $\underline{R}(\{b,c,d\}) = \{b,c,d\}$, and hence $\tau_R^* = \{X,\phi,\{b\},\{c\},$

$\{d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, c\}, \{b, c, d\}, \{a, c, d\}, \{a, b, c\}\}$. It's clear that β is a basis for τ_R^*.

Lemma 3.13. *If (X, τ) is Alexandrov T_1-space. Then (X, τ) is the discrete space.*

Proof. We want to prove that every subset of X is closed.

$$\begin{aligned}\overline{A} &= \cup_{x\in A}\overline{\{x\}}\ ((X,\tau)\ is\ Alexandrov\ topology)\\ &= \cup_{x\in A}\{x\}\ ((X,\tau)\ is\ T_1\ Space)\\ &= A.\end{aligned}$$

□

In the following theorem, we have non discrete topological spaces generated by relations and is T_1 space, which are not found before.

Theorem 3.14. *Let R be a reflexive relation on X and $\mathcal{I}_f$ be an ideal of finite subsets of X. Then the topological space(X, τ_R^*) is T_1 space.*

Proof. We want to prove that for every $x \in X$, $\{x\}$ is closed. Since $R^*(\{x\}) = \phi$. It follows that $\overline{R}(\{x\}) = \{x\}\ \forall x \in X$. □

4. Some important examples

Example 4.1. Let X be an infinite set and $R = X \times X$. If $\mathcal{I}_f$ is an ideal of finite subsets of X, then

$$\overline{R}(A) = \begin{cases} X & \text{if } A \notin \mathcal{I}_f, \\ \text{A} & \text{if } A \in \mathcal{I}_f. \end{cases}$$

This means that the induced topology τ_R^*, defined by (3.8), is the cofinite topology.

Example 4.2. If $\mathcal{I}_c$ is an ideal of countable subsets of X, then

$$\overline{R}(A) = \begin{cases} X & \text{if } A \notin \mathcal{I}_c, \\ \text{A} & \text{if } A \in \mathcal{I}_c. \end{cases}$$

This means that the induced topology τ_R^*, defined by (3.8), is the cocountable topology.

Example 4.3. If $a \in X$ and $\mathcal{I}_{(X-\{a\})}$, then

$$\overline{R}(A) = \begin{cases} X & \text{if } A \notin \mathcal{I}_{(X-\{a\})}, \\ \text{A} & \text{if } A \in \mathcal{I}_{(X-\{a\})}. \end{cases}$$

This means that the induced topology τ_R^*, defined by (3.8), is the particular point topology.

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