Ranking method of trapezoidal intuitionistic fuzzy numbers

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Abstract. In this paper, considers the usage of intuitionistic fuzzy numbers in decision making. The values and ambiguities of the membership degree and the non-membership degree for trapezoidal intuitionistic fuzzy number are defined as well as the value-index and ambiguity-index. The proposed ranking method is easily implemented and has a natural interpretation.

1. Introduction

Bellman and Zadeh [3] initially proposed the basic model of fuzzy decision making based on the theory of fuzzy mathematics, FMADM has been receiving more and more attentions from researchers at home and abroad. For the comparison of fuzzy numbers, there are many different methods ([5],[9]). The ranking of fuzzy numbers is important in fuzzy multiattribute decision making (MADM). There exists a large amount of literature involving the ranking of fuzzy numbers ([1]-[6],[10]). Recently, the IFN receives little attention and different definitions of IFNs have been proposed as well as the corresponding ranking methods of IFNs. Mitchell [8] interpreted an IFN as an ensemble of ordinary fuzzy numbers and introduced a ranking method. However, these existing definitions of IFNs are complicated and the ranking methods of IFNs have tedious calculations. By adding a degree of non-membership, Shu et al [12] defined triangular intuitionistic fuzzy numbers (TIFNs), but not given the ranking of TIFNs. Moreover, Rezvani [11] proposed a new method for ranking in perimeters of two generalized trapezoidal fuzzy numbers. Also, Deng Feng Li et al. [7] proposed A Ranking Method of Triangular Intuitionistic Fuzzy Numbers and Application to Decision Making.

In this paper, considers the usage of intuitionistic fuzzy numbers in decision making. The values and ambiguities of the membership degree and the non-membership degree for trapezoidal intuitionistic fuzzy number are defined as well as the value-index and ambiguity-index. The proposed ranking method is easily implemented and has a natural interpretation.

2. Preliminaries

Generally, a generalized fuzzy number A is described as any fuzzy subset of the real line R, whose membership function μ_A satisfies the following conditions,

(i) μ_A is a continuous mapping from R to the closed interval [0,1],
(ii) $\mu_A(x) = 0, -\infty < x \leq a$,
(iii) $\mu_A(x) = L(x)$ is strictly increasing on $[a,b]$,
(iv) $\mu_A(x) = w, b \leq x \leq c$,
(v) $\mu_A(x) = R(x)$ is strictly decreasing on $[c,d]$,
(vi) $\mu_A(x) = 0, d \leq x < \infty$

where $0 < w \leq 1$ and a, b, c, and d are real numbers. We call this type of generalized fuzzy number a trapezoidal fuzzy number, and it is denoted by

$$(2.1) \qquad A = (a, b, c, d; w) \ .$$

A $A = (a, b, c, d; w)$ is a fuzzy set of the real line R whose membership function $\mu_A(x)$ is defined as

$$(2.2) \qquad \mu_A(x) = \begin{cases} w\frac{x-a}{b-a} & if & a \leq x \leq b \\ w & if & b \leq x \leq c \\ w\frac{d-x}{d-c} & if & c \leq x \leq d \\ 0 & & otherwise \end{cases}$$

For an universe of discourse defined by X, the intuitionistic fuzzy set A in X is characterized by a membership function $\mu_A(.)$ and a non-membership function $\nu_A(.)$, where $\mu_A : X \to [0,1]$, and $\nu_A : X \to [0,1]$. For each point x in X, $\mu_A(x)$ (resp.$\nu_A(x)$) is the degree of membership (resp. non-membership) of x in A, with $0 \leq \mu_A(x) + \nu_A(x)) \leq 1$. An intuitionistic fuzzy set becomes a fuzzy set if $\nu_A(x)) = 0$ for all x in A.

Let $a'_1 \leq a_1 \leq b_1 \leq c_1 \leq d_1 \leq d'_1$. A trapezoidal intuitionistic fuzzy number A in R, written as $(a_1, b_1, c_1, d_1; a'_1, b_1, c_1, d'_1)$, has the membership function

$$(2.3) \qquad \mu_A(x) = \begin{cases} \frac{x-a_1}{b_1-a_1} & if & a_1 \leq x \leq b_1 \\ 1 & if & b_1 \leq x \leq c_1 \\ \frac{d_1-x}{d_1-c_1} & if & c_1 \leq x \leq d_1 \\ 0 & & otherwise \end{cases}$$

and the non-membership function

$$(2.4) \qquad \nu_A(x) = \begin{cases} \frac{b_1-x}{b_1-a_1} & if & a'_1 \leq x \leq b_1 \\ 0 & if & b_1 \leq x \leq c_1 \\ \frac{x-c_1}{d_1-c_1} & if & c_1 \leq x \leq d'_1 \\ 1 & & otherwise \end{cases}$$

3. Arithmetic operations and cut sets on intuitionistic fuzzy number

The arithmetic operations on intuitionistic fuzzy number can be defined using the (α, β)-cut method. Let $\alpha, \beta \in [0, 1]$ be fixed numbers such that $\alpha + \beta \leq 1$. A set of (α, β)-cut generated by an IFS A is defined by:

(3.1) $\quad A_{\alpha,\beta} = \{(x, \mu_A(x), \nu_A(x)), \ x \in X, \ \mu_A(x) \geq \alpha, \ \nu_A(x) \leq \beta\}$.

The (α, β)-cut of an intuitionistic fuzzy number is given by

(3.2) $\quad A_{\alpha,\beta} = \{[A_1(\alpha), A_2(\alpha)], [A'_1(\beta), A'_2(\beta)]\}$,

where:

i) $A_1(\alpha)$ and $A'_2(\beta)$ are continuous, monotonic increasing functions of α, respective β;

ii) $A_2(\alpha)$ and $A'_1(\beta)$ are continuous, monotonic increasing functions of α, respective β;

iii) $A_1(1) = A_2(1)$ and $A'_1(0) = A'_2(0)$.

The (α, β)-cut trapezoidal intuitionistic fuzzy number is defined as usually, by

$$A_{\alpha,\beta} = \{[A_1(\alpha), A_2(\alpha)], [A'_1(\beta), A'_2(\beta)]\} ,$$

$$\alpha + \beta \leq 1, \quad \alpha, \beta \in [0, 1] ,$$

where

(3.3) $\quad A_1(\alpha) = a_1 + \alpha(b_1 - a_1)$,

(3.4) $\quad A_2(\alpha) = d_1 - \alpha(d_1 - c_1)$,

(3.5) $\quad A'_1(\beta) = b_1 - \beta(b_1 - a'_1)$,

(3.6) $\quad A'_2(\beta) = c_1 + \beta(d'_1 - c_1)$,

The above operations defined for trapezoidal intuitionistic fuzzy number:
1) If trapezoidal intuitionistic fuzzy number $A = (a_1, b_1, c_1, d_1; a'_1, b_1, c_1, d'_1)$, and $k > 0$, then the trapezoidal intuitionistic fuzzy number kA is given by

(3.7) $\quad (ka_1, kb_1, kc_1, kd_1; ka'_1, kb_1, kc_1, kd'_1)$.

2) If trapezoidal intuitionistic fuzzy number $A = (a_1, b_1, c_1, d_1; a'_1, b_1, c_1, d'_1)$, and $k < 0$, then the trapezoidal intuitionistic fuzzy number kA is given by

(3.8) $\quad (kd_1, kc_1, kb_1, ka_1; kd'_1, kc_1, kb_1, ka'_1)$.

3) If $A = (a_1, b_1, c_1, d_1; a'_1, b_1, c_1, d'_1)$, and $B = (a_2, b_2, c_2, d_2; a'_2, b_2, c_2, d'_2)$ are trapezoidal intuitionistic fuzzy numbers, then the trapezoidal intuitionistic fuzzy number $A \oplus B$ is defined by

(3.9) $\quad (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2; a'_1 + a'_2, b_1 + b_2, c_1 + c_2, d'_1 + d'_2)$.

4) If $A = (a_1, b_1, c_1, d_1; a'_1, b_1, c_1, d'_1)$, and $B = (a_2, b_2, c_2, d_2; a'_2, b_2, c_2, d'_2)$ are trapezoidal intuitionistic fuzzy numbers, then the trapezoidal intuitionistic fuzzy number $A \otimes B$ is defined by

(3.10) $\quad (a_1 a_2, b_1 b_2, c_1 c_2, d_1 d_2; a'_1 a'_2, b_1 b_2, c_1 c_2, d'_1 d'_2)$.

Definition 3.1. A α-cut sets of $A = (a_1, b_1, c_1, d_1; a_1', b_1, c_1, d_1')$ is a crisp subset of R, which is defined as follows:

$$(3.11) \qquad A_\alpha = \{x \mid \mu_A(x) \geq \alpha\} \ .$$

Using Eq. (2.2) and definition 3.1., it follows that A_α is a closed interval, denoted by $A_\alpha = [L_\alpha(A), R_\alpha(A)]$, which can be calculated as follows:

$$(3.12) \qquad [L_\alpha(A), R_\alpha(A)] = [a_1 + \alpha(b_1 - a_1), d_1 - \alpha(d_1 - c_1)] \ .$$

Definition 3.2. A β-cut sets of $A = (a_2, b_2, c_2, d_2; a_2', b_2, c_2, d_2')$ is a crisp subset of R, which is defined as follows:

$$(3.13) \qquad A_\beta = \{x \mid \nu_A(x) \leq \beta\} \ .$$

Where $\nu_A \leq \beta \leq 1$.

Using Eq. (2.3) and definition 3.2, it follows that A_β is a closed interval, denoted by $A_\beta = [L_\beta(A), R_\beta(A)]$, which can be calculated as follows:

$$(3.14) \qquad [L_\alpha(A), R_\alpha(A)] = [b_2 - \beta(b_2 - a_2'), c_2 + \beta(d_2' - c_2)] \ .$$

4. Value and ambiguity of an trapezoidal intuitionistic fuzzy number

Definition 4.1. Let A_α and A_β be any α-cut and β-cut set of a trapezoidal intuitionistic fuzzy number $A = (a, b, c, d; a', b, c, d')$, respectively. Then the values of the membership function a μ_A and the non-membership function ν_A for the trapezoidal intuitionistic fuzzy number A are defined as follows:

$$(4.1) \qquad V_\mu(A) = \int_0^1 (L_\alpha(A) + R_\alpha(A)) \ f(\alpha) \ d\alpha \ ,$$

and

$$(4.2) \qquad V_\nu(A) = \int_0^1 (L_\beta(A) + R_\beta(A)) \ f(\beta) \ d\beta \ ,$$

respectively.

The function $f(\alpha) = \alpha$ ($\alpha \in [0, w]$) gives different weights to elements in different α-cut sets. In fact, $f(\alpha)$ diminishes the contribution of the lower α-cut sets, which is reasonable since these cut sets arising from values of μ_A have a considerable amount of uncertainty. Obviously, $V_\mu A$ synthetically reflects the information on every membership degree, and may be regarded as a central value that represents from the membership function point of view. Similarly, the function $g(\beta) = 1 - \beta$ ($\beta \in [\nu, 1]$) has the effect of weighting on the different β-cut sets. $g(\beta)$ diminishes the contribution of the higher β-cut sets, which is reasonable since these cut sets arising from values of ν_A have a considerable amount of uncertainty. $V_\nu A$ synthetically reflects the information on every nonmembership degree and may be regarded as a central value that represents from the non-membership function point of view.

Definition 4.2. Let A_α and A_β be any α-cut and β-cut set of an trapezoidal intuitionistic fuzzy number $A = (a, b, c, d; a', b, c, d')$, respectively. Then the values of the

membership function a μ_A and the non-membership function ν_A for the trapezoidal intuitionistic fuzzy number A are defined as follows:

$$(4.3) \qquad\qquad G_\mu(A) = \int_0^1 (R_\alpha(A) - L_\alpha(A)) \, f(\alpha) \, d\alpha \;,$$

and

$$(4.4) \qquad\qquad G_\nu(A) = \int_0^1 (R_\beta(A) - L_\beta(A)) \, g(\beta) \, d\beta \;,$$

respectively.

It is easy to see that $R_\alpha(A) - L_\alpha(A)$ and $R_\beta(A) - L_\beta(A)$ are just about the lengths of the intervals A_α and A_β, respectively. Thus, $G_\mu(A)$ and $G_\nu(A)$ may be regarded as the global spreads of the membership function μ_A and the non-membership function ν_A. Obviously, $G_\mu(A)$ and $G_\nu(A)$ basically measure how much there is vagueness in the trapezoidal intuitionistic fuzzy number A.

Theorem 4.3. *The values of the membership function and the nonmembership function of the trapezoidal intuitionistic fuzzy number A are calculated as follows:*

$$(4.5) \qquad\qquad i) \; V_\mu(A) = \frac{(a + 2b + 2c + d)}{6} \;,$$

$$(4.6) \qquad\qquad ii) \; V_\nu(A) = \frac{(b + c)}{2} + \frac{(d' - c - b + a')}{6} \;.$$

Proof. (i): $V_\mu(A) = \int_0^1 (L_\alpha(A) + R_\alpha(A)) \, f(\alpha) \, d\alpha$

$$= \int_0^1 [(a + \alpha(b - a)) + (d - \alpha(d - c))] \, f(\alpha) \, d\alpha$$

$$= \int_0^1 [(a + \alpha(b - a)) + (d - \alpha(d - c))] \, \alpha \, d\alpha$$

$$= \int_0^1 [(a + d)\alpha + (b - a - d + c)\alpha^2] \, d\alpha$$

$$= \frac{(a + 2b + 2c + d)}{6}.$$

(ii): $V_\nu(A) = \int_0^1 (L_\alpha(A) + R_\alpha(A)) \, g(\beta) \, d\beta$

$$= \int_0^1 [(b - \beta(b - a')) + (c + \beta(d' - c))] \, g(\beta) \, d\beta$$

$$= \int_0^1 [(b - \beta(b - a')) + (c + \beta(d' - c))] \, (1 - \beta) \, d\beta$$

$$= \int_0^1 [(b + c)(1 - \beta) + (d' - c - b + a')\beta(1 - \beta)] \, d\beta$$

$$= \int_0^1 [(b + c) - (b + c)\beta + (d' - c - b + a')\beta - (d' - c - b + a')\beta^2] \, d\beta$$

$$= (b + c)\beta]_0^1 - \frac{(b + c)\beta^2}{2}]_0^1 + \frac{(d' - c - b + a')\beta^2}{2}]_0^1 - \frac{(d' - c - b + a')\beta^3}{3}]_0^1$$

$$= \frac{(b + c)}{2} + \frac{(d' - c - b + a')}{6}. \qquad\qquad \square$$

Theorem 4.4. *Let*

$$A = (a_1, b_1, c_1, d_1; a_1', b_1, c_1, d_1') \; and \; B = (a_2, b_2, c_2, d_2; a_2', b_2, c_2, d_2')$$

be two trapezoidal intuitionistic fuzzy number. If $a_1 > d_2$, then $A > B$.

Proof. $V_\mu(A) = \int_0^1 (L_\alpha(A) + R_\alpha(A)) \ f(\alpha) \ d\alpha$

$$= \int_0^1 [(a_1 + \alpha(b_1 - a_1)) + (d_1 - \alpha(d_1 - c_1))] \ \alpha \ d\alpha$$

$$\geq \int_0^1 2a_1 \ \alpha \ d\alpha = a_1$$

and

$V_\mu(B) = \int_0^1 (L_\alpha(B) + R_\alpha(B)) \ f(\alpha) \ d\alpha$

$$= \int_0^1 [(a_2 + \alpha(b_2 - a_2)) + (d_2 - \alpha(d_2 - c_2))] \ \alpha \ d\alpha$$

$$\leq \int_0^1 2d_2 \ \alpha \ d\alpha = d_2.$$

Combining with both $a_1 > d_2$, we have $V_\lambda(A) > V_\lambda(B)$, then $A > B$.

Again

$V_\nu(A) = \int_0^1 (L_\alpha(A) + R_\alpha(A)) \ g(\beta) \ d\beta$

$$= \int_0^1 [(b_1 - \beta(b_1 - a'_1)) + (c + \beta(d'_1 - c_1))] \ (1 - \beta) \ d\beta$$

$$\geq \int_0^1 2a'_1(1 - \beta) \ d\beta = a'_1$$

and

$V_\nu(B) = \int_0^1 (L_\alpha(A) + R_\alpha(A)) \ g(\beta) \ d\beta$

$$= \int_0^1 [(b_2 - \beta(b_2 - a'_2)) + (c + \beta(d'_2 - c_2))] \ (1 - \beta) \ d\beta$$

$$\leq \int_0^1 2d'_2(1 - \beta) \ d\beta = d'_2.$$

Combining with both $a'_1 > d'_2$, then $A > B$. □

Theorem 4.5. *Let*

$$A = (a_1, b_1, c_1, d_1; a'_1, b_1, c_1, d'_1), \ B = (a_2, b_2, c_2, d_2; a'_2, b_2, c_2, d'_2)$$

and $C = (a_3, b_3, c_3, d_3; a'_3, b_3, c_3, d'_3)$ *be three trapezoidal intuitionistic fuzzy number. If* $A > B$, *then* $A + C > B + C$.

Proof. $V_\mu(A + C) = \int_0^1 (L_\alpha(A) + R_\alpha(A) + L_\alpha(C) + R_\alpha(C)) \ f(\alpha) \ d\alpha$

$$= \int_0^1 (L_\alpha(A) + R_\alpha(A)) \ f(\alpha) \ d\alpha + \int_0^1 (L_\alpha(C) + R_\alpha(C)) \ f(\alpha) \ d\alpha$$

$$= \int_0^1 ((a_1 + \alpha(b_1 - a_1)) + (d_1 - \alpha(d_1 - c_1))) \ \alpha \ d\alpha$$

$$+ \int_0^1 (b_3 - a_3)) + (d_3 - \alpha(d_3 - c_3))) \ \alpha \ d\alpha$$

$$= \frac{(a_1 + 2b_1 + 2c_1 + d_1)}{6} + \frac{(a_3 + 2b_3 + 2c_3 + d_3)}{6}$$

$$= \frac{[(a_1 + a_3) + 2(b_1 + b_3) + 2(c_1 + c_3) + (d_1 + d_3)]}{6}$$

and

$V_\mu(B + C) = \int_0^1 (L_\alpha(B) + R_\alpha(B) + L_\alpha(C) + R_\alpha(C)) \ f(\alpha) \ d\alpha$

$$= \int_0^1 (L_\alpha(B) + R_\alpha(B)) \ f(\alpha) \ d\alpha + \int_0^1 (L_\alpha(C) + R_\alpha(C)) \ f(\alpha) \ d\alpha$$

$$= \int_0^1 ((a_2 + \alpha(b_2 - a_2)) + (d_2 - \alpha(d_2 - c_2))) \ \alpha \ d\alpha$$

$$+ \int_0^1 (b_3 - a_3)) + (d_3 - \alpha(d_3 - c_3))) \ \alpha \ d\alpha$$

$$= \frac{(a_2+2b_2+2c_2+d_2)}{6} + \frac{(a_3+2b_3+2c_3+d_3)}{6}$$

$$= \frac{[(a_2+a_3)+2(b_2+b_3)+2(c_2+c_3)+(d_2+d_3)]}{6}.$$

Because $A > B$, then

$$\frac{(a_1+2b_1+2c_1+d_1)}{6} > \frac{(a_2+2b_2+2c_2+d_2)}{6}$$

$$\Rightarrow \int_0^1 (L_\alpha(A) + R_\alpha(A)) \ \alpha \ d\alpha > \int_0^1 (L_\alpha(B) + R_\alpha(B)) \ \alpha \ d\alpha.$$

Then

$$\frac{[(a_1+a_3)+2(b_1+b_3)+2(c_1+c_3)+(d_1+d_3)]}{6} > \frac{[(a_2+a_3)+2(b_2+b_3)+2(c_2+c_3)+(d_2+d_3)]}{6}$$

$$\Rightarrow \int_0^1 (L_\alpha(A) + R_\alpha(A)) \ f(\alpha) \ d\alpha + \int_0^1 (L_\alpha(C) + R_\alpha(C)) \ f(\alpha) \ d\alpha$$

$$> \int_0^1 (L_\alpha(B) + R_\alpha(B)) \ f(\alpha) \ d\alpha + \int_0^1 (L_\alpha(C) + R_\alpha(C)) \ f(\alpha) \ d\alpha$$

$$\Rightarrow V_\mu(A + C) > V_\mu(B + C).$$

Again

$$V_\nu(A + C) = \int_0^1 (L_\alpha(A) + R_\alpha(A) + L_\alpha(C) + R_\alpha(C)) \ g(\beta) \ d\beta$$

$$= \int_0^1 (L_\alpha(A) + R_\alpha(A)) \ g(\beta) \ d\beta + \int_0^1 (L_\alpha(C) + R_\alpha(C)) \ g(\beta) \ d\beta$$

$$= \int_0^1 [(b_1 - \beta(b_1 - a_1')) + (c + \beta(d_1' - c_1))] \ (1 - \beta) \ d\beta +$$

$$\int_0^1 [(b_3 - \beta(b_3 - a_3')) + (c + \beta(d_3' - c_3))] \ (1 - \beta) \ d\beta$$

$$= \frac{(b_1 + c_1)}{2} + \frac{(d_1' - c_1 - b_1 + a_1')}{6} + \frac{(b_2 + c_2)}{2} + \frac{(d_2' - c_2 - b_2 + a_2')}{6}$$

$$= \frac{(b_1 + b_2) + (c_1 + c_2)}{2} + \frac{(d_1' + d_2') - (c_1 + c_2) - (b_1 + b_2) + (a_1' + a_2')}{6}$$

Because $A > B$, then

$$\frac{(b_1 + c_1)}{2} + \frac{(d_1' - c_1 - b_1 + a_1')}{6} > \frac{(b_2 + c_2)}{2} + \frac{(d_2' - c_2 - b_2 + a_2')}{6}$$

$$\Rightarrow \int_0^1 (L_\alpha(A) + R_\alpha(A)) \ (1 - \beta) \ d\beta > \int_0^1 (L_\alpha(B) + R_\alpha(B)) \ (1 - \beta) \ d\beta$$

Then

$$\frac{(b_1 + b_3) + (c_1 + c_3)}{2} + \frac{(d_1' + d_3') - (c_1 + c_3) - (b_1 + b_3) + (a_1' + a_3')}{6}$$

$$> \frac{(b_2 + b_3) + (c_2 + c_3)}{2} + \frac{(d_2' + d_3') - (c_2 + c_3) - (b_2 + b_3) + (a_2' + a_3')}{6}$$

$$\int_0^1 \left(L_\alpha(A) + R_\alpha(A)\right) g(\beta) \, d\beta + \int_0^1 \left(L_\alpha(C) + R_\alpha(C)\right) g(\beta) \, d\beta$$

$$> \int_0^1 \left(L_\alpha(B) + R_\alpha(B)\right) g(\beta) \, d\beta + \int_0^1 \left(L_\alpha(C) + R_\alpha(C)\right) g(\beta) \, d\beta$$

$$\Rightarrow V_\nu(A + C) > V_\nu(B + C)$$

Hence $A + C > B + C$. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\square$

5. Examples

Example 5.1. Let $A = (0.2, 0.3, 0.4, 0.5)$ and $B = (0.1, 0.2, 0.3, 0.4)$ be two trapezoidal intuitionistic fuzzy number. Then

$$V_\mu(A) = \frac{0.2 + 0.6 + 0.8 + 0.5}{6} = 0.35$$

and

$$V_\mu(B) = \frac{0.1 + 0.4 + 0.6 + 0.4}{6} = 0.25$$

Then $V_\mu(A) > V_\mu(B) \Rightarrow A > B$.

Example 5.2. Let $A = (0.2, 0.3, 0.4, 0.5)$ and $B = (0.1, 0.2, 0.3, 0.4)$ be two trapezoidal intuitionistic fuzzy number. Then

$$V_\mu(A) = \frac{0.25 + 0.8 + 1.1 + 0.6}{6} = 0.458$$

and

$$V_\mu(B) = \frac{0.33 + 1.12 + 1.28 + 0.67}{6} = 0.57$$

Then $V_\mu(A) < V_\mu(B) \Rightarrow A < B$.

Example 5.3. Let $A = (0.05, 0.1, 0.2, 0.4)$ and $B = (0.25, 0.2, 0.25, 0.3)$ and $C = (0.01, 0.05, 0.07, 0.09)$ be three trapezoidal intuitionistic fuzzy number. Then

$$V_\mu(A) = \frac{0.05 + 0.2 + 0.4 + 0.4}{6} = 0.175$$

and

$$V_\mu(B) = \frac{0.25 + 0.4 + 0.5 + 0.3}{6} = 0.242$$

and

$$V_\mu(C) = \frac{0.01 + 0.1 + 0.14 + 0.09}{6} = 0.057$$

Then $V_\mu(B) > V_\mu(A) > V_\mu(C) \Rightarrow B > A > C$.

References

[1] S. Abbasbandy and M. Amirfakhrian, The nearest trapezoidal form of a generalized left right fuzzy number, Internat. J. Approx. Reason. 43(2) (2006) 166–178.

[2] S. Abbasbandy and B. Asady, Ranking of fuzzy numbers by sign distance, Inform. Sci. 176 (2006) 2405–2416.

[3] R. E. Bellman and L. A. Zadeh, Decision-making in a fuzzy environment, Management Sci. 17 (1970/71) B141–B164.

[4] T. Chu and C. Tsao, Ranking fuzzy numbers with an area between the centroid point and original point, Comput. Math. Appl. 43 (2002) 111–117.

[5] D. Dubois and H. Prade, Ranking fuzzy numbers in the setting of possibility theory, Inform. Sci. 30 (1983) 183–224.

[6] C. Kahraman and A. C. Tolga, An alternative ranking approach and its usage in multi-criteria decision-making, International Journal of Computational Intelligence Systems 2 (2009) 219–235.

[7] Deng Feng Li, Jiang Xia Nan and Mao Jun Zhang, A ranking method of triangular intuitionistic fuzzy numbers and application to decision making, International Journal of Computational Intelligence Systems 3(5) (2010) 522–530.

[8] H. B. Mitchell, Ranking intuitionistic fuzzy numbers, Internat. J. Uncertain. Fuzziness Knowledge-Based Systems 12(3) (2004) 377–386.

[9] S. Rezvani, Three-tier FMCDM problems with trapezoidal fuzzy number, World Applied Sciences Journal 10(9) (2010) 1106–1113.

[10] S. Rezvani, Multiplication operation on trapezoidal fuzzy numbers, J. Phys. Sci. 15 (2011) 17–26.

[11] S. Rezvani, A new method for ranking in perimeters of two generalized trapezoidal fuzzy numbers, International Journal of Applied Operational Research 2(3) (2012) 83–90.

[12] M. H. Shu, C. H. Cheng and J. R. Chang, Using intuitionistic fuzzy sets for fault-tree analysis on printed circuit board assembly, Microelectronics Reliability 46 (2006) 2139–2148.

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