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Construction of normal fuzzy numbers: case studies with stock exchange data

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Abstract. The mathematics of partial presence leads to a Randomness-Fuzziness Consistency Principle which states that two laws of randomness in $[\alpha,\ \beta]$ and $[\beta,\ \gamma]$ are necessary and sufficient to define a normal fuzzy number $[\alpha,\ \beta,\ \gamma]$. In this article, we have shown using daily data for a short duration available from the BSE/NSE (Mumbai Stock Exchange or National Stock Exchange) Derivatives with reference to the minimum and the maximum prices of stock of various companies can lead to construction of normal fuzzy numbers following the aforesaid principle. It has been found that for the companies considered, the uncertainties in the prices concerned can be approximately defined by triangular fuzzy numbers.

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1. Introduction

Construction of normal fuzzy number has been discussed in ([1],[2]) based on the Randomness-Fuzziness Consistency Principle deduced by Baruah ([3],[4],[5]). A fuzzy real number $[\alpha,\ \beta,\ \gamma]$ is an interval around the real number β with the elements in the interval being partially present. Partial presence of an element in a fuzzy set is defined by the name *membership function*. Based on the aforesaid principle, in this article we have constructed normal fuzzy numbers using finance related daily data obtained from Stocks on BSE/NSE (Mumbai Stock Exchange or National Stock Exchange) derivatives in lowest and highest stock prices of different companies considered for a small number of days. If the lowest stock price is assumed to follow a law of probability in the interval $[\alpha,\ \beta]$, and if the highest stock price is assumed to follow another law of probability in the interval $[\beta,\ \gamma]$, then according to the aforesaid principle, we are in a fuzzy situation.

2. Methodology

The basic problem in constructing normal fuzzy numbers was the misunderstanding as to how exactly to define partial presence of an element in an interval. Indeed various explanations regarding the possible relationship between probability and fuzziness have come up, and no concrete conclusion could be arrived at. Baruah ([3],[4],[5]) has recently shown that two laws of randomness, and therefore two laws of probability, can define a normal law of fuzziness. This has led to a proper measure theoretic explanation of partial presence, and construction of fuzzy numbers can be based on that.

We need to understand that if a variable X assumes values in an interval $[L,\ U]$ where L follows a law of randomness in the interval $[\alpha,\ \beta]$ while U follows another law of randomness in the interval $[\beta,\ \gamma]$, then we are in a situation defining fuzzy uncertainty. In such a case, Baruah's principle states that the distribution function of L in the interval $[\alpha,\ \beta]$ together with the complementary distribution function of U in the interval $[\beta,\ \gamma]$, would give us the membership function of a normal fuzzy number $[\alpha,\ \beta\ ,\ \gamma]$. The two concerned laws of randomness may or may not be geared to laws of probability because measure theoretically speaking the notion of probability need not actually appear in the definition of randomness in the sense that a probabilistic variable is necessarily random while a random variable need not be probabilistic.

We collected data for three companies on their stock prices for a period of 5 days starting from 07^{th} February, 2012 to 11^{th} February, 2012 from The Economic Times([6]). We collected the values taken by L and U on those 5 days. For any particular company, these values were say (a_1, a_2, a_3, a_4, a_5) and (b_1, b_2, b_3, b_4, b_5) respectively. For any particular day, the occurrences in every interval (a_i, b_i) for $i = 1, 2,...,5$ were obviously random supported by a probability law. We are not referring to those probability laws. It is obvious that in our case, the occurrences of L and U were probabilistic. So following the operation of set superimposition defined in ([3],[4],[5]), we may proceed to construct normal fuzzy numbers which would define the uncertainty associated with share price variations using the mathematics of partial presence. We have used data for a short duration because the condition $\max(a_i) \le \min(b_i)$ has to be satisfied. Indeed, for data of a long duration, this condition may not be satisfied.

Suppose $a_{(1)}$, $a_{(2)}$,...,$a_{(5)}$ are values of a_1, a_2,...,a_5 arranged in increasing order of magnitude, and $b_{(1)}$, $b_{(2)}$,...,$b_{(5)}$ are values of b_1, b_2,...,b_5 arranged in increasing order of magnitude. We shall now have to superimpose the 5 intervals thus found in every case, and shall have to normalize the frequency of occurrences by dividing by the total frequency. Superimposing the intervals $[a_1,\ b_1]$, $[a_2,\ b_2]$,...,$[a_5,\ b_5]$ and thereafter normalizing in the aforesaid manner is equivalent to superimposing the fuzzy intervals $[a_1, b_1]^{(1/5)}, [a_2, b_2]^{(1/5)}, ..., [a_5, b_5]^{(1/5)}$ with constant fuzzy membership value $\frac{1}{5}$ for every fuzzy interval. We shall thus get, subject to the condition that $[a_1,\ b_1] \cap [a_2,\ b_2] \cap [a_3,\ b_3] \cap [a_4,\ b_4] \cap [a_5,\ b_5]$ is not void,

$$
\begin{aligned}
&[a_1, b_1]^{(\frac{1}{5})}(S)[a_2, b_2]^{(\frac{1}{5})}(S)[a_3, b_3]^{(\frac{1}{5})}(S)[a_4, b_4]^{(\frac{1}{5})}(S)[a_5, b_5]^{(\frac{1}{5})}\\
&= [a_{(1)}, a_{(2)}]^{(\frac{1}{5})} \cup [a_{(2)}, a_{(3)}]^{(\frac{2}{5})} \cup [a_{(3)}, a_{(4)}]^{(\frac{3}{5})} \cup [a_{(4)}, a_{(5)}]^{(\frac{4}{5})} \cup [a_{(5)}, b_{(1)}]^{(1)}\\
&\quad \cup [b_{(1)}, b_{(2)}]^{(\frac{4}{5})} \cup [b_{(2)}, b_{(3)}]^{(\frac{3}{5})} \cup [b_{(3)}, b_{(4)}]^{(\frac{2}{5})} \cup [b_{(4)}, b_{(5)}]^{(\frac{1}{5})},
\end{aligned}
$$

where for example, $[b_{(1)}, b_{(2)}]^{(4/5)}$ represents the uniformly fuzzy interval $[b_{(1)}, b_{(2)}]$ with membership value 4/5 in the entire interval.

We shall thereafter use the following theorem which actually is the Randomness-Fuzziness Consistency Principle that explains fuzziness measure theoretically. In our special case, the two laws of randomness are in fact two independent laws of probability. Hence in our case, the theorem stated below is actually the Probability-Possibility Consistency Principle.

Theorem 2.1. *Two densities* $\frac{d}{dx}\psi_1(x)$ *and* $\frac{d}{dx}(1-\psi_2(x))$ *in the intervals* $[\alpha, \beta]$ *and* $[\beta, \gamma]$ *respectively can define a normal fuzzy number* $N = [\alpha, \beta, \gamma]$ *with membership function*

$$
\mu_N(x) = \begin{cases} \psi_1(x), & \alpha \le x \le \beta \\ \psi_2(x), & \beta \le x \le \gamma \\ 0, & otherwise \end{cases}
$$

Common statistical understanding is enough to say that if the intervals $[a_{(1)}, a_{(2)}]$, $[a_{(2)}, a_{(3)}]$, $[a_{(3)}, a_{(4)}]$, $[a_{(4)}, a_{(5)}]$ *are more or less equal in length, the underlying probability mass function would have to be approximately uniform, and if the intervals are not of equal length, then the underlying probability mass function would be something other than uniform. What we mean is that, in the limiting case,* $\frac{d}{dx}\psi_1(x)$ *would be a constant. The same analysis would approximately apply for the intervals* $[b_{(1)}, b_{(2)}], [b_{(2)}, b_{(3)}], [b_{(3)}, b_{(4)}], [b_{(4)}, b_{(5)}]$.

3. Case Studies

A case study was carried out with data on stock prices obtained from Stocks on BSE/NSE Derivatives based on Mumbai Stock Exchange data. The data for the study were collected for three companies, viz, Bata India Ltd., Biocon Ltd. and Bombay Dyeing & Manufacturing Company Ltd. for a period of 5 days.

3.1. **Derivation of Membership Function: Bata India Ltd.** For the Bata India Ltd., we collected the data regarding the lowest and the highest stock prices (in INR) for 5 consecutive days. The data are

$$[691.05, 715.6], [677.5, 697.25], [678.55, 701.8], [685, 707.7], [693.6, 725.9]$$

The minimum prices in increasing order of magnitude are

$$677.5, 678.55, 685, 691.05, 693.6$$

and similarly the maximum prices in increasing order of magnitude are

$$697.25, 701.8, 707.7, 715.6, 725.9$$

We then superimposed the intervals making them equally fuzzy with constant membership equal to 1/5 in every case. Accordingly,

$$[691.05, 715.6]^{(\frac{1}{5})}(S)[677.5, 697.25]^{(\frac{1}{5})}(S)[678.55, 701.8]^{(\frac{1}{5})}(S)[685, 707.7]^{(\frac{1}{5})}(S)[693.6, 725.9]^{(\frac{1}{5})}$$
$$= [677.5, 678]^{(\frac{1}{5})} \cup [678.55, 685]^{(\frac{2}{5})} \cup [685, 691.05]^{(\frac{3}{5})} \cup [691.05, 693.6]^{(\frac{4}{5})} \cup [693.6, 697.25]^{(\frac{5}{5})} \cup [697.25, 701.8]^{(\frac{4}{5})} \cup [701.8, 707.7]^{(\frac{3}{5})} \cup [707.7, 715.6]^{(\frac{2}{5})} \cup [715.6, 725.9]^{(\frac{1}{5})}$$

The following table depicts the minimum and the maximum prices for Bata India Ltd. with their respective probabilities. The probabilities shown are based on the principle mentioned above.

Table 1. Minimum and Maximum Prices in Ascending Order of Magnitude for Bata India Ltd.

Values of $a_{(i)}$, $i = 1, 2, ..., 5$	Probability
677.5	0.20
678.55	0.40
685	0.60
691.05	0.80
Average of 693.6 and 697.25	1.00

Values of $b_{(i)}$, $i = 1, 2, ..., 5$	Probability
Average of 693.6 and 697.25	1
701.8	0.80
707.7	0.60
715.6	0.40
725.9	0.20

The values given in Table1 were plotted in line diagram which gives us a continuous curve approximated by the points. After plotting the values (Figure 3.1), we have seen that the minimum stock price follows an approximately uniform probability law in the interval $[677.5, 695.43]$ while the maximum price follows another approximately uniform law of probability in the interval $[695.43, 725.9]$. Thus, according to the principle, the distribution function of minimum prices in $[677.5, 695.43]$ and the complementary distribution function of maximum prices in $[695.43, 725.9]$ together define an approximately triangular normal fuzzy number $[677.5, 695.43, 725.9]$, which is clear from the diagram given below:

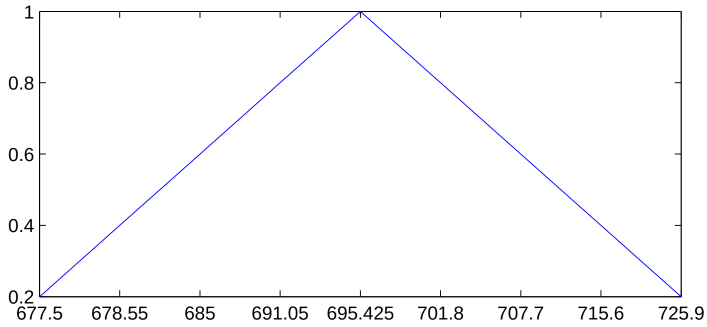

Figure 1. Membership Curve of Stock Prices for Bata India Ltd.

Thus, the fuzzy membership function in this example can be approximated as

$$\mu_X(x) = \begin{cases} \frac{x-677.5}{695.43-677.5}, 677.5 \leq x \leq 695.43 \\ \frac{x-725.9}{695.43-725.9}, 695.43 \leq x \leq 725.9 \\ 0, \qquad\qquad otherwise \end{cases}$$

Here

$$L(x) = \frac{x - 677.5}{695.43 - 677.5}$$

and

$$R(x) = \frac{x - 725.9}{695.43 - 725.9}$$

are the left and right reference functions according to Dubois and Prade. $L(x)$ is a non-decreasing from 0 to 1, and $R(x)$ is non-increasing from 1 to 0 for $677.5 \leq x \leq 695.43$ and $695.43 \leq x \leq 725.9$ respectively. According to our findings,

$$F(x) = L(x) = \frac{x - 677.5}{695.43 - 677.5}$$

is a probability distribution function in $677.5 \leq x \leq 695.43$, and

$$G(x) = 1 - R(x) = 1 - \frac{x - 725.9}{695.43 - 725.9}$$

is another probability distribution function in $695.43 \leq x \leq 725.9$.

The probability density function for $F(x)$ is given by

$$f(x) = \frac{d}{dx}F(x) = \frac{1}{17.93}, 677.5 \leq x \leq 695.43$$

Similarly, the probability density function for $G(x)$ is given by

$$g(x) = \frac{d}{dx}G(x) = \frac{1}{30.47}, 695.43 \leq x \leq 725.9$$

3.2. **Derivation of Membership Function: Biocon Ltd.** The data collected for Biocon Ltd. regarding the lowest and the highest stock prices (in INR) for 5 consecutive days are

$$[282, 290.85], [276.75, 287.95], [277, 286.7], [281, 291.4], [282.55, 294.75]$$

The minimum prices in increasing order of magnitude are

$$276.75, 277, 281, 282, 282.55$$

and similarly the maximum prices in increasing order of magnitude are

$$286.7, 287.95, 290.85, 291.4, 294.75$$

We then superimposed the intervals making them equally fuzzy with constant membership equal to 1/5 in every case. Thus we get,

$$\begin{aligned}
&[282, 290.85]^{(\frac{1}{5})}(S)[276.75, 287.95]^{(\frac{1}{5})}(S)[277, 286.7]^{(\frac{1}{5})}(S)[281, 291.4]^{(\frac{1}{5})}\\
&\quad(S)[282.55, 294.75]^{(\frac{1}{5})}\\
&= [276.75, 277]^{(\frac{1}{5})} \cup [277, 281]^{(\frac{2}{5})} \cup [281, 282]^{(\frac{3}{5})} \cup [282, 282.55]^{(\frac{4}{5})}\cup\\
&\quad[282.55, 286.7]^{(\frac{5}{5})} \cup [286.7, 287.95]^{(\frac{4}{5})} \cup [287.95, 290.85]^{(\frac{3}{5})} \cup [290.85, 291.4]^{(\frac{2}{5})}\\
&\quad\cup[291.4, 294.75]^{(\frac{1}{5})}
\end{aligned}$$

Table2 depicts the minimum and the maximum prices for Biocon Ltd. with their respective probabilities.

Table 2. Minimum and Maximum Prices in Ascending Order of Magnitude for Biocon Ltd.

Values of $a_{(i)}$, $i = 1, 2, ..., 5$	Probability
276.75	0.20
277	0.40
281	0.60
282	0.80
Average of 282.55 and 286.7	1.00

Values of $b_{(i)}$, $i = 1, 2, ..., 5$	Probability
Average of 282.55 and 286.7	1
287.95	0.80
290.85	0.60
291.4	0.40
294.75	0.20

After plotting the values in a line diagram (Figure 3.2) approximated by a continuous curve, we have seen that the minimum stock price follows approximately a uniform probability law in the interval [276.75, 284.63] while the maximum price follows another approximately uniform law of probability in the interval [284.63, 294.75]. Thus, according to the principle, the distribution function of minimum prices in [276.75, 284.63] and the complementary distribution function of maximum prices in

$[284.63, 294.75]$ together define an approximately triangular normal fuzzy number $[276.75, 284.63, 294.75]$, which is clear from the diagram given below:

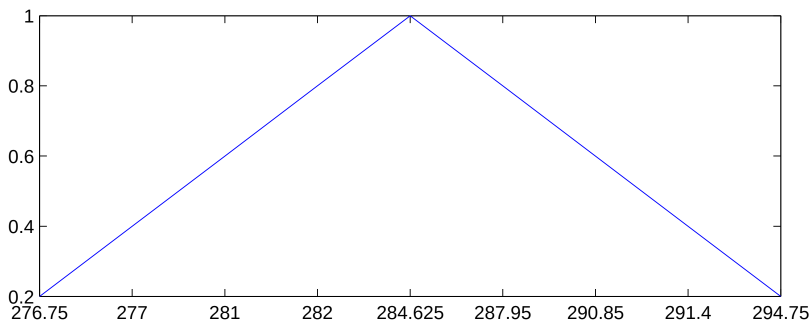

FIGURE 2. Membership Curve of Stock Prices for Biocon Ltd.

Thus, the fuzzy membership function in this example can be approximated as

$$\mu_X(x) = \begin{cases} \frac{x-276.75}{284.63-276.75}, 276.75 \le x \le 284.63 \\ \frac{x-294.75}{284.63-294.75}, 284.63 \le x \le 294.75 \\ 0, \qquad\qquad otherwise \end{cases}$$

Here,

$$L(x) = \frac{x - 276.75}{284.63 - 276.75}$$

and

$$R(x) = \frac{x - 294.75}{284.63 - 294.75}$$

are the left and right reference functions according to Dubois and Prade. $L(x)$ is a non-decreasing from 0 to 1, and $R(x)$ is non-increasing from 1 to 0 for $276.75 \le x \le 284.63$ and $284.63 \le x \le 294.75$ respectively. According to our findings,

$$F(x) = L(x) = \frac{x - 276.75}{284.63 - 276.75}$$

is a probability distribution function in $276.75 \le x \le 284.63$, and

$$G(x) = 1 - R(x) = 1 - \frac{x - 294.75}{284.63 - 294.75}$$

is another probability distribution function in $284.63 \le x \le 294.75$.

The probability density function for $F(x)$ is given by

$$f(x) = \frac{d}{dx}F(x) = \frac{1}{7.88}, 276.75 \le x \le 284.63$$

Similarly, the probability density function for G(x) is given by

$$g(x) = \frac{d}{dx}G(x) = \frac{1}{10.12}, 284.63 \le x \le 294.75$$

3.3. **Derivation of Membership Function: Bombay Dyeing & Manufacturing Company Ltd.** For the Bombay Dyeing & Manufacturing Company Ltd., the stock prices (in INR) regarding the lowest and the highest order collected for 5 consecutive days are

$$[427.1, 446.05], [432, 452], [442.1, 454], [443.05, 453.3], [446.25, 463.7]$$

The minimum prices in increasing order of magnitude are

$$427.1, 432, 442.1, 443.05, 446.25$$

and similarly the maximum prices in increasing order of magnitude are

$$446.05, 452, 453.3, 454, 463.7$$

We then superimposed the intervals making them equally fuzzy with constant membership equal to 1/5 in every case. Accordingly,

$$\begin{aligned}
&[427.1, 446.05]^{(\frac{1}{5})}(S)[432, 452]^{(\frac{1}{5})}(S)[442.1, 454]^{(\frac{1}{5})}(S)[443.05, 453.3]^{(\frac{1}{5})}\\
&(S)[446.25, 463.7]^{(\frac{1}{5})}\\
&= [427.1, 432]^{(\frac{1}{5})} \cup [432, 442.1]^{(\frac{2}{5})} \cup [442.1, 454]^{(\frac{3}{5})} \cup [443.05, 446.25]^{(\frac{4}{5})} \cup\\
&\quad [446.25, 446.05]^{(\frac{5}{5})} \cup [446.05, 452]^{(\frac{4}{5})} \cup [452, 453.3]^{(\frac{3}{5})} \cup [453.3, 454]^{(\frac{2}{5})}\\
&\quad \cup [454, 463.7]^{(\frac{1}{5})}
\end{aligned}$$

The table3 depicts the minimum and the maximum prices for Bombay Dyeing & Manufacturing Company Ltd. with their respective probabilities.

Table 3. Minimum and Maximum Prices in Ascending Order of Magnitude for Bombay Dyeing & Manufacturing Company Ltd.

Values of $a_{(i)}$, $i = 1, 2, ..., 5$	Probability
427.1	0.20
432	0.40
442.1	0.60
443.05	0.80
Average of 446.25 and 446.05	1.00

Values of $b_{(i)}$, $i = 1, 2, ..., 5$	Probability
Average of 446.25 and 446.05	1
452	0.80
453.3	0.60
454	0.40
463.7	0.20

After plotting the values in a line diagram (Figure 3.3) approximated by a continuous curve, we have seen that the minimum stock price follows approximately a uniform probability law in the interval [427.1, 446.15] while the maximum price follows another approximately uniform law of probability in the interval [446.15, 463.7]. Thus, according to the principle, the distribution function of minimum prices in

$[427.1, 446.15]$ and the complementary distribution function of maximum prices in $[446.15, 463.7]$ together define an approximately triangular normal fuzzy number $[427.1, 446.15, 463.7]$, which is clear from the diagram given below:

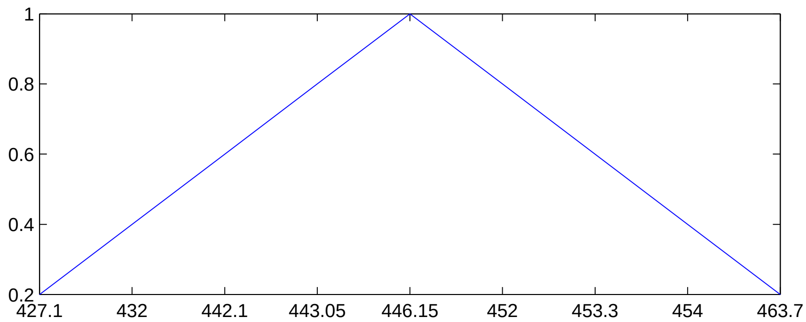

FIGURE 3. Membership Curve of Stock Prices for Bombay Dyeing & Manufacturing Company Ltd.

Thus, the fuzzy membership function in this example can be approximated as

$$\mu_X(x) = \begin{cases} \frac{x-427.1}{446.15-427.1}, 427.1 \leq x \leq 446.15 \\ \frac{x-463.7}{446.15-463.7}, 446.15 \leq x \leq 463.7 \\ 0, \qquad\qquad otherwise \end{cases}$$

Here

$$L(x) = \frac{x - 427.1}{446.15 - 427.1}$$

and

$$R(x) = \frac{x - 463.7}{446.15 - 463.7}$$

are the left and right reference functions according to Dubois and Prade. $L(x)$ is a non-decreasing from 0 to 1, and $R(x)$ is non-increasing from 1 to 0 for $427.1 \leq x \leq 446.15$ and $446.15 \leq x \leq 463.7$ respectively. According to our findings,

$$F(x) = L(x) = \frac{x - 427.1}{446.15 - 427.1}$$

is a probability distribution function in $427.1 \leq x \leq 446.15$, and

$$G(x) = 1 - R(x) = 1 - \frac{x - 463.7}{446.15 - 463.7}$$

is another probability distribution function in $446.15 \leq x \leq 463.7$.

The probability density function for $F(x)$ is given by

$$f(x) = \frac{d}{dx}F(x) = \frac{1}{19.05}, 427.1 \leq x \leq 446.15$$

Similarly, the probability density function for $G(x)$ is given by

$$g(x) = \frac{d}{dx}G(x) = \frac{1}{17.55}, 446.15 \leq x \leq 463.7$$

4. Conclusions

Variations of the lowest and the highest prices of a commodity noted everyday for a small number of days lead to defining approximately the uncertainty in terms of a normal fuzzy number constructed on the assumption that the lowest and the highest prices follow two independent probability laws. We have gone for case studies regarding the lowest and the highest prices of stocks of three Indian companies, and have found that triangular normal fuzzy numbers can be used to define approximately the uncertainties concerned. Accordingly, the possibility that stock price of a company assumes a particular value can be evaluated using the mathematics of partial presence of an element in an interval. The Randomness-Fuzziness Consistency Principle is applicable for large number of sample points, where the condition $max(a_i) \leq min(b_i)$ should be satisfied. Here in our case, considering the aforesaid condition, we used data, collected in a short duration, which will give us the concerned probability mass functions. Certain approximations have been done to obtain the membership curves of the companies, resulting that the uncertainties concerned are approximately triangular fuzzy numbers.

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