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L-fuzzifying soft topological spaces and L-fuzzifying soft interior operators

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Abstract. In this paper, we first introduce definitions of L-fuzzifying soft topological spaces and L-fuzzifying soft interior spaces . Then let LF-**STOP** the category L-fuzzifying soft topological spaces and their continuous mappings, and LF-**SIS** the category L-fuzzifying soft interior spaces and their continuous mappings, we show that LF-**STOP** is isomorphic to LF-**SIS**.

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1. Introduction

The real world is too complex for our immediate and direct understanding. We create "models" of reality that are simplifications of aspects of the real word.In 1999 D.Molodtsov [15] introduced the concept of a soft set and started to develop basic of the theory as a new approach for modeling uncertainties. Research works on soft set theory and its applications in various fields are progressing rapidly ([1],[3]-[10],[12], [13],[16]-[23]). In [17], Shabir and Naz introduced soft topological spaces. The notions of soft open sets, soft closed sets, soft closure, soft interior points, soft interior of a point and soft separation axioms were introduced and their basic properties were investigated. Finally, soft T_i-spaces ($i = 1, 2, 3, 4$) and notions of soft normal and soft regular spaces were discussed in detail. In paper [14], W. K. Min firstly pointed out some errors in Remark 4 and Example 9 of paper [17], and secondly investigated properties of soft separation axioms defined in paper [17]. In paper [24], the authors introduced some new concepts in soft topological spaces such as soft point, interior point, interior, interior, continuity, and compactness. Based on paper [24], in this paper, we first introduce definitions of L-fuzzifying soft topological spaces and L-fuzzifying soft interior spaces. Then let LF-**STOP** the category ([2]) L-fuzzifying soft topological spaces and their continuous mappings, and LF-**SIS** the category L-fuzzifying soft interior spaces and their continuous mappings, we show that LF-**STOP** is isomorphic to LF-**SIS**.

2. Preliminaries

In this paper, let X be a set and L a Hutton algebra. If L is a completely distributive lattice and $x \lhd \bigvee_{t\in T} y_t$, then there must be $t^\star \in T$ such that $x \lhd y_{t^\star}$ (here $x \lhd a$ means: $K \subset L, a \leq \bigvee K \Rightarrow \exists y \in K$ such that $x \leq y$), some more properties about $\lhd$ can be found in [11].

Definition 2.1 ([15]). (1) A soft set on a set X is a pair (M, E, X) (here E is a nonempty a parameter set), and $M : E \longrightarrow 2^X$ (the set of all subset of X) is a mapping. The set of all soft sets on X is denoted by $\mathbf{S(X, E)}$.

(2) For two given subsets $(M, E, X), (N, E, X) \in \mathbf{S(X, E)}$, we say that$(M, E, X)$ is a soft subset of (N, E, X) if $M(e) \subseteq N(e)$ $(\forall e \in E)$,we denote it by $(M, E, X)\widetilde{\subseteq}(N, E, X)$. If $(M, E, X)\widetilde{\subseteq}(N, E, X)$ and $(M, E, X)\widetilde{\supseteq}(N, E, X)$, we say (M, E, X) and (N, E, X) be soft equal. We denote it by $(M, E, X) = (N, E, X)$

Definition 2.2 ([13]). Let (F, A) and (G, B) are two soft sets on X, union of two soft sets (F, A) and (G, B) is the soft set (H, C), where $C = A \cup B$ and

$$H(e) = \begin{cases} F(e), & \text{if } e \in A - B, \\ G(e), & \text{if } e \in B - A, \forall e \in C, \\ F(e) \cup G(e). & \text{if } e \in A \cap B. \end{cases}$$

We write $(F, A)\widetilde{\bigcup}(G, B) = (H, C)$.

Definition 2.3 ([17]). Let (F, A) and (G, B) are two soft sets on X, intersection of two soft sets (F, A) and (G, B) is the soft set (H, C), where $C = A \cap B$ and $H(e) = F(e) \cap G(e)$ $(\forall e \in C)$. We write $(F, A)\widetilde{\bigcap}(G, B) = (H, C)$.

Definition 2.4 ([17]). (1) For each $A \in 2^X$, $(\widetilde{A}, E, X) \in \mathbf{S(X, E)}$ is defined by $\widetilde{A}(e) = A$ for each $e \in E$; we identify $\widetilde{\{x\}}$ with $\widetilde{x}$ for each $x \in X$. For each $(M, E, X) \in \mathbf{S(X, E)}$, $(M', E, X) \in \mathbf{S(X, E)}$ is defined by $M'(e) = X - M(e)$ $(\forall e \in E)$; sometimes we use $(M, E, X)'$ (resp.$\widetilde{A}$)to replace (M', E, X) (resp.$(\widetilde{A}, E, X)$).

(2) For a given subset $\{(H_j, E, X)\}_{j\in J} \subseteq \mathbf{S(X, E)}$, we call members $(M, E, X) = \widetilde{\bigcup}_{j\in J}(H_j, E, X)$ and $(N, E, X) = \widetilde{\bigcap}_{j\in J}(H_j, E, X)$ of $\mathbf{S(X, E)}$ union and intersection of the family $\{(H_j, E, X)\}_{j\in J}$ respectively, which are defined by

$$M(e) = \bigcup_{j\in J} H_j(e) \ (\forall e \in E) \quad \text{and} \quad N(e) = \bigcap_{j\in J} H_j(e) \ (\forall e \in E).$$

(3) For a given subset $(H, E, X) \in \mathbf{S(X, E)}$, and $x \in X$,we say that $x \in (H, E, X)$ whenever $x \in H(e)$ for each $e \in E$. If $x \notin H(e)$ for some $e \in E$, we say $x \notin (H, E, X)$.

(4) For two given subsets $(M, E, X), (N, E, X) \in \mathbf{S(X, E)}$,then

(i) $((M, E, X)\widetilde{\cup}(N, E, X))' = (M, E, X)'\widetilde{\cap}(N, E, X)'$,

(ii) $((M, E, X)\widetilde{\cap}(N, E, X))' = (M, E, X)'\widetilde{\cup}(N, E, X)'$.

Definition 2.5 ([19]). Defined soft function $(f,g):\mathbf{S(X,E)}\longrightarrow\mathbf{S(Y,F)}$ by

$$(f,g)(M,E,X)=(g^{\rightarrow}(M),f(E),Y)$$

for each $(M,E,X)\in\mathbf{S(X,E)}$ and

$$(f,g)^{-1}(N,F,Y)=(g^{\leftarrow}\circ N\circ f,f^{-1}(F),X)$$

for each $(N,F,Y)\in\mathbf{S(Y,F)}$, where

$$g^{\rightarrow}(M)(\alpha)=\bigcup_{f(e)=\alpha}g(M(e))\ (\forall\alpha\in f(E)),$$

$$(g^{\leftarrow}\circ N\circ f)(e)=g^{\leftarrow}(N(f(e)))(\forall e\in f^{-1}(F))$$

$f(E)$ is the image of E in the category **SET**, $f^{-1}(F)$ is the preimage of F in the category **SET**. $g^{\rightarrow}(M)$ is defined by the Zadeh extension principle, $g^{\leftarrow}(M)$ is the backward operator induced by the mapping $g:X\longrightarrow Y$.

Definition 2.6 ([17]). Let X be a set, and $\mathscr{T}\subseteq\mathbf{S(X,E)}$. $(X,\mathscr{T},E)$ is called a soft topological space over X if $\mathscr{T}$ is closed under the operations of arbitrary unions and nonempty finite intersections (it thus contains $(\widetilde{\varnothing},E,X)$); $\mathscr{T}$ is called a soft topology on X , members of $\mathscr{T}$ are called soft open sets, (M',E,X) is called a soft closed set for each $(M,E,X)\in\mathscr{T}$.

Definition 2.7 ([24]). (1) The soft set $(M,E,X)\in\mathbf{S(X,E)}$ is called a soft point in $\widetilde{X}$, denoted by e_M, if for the element $e\in E$, $M(e)\neq\varnothing$ and $M(e')=\varnothing$ for all $e'\in E-\{e\}$.

(2) The soft point e_M is said to be in the soft set (N,E,X),for each $e\in E$,we have $M(e)\subseteq N(e)$.

Proposition 2.8 ([24]). *Let $e_M\in\widetilde{X}$ and $(N,E,X)\widetilde{\subseteq}\widetilde{X}$. If $e_M\in(N,E,X)$,then $e_M\notin(N,E,X)'$.*

Remark 2.9 ([24]). The converse of the above proposition is not true in general.

More knowledge about the soft point can be founded in paper [24]. $\mathbf{SP(X)}$ denoted the set of all soft points in $\widetilde{X}$. Obviously, if $e_M\in\mathbf{SP(X)}$, then $(id_E,g)(e_M)\in\mathbf{SP(Y)}$.

Remark 2.10. For every set $A\subseteq X$, define its indicator function χ_A as follows:

$$\chi_A(x)=\begin{cases}1, & \text{if } x\in A,\\ 0, & \text{if } x\notin A.\end{cases}$$

This correspondence between a set and its indicator function is obviously one-to-one correspondence. We have known that Zadeh's fuzzy set may be considered as a special case of the soft set (see [15] or Example 2.10 of [24]), so soft sets are generalizations of ordinary sets. In fact, let $A,B\in 2^X$, we have $\widetilde{A},\widetilde{B}\in\mathbf{S(X,E)}$, and we easily know $\widetilde{A}\widetilde{\cap}\widetilde{B}=\widetilde{(A\cap B)}$, $\widetilde{A}\widetilde{\cup}\widetilde{B}=\widetilde{(A\cup B)}$ and $(id_E,g)(\widetilde{A})=\widetilde{g(A)}$. So soft union (resp.intersection) are generalizations of union (resp. intersection) of ordinary sets and soft functions between soft sets are generalizations of functions between ordinary sets. More reasonable explanation about these reasons, please see [13, 17, 19].

3. Basic concepts

Definition 3.1. An L-fuzzifying soft topology on a set X is a mapping $\mathcal{T} : \mathbf{S(X,E)} \rightarrow L$ such that

(LFST1) $\mathcal{T}(\widetilde{\varnothing}) = \mathcal{T}(\widetilde{X}) = 1$.

(LFST2) $\forall (F,E,X), (G,E,X) \in \mathbf{S(X,E)}$,

$$\mathcal{T}\left((F,E,X)\widetilde{\bigcap}(G,E,X)\right) \geq \mathcal{T}(F,E,X) \bigwedge \mathcal{T}(G,E,X).$$

(LFST3) $\forall \{(F_j,E,X)\}_{j\in J} \subseteq \mathbf{S(X,E)}$,

$$\mathcal{T}\left(\widetilde{\bigcup}_{j\in J}(F_j,E,X)\right) \geq \bigwedge_{j\in J} \mathcal{T}(F_j,E,X).$$

$\mathcal{T}(F,E,X)$ can be interpreted as the degree to which (F,E,X) is an open soft set, $\mathcal{T}^\star(F,E,X) = \mathcal{T}(F^{'},E,X)$ will be called the degree of closedness, the triple $(X,\mathcal{T},E)$ is called L-fuzzifying soft topological space. A mapping $g : X \longrightarrow Y$ from an L-fuzzifying soft topological space $(X,\mathcal{T}_1,E)$ to another L-fuzzifying soft topological space $(Y,\mathcal{T}_2,E)$ is said to be continuous if

$$\mathcal{T}_1((id_E,g)^{-1}(F,E,Y)) \geq \mathcal{T}_2((F,E,Y)) \ (\forall (F,E,Y) \in \mathbf{S(Y,E)}).$$

The category of all L-fuzzifying soft topological spaces and their continuous mappings is denoted by LF-**STOP**.

If $(X,\mathscr{T},E)$ is a soft topological space over X, define $\chi_{\mathscr{T}} : \mathbf{S(X,E)} \longrightarrow L$ as follows: $\chi_{\mathscr{T}}(F,E,X) = 1$, if $(F,E,X) \in \mathscr{T}$; if not, $\chi_{\mathscr{T}}(F,E,X) = 0$. Obviously, $\chi_{\mathscr{T}}$ is a special L-fuzzifying soft topology.

Definition 3.2. An L-fuzzifying soft interior operator on a set X is a mapping $\mathcal{I} : \mathbf{S(X,E)} \longrightarrow L^{\mathbf{SP(X)}}$, satisfying the following conditions:

(LFSI1)

$\mathcal{I}(\widetilde{X})(e_M) = 1 \ (\forall\ e_M \in \mathbf{SP(X)})$ and $\mathcal{I}(F,E,X)(e_M) = 0 \ (\forall\ e_M \notin (F,E,X))$,

(LFSI2) $\forall\ e_M \in \mathbf{SP(X)}, (F,E,X) \in \mathbf{S(X,E)}$

$$\mathcal{I}(F,E,X)(e_M) = \bigwedge_{e_N \widetilde{\subseteq} e_M} \mathcal{I}(F,E,X)(e_N),$$

(LFSI3) $\forall\ (F,E,X), (G,E,X) \in \mathbf{S(X,E)}$,

$$\mathcal{I}\left((F,E,X)\widetilde{\bigcap}(G,E,X)\right) = \mathcal{I}(F,E,X) \bigwedge \mathcal{I}(G,E,X),$$

(LFSI4) $\forall (F,E,X) \in \mathbf{S(X,E)}$, $\forall a \in L - \{1\}$.

$$[\mathcal{I}(F,E,X)]^{(a)} \widetilde{\subseteq} \left[\mathcal{I}\left(\widetilde{\bigcup}[\mathcal{I}(F,E,X)]^{(a)}\right)\right]^{(a)},$$

where $[\mathcal{I}(F,E,X)]^{(a)} = \{e_M \in \mathbf{SP(X)} \mid (\mathcal{I}(F,E,X))(e_M) \not\leq a\}$, $\mathcal{I}(F,E,X)(e_M)$ be called the degree to which e_M belongs to the interior of (F,E,X), the triple $(X,\mathcal{I},E)$ is called L-fuzzifying soft interior spaces. A mapping $g : X \longrightarrow Y$ from an

L-fuzzifying soft interior space $(X,\mathcal{I}_1,E)$ to another L-fuzzifying soft interior space $(Y,\mathcal{I}_2,E)$ is said to be continuous if

$$\mathcal{I}_2(G,E,Y)\left((id_E,g)(e_M)\right)\leq \mathcal{I}_1((id_E,g)^{-1}(G,E,Y))(e_M)$$

for each $(G,E,Y)\in \mathbf{S(Y,E)}$ and $e_M\in\mathbf{SP(X)}$. The category of all L-fuzzifying soft interior spaces and their continuous mappings is denoted by LF-**SIS**.

The main result is as follows:

Theorem 3.3. *LF-**STOP** is isomorphic to LF-**SIS**.*

4. Proof of the main results

Proof of theorem 3.3 Step 1 Define $\mathcal{I}_{\mathcal{T}}:\mathbf{S(X,E)}\longrightarrow L^{\mathbf{SP(X)}}$ as follows:

$$\mathcal{I}_{\mathcal{T}}(F,E,X)(e_M)=\bigvee_{e_M\in(G,E,X)\widetilde{\subseteq}(F,E,X)}\mathcal{T}(G,E,X).$$

Then $\mathcal{I}_{\mathcal{T}}$ is an L-fuzzifying soft interior operator on X. In fact,

(LFSI1)-(LFSI2) are true trivially.

(LFSI3) $\forall\ (F,E,X),(G,E,X)\in\mathbf{S(X,E)},\forall\ e_M\in\mathbf{SP(X)}$, by the definition of $\mathcal{I}_{\mathcal{T}}$, we easily show that

$$\mathcal{I}_{\mathcal{T}}\left((F,E,X)\widetilde{\bigcap}(G,E,X)\right)\leq\mathcal{I}_{\mathcal{T}}(F,E,X)\bigwedge\mathcal{I}_{\mathcal{T}}(G,E,X).$$

In order to prove

$$\mathcal{I}_{\mathcal{T}}\left((F,E,X)\widetilde{\bigcap}(G,E,X)\right)\geq\mathcal{I}_{\mathcal{T}}(F,E,X)\bigwedge\mathcal{I}_{\mathcal{T}}(G,E,X),$$

let $a\lhd(\mathcal{I}_{\mathcal{T}}(F,E,X)\bigwedge\mathcal{I}_{\mathcal{T}}(G,E,X))(e_M)=\mathcal{I}_{\mathcal{T}}(F,E,X)(e_M)\bigwedge\mathcal{I}_{\mathcal{T}}(G,E,X)(e_M)$, we have

$$a\lhd\mathcal{I}_{\mathcal{T}}(F,E,X)(e_M)=\bigvee_{e_M\in(A,E,X)\widetilde{\subseteq}(F,E,X)}\mathcal{T}(A,E,X)$$

and

$$a\lhd\mathcal{I}_{\mathcal{T}}(G,E,X)(e_M)=\bigvee_{e_M\in(B,E,X)\widetilde{\subseteq}(G,E,X)}\mathcal{T}(B,E,X).$$

Hence there are (A,E,X) and (B,E,X) such that

$$e_M\in(A,E,X)\widetilde{\subseteq}(F,E,X),e_M\in(B,E,X)\widetilde{\subseteq}(G,E,X)$$

and

$$a\leq\mathcal{T}(A,E,X),a\leq\mathcal{T}(B,E,X).$$

Thus

$$e_M\in(A,E,X)\widetilde{\bigcap}(B,E,X)\widetilde{\subseteq}(F,E,X)\widetilde{\bigcap}(G,E,X),$$

and

$$a\leq\mathcal{T}(A,E,X)\bigwedge\mathcal{T}(B,E,X)\leq\mathcal{T}\left((A,E,X)\widetilde{\bigcap}(B,E,X)\right).$$

Hence

$$a \leq \mathcal{T}\left((A,E,X)\widetilde{\bigcap}(B,E,X)\right) \leq \bigvee_{e_M \in (C,E,X)\widetilde{\subseteq}(F,E,X)\widetilde{\bigcap}(G,E,X)} \mathcal{T}(C,E,X)$$

$$(\text{By the definition of } \mathcal{I}_{\mathcal{T}}) = \mathcal{I}_{\mathcal{T}}\left((F,E,X)\widetilde{\bigcap}(G,E,X)\right)(e_M).$$

From the above prove, we have

$$\mathcal{I}_{\mathcal{T}}\left((F,E,X)\widetilde{\bigcap}(G,E,X)\right)(e_M) \geq (\mathcal{I}_{\mathcal{T}}(F,E,X) \bigwedge \mathcal{I}_{\mathcal{T}}(G,E,X))(e_M).$$

Note that the arbitrariness of soft point e_M, we easily know

$$\mathcal{I}_{\mathcal{T}}\left((F,E,X)\widetilde{\bigcap}(G,E,X)\right) \geq \mathcal{I}_{\mathcal{T}}(F,E,X) \bigwedge \mathcal{I}_{\mathcal{T}}(G,E,X).$$

(LFSI4) We first show that

$$\bigwedge_{e_N \in (G,E,X)} \mathcal{I}_{\mathcal{T}}(G,E,X)(e_N) = \mathcal{T}(G,E,X). \qquad (1)$$

By

$$\mathcal{I}_{\mathcal{T}}(G,E,X)(e_N) = \bigvee_{e_N \in (A,E,X)\widetilde{\subseteq}(G,E,X)} \mathcal{T}(A,E,X),$$

we easily obtain

$$\bigwedge_{e_N \in (G,E,X)} \mathcal{I}_{\mathcal{T}}(G,E,X)(e_N) \geq \mathcal{T}(G,E,X).$$

On the other hand, let $a \lhd \bigwedge_{e_N \in (G,E,X)} \mathcal{I}_{\mathcal{T}}(G,E,X)(e_N)$, we have $a \lhd \mathcal{I}_{\mathcal{T}}(G,E,X)(e_N)$ for each $e_N \in (G,E,X)$. Further, there exists $(G,E,X)_{e_N} \in \mathbf{S}(\mathbf{X},\mathbf{E})$ such that $e_N \in (G,E,X)_{e_N} \widetilde{\subseteq} (G,E,X)$ and $a \lhd \mathcal{T}(G,E,X)_{e_N}$. Obviously,

$$(G,E,X) = \widetilde{\bigcup}_{e_N \in (G,E,X)} (G,E,X)_{e_N}.$$

Therefore

$$\mathcal{T}(G,E,X) = \mathcal{T}(\widetilde{\bigcup}_{e_N \in (G,E,X)} (G,E,X)_{e_N}) \geq \bigwedge_{e_N \in (G,E,X)} \mathcal{T}(G,E,X)_{e_N} \geq a.$$

Now, if $e_M \in (\mathcal{I}_{\mathcal{T}}(F,E,X))^{(a)}$, then

$$\mathcal{I}_{\mathcal{T}}(F,E,X)(e_M) = \bigvee_{e_M \in (G,E,X)\widetilde{\subseteq}(F,E,X)} \mathcal{T}(G,E,X) \not\leq a,$$

there exists $(G,E,X) \in \mathbf{S}(\mathbf{X},\mathbf{E})$ such that $e_M \in (G,E,X)\widetilde{\subseteq}(F,E,X)$ and $\mathcal{T}(G, E,X) \not\leq a$. By (1), for each $e_N \in (G,E,X)$, we have $\mathcal{I}_{\mathcal{T}}(G,E,X)(e_N) \not\leq a$, i.e. $e_N \in [\mathcal{I}_{\mathcal{T}}(G,E,X)]^{(a)}$, by (LFSI3), we have

$$e_M \in (G,E,X) = \widetilde{\bigcup}\{e_N \in \mathbf{SP}(\mathbf{X}) \mid e_N \in (G,E,X)\} \widetilde{\subseteq} \widetilde{\bigcup}[\mathcal{I}_{\mathcal{T}}(G,E,X)]^{(a)}$$

$$\widetilde{\subseteq} \widetilde{\bigcup}[\mathcal{I}_{\mathcal{T}}(F,E,X)]^{(a)}.$$

This implies

$$\mathcal{I}_{\mathcal{T}}\left(\widetilde{\bigcup}[\mathcal{I}_{\mathcal{T}}(F,E,X)]^{(a)}\right)(e_M) = \bigvee_{e_M\in(G,E,X)\widetilde{\subseteq}\widetilde{\bigcup}[\mathcal{I}_{\mathcal{T}}(F,E,X)]^{(a)}} \mathcal{T}(G,E,X) \not\leq a.$$

Therefore,

$$e_M \in \left[\mathcal{I}_{\mathcal{T}}\left(\widetilde{\bigcup}[\mathcal{I}_{\mathcal{T}}(F,E,X)]^{(a)}\right)\right]^{(a)}.$$

Step 2 Define $\mathcal{T}_{\mathcal{I}} : \mathbf{S}(\mathbf{X},\mathbf{E}) \longrightarrow L$ as follows:

$$\mathcal{T}_{\mathcal{I}}(F,E,X) = \bigwedge_{e_M\in(F,E,X)} \mathcal{I}(F,E,X)(e_M).$$

Then $\mathcal{T}_{\mathcal{I}}$ is an L-fuzzifying soft topology on X and $\mathcal{I}_{\mathcal{T}_{\mathcal{I}}} = \mathcal{I}$. In fact, (LFST1) is trivial from (LFSI1).

(LFSI2) $\forall (F,E,X),(G,E,X) \in \mathbf{S}(\mathbf{X},\mathbf{E})$,

$$\begin{aligned}\mathcal{T}_{\mathcal{I}}\left((F,E,X)\widetilde{\bigcap}(G,E,X)\right) &= \bigwedge_{e_M\in(F,E,X)\widetilde{\bigcap}(G,E,X)} \mathcal{I}((F,E,X)\widetilde{\bigcap}(G,E,X))(e_M)\\ &= \bigwedge_{e_M\in(F,E,X)\widetilde{\bigcap}(G,E,X)} \mathcal{I}((F,E,X))(e_M)\widetilde{\bigcap}\,\mathcal{I}(G,E,X))(e_M)\\ &\geq \left(\bigwedge_{e_M\in(F,E,X)} \mathcal{I}((F,E,X))(e_M)\right)\bigwedge\left(\bigwedge_{e_M\in(F,E,X)} \mathcal{I}((G,E,X))(e_M)\right)\\ &= \mathcal{T}_{\mathcal{I}}(F,E,X)\bigwedge\mathcal{T}_{\mathcal{I}}(G,E,X).\end{aligned}$$

(LFSI3) $\forall \{(F_j,E,X)\}_{j\in J} \subseteq \mathbf{S}(\mathbf{X},\mathbf{E})$,

$$\begin{aligned}\mathcal{T}_{\mathcal{I}}\left(\widetilde{\bigcup}_{j\in J}(F_j,E,X)\right) &= \bigwedge_{e_M\in\widetilde{\bigcup}_{j\in J}(F_j,E,X)} \mathcal{I}(\widetilde{\bigcup}_{j\in J}(F_j,E,X))(e_M)\\ &\geq \bigwedge_{j\in J}\bigwedge_{e_M\in(F_j,E,X)} \mathcal{I}(F_j,E,X)(e_M) = \bigwedge_{j\in J}\mathcal{T}_{\mathcal{I}}(F_j,E,X).\end{aligned}$$

For each $(F,E,X) \in \mathbf{S}(\mathbf{X},\mathbf{E})$ and $e_M \in \mathbf{SP}(\mathbf{X})$, we have

$$\begin{aligned}\mathcal{I}_{\mathcal{T}_{\mathcal{I}}}(F,E,X)(e_M) &= \bigvee_{e_M\in(G,E,X)\widetilde{\subseteq}(F,E,X)} \mathcal{T}_{\mathcal{I}}(G,E,X)\\ &= \bigvee_{e_M\in(G,E,X)\widetilde{\subseteq}(F,E,X)}\bigwedge_{e_N\in(G,E,X)} \mathcal{I}(G,E,X)(e_N).\end{aligned}$$

In order to show that $\mathcal{I}_{\mathcal{T}_{\mathcal{I}}} = \mathcal{I}$, we need to show that

$$\bigvee_{e_M\in(G,E,X)\widetilde{\subseteq}(F,E,X)}\bigwedge_{e_N\in(G,E,X)} \mathcal{I}(G,E,X)(e_N) = \mathcal{I}(F,E,X)(e_M).$$

In fact, on the one hand, suppose that $b \in L$ and $\mathcal{I}(F,E,X)(e_M) \not\leq b$, then there exists $a \in \alpha(b)$ (here $\alpha(b)$ is the largest maximal set of b (see [11])) such that $\mathcal{I}(F,E,X)(e_M) \not\leq a$. By (LFSI4), we have

$$e_M \in [\mathcal{I}(F,E,X)]^{(a)} \widetilde{\subseteq} \left[\mathcal{I}\left(\widetilde{\bigcup}[\mathcal{I}(F,E,X)]^{(a)}\right)\right]^{(a)}.$$

If $(G_0,E,X) = \widetilde{\bigcup}[\mathcal{I}(F,E,X)]^{(a)}$, then $e_M \in (G_0,E,X)\widetilde{\subseteq}(F,E,X)$. Note that $[\mathcal{I}(F,E,X)]^{(a)} = \{e_M \in \mathbf{SP}(\mathbf{X}) \mid (\mathcal{I}(F,E,X))(e_M) \not\leq a\}$, so there exists $e_P \in [\mathcal{I}(F,E,X)]^{(a)}$ such that $e_P \widetilde{\supseteq} e_N$ for each $e_N \in (G_0,E,X)$, by the definition of (G_0,E,X) and the results above proved, we have

$$e_P \in [\mathcal{I}(F,E,X)]^{(a)} \widetilde{\subseteq} \left[\mathcal{I}\left(\widetilde{\bigcup}[\mathcal{I}(F,E,X)]^{(a)}\right)\right]^{(a)} = [\mathcal{I}(G_0,E,X)]^{(a)}.$$

This means that $\mathcal{I}(G_0,E,X)(e_P) \not\leq a$ for each $e_N \widetilde{\subseteq} e_P$, by (LFSI2), we have

$$\mathcal{I}(G_0,E,X)(e_P) \leq \mathcal{I}(G_0,E,X)(e_N).$$

So $\mathcal{I}(G_0,E,X)(e_N) \not\leq a$. By the definition of $\alpha(b)$, we have

$$\bigwedge_{e_N \in (G_0,E,X)} \mathcal{I}(G_0,E,X)(e_N) \not\leq b.$$

By

$$\bigwedge_{e_N \in (G_0,E,X)} \mathcal{I}(G_0,E,X)(e_N) \leq \bigvee_{e_M \in (G,E,X) \widetilde{\subseteq} (F,E,X)} \bigwedge_{e_N \in (G,E,X)} \mathcal{I}(G,E,X)(e_N).$$

This implies

$$\bigvee_{e_M \in (G,E,X) \widetilde{\subseteq} (F,E,X)} \bigwedge_{e_N \in (G,E,X)} \mathcal{I}(G,E,X)(e_N) \not\leq b.$$

Therefore,

$$\bigvee_{e_M \in (G,E,X) \widetilde{\subseteq} (F,E,X)} \bigwedge_{e_N \in (G,E,X)} \mathcal{I}(G,E,X)(e_N) \geq \mathcal{I}(F,E,X)(e_M).$$

On the other hand, suppose that $b \in L$ and

$$\bigvee_{e_M \in (G,E,X) \widetilde{\subseteq} (F,E,X)} \bigwedge_{e_N \in (G,E,X)} \mathcal{I}(G,E,X)(e_N) \not\leq b,$$

there exists $a \in \alpha(b)$ such that

$$\bigvee_{e_M \in (G,E,X) \widetilde{\subseteq} (F,E,X)} \bigwedge_{e_N \in (G,E,X)} \mathcal{I}(G,E,X)(e_N) \not\leq a.$$

Furthermore, there exists $(G,E,X) \in \mathbf{S}(\mathbf{X},\mathbf{E})$ such that $e_M \in (G,E,X)\widetilde{\subseteq}(F,E,X)$ and

$$\bigwedge_{e_N \in (G,E,X)} \mathcal{I}(G,E,X)(e_N) \not\leq a.$$

In particular, $\mathcal{I}(G,E,X)(e_P) \not\leq a$ $(\forall e_P \widetilde{\subseteq} e_M)$, by the definition of $\alpha(b)$, we have

$$\bigwedge_{e_P \widetilde{\subseteq} e_M} \mathcal{I}(G,E,X)(e_P) \not\leq b.$$

Furthermore, by the definition of (LFSI2), we have

$$\mathcal{I}(F,E,X)(e_M) = \bigwedge_{e_P \widetilde{\subseteq} e_M} \mathcal{I}(F,E,X)(e_P) \geq \bigwedge_{e_P \widetilde{\subseteq} e_M} \mathcal{I}(G,E,X)(e_P) \not\leq b.$$

This implies

$$\bigvee_{e_M \in (G,E,X) \widetilde{\subseteq} (F,E,X)} \bigwedge_{e_N \in (G,E,X)} \mathcal{I}(G,E,X)(e_N) \leq \mathcal{I}(F,E,X)(e_M).$$

Therefore, $\mathcal{I}_{\mathcal{T}_{\mathcal{I}}} = \mathcal{I}$.

Step 3 For each $(F,E,X) \in \mathbf{S}(\mathbf{X},\mathbf{E})$, by the definition of $\mathcal{T}_{\mathcal{I}}$ and (1) , we have

$$\mathcal{T}_{\mathcal{I}_{\mathcal{T}}}(F,E,X) = \bigwedge_{e_M \in (F,E,X)} \mathcal{I}_{\mathcal{T}}(F,E,X)(e_M) = \mathcal{T}(F,E,X).$$

Hence $\mathcal{T}_{\mathcal{I}_{\mathcal{T}}} = \mathcal{T}$.

Step 4 If $f : (X,\mathcal{T}_1,E) \longrightarrow (Y,\mathcal{T}_2,E)$ is continuous with respect to L-fuzzifying soft topologies $\mathcal{T}_1$ and $\mathcal{T}_2$, then

$$\mathcal{T}_1((id_E,g)^{-1}(B,E,Y)) \geq \mathcal{T}_2((B,E,Y)) \ (\forall (B,E,Y) \in \mathbf{S}(\mathbf{Y},\mathbf{E})).$$

Hence

$$\begin{aligned} \mathcal{I}_{\mathcal{T}_2}(A,E,Y)\,((id_E,g)(e_M)) &= \bigvee_{(id_E,g)(e_M) \in (B,E,Y) \widetilde{\subseteq} (A,E,Y)} \mathcal{T}_2(B,E,Y) \\ &\leq \bigvee_{e_M \in (id_E,g)^{-1}(B,E,Y) \widetilde{\subseteq} (id_E,g)^{-1}(A,E,Y)} \mathcal{T}_1((id_E,g)^{-1}(B,E,Y)) \\ &\leq \mathcal{I}_{\mathcal{T}_1}((id_E,g)^{-1}(A,E,Y))(e_M). \end{aligned}$$

Therefore $f : (X,\mathcal{I}_{\mathcal{T}_1},E) \longrightarrow (Y,\mathcal{I}_{\mathcal{T}_2},E)$ is continuous with respect to L-fuzzifying soft interior operators $\mathcal{I}_{\mathcal{T}_1}$ and $\mathcal{I}_{\mathcal{T}_2}$.

Step 5 If $f : (X,\mathcal{I}_1,E) \longrightarrow (Y,\mathcal{I}_2,E)$ is continuous with respect to L-fuzzifying soft interior operators $\mathcal{I}_1$ and $\mathcal{I}_1$, then

$$\begin{aligned} &\mathcal{I}_2(A,E,Y)\,((id_E,g)(e_M)) \\ &\quad \leq \mathcal{I}_1((id_E,g)^{-1}(A,E,Y))(e_M) \ (\forall (G,E,Y) \in \mathbf{S}(\mathbf{Y},\mathbf{E}), e_M \in \mathbf{SP}(\mathbf{X})). \end{aligned}$$

Hence

$$\begin{aligned} \mathcal{T}_{\mathcal{I}_2}(A,E,Y) &= \bigwedge_{e_N \in (A,E,Y)} \mathcal{I}_2(A,E,Y)(e_N) \\ &\leq \bigwedge_{(id_E,g)(e_M) \in (A,E,Y)} \mathcal{I}_2(A,E,Y)((id_E,g)(e_M)) \\ &= \bigwedge_{e_M \in (id_E,g)^{-1}(A,E,Y)} \mathcal{I}_2(A,E,Y)((id_E,g)(e_M)) \\ &\leq \bigwedge_{e_M \in (id_E,g)^{-1}(A,E,Y)} \mathcal{I}_1((id_E,g)^{-1}(A,E,Y))(e_M) \end{aligned}$$

$$= \mathcal{T}_{\mathcal{I}_1}((id_E, g)^{-1}(A, E, Y)).$$

Therefore $f : (X, \mathcal{T}_{\mathcal{I}_1}, E) \longrightarrow (Y, \mathcal{T}_{\mathcal{I}_2}, E)$ is continuous with respect to L-fuzzifying soft topologies $\mathcal{T}_{\mathcal{I}_1}$ and $\mathcal{T}_{\mathcal{I}_1}$.

From Step 1 to Step 5, we can obtain LF-**STOP** is isomorphic to LF-**SIS**.

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