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Some properties on the cut sets of intuitionistic fuzzy sets

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Abstract. In this paper, the notions of convex (concave) intuitionistic fuzzy sets and the binary operation between two intuitionistic fuzzy sets are introduced. Their properties and characterizations are presented by using the cut sets (upper cut sets and lower cut sets) of intuitionistic fuzzy sets.

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1. Introducoion

The concept of fuzzy sets was introduced by Zadeh [13] in 1965. And in 1983, Atanassov [1] gave the notion of intuitionistic fuzzy sets (IFSs) which is characterized by a membership function and a non-membership function. Since then, the theories and applications of IFSs are developed rapidly [4, 5, 7, 9, 11, 12, 14]. As is well known, the cut set of fuzzy sets are the bridge between the fuzzy sets and the crisp sets. Li [6] and Zou [15] gave the concepts of upper cut sets and lower cut sets of IFSs respectively, and they also discussed the decomposition theorem, representation theorem of IFSs by using the cut sets. Because the convexity plays an important role in operational research and applied mathematics, the convexity of fuzzy sets received much attention. Syau [8], Chen [2] and Wu[10] described the notion of convex fuzzy sets and investigated the properties. Then it was generalized to the L-fuzzy convex sets where L is a completely distributive lattice by Huang [3]. In this paper, we generalized the notion of convex fuzzy sets to convex (concave) IFSs. We presented some of their equivalent characterizations in terms of the cut sets of IFSs. Moreover, the binary operation between two IFSs was defined, and its properties are also discussed.

2. Preliminaries

Definition 2.1 ([1])**.** Let X be a set and $\mu_A : X \to [0,1]$, $\nu_A : X \to [0,1]$ be two mappings. If $\mu_A(x)+\nu_A(x) \leq 1, \forall x \in X$, then we call $A = \{(x, \mu_A(x), \nu_A(x))|x \in X\}$ an intuitionistic fuzzy sets over X, and denote $A^c = \{(x, \nu_A(x), \mu_A(x))|x \in X\}$, and $A(x) = (\mu_A(x), \nu_A(x))$.

Definition 2.2 ([6])**.** Let $A = \{(x, \mu_A(x), \nu_A(x))|x \in X\}$ be an intuitionistic fuzzy set and $\alpha, \beta \in [0,1]$, $\alpha + \beta \leq 1$. Then we call

$$A_{[\alpha,\beta]} = \{x|\mu_A(x) \geq \alpha, \nu_A(x) \leq \beta\}, \quad A_{(\alpha,\beta)} = \{x|\mu_A(x) > \alpha, \nu_A(x) < \beta\},$$
$$A_{(\alpha,\beta]} = \{x|\mu_A(x) > \alpha, \nu_A(x) \leq \beta\}, \quad A_{[\alpha,\beta)} = \{x|\mu_A(x) \geq \alpha, \nu_A(x) < \beta\}$$

the $[\alpha, \beta]-$ upper cut set, the $(\alpha, \beta)-$ upper cut set, the $(\alpha, \beta]-$ upper cut set, the $[\alpha, \beta)-$ upper cut set of A, respectively.

Definition 2.3 ([15])**.** Let $A = \{(x, \mu_A(x), \nu_A(x))|x \in X\}$ be an intuitionistic fuzzy set and $\alpha, \beta \in [0,1]$, $\alpha + \beta \leq 1$. Then we call

$$A^{[\alpha,\beta]} = \{x|\mu_A(x) \leq \alpha, \nu_A(x) \geq \beta\}, \quad A^{(\alpha,\beta)} = \{x|\mu_A(x) < \alpha, \nu_A(x) > \beta\},$$
$$A^{(\alpha,\beta]} = \{x|\mu_A(x) < \alpha, \nu_A(x) \geq \beta\}, \quad A^{[\alpha,\beta)} = \{x|\mu_A(x) \leq \alpha, \nu_A(x) > \beta\}$$

the $[\alpha, \beta]-$ lower cut set, the $(\alpha, \beta)-$ lower cut set, the $(\alpha, \beta]-$ lower cut set, the $[\alpha, \beta)-$ lower cut set of A, respectively.

3. The properties on the cut sets of IFSs

In this paper, let $L = \{(\alpha, \beta)|\alpha, \beta \in [0,1], \alpha + \beta \leq 1\}$. We define in L that: $(\alpha_1, \beta_1) \leq (\alpha_2, \beta_2) \Leftrightarrow \alpha_1 \leq \alpha_2$, $\beta_1 \geq \beta_2$.

Definition 3.1. Let A be a crisp set, $\forall(\alpha, \beta) \in L$, define the operators "$\circ$" and "$\bullet$" as follows:

$$(\alpha,\beta) \circ A = \begin{cases} (\alpha,\beta), & x \in A, \\ (0,1), & x \notin A. \end{cases} \quad (\alpha,\beta) \bullet A = \begin{cases} (\beta,\alpha), & x \in A, \\ (0,1), & x \notin A. \end{cases}$$

Theorem 3.2. *Let A be an IFS, then $\forall\,(\alpha, \beta)$, $(\alpha_t, \beta_t) \in L$, we have*

(1) $$A = \bigcup_{(\alpha,\beta)\in L} \Big((\alpha,\beta) \circ A_{[\alpha,\beta]}\Big) = \bigcup_{(\alpha,\beta)\in L} \Big((\alpha,\beta) \circ A_{(\alpha,\beta)}\Big) = \bigcup_{(\alpha,\beta)\in L} \Big((\alpha,\beta) \circ A_{(\alpha,\beta]}\Big) = \bigcup_{(\alpha,\beta)\in L} \Big((\alpha,\beta) \circ A_{[\alpha,\beta)}\Big)$$

(2) $$A_{[\alpha,\beta]} = \bigcap_{(\alpha_t,\beta_t)<(\alpha,\beta)} A_{(\alpha_t,\beta_t)} = \bigcap_{(\alpha_t,\beta_t)<(\alpha,\beta)} A_{[\alpha_t,\beta_t]},$$

(3) $$A_{(\alpha,\beta)} = \bigcup_{(\alpha_t,\beta_t)>(\alpha,\beta)} A_{(\alpha_t,\beta_t)} = \bigcup_{(\alpha_t,\beta_t)>(\alpha,\beta)} A_{[\alpha_t,\beta_t]}.$$

Proof. (1) For $x \in X$, by

$$\begin{aligned} \bigcup_{(\alpha,\beta)\in L} \Big((\alpha,\beta) \circ A_{[\alpha,\beta]}\Big)(x) &= \bigvee_{(\alpha,\beta)\in L} \{(\alpha,\beta)|A_{[\alpha,\beta]}(x) = 1\} \\ &= \bigvee_{(\alpha,\beta)\in L} \{(\alpha,\beta)|\mu_A(x) \geq \alpha,\ \nu_A(x) \leq \beta\} \\ &= (\mu_A(x), \nu_A(x)) = A(x), \end{aligned}$$

we can prove the first equation in (1). The other equations in (1) can be proved in the same way.

(2) For $(\alpha_t,\beta_t)<(\alpha,\beta)$, we have $A_{[\alpha,\beta]}\subseteq A_{(\alpha_t,\beta_t)}\subseteq A_{[\alpha_t,\beta_t]}$, then we can obtain

$$A_{[\alpha,\beta]}\subseteq \bigcap_{(\alpha_t,\beta_t)<(\alpha,\beta)} A_{(\alpha_t,\beta_t)}\subseteq \bigcap_{(\alpha_t,\beta_t)<(\alpha,\beta)} A_{[\alpha_t,\beta_t]}.$$

On the other hand,

$$\begin{aligned}\forall x\in \bigcap_{(\alpha_t,\beta_t)<(\alpha,\beta)} A_{[\alpha_t,\beta_t]} &\Rightarrow \forall(\alpha_t,\beta_t)<(\alpha,\beta), x\in A_{[\alpha_t,\beta_t]}\\ &\Rightarrow (\mu_A(x),\nu_A(x))\geq(\alpha_t,\beta_t)\\ &\Rightarrow (\mu_A(x),\nu_A(x))\geq sup\{(\alpha_t,\beta_t|(\alpha_t,\beta_t)<(\alpha,\beta))\}=(\alpha,\beta)\\ &\Rightarrow x\in A_{[\alpha,\beta]}.\end{aligned}$$

Thus, $A_{[\alpha,\beta]}=\bigcap\limits_{(\alpha_t,\beta_t)<(\alpha,\beta)} A_{(\alpha_t,\beta_t)}=\bigcap\limits_{(\alpha_t,\beta_t)<(\alpha,\beta)} A_{[\alpha_t,\beta_t]}$.

(3) For $(\alpha_t,\beta_t)>(\alpha,\beta)$, we have $A_{(\alpha_t,\beta_t)}\subseteq A_{[\alpha_t,\beta_t]}\subseteq A_{(\alpha,\beta)}$, then we can obtain

$$\bigcup_{(\alpha_t,\beta_t)>(\alpha,\beta)} A_{(\alpha_t,\beta_t)}\subseteq \bigcup_{(\alpha_t,\beta_t)>(\alpha,\beta)} A_{[\alpha_t,\beta_t]}\subseteq A_{(\alpha,\beta)}.$$

On the other hand,

$$\begin{aligned}\forall x\in A_{(\alpha,\beta)} &\Rightarrow (\mu_A(x),\nu_A(x))>(\alpha,\beta)\\ &\Rightarrow \exists(\alpha_i,\beta_i)\in L, \text{such that}(\alpha_i,\beta_i)>(\alpha,\beta), \text{and } (\mu_A(x),\nu_A(x))>(\alpha_i,\beta_i)\\ &\Rightarrow x\in A_{(\alpha_i,\beta_i)}\\ &\Rightarrow x\in \bigcup_{(\alpha_t,\beta_t)>(\alpha,\beta)} A_{(\alpha_t,\beta_t)}.\end{aligned}$$

Thus, $A_{(\alpha,\beta)}=\bigcup\limits_{(\alpha_t,\beta_t)>(\alpha,\beta)} A_{(\alpha_t,\beta_t)}=\bigcup\limits_{(\alpha_t,\beta_t)>(\alpha,\beta)} A_{[\alpha_t,\beta_t]}$. □

Analogously, we can obtain the following theorem.

Theorem 3.3. *Let A be an IFS, then $\forall\,(\alpha,\beta)$, $(\alpha_t,\beta_t)\in L$, we have*

(1) $A^c=\bigcup\limits_{(\alpha,\beta)\in L}\Big((\alpha,\beta)\bullet A^{[\alpha,\beta]}\Big)=\bigcup\limits_{(\alpha,\beta)\in L}\Big((\alpha,\beta)\bullet A^{(\alpha,\beta)}\Big)$
$=\bigcup\limits_{(\alpha,\beta)\in L}\Big((\alpha,\beta)\bullet A^{(\alpha,\beta]}\Big)=\bigcup\limits_{(\alpha,\beta)\in L}\Big((\alpha,\beta)\bullet A^{[\alpha,\beta)}\Big)$

(2) $A^{[\alpha,\beta]}=\bigcap\limits_{(\alpha_t,\beta_t)>(\alpha,\beta)} A^{(\alpha_t,\beta_t)}=\bigcap\limits_{(\alpha_t,\beta_t)>(\alpha,\beta)} A^{[\alpha_t,\beta_t]}$,

(3) $A^{(\alpha,\beta)}=\bigcup\limits_{(\alpha_t,\beta_t)<(\alpha,\beta)} A^{(\alpha_t,\beta_t)}=\bigcup\limits_{(\alpha_t,\beta_t)<(\alpha,\beta)} A^{[\alpha_t,\beta_t]}$.

Since the concept of "convex" plays an important part in operational research and applied mathematics, we introduce the concepts of convex intuitionistic fuzzy sets (convex IFSs) and concave intuitionistic fuzzy sets (concave IFSs), then give some equivalent characterizations in terms of the cut sets of IFSs.

Definition 3.4. IFS A is called a convex IFS if $\forall x,y\in X$, $\forall\lambda\in[0,1]$,

$$\mu_A(\lambda x+(1-\lambda)y)\geq\mu_A(x)\wedge\mu_A(y) \text{ and } \nu_A(\lambda x+(1-\lambda)y)\leq\nu_A(x)\vee\nu_A(y).$$

Definition 3.5. IFS A is called a concave IFS if $\forall x, y \in X$, $\forall \lambda \in [0,1]$,

$$\mu_A(\lambda x + (1-\lambda)y) \leq \mu_A(x) \vee \mu_A(y) \text{ and } \nu_A(\lambda x + (1-\lambda)y) \geq \nu_A(x) \wedge \nu_A(y).$$

It is clear that A is a convex IFS $\Leftrightarrow$ A^c is a concave IFS.

Theorem 3.6. *If A is an IFS, then $\forall(\alpha,\beta) \in L$, the following conditions are equivalent:*
(1) *A is a convex IFS;*
(2) *$A_{[\alpha,\beta]}$ is a convex set;*
(3) *$A_{(\alpha,\beta)}$ is a convex set;*
(4) *$A_{(\alpha,\beta]}$ is a convex set;*
(5) *$A_{[\alpha,\beta)}$ is a convex set.*

Proof. We only give the proof of (1)$\Leftrightarrow$ (2), the others are analogous.
(1) $\Rightarrow$ (2) Suppose A is a convex IFS, then $\forall x, y \in A_{[\alpha,\beta]}$, $\forall \lambda \in [0,1]$, we have

$$\mu_A(\lambda x + (1-\lambda)y) \geq \mu_A(x) \wedge \mu_A(y) \geq \alpha,$$

$$\nu_A(\lambda x + (1-\lambda)y) \leq \nu_A(x) \vee \nu_A(y) \leq \beta.$$

This implies $\lambda x + (1-\lambda)y \in A_{[\alpha,\beta]}$. Therefore $A_{[\alpha,\beta]}$ is a convex set.
(2) $\Rightarrow$ (1) Assume that $\forall(\alpha,\beta) \in L$, $A_{[\alpha,\beta]}$ is a convex set. For any $x, y \in X$ and any $(\alpha,\beta) \in L$ with $\mu_A(x) \wedge \mu_A(y) \geq \alpha$, $\nu_A(x) \vee \nu_A(y) \leq \beta$, we have $x \in A_{[\alpha,\beta]}$ and $y \in A_{[\alpha,\beta]}$. By convexity of $A_{[\alpha,\beta]}$ we know that $\forall \lambda \in [0,1]$, $\lambda x + (1-\lambda)y \in A_{[\alpha,\beta]}$, that is

$$\mu_A(\lambda x + (1-\lambda)y) \geq \alpha, \quad \nu_A(\lambda x + (1-\lambda)y) \leq \beta.$$

This shows that $\mu_A(x) \wedge \mu_A(y) \geq \alpha$ implies $\mu_A(\lambda x + (1-\lambda)y) \geq \alpha$, and $\nu_A(x) \vee \nu_A(y) \leq \beta$ implies $\nu_A(\lambda x + (1-\lambda)y) \leq \beta$. So we obtain

$$\mu_A(\lambda x + (1-\lambda)y) \geq \mu_A(x) \wedge \mu_A(y) \text{ and } \nu_A(\lambda x + (1-\lambda)y) \leq \nu_A(x) \vee \nu_A(y).$$

This proves that A is a convex IFS. □

Theorem 3.7. *If A is an IFS, then $\forall(\alpha,\beta) \in L$, the following conditions are equivalent:*
(1) *A is a concave IFS;*
(2) *$A^{[\alpha,\beta]}$ is a convex set;*
(3) *$A^{(\alpha,\beta)}$ is a convex set;*
(4) *$A^{(\alpha,\beta]}$ is a convex set;*
(5) *$A^{[\alpha,\beta)}$ is a convex set.*

4. The binary operation between the IFSs

Some operations of IFSs can be generalized as follows:

Definition 4.1. Assume that $*$ is a binary operation. For IFSs A and B, define an extension of $*$ as

$$A * B = \{(z, \mu_{A*B}(z), \nu_{A*B}(z) | z = x * y \in X\},$$

where $\mu_{A*B}(z) = \bigvee_{z=x*y} \Big(\mu_A(x) \wedge \mu_B(y)\Big)$, $\nu_{A*B}(z) = \bigwedge_{z=x*y} \Big(\nu_A(x) \vee \nu_B(y)\Big)$.

If the operation $*$ is substituted by the operation "+", "−", "×" and "÷" respectively, then we can obtain the following operations:

$$A + B = \{(z, \mu_{A+B}(z), \nu_{A+B}(z)|z = x + y \in X\},$$
$$A - B = \{(z, \mu_{A-B}(z), \nu_{A-B}(z)|z = x - y \in X\},$$
$$A \times B = \{(z, \mu_{A\times B}(z), \nu_{A\times B}(z)|z = x \times y \in X\},$$
$$A \div B = \{(z, \mu_{A\div B}(z), \nu_{A\div B}(z)|z = x \div y \in X\}.$$

It is easily to verify that $A * B$ is still an IFS.

Theorem 4.2. *Let A and B are IFSs, $\forall(\alpha, \beta) \in L$, we have*
(1) $A_{[\alpha,\beta]} + B_{[\alpha,\beta]} \subseteq (A + B)_{[\alpha,\beta]}$*;*
(2) $A_{(\alpha,\beta)} + B_{(\alpha,\beta)} \subseteq (A + B)_{(\alpha,\beta)}$*;*
(3) *if $\alpha+\beta = 1$, then $A_{[\alpha,\beta]}+B_{[\alpha,\beta]} = (A+B)_{[\alpha,\beta]}$, $A_{(\alpha,\beta)}+B_{(\alpha,\beta)} = (A+B)_{(\alpha,\beta)}$.*

Proof. (1) $\forall z \in A_{[\alpha,\beta]} + B_{[\alpha,\beta]}$, there exist $x \in A_{[\alpha,\beta]}$ and $y \in A_{[\alpha,\beta]}$ such that $x + y = z$. Then $\mu_A(x) \geq \alpha$, $\mu_B(y) \geq \alpha$, $\nu_A(x) \leq \beta$ and $\nu_B(y) \leq \beta$. Furthermore, we can obtain that

$$\mu_{A+B}(z) = \bigvee_{z=x+y} \Big(\mu_A(x) \wedge \mu_B(y)\Big) \geq \alpha,$$
$$\nu_{A+B}(z) = \bigwedge_{z=x+y} \Big(\nu_A(x) \vee \mu_B(y)\Big) \leq \beta,$$

by Definition 4.1. This shows that $z \in (A + B)_{[\alpha,\beta]}$.

Analogously we can prove (2).

(3) By (1), we know that $A_{[\alpha,\beta]} + B_{[\alpha,\beta]} \subseteq (A + B)_{[\alpha,\beta]}$. In order to prove the first equation in (3), we only need to show that $(A+B)_{[\alpha,\beta]} \subseteq A_{[\alpha,\beta]}+B_{[\alpha,\beta]}$. When $\alpha + \beta = 1$, then $\forall z \in (A + B)_{[\alpha,\beta]}$, we have

$$\mu_{A+B}(z) = \bigvee_{z=x+y} \Big(\mu_A(x) \wedge \mu_B(y)\Big) \geq \alpha.$$

So there exist x_0, y_0 satisfying $x_0 + y_0 = z$ such that $\mu_A(x_0) \wedge \mu_B(y_0) \geq \alpha$. This implies that $\mu_A(x_0) \geq \alpha$, $\mu_B(y_0) \geq \alpha$, $\nu_A(x_0) \leq 1 - \mu_A(x_0) \leq 1 - \alpha = \beta$ and $\nu_B(y_0) \leq 1 - \mu_B(y_0) \leq 1 - \alpha = \beta$. Therefore, we have $x_0 \in A_{[\alpha,\beta]}$ and $y_0 \in B_{[\alpha,\beta]}$. Thus, $z = x_0 + y_0 \in A_{[\alpha,\beta]} + B_{[\alpha,\beta]}$. This completes the proof. □

Remark 4.3. In general, if $\alpha + \beta < 1$, (3) in Theorem 4.2 is not true. This can be seen from the following example.

Example 4.4. Let

$$A = \{(x_1, 0.3, 0.6),\ (x_2, 0.3, 0.5)\},\ B = \{(y_1, 0.5, 0.4),\ (y_2, 0.2, 0.4)\}$$

and $x_1 + y_1 = x_2 + y_2$. By Definition 4.1, we have

$$A + B = \{(x_1 + y_1, 0.3, 0.5),\ (x_1 + y_2, 02, 0.6),\ (x_2 + y_1, 0.3, 0.5)\}.$$

Then choose $(\alpha, \beta) = (0.3, 0.5)$, we can obtain that

$$(A + B)_{[0.3,0.5]} = \{x_1 + y_1,\ x_2 + y_1\},$$

but

$$A_{[0.3,0.5]} = \{x_2\},\quad B_{[0.3,0.5]} = \{y_1\}.$$

It can be easily seen that $A_{[\alpha,\beta]} + B_{[\alpha,\beta]} \subsetneq (A+B)_{[\alpha,\beta]}$.

Theorem 4.5. *Let A and B are IFSs,* $\forall(\alpha,\beta) \in L$, *we have*

(1) $(A^c)^{[\alpha,\beta]} + (B^c)^{[\alpha,\beta]} \subseteq [(A+B)^c]^{[\alpha,\beta]}$;

(2) $(A^c)^{(\alpha,\beta)} + (B^c)^{(\alpha,\beta)} \subseteq [(A+B)^c]^{(\alpha,\beta)}$;

(3) *if* $\alpha+\beta = 1$, *then* $(A^c)^{[\alpha,\beta]} + (B^c)^{[\alpha,\beta]} = [(A+B)^c]^{[\alpha,\beta]}$, $(A^c)^{(\alpha,\beta)} + (B^c)^{(\alpha,\beta)} = [(A+B)^c]^{(\alpha,\beta)}$.

Remark 4.6. In Theorem 4.2 and Theorem 4.5, "+" can be substituted by "−", "×" and "÷" respectively.

5. Conclusion

In this paper, the properties of the cut sets of IFSs are studied. By defining the concepts of convex IFSs and concave IFSs, some equivalent characterizations between convex IFSs (concave IFSs) and convex sets are given in terms of the cut sets of IFSs. Finally, the binary operation between the IFSs and its properties are introduced and discussed.

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