**A**nnals of **F**uzzy **M**athematics and **I**nformatics
Volume 5, No. 2, (March 2013), pp. 441–450
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr



# A note on gradation of RS-compactness and S*-closed spaces in L-topological spaces

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Received 28 June 2012; Revised 11 July 2012; Accepted 6 August 2012

Abstract. In this paper we define gradation of RS-compactness and S*-closed spaces in L-topological spaces.We generalize the degrizations of RS-compactness and S*-closedness to the L-topological space and give the positions of them under weak forms of L-continuity.

2010 AMS Classification: 54A40, 06D72

Keywords: Fuzzy topology, Fuzzy RS-compactness, Fuzzy S-closedness, Fuzzy continuity.

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## 1. Introduction

Zadeh in [11] introduced the fundamental concept of a fuzzy set. The notions of compactness plays an important role in topological spaces.As in defined in [5, 7, 8, 10], it is a natural problem to consider some degree of compactness in L-topological spaces. In [2, 8] some weaker forms of gradation of compactness in L-topological spaces.The present paper studies the degrizations of RS-compactness and S*-closedness notions based on the quadruble $M = (L, \leq, \otimes, *)$ where $(L, \leq), \otimes$ and $*$ respectively denote a complete lattice and binary operations on L, was introduced by Höhle and Sostak [6, 9, 10].

## 2. Preliminaries

The notions L-co topological space and L-closure operator are given from Demirci [6] in his paper as a dual form of the binary operation $\otimes$ on L. In this study, we always assume that $(L, \leq)$ is a complete lattice, where $\wedge, \vee, \top, \perp$ respectively denote the meet operation, join operation, the greatest element of L and the least element of L.

The quadruble $M = (L, \leq, \otimes, *)$ consists of an integral, commutative cl-monoid $(L, \leq, *)$ and a cl-quasi-monoid $M = (L, \leq, \otimes)$. In any integral, commutative cl-monoid $M = (L, \leq, *)$, there exists a further binary operation $\longrightarrow$ on L, called the residuum operation on L, such that

$$\alpha * \beta \leq \gamma \Longleftrightarrow \alpha \leq \beta \longrightarrow \gamma, \forall \alpha, \beta, \gamma \in L.$$

The residuum operation $\longrightarrow$ is explicitly given by the formula

$$\alpha \longrightarrow \beta = \vee\{\lambda \in L : \alpha * \lambda \leq \beta\}, \forall \alpha, \beta \in L.$$

A mapping $f : X \longmapsto L$ is called an L-fuzzy set of $X$. The set of all L-fuzzy sets of $X$ is denoted by $L^X$. In any integral, commutative cl-monoid $(L, \leq, *)$, the negation is defined as an unary operation in the sense of $\urcorner : L \longmapsto L$ by $\urcorner(\alpha) = \alpha \longrightarrow \bot, \forall \alpha \in L$. [2, 6, 9]

**Definition 2.1** ([9])**.** A subset $\tau$ of $L^X$, is called an L-topology on $X$ iff $\tau$ satisfies the following conditions:
L01. $1_X, 1_\varnothing \in \tau$,
L02. $f, g \in \tau \Rightarrow f \otimes g \in \tau$, for each $f, g \in L^X$,
L03. $\{f_i : i \in I\} \subseteq \tau \Rightarrow \vee_{i \in I} f_i \in \tau, \forall \{f_i : i \in I\} \subseteq L^X$.
For a given L-topology $\tau$ on $X$, the pair $(X, \tau)$ is called an L-topological space.

**Definition 2.2** ([6])**.** A map $I : L^X \longmapsto L^X$ is called an L-interior operator on $X$ iff $I$ satisfies the next conditions :
I1. $I(1_X) = 1_X$,
I2. $f \leq g \Rightarrow I(f) \leq I(g), \forall f, g \in L^X$,
I3. $I(f) \otimes I(g) \leq I(f \otimes g), \forall f, g \in L^X$,
I4. $I(f) \leq f, \forall f \in L^X$,
I5. $I(f) \leq I(I(f)), \forall f \in L^X$.

**Remark 2.3** ([6])**.** Each L-topology $\tau$ on $X$ induces an L-interior operator $I_r$ by $I_r(f) = \vee\{g \in \tau : g \leq f\}$ for each $f \in L^X$. Conversely, each L-interior operator $I$ induces and L-topology $\tau_I$ by the $\tau_I = \{g \in L^X : g \leq I(g)\}$.

**Definition 2.4** ([6])**.** Let $(X, \tau)$ be an L-topological space and $f \in L^X$. Then
(i) f is said to be $\tau$-closed iff $\urcorner(f) \in \tau$
(ii) $\tau$ closure $\bar{f}$ of $f$ is defined by $\bar{f} = \wedge\{h \in L^X : \urcorner(h) \in \tau, f \leq h\}$.

**Definition 2.5** ([2])**.** A L-fuzzy set $f \in L^X$ in an L-topological space $(X, \tau)$ is said to be
(i) regular L-open iff $f = (\bar{f})^0$,
(ii) regular L-closed iff $f = (\overline{f^0})$

**Definition 2.6** ([6])**.** The map $\widetilde{\subseteq} : L^X \times L^X \longmapsto L$ defined by

$$\widetilde{\subseteq}(f, g) = \wedge_{x \in X} f(x) \rightarrow g(x)$$

for each $f, g \in L^X$, is called the L-fuzzy inclusion relation on $L^X$. The element $\widetilde{\subseteq}(f, g)$ of L can be conceived as the degree for which $f$ is included by $g$.

**Definition 2.7.** ([1, 2, 3]) Let $(X, \tau)$ and $(Y, \delta)$ be L-topological spaces. A function $\Phi : (X, \tau) \longmapsto (Y, \delta)$ is called L-continuous iff for each $g \in \delta, \Phi^{-1}(g) \in \tau$

**Definition 2.8.** ([1, 2]) Let $(X,\tau)$ and $(Y,\delta)$ be L-topological spaces. A function $\Phi:(X,\tau)\longmapsto(Y,\delta)$ is called almost L-continuous iff for each $(\bar{f})^0\in\delta, \Phi^{-1}((\bar{f})^0)\in\tau$

**Definition 2.9** ([2])**.** Let $(X,\tau)$ be an L-topological space.

(i) $(X,\tau)$ is L-compact iff for every family $\{f_i : i\in I\}$ of $\tau$ such that $\vee_{i\in I} f_i(x)=\top, \forall x\in X$, there exists a finite subset $I_0\subseteq I$ such that $\vee_{i\in I_0} f_i(x)=\top, \forall x\in X$.

(ii) Let $f\in L^X$. The L-fuzzy set $f$ is said to be L-compact iff for every family $\{f_i : i\in I\}$ of $\tau$ such that $f(x)\le\vee_{i\in I} f_i(x), \forall x\in X$, there exists a finite subset $I_0\subseteq I$ such that $f(x)\le\vee_{i\in I_0} f_i(x), \forall x\in X$.

**Definition 2.10** ([2])**.** Let $(X,\tau)$ be an L-topological space and $f$ be an L-fuzzy set in $X$. The element $c(f)$ of L is defined by $c(f)=\wedge\{\widetilde{\subseteq}(f,\vee\vartheta)\to[\vee\{\widetilde{\subseteq}(f,\vee_{\vartheta\in\vartheta_0}\vartheta) : \vartheta_0\subseteq'\vartheta\}] : \vartheta\subseteq\tau\}$ is called the degree of compactness of $f$, where $\vartheta_0\subseteq'\vartheta$ means that $\vartheta_0$ is a finite subfamily of $\vartheta$.

**Definition 2.11.** [[2]] Let $(X,\tau)$ be an L-topological space and $f$ be an L-fuzzy set in $X$.

(i) $f$ is said to be almost L-compact iff for every family $\{f_i : i\in I\}$ of $\tau$ such that $f\le\vee_{i\in I} f_i$, there exists a finite subset $I_0\subseteq I$ such that $f\le\vee_{i\in I_0}\bar{f_i}$ where '-' denotes the closure of $f$ in $(X,\tau)$.

(ii) $f$ is said to be nearly L-compact iff for every family $\{f_i : i\in I\}$ of $\tau$ such that $f\le\vee_{i\in I} f_i$, there exists a finite subset $I_0\subseteq I$ such that $f\le\vee_{i\in I_0}\bar{f_i}^{\circ}$, where '$\circ$' denotes the interior operation in $(X,\tau)$.

**Definition 2.12** ([2])**.** Let $(X,\tau)$ be an L-topological space and $f$ be an L-fuzzy set in $X$.

(i) The element $ac(f)=\wedge\{\widetilde{\subseteq}(f,\vee\vartheta)\to[\vee\{\widetilde{\subseteq}(f,\vee_{\vartheta\in\vartheta_0}\bar{\vartheta}) : \vartheta_0\subseteq'\vartheta\}] : \vartheta\subseteq\tau\}$ is called the degree of almost compactness of $f$.

(ii) The element $nc(f)=\wedge\{\widetilde{\subseteq}(f,\vee\vartheta)\to[\vee\{\widetilde{\subseteq}(f,\vee_{\vartheta\in\vartheta_0}\bar{\vartheta}^{\circ}) : \vartheta_0\subseteq'\vartheta\}] : \vartheta\subseteq\tau\}$ is called the degree of near compactness of $f$.

**Proposition 2.13** ([2])**.** *Let $(X,\tau)$ be an L-topological space and $f$ be an L-fuzzy set in $X$. Then $c(f)\le nc(f)\le ac(f)$.*

**Proposition 2.14** ([2])**.** *Let $(X,\tau)$ and $(Y,\delta)$ be L-topological spaces and $\Phi:(X,\tau)\to(Y,\delta)$ be a function. If $\Phi$ is L-continuous, then $c(f)\le c(\Phi(f))$.*

**Definition 2.15** ([4])**.** Let $(X,\tau)$ be a L-topological space. Then $f\in L^X$ is called semiopen if there exists $g\in\tau$ such that $g\le f\le(\bar{g})$ ($f$ is semiclosed if $\neg(f)$ is semiopen).

**Definition 2.16** ([1])**.** Let $(X,\tau)$ be a L-topological space. Then $(X,\tau)$ is called regular iff each fuzzy open set of $X$ is a union of fuzzy open sets of $X$ such that $\bar{f_i}\le f$, for each $i$.

**Definition 2.17** ([4])**.** A L-fuzzy set $f\in L^X$ in an L-topological space $(X,\tau)$ is said to be

(i) regular semiopen if $f=sIntsClf$,

(ii) regular semiclosed if $f=sClsIntf$.

Let $(X,\tau)$ be an L-topological space and $f\in L^X$. Then

$$IntClf\le sIntsClf\le ClIntClf$$

and $IntClf \leq sClf$ (see [4]).

**Definition 2.18** ([2])**.** Let $(X, \tau)$ be an L-topological space and $f$ be an L-fuzzy set in $X$. $f$ is said to be strong L-compact iff for every subfamily $\vartheta$ of $\tau$ such that $f \leq \vee\vartheta$, there exists a finite subfamily Pre-L-open $\vartheta_0 \subseteq \vartheta$ such that $f \leq \vee_{\vartheta \in \vartheta_0}\vartheta$.

## 3. Gradation of RS-compactness in L-topological spaces

In this section we shall generalize the degrization of RS-compactness in L-topological spaces.

**Definition 3.1.** Let $(X, \tau)$ be an L-topological space.

(i) $(X, \tau)$ is RS L-compact iff for every regular semiopen family $\{f_i : i \in I\}$ of L-fuzzy sets with $\vee_{i \in I} f_i = \top$, there exists a finite subset $I_0 \subset I$ such that $\vee_{i \in I_0} f_i^\circ = \top$ for each $x \in X$, where '$\circ$' denotes the interior of $f$ in $(X, \tau)$.

(ii) Let $f \in L^X$. The L-fuzzy set $f$ is said to be RS L-compact iff for every family $\{f_i : i \in I\}$ of regular semiopen L-fuzzy sets with $f(x) \leq \vee_{i \in I} f_i(x), \forall x \in X$, there exist a finite subset $I_0 \subset I$ such that $f(x) \leq \vee_{i \in I_0} f_i^\circ(x), \forall x \in X$.

(iii) $(X, \tau)$ is weakly RS L-compact iff for every regular semiopen family $\{f_i : i \in I\}$ of L-fuzzy sets with $\vee_{i \in I} f_i = \top$, there exists a finite subset $I_0 \subset I$ such that $\vee_{i \in I_0} f_i(x) = \top, \forall x \in X$.

(iv) Let $f \in L^X$. The L-fuzzy set $f$ is said to be weakly RS L-compact iff for every family $\{f_i : i \in I\}$ of regular semiopen L-fuzzy sets with $f(x) \leq \vee_{i \in I} f_i(x), \forall x \in X$, there exists a finite subset $I_0 \subset I$ such that $f(x) \leq \vee_{i \in I_0} f_i(x), \forall x \in X$.

Obviously every RS L-compact set is WRS L-compact set.

**Definition 3.2.** Let $(X, \tau)$ be an L-topological space and $f$ be an L-fuzzy set in $X$. Then

(i) The element $rsc(f)$ of L is defined by

$$rsc(f) = \wedge\{\widetilde{\subseteq}(f, \vee\vartheta) \to [\vee\{\widetilde{\subseteq}(f, \vee_{\vartheta \in \vartheta_0}\vartheta^\circ) : \vartheta_0 \subseteq' \vartheta\}] : \vartheta \in RSO(X)\}$$

is called the degree of RS-compactness of $f$, where $\vartheta_0 \subseteq' \vartheta$ means that $\vartheta_0$ is a finite subfamily of $\vartheta$ and $\vartheta \in RSO(X)$ means that $\vartheta$ is a regular semiopen family of $L^X$.

(ii) The element $wrsc(f)$ of L is defined by

$$wrsc(f) = \wedge\{\widetilde{\subseteq}(f, \vee\vartheta) \to [\vee\{\widetilde{\subseteq}(f, \vee_{\vartheta \in \vartheta_0}\vartheta) : \vartheta_0 \subseteq' \vartheta\}] : \vartheta \in RSO(X)\}$$

is called the degree of weakly RS-compactness of $f$, where $\vartheta_0 \subseteq' \vartheta$ means that $\vartheta_0$ is a finite subfamily of $\vartheta$ and $\vartheta \in RSO(X)$ means that $\vartheta$ is a regular semiopen family of $L^X$.

**Proposition 3.3.** *Let $(X, \tau)$ be an L-topological space and $f$ be an L-fuzzy set in $X$. Then the following implications are valid:*

(i) *$f$ is RS L-compact $\Rightarrow rsc(f) = \top$,*

(ii) *$f$ is WRS L-compact $\Rightarrow wrsc(f) = \top$.*

*Proof.* The proofs of (i) and (ii) follow immediately from the Definition 3.2 □

**Proposition 3.4.** *Let $(X, \tau)$ be an L-topological space and $f$ be an L-fuzzy set in $X$. Then $rsc(f) \leq wrsc(f)$.*

*Proof.* Let $\vartheta$ be regular semiopen family of $L^X$. Since $\vee_{\vartheta\in\vartheta_0}\vartheta^0 \leq \vee_{\vartheta\in\vartheta_0}\vartheta$ and from the isotonity of the residum operation, we have

$$f \rightarrow \vee_{\vartheta\in\vartheta_0}\vartheta^0 \leq f \rightarrow \vee_{\vartheta\in\vartheta_0}\vartheta.$$

Moreover we obtain $\wedge(f \rightarrow \vee_{\vartheta\in\vartheta_0}\vartheta^0) \leq \wedge(f \rightarrow \vee_{\vartheta\in\vartheta_0}\vartheta)$. Thus

$$\widetilde{\subseteq}(f, \vee\vartheta_0^\circ) \leq \widetilde{\subseteq}(f, \vee\vartheta_0)$$

for every $\vartheta_0 \subseteq' \vartheta$. Hence

$$[\vee\{\widetilde{\subseteq}(f, \vee_{\vartheta\in\vartheta_0}\vartheta^\circ) : \vartheta_0 \subseteq' \vartheta\} \leq \{\widetilde{\subseteq}(f, \vee_{\vartheta\in\vartheta_0}\vartheta) : \vartheta_0 \subseteq' \vartheta\}].$$

We conclude that $rsc(f) \leq wrsc(f)$. □

**Definition 3.5.** (i) Let $(X, \tau)$ be an L-topological space. $(X, \tau)$ is SL-closed iff for every family $\{f_i : i \in I\}$ of semiopen L-fuzzy sets with $\vee_{i\in I} f_i = \top$, there exist a finite subset $I_0 \subseteq I$ such that $\vee_{i\in I}\bar{f}_i = \top$, for each $x \in X$.

(ii) Let $(X, \tau)$ be an L-topological space and $f$ be an L-fuzzy set in $X$. $f$ is said to be SL-closed if for every family $\{f_i : i \in I\}$ of semiopen L-fuzzy sets with $f(x) \leq \vee_{i\in I} f_i(x)$, there exist a finite subset $I_0 \subseteq I$ such that $f(x) \leq \vee_{i\in I}\bar{f}_i(x)$ for each $x \in X$.

**Definition 3.6.** Let $(X, \tau)$ be an L-topological space and $f$ be an L-fuzzy set in $X$. Then the element $s\text{-}cl(f)$ of L is defined by

$$s\text{-}cl(f) = \wedge\{\widetilde{\subseteq}(f, \vee\vartheta) \rightarrow [\vee\{\widetilde{\subseteq}(f, \vee_{\vartheta\in\vartheta_0}\bar{\vartheta}) : \vartheta_0 \subseteq' \vartheta\}] : \vartheta \in SO(X)\}$$

is called the degree of S-closedness of $f$, where $\vartheta \in SO(X)$ means that $\vartheta$ is a semiopen family of $L^X$.

**Theorem 3.7.** *Let $(X, \tau)$ be an L-topological space and $f$ be an L-fuzzy set in $X$. Then the following implications are valid:*

$$\begin{array}{ccccccc}
 & \nearrow & f \text{ is nearly L-compact} & \nwarrow & & & \\
 & & \downarrow & & & & \\
f \text{ is L-compact} & \rightarrow & f \text{ is almost L-compact} & \leftarrow & f \text{ is WRS L-compact} & \leftarrow & f \text{ is RS L-compact} \\
 & & \uparrow & \swarrow & & & \\
 & & f \text{ is SL-closed} & & & &
\end{array}$$

*Proof.* It is clear from Definition 3.2, Definition 3.5 and Definition 2.11. □

**Theorem 3.8.** *Let $(X, \tau)$ and $(Y, \delta)$ be L- topological spaces and $\Phi : (X, \tau) \longmapsto (Y, \delta)$ be a function. If $\Phi$ is almost L-continuous, then $wrsc(f) \leq wrsc(\Phi(f))$.*

*Proof.* Let

$$wrsc(\Phi(f)) = \wedge\{\widetilde{\subseteq}(\Phi(f), \vee\vartheta) \rightarrow [\vee\{\widetilde{\subseteq}(\Phi(f)), \vee_{\vartheta\in\vartheta_0}\vartheta) : \vartheta_0 \subseteq' \vartheta\}] : \vartheta \in RSO(X)\}.$$

Conversely, we suppose that $wrsc(f) > wrsc(\Phi(f))$. Then there exists $\vartheta \in RSO(X)$ such that

$$\widetilde{\subseteq}(\Phi(f), \vee\vartheta) \rightarrow [\vee\{\widetilde{\subseteq}(\Phi(f)), \vee_{\vartheta\in\vartheta_0}\vartheta) : \vartheta_0 \subseteq' \vartheta\}] < wrsc(f).$$

Furthermore, from the almost L-continuity of $\Phi$ given us $\Phi^{-1}(\vartheta) \in \tau$. From the definition weakly RS-compactness degree of $f$ is

$$wrsc(f) = \wedge\{\widetilde{\subseteq}(f, \vee\vartheta) \to [\vee\{\widetilde{\subseteq}(f, \vee_{\upsilon\in\upsilon_0}\upsilon) : \upsilon_0 \subseteq' \upsilon\}] : \upsilon \subseteq \tau\}.$$

Now we take the family $\upsilon = \phi^{-1}(\vartheta)$. Hence

$$[\vee\{\widetilde{\subseteq}(\Phi(f)), \vee_{\vartheta\in\vartheta_0}\vartheta) : \vartheta_0 \subseteq' \vartheta\}] < [\vee\{\widetilde{\subseteq}(f, \vee_{\upsilon\in\upsilon_0}\upsilon) : \upsilon_0 \subseteq' \upsilon\}].$$

It follows that there exists $\vartheta_1, \vartheta_2, \dots, \vartheta_n \in \vartheta$ such that

$$[\vee\{\widetilde{\subseteq}(\Phi(f)), \vee_{\vartheta\in\vartheta_0}\vartheta) : \vartheta_0 \subseteq' \vartheta\}] < \widetilde{\subseteq}(f, \vee\Phi^{-1}(\vartheta_i)).$$

Thus $\widetilde{\subseteq}(\Phi(f), \vee_{i=1}^{n}\vartheta_i) < \widetilde{\subseteq}(f, \vee_{i=1}^{n}\Phi^{-1}(\vartheta_i))$, which is a contradiction with Proposition 3.3.(vi) [6], we have

$$\widetilde{\subseteq}(f, \vee_{i=1}^{n}\Phi^{-1}(\vartheta_i)) \leq \widetilde{\subseteq}(\Phi(f), \vee_{i=1}^{n}\Phi(\Phi^{-1}(\vartheta_i))) \leq \widetilde{\subseteq}(\Phi(f), \vee_{i=1}^{n}\vartheta_i).$$

Hence $wrsc(f) \leq wrsc(\Phi(f))$. □

**Proposition 3.9.** *Let $(X, \tau)$ and $(Y, \delta)$ be L-topological spaces and $\Phi : (X, \tau) \longmapsto (Y, \delta)$ be a function. If $\Phi$ is weakly L-continuous, then $rsc(f) \leq rsc(\Phi(f))$.*

*Proof.* It can be proved to the previous theorem, similarly. □

**Proposition 3.10.** *Let $(X, \tau)$ be an L-topological space and $f$ be an L-fuzzy set in $X$. Then $f$ is SL-closed$\Rightarrow$ s-cl$(f) = \top$.*

*Proof.* Trivial. □

**Proposition 3.11.** *Let $(X, \tau)$ be an L-topological space and $f$ be an L-fuzzy set in $X$. Then $rsc(f) \leq wrsc(f) \leq$ s-cl$(f) \leq ac(f)$.*

*Proof.* This follows a similar procedure to the Proposition 3.4 □

**Theorem 3.12.** *Let $(X, \tau)$ be an L-topological space and let $X$ be an extremally disconnected space (i.e $\bar{f} \in \tau$ for every $f \in \tau$). Then the following conditions are equivalent:*

(i) *$f \in L^X$ is almost L-compact,*
(ii) *$f \in L^X$ is nearly L-compact,*
(iii) *$f \in L^X$ is SL-closed,*
(iv) *$f \in L^X$ is weakly RS L-compact.*

*Proof.* It is clear from the previous definitions. □

**Proposition 3.13.** *Let $(X, \tau)$ and $(Y, \delta)$ be L-topological spaces and $\Phi : (X, \tau) \longmapsto (Y, \delta)$ be a function. If $\Phi$ is weakly L-continuous and almost L-open mapping, then*

$$s\text{-}cl(f) \leq s\text{-}cl(\Phi(f)).$$

*Proof.* It can be proved to the Proposition 3.4, similarly. □

**Proposition 3.14.** *Let $(X, \tau)$ be an extremally disconnected L-topological space and L-semiregular topological space. Then $c(f) =$ s-cl$(f)$.*

*Proof.* Similar to Proposition 3.4. □

**Corollary 3.15.** *Let $(X, \tau)$ be an extremally disconnected L-topological space and L-semiregular topological space. Then $c(f) = nc(f) = ac(f) = s\text{-}cl(f) = wrsc(f)$.*

*Proof.* This result follows from Proposition 3.11 and Proposition 3.14. □

## 4. Gradation of S*-closed spaces in L-topological spaces

In this section we shall generalize the degrization of S*-closedness in L-topological spaces.

**Definition 4.1.** Let $(X, \tau)$ be an L-topological space.

(i) $(X, \tau)$ is SL-compact iff for every semiopen family $\{f_i : i \in I\}$ of L-fuzzy sets with $\vee_{i\in I} f_i = \top$, there exists a finite subset $I_0 \subseteq I$ such that $\vee_{i\in I_0} f_i(x) = \top, \forall x \in X$.

(ii) Let $f \in L^X$. The L-fuzzy set $f$ is said to be SL-compact iff for every family $\{f_i : i \in I\}$ of semiopen L-fuzzy sets with $f(x) \leq \vee_{i\in I} f_i(x), \forall x \in X$, there exist a finite subset $I_0 \subseteq I$ such that $f(x) \leq \vee_{i\in I_0} f_i(x), \forall x \in X$.

Obviuosly every SL-compact L-topological space is SL-closed.

**Definition 4.2.** Let $(X, \tau)$ be an L-topological space.

(i) $(X, \tau)$ is S*L-closed iff for every semiopen family $\{f_i : i \in I\}$ of L-fuzzy sets with $\vee_{i\in I} f_i = \top$, there exists a finite subset $I_0 \subseteq I$ such that $\vee_{i\in I_0} scl f_i(x) = \top, \forall x \in X$.

(ii) Let $f \in L^X$. The L-fuzzy set $f$ is said to be S*L-closed iff for every family $\{f_i : i \in I\}$ of semiopen L-fuzzy sets with $f(x) \leq \vee_{i\in I} f_i(x), \forall x \in X$, there exist a finite subset $I_0 \subseteq I$ such that $f(x) \leq \vee_{i\in I_0} scl f_i(x), \forall x \in X$.

**Definition 4.3.** Let $(X, \tau)$ be an L-topological space and $f$ be an L-fuzzy set in $X$. Then

(i) The element $sc(f)$ of L is defined by

$$sc(f) = \wedge\{\widetilde{\subseteq}(f, \vee\vartheta) \to [\vee\{\widetilde{\subseteq}(f, \vee_{\vartheta\in\vartheta_0}\vartheta) : \vartheta_0 \subseteq' \vartheta\}] : \vartheta \in SO(X)\}$$

is called the degree of S-compactness of $f$, where $\vartheta_0 \subseteq' \vartheta$ means that $\vartheta_0$ is a finite subfamily of $\vartheta$ and $\vartheta \in SO(X)$ means that $\vartheta$ is a semiopen family of $L^X$.

(ii) The element $s^*cl(f)$ of L is defined by

$$s^*cl(f) = \wedge\{\widetilde{\subseteq}(f, \vee\vartheta) \to [\vee\{\widetilde{\subseteq}(f, \vee_{\vartheta\in\vartheta_0} scl\vartheta) : \vartheta_0 \subseteq' \vartheta\}] : \vartheta \in SO(X)\}$$

is called the degree of S*-closedness of $f$, where $\vartheta_0 \subseteq' \vartheta$ means that $\vartheta_0$ is a finite subfamily of $\vartheta$ and $\vartheta \in SO(X)$ means that $\vartheta$ is a semiopen family of $L^X$.

**Proposition 4.4.** *Let $(X, \tau)$ be an L-topological space and $f$ be an L-fuzzy set in $X$. Then*

(i) *$f$ is S*L-closed$\Rightarrow S^*cl(f) = \top$,*

(ii) *$f$ is SL-compact$\Rightarrow sc(f) = \top$.*

*Proof.* It is clear from the previous definitions. □

**Proposition 4.5.** *Let $(X, \tau)$ be an L-topological space and $f$ be an L-fuzzy set in $X$. Then*

$$nc(f) \leq S^*cl(f).$$

*Proof.* Let $\vartheta$ be open family of $L^X$. Since $\vee_{\vartheta\in\vartheta_0} IntCl(\vartheta) \leq \vee_{\vartheta\in\vartheta_0} sCl(\vartheta)$ and from the isotonity of the residum operation, we have

$$f \to \vee_{\vartheta\in\vartheta_0} IntCl(\vartheta) \leq f \to \vee_{\vartheta\in\vartheta_0} sCl(\vartheta).$$

Moreover we obtain $\wedge(f \to \vee_{\vartheta\in\vartheta_0} IntCl(\vartheta)) \leq \wedge(f \to \vee_{\vartheta\in\vartheta_0} sCl(\vartheta))$. Thus

$$\widetilde{\subseteq}(f, \vee IntCl(\vartheta_0)) \leq \widetilde{\subseteq}(f, \vee sCl(\vartheta_0))$$

for every $\vartheta_0 \subseteq' \vartheta$. Hence

$$[\vee\{\widetilde{\subseteq}(f, \vee_{\vartheta\in\vartheta_0} IntCl(\vartheta_0)) : \vartheta_0 \subseteq' \vartheta\}] \leq \{\widetilde{\subseteq}(f, \vee_{\vartheta\in\vartheta_0} sCl(\vartheta)) : \vartheta_0 \subseteq' \vartheta\}.$$

We conclude that $nc(f) \leq S^*cl(f)$. □

**Theorem 4.6.** *Let $(X, \tau)$ be an L-topological space and $f$ be an L-fuzzy set in $X$. Then the following implications are valid:*

$$f \text{ is nearly L-compact} \Rightarrow f \text{ is } S^* L\text{-closed} \Rightarrow f \text{ is almost L-compact.}$$

*Proof.* This immediately follows from Definition 4.2 and Definition 2.11. □

**Proposition 4.7.** *Let $(X, \tau)$ be an extremally disconnected topological space and $f$ be an L-fuzzy set in $X$. Then $nc(f) = S^*cl(f)$.*

*Proof.* Similar to Proposition 4.5. □

**Proposition 4.8.** *Let $(X, \tau)$ be an L-topological space and $f$ be an L-fuzzy set in $X$. Then*

$$f \text{ is strongly L-compact} \Rightarrow f \text{ is L-compact.}$$

*Proof.* It is clear from the previous definitions. □

**Corollary 4.9.** *Let $(X, \tau)$ be an L-topological space and let $X$ be an extremally disconnected space (i.e. $\bar{f} \in \tau$ for every $f \in \tau$). Then the following conditions are equivalent:*

(i) $f \in L^X$ *is almost L-compact,*
(ii) $f \in L^X$ *is nearly L-compact,*
(iii) $f \in L^X$ *is SL-closed,*
(iv) $f \in L^X$ *is weakly RS L-compact,*
(v) $f \in L^X$ *is $S^*$L-closed,*
(vi) $f \in L^X$ *is strongly L-compact.*

*Proof.* It is clear from the previous definitions. □

**Definition 4.10.** Let $(X, \tau)$ and $(Y, \delta)$ be L-topological spaces. A function $\Phi : (X, \tau) \longmapsto (Y, \delta)$ is called L-semicontinuous iff for each $f \in \delta$, $\Phi^{-1}(f)$ is a fuzzy semiopen set of $L^X$.

**Theorem 4.11.** *Let $(X, \tau)$ and $(Y, \delta)$ be L-topological spaces and $\Phi : (X, \tau) \longmapsto (Y, \delta)$ be a function. If $\Phi$ is L-semicontinuous, then $sc(f) \leq c(\Phi(f))$.*

*Proof.* Let $sc(f) = \wedge\{\widetilde{\subseteq}(f, \vee\vartheta) \to [\vee\{\widetilde{\subseteq}(f, \vee_{\vartheta\in\vartheta_0}\vartheta) : \vartheta_0 \subseteq' \vartheta\}] : \vartheta \in SO(X)\}$ Conversely, we suppose that $sc(f) > c(\Phi(f))$. Then there exists $\vartheta \subseteq \delta$ such that

$$\widetilde{\subseteq}(\Phi(f), \vee\vartheta) \to [\vee\{\widetilde{\subseteq}(\Phi(f), \vee_{\vartheta\in\vartheta_0}\vartheta) : \vartheta_0 \subseteq' \vartheta\}] < sc(f).$$

Furthermore, from the L-semicontinuity of $\Phi$, $\Phi^{-1}(f)$ is a fuzzy semiopen set of $L^X$. From the definition of S-compactness degree of $f$ is

$$sc(f) = \wedge\{\widetilde{\subseteq}(f, \vee\vartheta) \to [\vee\{\widetilde{\subseteq}(f, \vee_{\upsilon\in\upsilon_0}\upsilon) : \upsilon_0 \subseteq' \upsilon\}] : \upsilon \in SO(X)\}.$$

Now we take the family $\upsilon = \Phi(\vartheta)$. Hence

$$[\vee\{\widetilde{\subseteq}(\Phi(f), \vee_{\vartheta\in\vartheta_0}\vartheta) : \vartheta_0 \subseteq' \vartheta\}] < [\vee\{\widetilde{\subseteq}(f, \vee_{\upsilon\in\upsilon_0}\upsilon) : \upsilon_0 \subseteq' \upsilon\}].$$

It follows that there exists $\vartheta_1, \vartheta_2, \ldots, \vartheta_n \in \vartheta$ such that

$$[\vee\{\widetilde{\subseteq}(\Phi(f), \vee_{\vartheta\in\vartheta_0}\vartheta) : \vartheta_0 \subseteq' \vartheta\}] < \widetilde{\subseteq}(f, \vee\Phi^{-1}(\vartheta_i)).$$

Thus $\widetilde{\subseteq}(\Phi(f), \vee_{i=1}^n\vartheta_i) < \widetilde{\subseteq}(f, \vee_{i=1}^n\Phi^{-1}(\vartheta_i))$, which is a contradiction with Proposition 3.3.(vi)[6], we have

$$\widetilde{\subseteq}(f, \vee_{i=1}^n\Phi^{-1}(\vartheta_i)) \leq \widetilde{\subseteq}(\Phi(f), \vee_{i=1}^n\Phi(\Phi^{-1}(\vartheta_i))) \leq \widetilde{\subseteq}(\Phi(f), \vee_{i=1}^n\vartheta_i).$$

Hence $sc(f) < c(\Phi(f))$. □

**Definition 4.12.** Let $(X, \tau)$ and $(Y, \delta)$ be L-topological spaces. A function $\Phi : (X, \tau) \longmapsto (Y, \delta)$ is called L-irresolute iff for each semiopen set of $f \in L^Y$, $\Phi^{-1}(f)$ is a fuzzy semiopen set of $L^X$.

**Theorem 4.13.** *Let $(X, \tau)$ and $(Y, \delta)$ be L-topological spaces and $\Phi : (X, \tau) \longmapsto (Y, \delta)$ be a function. If $\Phi$ is L-irresolute, then $S^*cl(f) \leq S^*cl(\Phi(f))$.*

*Proof.* It can be proved to the Theorem 4.11.,similarly. □

**Theorem 4.14.** *Let $(X, \tau)$ and $(Y, \delta)$ be L-topological spaces and $\Phi : (X, \tau) \longmapsto (Y, \delta)$ be a function. If $\Phi$ is L-semicontinuous, then $s^*cl(f) \leq ac(\Phi(f))$.*

*Proof.* It can be proved to the Theorem 4.11 similarly. □

**Definition 4.15.** Let $(X, \tau)$ and $(Y, \delta)$ be L-topological spaces. A function $\Phi : (X, \tau) \longmapsto (Y, \delta)$ is called semi weakly L-continuous iff for each $f \in L^Y$ semiopen set, we have $\Phi^{-1}(f) \leq slnt[\Phi^{-1}(sCl(f))]$.

**Theorem 4.16.** *Let $(X, \tau)$ and $(Y, \delta)$ be L-topological spaces and $\Phi : (X, \tau) \longmapsto (Y, \delta)$ be a function. If $\Phi$ is semi weakly L-continuous, then $sc(f) \leq s^*cl(\Phi(f))$.*

*Proof.* This is analogous to the proof of Theorem 4.11. □

**Acknowledgements.** The author is very grateful to the editor and the reviewers for their valuable suggestions.

## References

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