Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 5, No. 2, (March 2013), pp. 417–427
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr

Uncertainty analysis in atmospheric dispersion using Shannon entropy

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Received 8 May 2012; Accepted 12 July 2012

Abstract. In this article, the uncertainty analysis based on the *Randomness-Fuzziness Consistency Principle* by using Shannon Entropy as a measure has been studied. The Randomness-Fuzziness Consistency Principle leads to defining a normal law of fuzziness using two different laws of randomness. For the two laws of randomness defined for every normal law of fuzziness, we can therefore have a pair of Shannon entropies. We have found that the pair of Shannon entropies for the atmospheric dispersion defined by the Gaussian plume model decreases exponentially with increase in the distance along the downwind direction. In other words, the effect of fuzziness decreases as the distance along the downwind direction increases.

2010 AMS Classification: 03E72, 60A86, 62A86, 70H03

Keywords: Uncertainty Analysis, Gaussian Plume Model, Randomness-Fuzziness Consistency Principle, Shannon Entropy.

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1. Introduction

Atmospheric dispersion is a phenomenon based on uncertainties, and in general, the concentration of pollutants observed at a given time and location downwind of a source cannot be predicted precisely [8]. Uncertainty here refers to lack of knowledge or information about an unknown quantity whose true value could be established if a perfect measurement device were available. Uncertainty is the measure of the reliability associated with a particular set of results and can be expressed in probabilistic terms. Uncertainty in atmospheric dispersion model predictions is associated with: (a) "data" or "parameter" uncertainty resulting from errors in the data used to execute and evaluate the model, uncertainties in empirical model parameters, and initial and boundary conditions; (b) "model" or "structural" uncertainty arising from inaccurate treatment of dynamical and chemical processes, approximate numerical solutions, and internal model errors; and (c) "stochastic" uncertainty, which results from the turbulent nature of the atmosphere as well as from unpredictability of human activities related to emissions [11].

The Gaussian plume model [13] is the most widely used method of estimating downwind concentration of airborne material released to the atmosphere. Sutton ([14], [15]) derived an air pollutant plume dispersion equation which included the assumption of Gaussian distribution for the vertical and crosswind dispersion of the plume and also included the effect of ground reflection of the plume. The Gaussian plume model that provides the time integrated air concentration at any downwind distance is given by

$$C(x,y,z) = \frac{Q}{u2\pi\sigma_y\sigma_z}\exp(-\frac{y^2}{2\sigma_y^2})[\exp(-\frac{(z-h)^2}{2\sigma_z^2}) + \exp(-\frac{(z+h)^2}{2\sigma_z^2})]$$

where $C(x,y,z)$ is the concentration of the pollutant (mcg/m^3) at any point x metres downwind of the source, y metres crosswind from the emission plume centreline and z metres above ground level, Q is the quantity or mass of the pollutant (gm/s), u is the average wind speed (m/s), σ_z is standard deviation in the vertical direction (m), σ_y is the standard deviation in the crosswind direction (m), h is the effective stack height of the source above ground level (m).

The values of horizontal and vertical dispersion co-efficients (σ_y and σ_z) here can be seen to be

$$\sigma_y = a_y x^{0.9071}$$
$$\sigma_z = a_z x^{b_z} + c_z$$

where the co-efficient a_y, a_z, b_z and c_z can be obtained from the table of parameters for Pasquill-Gifford σ_y and σ_z [9]. Based on the temperature gradient, atmospheric conditions are categorized into six classes, so called the Pasquill stability classes [10].

The plume rise is added to the height of the plume's source point to obtain the so-called 'effective stack height', h. The plume rise equation due to Moses and Carson [16] is as follows:

$$\triangle h = A\frac{V_s d}{u} + B\frac{\sqrt{Q_h}}{u}$$

where, V_s stack gas exit speed (m/s), d is the stack diameter (m), u is the average wind speed (m) and Q_h is the stack heat emission rate (C_i). The parameters A and B are different for different atmospheric conditions, such as super adiabatic condition, neutral condition and sub-adiabatic condition, and they are $A = 3.47$, $B = 5.15$; $A = 0.35$, $B = 2.64$ and $A = -1.04$, $B = 2.24$ respectively.

The methodology used in this article is based on a result linking fuzziness with randomness. The existence of two laws of randomness is required to define a law of fuzziness ([1], [2], [3], [4], [5], [6], [7]). The principle states that the left reference function of any normal fuzzy number is actually a probability distribution function, and that the right reference function is actually a complementary probability distribution function, for which however one needs to look into the matters through application of the Glivenko-Centelli theorem of Order Statistics on superimposed uniformly fuzzy intervals.

2. Fuzzification with reference to concentration at different downwind distances of the model

Basically, we have obtained the membership functions of the atmospheric dispersion defined by the Gaussian plume model $C(x, y, 0)$ for different distances along the downwind direction [x: 1600 m, 3000 m, 7000 m, 14000 m and 30000 m] and along the crosswind direction for $y = 2$ m, for the extremely unstable atmospheric condition (category A) under the super-adiabatic condition of plume rise due to Moses-Carson equation. Here the membership function of $C(x, y, 0)$ has been found by using Lagrangian polynomial on discretized values of the α-cuts of $C(x, y, 0)$. We had to do this because here in this case the method of α-cuts would fail to supply the results for the reason that we here have a non-invertible function. So the only alternative is to use a method of interpolation. To make the matters simple, we have considered Lagrangian polynomial of degree four. Here the parameters quantity of emission, average wind speed, stack gas exit speed and stack heat emission rate are considered as triangular fuzzy numbers (TFN).

Input data: Quantity of emission $= TFN[100, 500, 1000]$, Average wind speed $= TFN[2, 4, 6]$, Stack gas exit speed $= TFN[1.2, 3.4, 6.3]$, Stack diameter = 5, Stack heat emission rate $= TFN[100, 500, 1000]$ and Physical stack height = 100.

2.1. Fuzzification with reference to Concentration at $x = 1600$ m.

The membership function and membership curve of pollutant concentration of the Gaussian Plume Model at downwind distance $x = 1600$ m are as follows

$$\mu_{C(x,y,0)}(X) = \begin{cases} F(X), & 5.626 \times 10^{-06} \le X \le 1.09906 \times 10^{-04} \\ G(X), & 1.09906 \times 10^{-04} \le X \le 0.001063421 \\ 0, & \text{otherwise} \end{cases}$$

where

$$\begin{aligned} F(X) = & \frac{(X-5.626\times10^{-06})(X-3.1339\times10^{-05})(X-5.8788\times10^{-05})(X-1.09906\times10^{-04})0.25}{-6.386283997\times10^{-19}} \\ & + \frac{(X-5.626\times10^{-06})(X-1.5358\times10^{-05})(X-5.8788\times10^{-05})(X-1.09906\times10^{-04})0.5}{8.861829681\times10^{-19}} \\ & + \frac{(X-5.626\times10^{-06})(X-1.5358\times10^{-05})(X-3.1339\times10^{-05})(X-1.09906\times10^{-04})0.75}{-3.239600977\times10^{-18}} \\ & + \frac{(X-5.626\times10^{-06})(X-1.5358\times10^{-05})(X-3.1339\times10^{-05})(X-5.8788\times10^{-05})}{3.959746586\times10^{-17}} \\ & \qquad 5.626 \times 10^{-06} \le X \le 1.09906 \times 10^{-04} \end{aligned}$$

and

$$\begin{aligned} G(X) = & \frac{(X-0.001063421)(X-3.17637\times10^{-04})(X-1.86548\times10^{-04})(X-1.09906\times10^{-04})0.25}{-2.051676691\times10^{-14}} \\ & + \frac{(X-0.001063421)(X-5.60057\times10^{-04})(X-1.86548\times10^{-04})(X-1.09906\times10^{-04})0.5}{4.923194348\times10^{-15}} \\ & + \frac{(X-0.001063421)(X-5.60057\times10^{-04})(X-3.17637\times10^{-04})(X-1.09906\times10^{-04})0.75}{-3.290524903\times10^{-15}} \\ & + \frac{(X-0.001063421)(X-5.60057\times10^{-04})(X-3.17637\times10^{-04})(X-1.86548\times10^{-04})}{6.833523815\times10^{-15}} \\ & \qquad 1.09906 \times 10^{-04} \le X \le 0.001063421 \end{aligned}$$

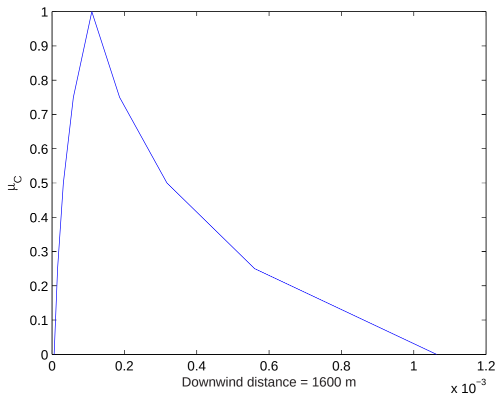

FIGURE 1. Membership Curve of Pollutant Concentration of Gaussian Plume Model $x = 1600$ m

2.2. **Fuzzification with reference to Concentration at $x = 3000$ m.** At downwind distance $x = 3000$ m, the membership function and curve of pollutant concentration of the Gaussian Plume Model are obtained as follows

$$\mu_{C(x,y,0)}(X) = \begin{cases} F(X), & 8.71483 \times 10^{-07} \leq X \leq 1.66736 \times 10^{-05} \\ G(X), & 1.66736 \times 10^{-05} \leq X \leq 0.000170023 \\ 0, & \text{otherwise} \end{cases}$$

where

$$\begin{aligned} F(X) = & \frac{(X-8.71483\times10^{-07})(X-4.60957\times10^{-06})(X-8.71885\times10^{-06})(X-1.66736\times10^{-05})0.25}{-3.04034\times10^{-22}} \\ & + \frac{(X-8.71483\times10^{-07})(X-2.27624\times10^{-06})(X-8.71885\times10^{-06})(X-1.66736\times10^{-05})0.5}{4.32399\times10^{-22}} \\ & + \frac{(X-8.71483\times10^{-07})(X-2.27624\times10^{-06})(X-4.60957\times10^{-06})(X-1.66736\times10^{-05})0.75}{-1.65265\times10^{-21}} \\ & + \frac{(X-8.71483\times10^{-07})(X-2.27624\times10^{-06})(X-4.60957\times10^{-06})(X-8.71885\times10^{-06}}{2.18334\times10^{-20}} \\ & \qquad 8.71483 \times 10^{-07} \leq X \leq 1.66736 \times 10^{-05} \end{aligned}$$

and

$$\begin{aligned} G(X) = & \frac{(X-0.000170023)(X-4.88349\times10^{-05})(X-2.84241\times10^{-05})(X-1.66736\times10^{-05})0.25}{-1.32751\times10^{-17}} \\ & + \frac{(X-0.000170023)(X-8.73736\times10^{-05})(X-2.84241\times10^{-05})(X-1.66736\times10^{-05})0.5}{3.06586\times10^{-18}} \\ & + \frac{(X-0.000170023)(X-8.73736\times10^{-05})(X-4.88349\times10^{-05})(X-1.66736\times10^{-05})0.75}{-2.002\times10^{-18}} \\ & + \frac{(X-0.000170023)(X-8.73736\times10^{-05})(X-4.88349\times10^{-05})(X-2.84241\times10^{-05})}{4.09722\times10^{-18}} \\ & \qquad 1.66736 \times 10^{-05} \leq X \leq 0.000170023 \end{aligned}$$

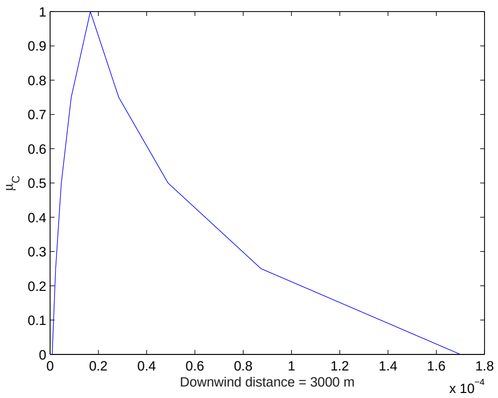

FIGURE 2. Membership Curve of Pollutant Concentration of Gaussian Plume Model $x = 3000$ m

2.3. **Fuzzification with reference to Concentration at $x = 7000$ m.** The membership function and curve of pollutant concentration of the Gaussian Plume Model at downwind distance $x = 7000$ m are as follows

$$\mu_{C(x,y,0)}(X) = \begin{cases} F(X), & 6.31493 \times 10^{-08} \leq X \leq 1.30963 \times 10^{-06} \\ G(X), & 1.30963 \times 10^{-06} \leq X \leq 1.45493 \times 10^{-05} \\ 0, & \text{otherwise} \end{cases}$$

where

$$\begin{aligned} F(X) = & \frac{(X-6.31493\times10^{-08})(X-3.40614\times10^{-07})(X-6.58749\times10^{-07})(X-1.30963\times10^{-06})0.25}{-1.01198\times10^{-26}} \\ & + \frac{(X-6.31493\times10^{-08})(X-1.65945\times10^{-07})(X-6.58749\times10^{-07})(X-1.30963\times10^{-06})0.5}{1.49405\times10^{-26}} \\ & + \frac{(X-6.31493\times10^{-08})(X-1.65945\times10^{-07})(X-3.40614\times10^{-07})(X-1.30963\times10^{-06})0.75}{-6.07773\times10^{-26}} \\ & + \frac{(X-6.31493\times10^{-08})(X-1.65945\times10^{-07})(X-3.40614\times10^{-07})(X-6.58749\times10^{-07})}{8.99134\times10^{-25}} \\ & \qquad 6.31493 \times 10^{-08} \leq X \leq 1.30963 \times 10^{-06} \end{aligned}$$

and

$$\begin{aligned} G(X) = & \frac{(X-1.45493\times10^{-05})(X-3.93214\times10^{-06})(X-2.2554\times10^{-06})(X-1.30963\times10^{-06})0.25}{-6.97534\times10^{-22}} \\ & + \frac{(X-1.45493\times10^{-05})(X-7.19483\times10^{-06})(X-2.2554\times10^{-06})(X-1.30963\times10^{-06})0.5}{1.52324\times10^{-22}} \\ & + \frac{(X-1.45493\times10^{-05})(X-7.19483\times10^{-06})(X-3.93214\times10^{-06})(X-1.30963\times10^{-06})0.75}{-2.002\times10^{-23}} \\ & + \frac{(X-1.45493\times10^{-05})(X-7.19483\times10^{-06})(X-3.93214\times10^{-06})(X-2.2554\times10^{-06})}{1.9326\times10^{-22}} \\ & \qquad 1.30963 \times 10^{-06} \leq X \leq 1.45493 \times 10^{-05} \end{aligned}$$

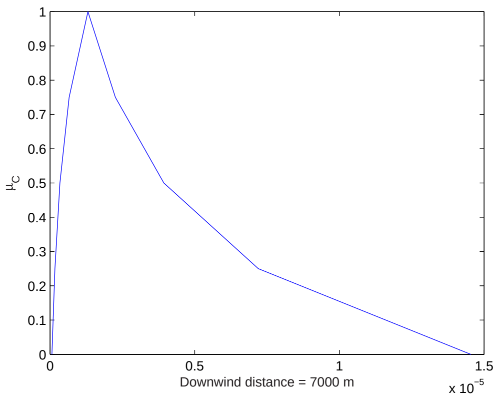

FIGURE 3. Membership Curve of Pollutant Concentration of Gaussian Plume Model $x = 7000$ m

2.4. **Fuzzification with reference to Concentration at $x = 14000$ m.** At downwind distance $x = 14000$ m, the membership function and curve of pollutant concentration of the Gaussian Plume Model are as follows

$$\mu_{C(x,y,0)}(X) = \begin{cases} F(X), & 7.3366 \times 10^{-09} \leq X \leq 1.63537 \times 10^{-07} \\ G(X), & 1.63537 \times 10^{-07} \leq X \leq 1.95263 \times 10^{-06} \\ 0, & \text{otherwise} \end{cases}$$

where

$$\begin{aligned} F(X) = & \frac{(X-7.3366\times10^{-09})(X-4.03631\times10^{-08})(X-7.95299\times10^{-08})(X-1.63537\times10^{-07})0.25}{-2.19262\times10^{-30}} \\ & + \frac{(X-7.3366\times10^{-09})(X-1.94344\times10^{-08})(X-7.95299\times10^{-08})(X-1.63537\times10^{-07})0.5}{3.33458\times10^{-30}} \\ & + \frac{(X-7.3366\times10^{-09})(X-1.94344\times10^{-08})(X-4.03631\times10^{-08})(X-1.63537\times10^{-07})0.75}{-1.42749\times10^{-29}} \\ & + \frac{(X-7.3366\times10^{-09})(X-1.94344\times10^{-08})(X-4.03631\times10^{-08})(X-7.95299\times10^{-08})}{2.3291\times10^{-28}} \\ & \qquad 7.3366 \times 10^{-09} \leq X \leq 1.63537 \times 10^{-07} \end{aligned}$$

and

$$\begin{aligned} G(X) = & \frac{(X-1.95263\times10^{-06})(X-5.00888\times10^{-07})(X-2.83937\times10^{-07})(X-1.63537\times10^{-07})0.25}{-2.20665\times10^{-25}} \\ & + \frac{(X-1.95263\times10^{-06})(X-9.33674\times10^{-07})(X-2.83937\times10^{-07})(X-1.63537\times10^{-07})0.5}{4.5984\times10^{-26}} \\ & + \frac{(X-1.95263\times10^{-06})(X-9.33674\times10^{-07})(X-5.00888\times10^{-07})(X-1.63537\times10^{-07})0.75}{-2.83206\times10^{-26}} \\ & + \frac{(X-1.95263\times10^{-06})(X-9.33674\times10^{-07})(X-5.00888\times10^{-07})(X-2.83937\times10^{-07})}{5.59641\times10^{-26}} \\ & \qquad 1.63537 \times 10^{-07} \leq X \leq 1.95263 \times 10^{-06} \end{aligned}$$

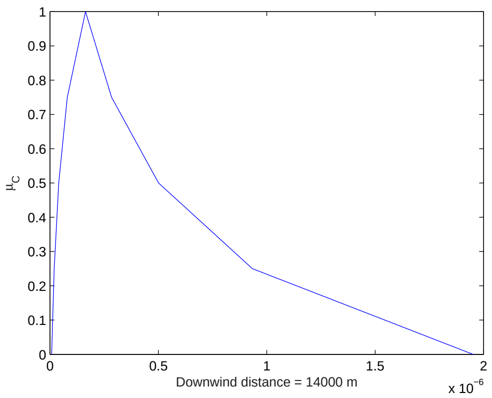

FIGURE 4. Membership Curve of Pollutant Concentration of Gaussian Plume Model $x = 14000$ m

2.5. **Fuzzification with reference to Concentration at $x = 30000$ m.** The membership function and curve of pollutant concentration of the Gaussian Plume Model at downwind distance $x = 30000$ m are as follows

$$\mu_{C(x,y,0)}(X) = \begin{cases} F(X), & 6.8785 \times 10^{-10} \leq X \leq 1.66052 \times 10^{-08} \\ G(X), & 1.66052 \times 10^{-08} \leq X \leq 2.14708 \times 10^{-07} \\ 0, & \text{otherwise} \end{cases}$$

where

$$\begin{aligned} F(X) = & \frac{(X-6.8785\times10^{-10})(X-3.86391\times10^{-09})(X-7.76877\times10^{-09})(X-1.66052\times10^{-08})0.25}{-2.04057\times10^{-34}} \\ & + \frac{(X-6.8785\times10^{-10})(X-1.83746\times10^{-09})(X-7.76877\times10^{-09})(X-1.66052\times10^{-08})0.5}{3.20216\times10^{-34}} \\ & + \frac{(X-6.8785\times10^{-10})(X-1.83746\times10^{-09})(X-3.86391\times10^{-09})(X-1.66052\times10^{-08})0.75}{-1.44918\times10^{-33}} \\ & + \frac{(X-6.8785\times10^{-10})(X-1.83746\times10^{-09})(X-3.86391\times10^{-09})(X-7.76877\times10^{-09})}{2.64652\times10^{-32}} \\ & \qquad 6.8785 \times 10^{-10} \leq X \leq 1.66052 \times 10^{-08} \end{aligned}$$

and

$$\begin{aligned} G(X) = & \frac{(X-2.14708\times10^{-07})(X-5.19287\times10^{-08})(X-2.90736\times10^{-08})(X-1.66052\times10^{-08})0.25}{-3.11018\times10^{-29}} \\ & + \frac{(X-2.14708\times10^{-07})(X-9.8772\times10^{-08})(X-2.90736\times10^{-08})(X-1.66052\times10^{-08})0.5}{6.15593\times10^{-30}} \\ & + \frac{(X-2.14708\times10^{-07})(X-9.8772\times10^{-08})(X-5.19287\times10^{-08})(X-1.66052\times10^{-08})0.75}{-3.68702\times10^{-30}} \\ & + \frac{(X-2.14708\times10^{-07})(X-9.8772\times10^{-08})(X-5.19287\times10^{-08})(X-2.90736\times10^{-08})}{7.16905\times10^{-30}} \\ & \qquad 1.66052 \times 10^{-08} \leq X \leq 2.14708 \times 10^{-07} \end{aligned}$$

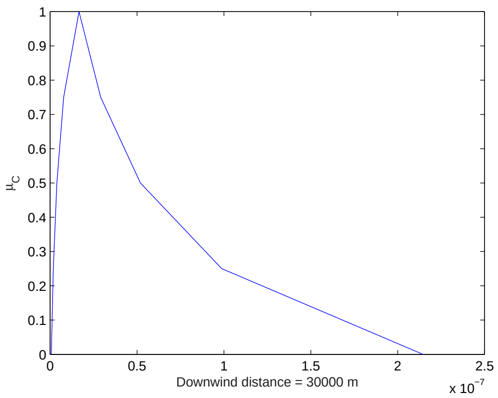

FIGURE 5. Membership Curve of Pollutant Concentration of Gaussian Plume Model $x = 30000$ m

3. Uncertainty analysis of the atmospheric dispersion defined by the gaussian plume model using Shannon entropy

The Shannon Diversity Index (H) [12] is an index that is commonly used to characterize species diversity in a community. The proportion (p_i) of species i relative to the total number of species is calculated, and then multiplied by the natural logarithm (ln p_i) of this proportion. The resulting product is summed across species, and multiplied by (-1) so as to get a positive value for H.

$$H = -\sum_{i=1}^{S} p_i ln p_i, \Sigma p_i = 1.$$

This diversity index, which later on came to be known as Shannon Entropy, can be interpreted as the uncertainty associated with a fuzzy event [17]. Now the Shannon measure defining entropy requires a probability law to be defined first. As for every law of fuzziness, there are two laws of randomness defining the law of fuzziness, for a fuzzy number we shall therefore have two laws of randomness which would lead to two Shannon Entropies. If we now find that the two Shannon Entropies increase with increase in the value of a parameter defining the two laws of randomness concerned, we would conclude that changes in the parameter are effective with reference to the two random variables concerned with variability increasing with increase in the parameter. Similarly, if the Shannon Entropies decrease with increase in the parameter, we would still say that changes in the parameter is effective with reference to the two laws of randomness; in this case however we would conclude that the uncertainty is pointing towards decreasing variability of the random variables concerned. In what follows, we could study uncertainty due to changes in using Shannon Entropy as a measure.

4. Shannon entropies with reference to concentration at different distances along the downwind direction of the model

It can be seen that from the left reference function, which is a distribution function, we can find the probabilities of occurrence of the random variable concerned in intervals of equal width. To use Shannon entropy by discretizing the distribution, we must consider equal width of the intervals. Similarly, from the right reference function, which is a complementary distribution function, we can compute another Shannon entropy. The Shannon entropies for the left reference function at different values of x are as given in Table 1: Taking the different values of distances along the

Table 1. Values of Shannon Entropies for the Left Reference Function

x	Shannon Entropy
1600	1.3509241
3000	1.2900427
7000	1.2640680
14000	1.2365850
30000	1.1854739

downwind direction as independent variable x and the values of Shannon entropy as dependent variable Y, we can fit a straight line which will give us an idea about the uncertainty analysis. However a straight line fit does not seem to be logical because it would give us negative values of entropy for very large x. A better and more logical fitting of curve is found to be exponential, which is given as

$$Y = a.e^{bx}$$

and the estimated values of a and b are 1.320840114 and -0.00000392907 respectively. To fit this equation we have used the method of iteration based on the Taylorian expansion. The pollutant concentration decreases exponentially along the downwind direction as given by the equation

$$Y = 1.320840114.e^{-0.00000392907x}$$

Similarly, for the right reference function at different values of x we get the Shannon entropies as given in Table 2. Taking the different values of distance along the

Table 2. Values of Shannon Entropies for the Right Reference Function

x	Shannon Entropy
1600	1.2548165
3000	1.2325232
7000	1.1791184
14000	1.0864665
30000	1.0138339

downwind direction as independent variable (x) and the values of Shannon entropies as dependent variable (Y), we have fitted another exponential equation of the type

$$Y = a.e^{bx}$$

For the entropies found from the right reference function, we have found that

$$Y = 1.249744111.e^{-0.00000750368x}$$

Uncertainties of pollutant concentration with reference to different distances along the downwind direction can be analyzed by Shannon entropy separately for the left and the right reference functions, as there are two probability laws in action. We have found that in both of the reference functions, the Shannon measure decrease exponentially with a very low rate of decay. As we can see, for a small value of x, the two laws of randomness concerned are very nearly uniform. As x increase, the uniformity decreases exponentially. This can actually be seen from the Table 3 below which includes the spreads of the fuzzy intervals, and spreads of the left and right reference functions. That fuzziness decrease with increase in x is obvious. However, use of Shannon entropy gives us a measure of the decrement in fuzziness of concentration with reference to increase in the distance from the source along the downwind direction.

Table 3. Length of Fuzzy Intervals

x	Spread of fuzziness	Spread of the Left Reference Function	Spread of the Right Reference Function
1600	0.001057782	0.000104283	0.001063407
3000	0.000169152	0.000015802	0.000153349
7000	0.000014486	0.000001246	0.000013240
14000	0.000001945	0.000000156	0.000001953
30000	0.000000214	0.000000016	0.000000198

5. Conclusions

A randomness based approach of analysing the uncertainty with the help of Shannon measure has been explained in this article. The Shannon measure defining entropy requires a probability law to be defined first. According to the Randomness-Fuzziness Consistency Principle as there are two laws of randomness defined for every normal law of fuzziness, we can therefore have a pair of Shannon entropies. Here, the uncertainty measures of atmospheric dispersion are considered for different downwind distances for the same set of fuzziness of input parameters in all the cases. For various values of distance along the downwind direction we can accordingly find out the pairs of Shannon entropies from the fuzzy membership functions of pollutant concentration of the Gaussian plume model. The Shannon entropies both for the left reference functions and for the right reference functions concerned decrease exponentially with increase in the values of distance along the downwind direction when we used the Gaussian plume model. We have seen that the spread of fuzziness given by the length of fuzzy intervals concerned defining the concentration of pollutant decreases with increase in downwind distance. Therefore, we can conclude that effect of fuzziness of the parameters concerned on concentration of pollutant reduces with increase in downwind distance. In other words, when downwind distance is considerably long, concentration of pollutant does not depend on whether the parameters concerned are fuzzy or not.

Acknowledgements. This work was carried out under a research project approved by the Board of Research in Nuclear Sciences, Bhabha Atomic Research Centre, Mumbai, sponsored by the Department of Atomic Energy, Government of India.

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