Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 5, No. 2, (March 2013), pp. 399–416
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr

Intuitionistic fuzzy G_δ-α-locally continuous functions

R. Narmada Devi, E. Roja, M. K. Uma

Received 4 May 2012; Accepted 17 July 2012

Abstract. The purpose of this paper is to introduce the concepts of an intuitionistic fuzzy G_δ set and intuitionistic fuzzy G_δ-α-locally closed sets. The concepts of an intuitionistic fuzzy εG_δ-α-locally quasi neighbourhood, intuitionistic fuzzy G_δ-α-locally continuous function, intuitionistic fuzzy G_δ-α-local T_2 space, intuitionistic fuzzy G_δ-α-local Urysohn space, intuitionistic fuzzy G_δ-α-local connected space and intuitionistic fuzzy G_δ-α-local compact space are introduced and interesting properties are established.

2010 AMS Classification: 54A40, 03E72

Keywords: Intuitionistic fuzzy G_δ set, Intuitionistic fuzzy G_δ-α-locally closed set, Intuitionistic fuzzy ε G_δ-α-locally quasi neighbourhood, Intuitionistic fuzzy G_δ-α-locally continuous function, Intuitionistic fuzzy G_δ-α-local T_2 space, Intuitionistic fuzzy G_δ-α-local Urysohn space, Intuitionistic fuzzy G_δ-α-local connected space and intuitionistic fuzzy G_δ-α-local compact space.

Corresponding Author: R. Narmada Devi (narmadadevi23@gmail.com)

1. Introduction

The concept of fuzzy sets was introduced by Zadeh [8] and later Atanassov [1] generalized the idea to intuitionistic fuzzy sets. On the otherhand, Coker [3] introduced the notions of an intuitionistic fuzzy topological spaces, intuitionistic fuzzy continuity and some other related concepts. The concept of an intuitionistic fuzzy α-closed set was introduced by H. Gurcay and D. Coker [6]. Ganster and Relly used locally closed sets [5] to define LC-continuity and LC-irresoluteness. G. Balasubramanian [2] introduced and studied the concept of fuzzy G_δ set in a fuzzy topological space. In this paper, the concepts of an intuitionistic fuzzy G_δ-α-locally closed set, intuitionistic fuzzy ε G_δ-α-locally quasi neighbourhood, intuitionistic fuzzy G_δ-α-locally continuous function, intuitionistic fuzzy G_δ-α-local T_2 space, intuitionistic fuzzy G_δ-α-local Urysohn space are introduced and studied. Some interesting properties among continuous functions are discussed. We also investigate some interesting properties of an intuitionistic fuzzy G_δ-α-local connected space and intuitionistic fuzzy G_δ-α-local compact space.

The concept of fuzzy sets was introduced by Zadeh [8] and later Atanassov [1] generalized the idea to intuitionistic fuzzy sets. On the otherhand, Coker [3] introduced the notions of an intuitionistic fuzzy topological spaces, intuitionistic fuzzy continuity and some other related concepts. The concept of an intuitionistic fuzzy α-closed set was introduced by H. Gurcay and D. Coker [6]. Ganster and Relly used locally closed sets [5] to define LC-continuity and LC-irresoluteness. G.Balasubramanian [2] introduced and studied the concept of fuzzy G_δ set in a fuzzy topological space. In this paper, the concepts of an intuitionistic fuzzy G_δ-α-locally closed set, intuitionistic fuzzy ε G_δ-α-locally quasi neighbourhood, intuitionistic fuzzy G_δ-α-locally continuous function, intuitionistic fuzzy G_δ-α-local T_2 space, intuitionistic fuzzy G_δ-α-local Urysohn space are introduced and studied. Some interesting properties among continuous functions are discussed. We also investigate some interesting properties of an intuitionistic fuzzy G_δ-α-local connected space and intuitionistic fuzzy G_δ-α-local compact space.

2. Preliminaries

Definition 2.1 ([1]). Let X be a nonempty fixed set and I be the closed interval [0,1]. An intuitionistic fuzzy set(IFS) A is an object of the following form $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$, where the mappings $\mu_A : X \longrightarrow I$ and $\gamma_A : X \longrightarrow I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\gamma_A(x)$) for each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each $x \in X$. Obviously,every fuzzy set A on a nonempty set X is an IFS of the following form, $A = \{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle : x \in X\}$. For the sake of simplicity, we shall use the symbol $A = \langle x, \mu_A, \gamma_A\rangle$ for the intuitionistic fuzzy set $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$.

Definition 2.2 ([1]). Let X be a nonempty set and the IFSs A and B in the form $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$, $B = \{\langle x, \mu_B(x), \gamma_B(x)\rangle : x \in X\}$. Then

(i) $A \subseteq B$ iff $\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for all $x \in X$;
(ii) $\overline{A} = \{\langle x, \gamma_A(x), \mu_A(x)\rangle : x \in X\}$;
(iii) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \gamma_A(x) \vee \gamma_B(x)\rangle : x \in X\}$;
(iv) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \gamma_A(x) \wedge \gamma_B(x)\rangle : x \in X\}$.

Definition 2.3 ([1]). The IFSs $0_\sim$ and $1_\sim$ are defined by $0_\sim = \{\langle x, 0, 1\rangle : x \in X\}$ and $1_\sim = \{\langle x, 1, 0\rangle : x \in X\}$.

Definition 2.4 ([3]). An intuitionistic fuzzy topology (IFT) in Coker's sense on a nonempty set X is a family τ of IFSs in X satisfying the following axioms:

(i) $0_\sim, 1_\sim \in \tau$;
(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$;
(iii) $\cup G_i \in \tau$ for arbitrary family $\{G_i \mid i \in I\} \subseteq \tau$.

In this paper by (X, τ) or simply by X we will denote the Coker's intuitionistic fuzzy topological space (IFTS). Each IFS in τ is called intuitionistic fuzzy open set

(IFOS) in X. The complement $\overline{A}$ of an IFOS A in X is called an intuitionistic fuzzy closed set (IFCS) in X.

Definition 2.5 ([3]). Let A be an IFS in IFTS X. Then

$int(A) = \bigcup\{G \mid G$ is an IFOS in X and $G \subseteq A\}$ is called an intuitionistic fuzzy interior of A;

$clA = \bigcap\{G \mid G$ is an IFCS in X and $G \supseteq A\}$ is called an intuitionistic fuzzy closure of A.

Proposition 2.6 ([1]). *For any IFS A in (X, τ) we have*

(i) $cl(\overline{A}) = \overline{int(A)}$

(ii) $int(\overline{A}) = \overline{cl(A)}$

Corollary 2.7 ([3]). *Let $A, A_i (i \in J)$ be IFSs in $X, B, B_j (j \in K)$ IFSs in Y and $f : X \to Y$ a function. Then*

(i) $A \subseteq f^{-1}(f(A))$ (If f is injective, then $A = f^{-1}(f(A))$),

(ii) $f(f^{-1}(B)) \subseteq B$ (If f is surjective, then $f(f^{-1}(B)) = B$),

(iii) $f^{-1}(\bigcup B_j) = \bigcup f^{-1}(B_j)$,

(iv) $f^{-1}(\bigcap B_j) = \bigcap f^{-1}(B_j)$,

(v) $f^{-1}(1_\sim) = 1_\sim$,

(vi) $f^{-1}(0_\sim) = 0_\sim$,

(vii) $f^{-1}(\overline{B}) = \overline{f^{-1}(B)}$.

Definition 2.8 ([4]). Let X be a nonempty set and $x \in X$ a fixed element in X. If $r \in I_0$, $s \in I_1$ are fixed real numbers such that $r + s \leq 1$, then the IFS $x_{r,s} = \langle x, x_r, 1 - x_{1-s}\rangle$ is called an intuitionistic fuzzy point(IFP) in X, where r denotes the degree of membership of $x_{r,s}$, s denotes the degree of nonmembership of $x_{r,s}$ and $x \in X$ the support of $x_{r,s}$. The IFP $x_{r,s}$ is contained in the IFS $A(x_{r,s} \in A)$ if and only if $r < \mu_A(x), s > \gamma_A(x)$.

Definition 2.9 ([6]). Let A be an IFS of an IFTS X. Then A is called an intuitionistic fuzzy α-open set(IFαOS) if $A \subseteq int(cl(int(A)))$. The complement of an intuitionistic fuzzy α-open set is called an intuitionistic fuzzy α-closed set(IFαCS).

Definition 2.10 ([7]). An IFS U of an IFTS X is called

(i) ε-nbd of an IFP $c(a, b)$, if there exists an IFOS G in X such that $c(a, b) \in G \leq U$.

(ii) εq-nbd of an IFP $c(a, b)$, if there exists an IFOS G in X such that $c(a, b)qG \leq U$.

Definition 2.11 ([3]). Let X and Y be two nonempty sets and $f : X \to Y$ be a function.

(i) If $B = \{\langle y, \mu_B(y), \gamma_B(y)\rangle : y \in Y\}$ is an IFS in Y, then the preimage of B under f (denoted by $f^{-1}(B)$) is defined by

$$f^{-1}(B) = \{\langle x, f^{-1}(\mu_B)(x), f^{-1}(\gamma_B)(x)\rangle : x \in X\}.$$

(ii) If $A = \{\langle x, \lambda_A(x), \vartheta_A(x)\rangle : x \in X\}$ is an IFS in X, then the image of A under f (denoted by $f(A)$) is defined by

$$f(A) = \{\langle y, f(\lambda_A(y)), (1 - f(1 - \vartheta_A))(y)\rangle : y \in Y\}.$$

Definition 2.12 ([3]). Let (X, τ) and (Y, ϕ) be two IFTSs and let $f : X \to Y$ be a function. Then f is said to be intuitionistic fuzzy continuous iff the preimage of each IFS in ϕ is an IFS in τ.

Definition 2.13 ([7]). Let $f : X \to Y$ be a function. The graph $g : X \to X \times Y$ of f is defined by: $g(x) = (x, f(x))\ \forall x \in X$.

Definition 2.14 ([3]). An intuitionistic fuzzy topological space (X, ψ) is called IFT_2 space iff for every IFPs $c(a, b)$ and $d(m, n)$ in X and $c \neq d$, there exist $G = \langle x, \mu_G, \gamma_G \rangle$, $H = \langle x, \mu_H, \gamma_H \rangle \in \psi$ with $\mu_G(c) = 0$, $\gamma_G(c) = 1$, $\mu_H(d) = 1$, $\gamma_H(d) = 0$ and $G \wedge H = 0_\sim$.

Definition 2.15 ([3]). Let (X, τ) be an IFTS. If a family $\{\langle x, \mu_{G_i}, \gamma_{G_i} \rangle : i \in J\}$ of IFOSs in X satisfies the condition $\bigcup\{\langle x, \mu_{G_i}, \gamma_{G_i} \rangle : i \in J\} = 1_\sim$ then it is called a fuzzy open cover of X.

Definition 2.16 ([3]). An IFTS (X, τ) is called fuzzy compact iff every fuzzy open cover of X has a finite subcover.

Definition 2.17 ([2]). Let (X, T) be a fuzzy topological space and λ be a fuzzy set in X. Then λ is called fuzy G_δ if $\lambda = \wedge_{i=1}^{\infty} \lambda_i$ where each $\lambda_i \in T$. The complement of fuzzy G_δ is fuzzy F_σ.

Definition 2.18 ([5]). A subset A of a space (X, τ) is called locally closed (briefly lc) if $A = C \cap D$,where C is open and D is closed in (X, τ).

3. Properties of an intuitionistic fuzzy G_δ-α-locally continuous functions

Definition 3.1. Let (X, T) be an intuitionistic fuzzy topological space. Let $A = \langle x, \mu_A, \gamma_A \rangle$ be an intuitionistic fuzzy set on an intuitionistic fuzzy topological space (X, T). Then A is said be an intuitionistic fuzzy locally closed set (in short, $IFlcs$) if $A = B \cap C$, where B is an intuitionistic fuzzy open set and C is an intuitionistic fuzzy closed set. The complement of an intuitionistic fuzzy locally closed set is said to be an intuitionistic fuzzy locally open set (in short, $IFlos$).

Definition 3.2. Let (X, T) be an intuitionistic fuzzy topological space. Let $A = \langle x, \mu_A, \gamma_A \rangle$ be an intuitionistic fuzzy set on an intuitionistic fuzzy topological space (X, T). Then A is said be an intuitionistic fuzzy G_δ set if $A = \bigcap_{i=1}^{\infty} A_i$, where $A_i = \langle x, \mu_{A_i}, \gamma_{A_i} \rangle$ is an intuitionistic fuzzy open set in an intuitionistic fuzzy topological space (X, T). The complement of an intuitionistic fuzzy G_δ set is said to be an intuitionistic fuzzy F_σ set.

Definition 3.3. Let (X, T) be an intuitionistic fuzzy topological space. Let $A = \langle x, \mu_A, \gamma_A \rangle$ be an intuitionistic fuzzy set on an intuitionistic fuzzy topological space (X, T). Then A is said be an intuitionistic fuzzy G_δ-locally closed set (in short, IFG_δ-lcs) if $A = B \cap C$, where B is an intuitionistic fuzzy G_δ set and C is an intuitionistic fuzzy closed set. The complement of an intuitionistic fuzzy G_δ-locally closed set is said to be an intuitionistic fuzzy G_δ-locally open set (in short, IFG_δ-los).

Definition 3.4. Let (X, T) be an intuitionistic fuzzy topological space. Let $A = \langle x, \mu_A, \gamma_A \rangle$ be an intuitionistic fuzzy set on an intuitionistic fuzzy topological space (X, T). Then A is said be an intuitionistic fuzzy G_δ-α-locally closed set (in short, $IF\ G_\delta$-α-lcs) if $A = B \cap C$, where B is an intuitionistic fuzzy G_δ set and C is an intuitionistic fuzzy α-closed set. The complement of an intuitionistic fuzzy G_δ-α-locally closed set is said to be an intuitionistic fuzzy G_δ-α-locally open set (in short, $IF\ G_\delta$-α-los).

Definition 3.5. Let (X, T) be an intuitionistic fuzzy topological space. Let $A = \langle x, \mu_A, \gamma_A \rangle$ be an intuitionistic fuzzy set on an intuitionistic fuzzy topological space (X, T). The intuitionistic fuzzy G_δ-α-locally closure of A is denoted and defined by IFG_δ-α-$lcl(A)$=$\bigcap\{B{:}B = \langle x, \mu_B, \gamma_B \rangle$ is an intuitionistic fuzzy G_δ-α-locally closed set in X and $A \subseteq B\}$.

Definition 3.6. Let (X, T) be an intuitionistic fuzzy topological space. Let $A = \langle x, \mu_A, \gamma_A \rangle$ be an intuitionistic fuzzy set on an intuitionistic fuzzy topological space (X, T). The intuitionistic fuzzy G_δ-α-locally interior of A is denoted and defined by IFG_δ-α-$lint(A)$=$\bigcup\{B{:}B = \langle x, \mu_B, \gamma_B \rangle$ is an intuitionistic fuzzy G_δ-α-locally open set in X and $B \subseteq A\}$.

Remark 3.7.
(i) IFG_δ-α-$lcl(A) = A$ if and only if A is an intuitionistic fuzzy G_δ-α-locally closed set.
(ii) IFG_δ-α-$lint(A) \subseteq A \subseteq IFG_\delta$-$\alpha$-$lcl(A)$.
(iii) IFG_δ-α-$lint(1_\sim) = 1_\sim$
(iv) IFG_δ-α-$lint(0_\sim) = 0_\sim$
(v) IFG_δ-α-$lcl(1_\sim) = 1_\sim$

Definition 3.8. Let (X, T) be an intuitionistic fuzzy topological space. Let $A = \langle x, \mu_A, \gamma_A \rangle$ be an intuitionistic fuzzy set in an intuitionistic fuzzy topological space (X, T). Then A is said to be an intuitionistic fuzzy ε G_δ-α-locally neighbourhood of an intuitionistic fuzzy point $x_{r,s}$ if there exists an intuitionistic fuzzy G_δ-α-locally open set B in an intuitionistic fuzzy topological space (X, T) such that $x_{r,s} \in B$, $B \subseteq A$. It is denoted by $IF\varepsilon G_\delta$-α-$lnbd$.

Definition 3.9. Let (X, T) be an intuitionistic fuzzy topological space. Let $A = \langle x, \mu_A, \gamma_A \rangle$ be an intuitionistic fuzzy set in an intuitionistic fuzzy topological space (X, T). Then A is said to be an intuitionistic fuzzy ε G_δ-α-locally quasi neighbourhood of an intuitionistic fuzzy point $x_{r,s}$ if there exists an intuitionistic fuzzy G_δ-α-locally open set B in an intuitionistic fuzzy topological space (X, T) such that $x_{r,s} q B$, $B \subseteq A$. It is denoted by $IF\varepsilon G_\delta$-α-$lqnbd$.

Remark 3.10.
(i) The family of all intuitionistic fuzzy ε G_δ-α-locally neighbourhood of an intuitionistic fuzzy point $x_{r,s}$ is denoted by $N_\varepsilon^{IFG_\delta\text{-}\alpha\text{-}l}(x_{r,s})$.
(ii) The family of all intuitionistic fuzzy ε G_δ-α-locally quasi neighbourhood of an intuitionistic fuzzy point $x_{r,s}$ is denoted by $N_\varepsilon^{IFG_\delta\text{-}\alpha\text{-}lq}(x_{r,s})$.

Definition 3.11. Let (X, T) and (Y, S) be any two intuitionistic fuzzy topological spaces. Let $f : (X, T) \to (Y, S)$ be an intuitionistic fuzzy mapping. Then f is said to be an intuitionistic fuzzy G_δ-α-locally continuous function, if for each intuitionistic fuzzy point $x_{r,s}$ in X and $B \in N_\varepsilon f(x_{r,s})$, there exists $A \in N_\varepsilon^{IFG_\delta\text{-}\alpha\text{-}lq}(x_{r,s})$ such that $f(A) \subseteq B$.

Proposition 3.12. *Let (X,T) and (Y,S) be any two intuitionistic fuzzy topological spaces. Let $f : (X,T) \to (Y,S)$ be an intuitionistic fuzzy mapping. Then the following are equivalent.*

(i) *f is an intuitionistic fuzzy G_δ-α-locally continuous function.*

(ii) *$f^{-1}(A)$ is an intuitionistic fuzzy G_δ-α-locally open set in an intuitionistic fuzzy topological space (X,T), for each intuitionistic fuzzy open set A in an intuitionistic fuzzy topological space (Y,S).*

(iii) *$f^{-1}(B)$ is an intuitionistic fuzzy G_δ-α-locally closed set in an intuitionistic fuzzy topological space (X,T), for each intuitionistic fuzzy closed set B in an intuitionistic fuzzy topological space (Y,S).*

(iv) *IFG_δ-α-$lcl(f^{-1}(A)) \subseteq f^{-1}(IFcl(A))$, for each intuitionistic fuzzy set A in an intuitionistic fuzzy topological space (Y,S).*

(v) *$f^{-1}(IFint(A)) \subseteq IFG_\delta$-$\alpha$-$lint(f^{-1}(A))$, for each intuitionistic fuzzy set A in an intuitionistic fuzzy topological space (Y,S).*

Proof.

(i)⇒(ii) Let A be an intuitionistic fuzzy open set in an intuitionistic fuzzy topological space (Y,S). Let $x_{r,s}$ be an intuitionistic fuzzy point in an intuitionistic fuzzy topological space (X,T) such that $x_{r,s} q f^{-1}(A)$. Since f is an intuitionistic fuzzy G_δ-α-locally continuous function, there exists $B \in N_\varepsilon^{IFG_\delta\text{-}\alpha\text{-}lq}(x_{r,s})$ such that $f(B) \subseteq A$. Then

$$x_{r,s} \in B \tag{3.1}$$

$$B \subseteq f^{-1}(f(B)). \tag{3.2}$$

Thus, $x_{r,s} \in B \subseteq f^{-1}(f(B)) \subseteq f^{-1}(A)$. This implies $x_{r,s} \in f^{-1}(A)$ which shows that $f^{-1}(A) \in N_\varepsilon^{IFG_\delta\text{-}\alpha\text{-}lq}(x_{r,s})$. Hence $f^{-1}(A)$ is an intuitionistic fuzzy G_δ-α-locally open set in an intuitionistic fuzzy topological space (X,T).

(ii)⇒(i) This can be proved by taking complement of (i)

(iii)⇒(iv) Let A be an intuitionistic fuzzy set in an intuitionistic fuzzy topological space (Y,S). Since $A \subseteq IFcl(A)$, $f^{-1}(A) \subseteq f^{-1}(IFcl(A))$. By (iii), $f^{-1}(IFcl(A))$ is an intuitionistic fuzzy G_δ-α-locally closed set in an intuitionistic fuzzy topological space (X,T). Thus, IFG_δ-α-$lcl(f^{-1}(A)) \subseteq f^{-1}(IFcl(A))$.

(iv)⇒(v) Using (iv), IFG_δ-α-$lcl(f^{-1}(A)) \subseteq f^{-1}(IFcl(A))$. Then,

$$\overline{IFG_\delta\text{-}\alpha\text{-}lcl(f^{-1}(A))} \supseteq \overline{f^{-1}(IFcl(A))}$$
$$IFG_\delta\text{-}\alpha\text{-}lint(\overline{f^{-1}(A)}) \supseteq f^{-1}(IFint(\overline{A}))$$
$$IFG_\delta\text{-}\alpha\text{-}lint(f^{-1}(\overline{A})) \supseteq f^{-1}(IFint(\overline{A}))$$

implies that $f^{-1}(IFint(\overline{A})) \subseteq IFG_\delta$-$\alpha$-$lint(\overline{f^{-1}(A)})$ putting $\overline{A} = A$, we have

$$f^{-1}(IFint(A)) \subseteq IFG_\delta\text{-}\alpha\text{-}lint(f^{-1}(A)).$$

(v)⇒(i) Let A be an intuitionistic fuzzy open set in an intuitionistic fuzzy topological space (Y,S). Then $IFintA = A$. Using (v), $f^{-1}(IFint(A)) \subseteq IFG_\delta$-$\alpha$-$lint(f^{-1}(A))$ implies that $f^{-1}(A) \subseteq IFG_\delta$-$\alpha$-$lint(f^{-1}(A))$. But, $IFG_\delta - \alpha -$

$lint(f^{-1}(A)) \subseteq f^{-1}(A)$ implies that $f^{-1}(A) = IFG_\delta$-α-$lint(f^{-1}(A))$ that is, $f^{-1}(A)$ is an intuitionistic fuzzy G_δ-α-locally open set in an intuitionistic fuzzy topological space (X, T). Let $x_{r,s}$ be any intuitionistic fuzzy point in $f^{-1}(A)$. Then $x_{r,s} \in f^{-1}(A)$.We have $x_{r,s}qf^{-1}(A)$ implies that $f(x_{r,s})qf(f^{-1}(A))$.But $f(f^{-1}(A)) \subseteq A$. Thus,for any intuitionistic fuzzy point $x_{r,s}$ and $A \in N_\varepsilon f(x_{r,s})$, there exists $B = f^{-1}(A) \in N_\varepsilon^{IFG_\delta\text{-}\alpha\text{-}lq}(x_{r,s})$ such that $f^{-1}(f(A)) \subseteq A$. Therefore, $f(B) \subseteq A$. Thus, f is an intuitionistic fuzzy G_δ-α-locally continuous function. □

Proposition 3.13. *Let (X, T) and (Y, S) be any two intuitionistic fuzzy topological spaces. Let $f : (X, T) \to (Y, S)$ be an intuitionistic fuzzy bijective function. Then f is an intuitionistic fuzzy G_δ-α-locally continuous function if and only if $IFint(f(A)) \subseteq f(IFG_\delta$-$\alpha$-$lint(A))$, for each intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X, T).*

Proof. Assume that f is an intuitionistic fuzzy G_δ-α-locally continuous function and A be an intuitionistic fuzzy set in an intuitionistic fuzzy topological space (X, T). Hence, $f^{-1}(IFint(f(A))$ is an intuitionistic fuzzy G_δ-α-locally open set in an intuitionistic fuzzy topological space (X, T). From Proposition(v) of (3.1)

$$f^{-1}(IFintf(A)) \subseteq IFG_\delta - \alpha - lint(f^{-1}(f(A)))$$

$$f^{-1}(IFintf(A)) \subseteq IFG_\delta - \alpha - lint(A)$$

Since f is an intuitionistic fuzzy surjective function,

$$f(f^{-1}(IFintf(A))) \subseteq f(IFG_\delta - \alpha - lint(A))$$

That is, $IFint(f(A)) \subseteq f(IFG_\delta$-$\alpha$-$lint(A))$.

Conversely, assume that $IFint(f(A)) \subseteq f(\ IFG_\delta$-$\alpha$-$lint(A))$, for each intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X, T). Let B be an intuitionistic fuzzy open set in an intuitionistic fuzzy topological space (Y, S). Then $B = IFint(B)$. Since f is an intuitionistic fuzzy surjective function,

$$B = IFint(B) = IFint(f(f^{-1}(B))) \subseteq f(IFG_\delta - \alpha - lint(f^{-1}(B)))$$

Since f is an intuitionistic fuzzy injective function,

$$f^{-1}(B) \subseteq f^{-1}(f(IFG_\delta - \alpha - lint(f^{-1}(B))))$$

From the fact that, f is an intuitionistic fuzzy injective function, we have

$$f^{-1}(B) \subseteq IFG_\delta\text{-}\alpha\text{-}lint(f^{-1}(B) \tag{3.3}$$

But

$$IFG_\delta\text{-}\alpha\text{-}lint(f^{-1}(B)) \subseteq f^{-1}(B) \tag{3.4}$$

From (3.3) and (3.4) implies that $f^{-1}(B) = IFG_\delta - \alpha - lint(f^{-1}(B)$ That is, $f^{-1}(B)$ is an intuitionistic fuzzy G_δ-α-locally open set in an intuitionistic fuzzy topological space (X, T). Thus, f is an intuitionistic fuzzy $G_\delta - \alpha$-locally continuous function. □

Proposition 3.14. *Let (X, T) and (Y, S) be any two intuitionistic fuzzy topological spaces. Let $f : (X, T) \to (Y, S)$ be an intuitionistic fuzzy bijective function. Then*

f is an intuitionistic fuzzy G_δ-α-locally continuous function if and only if $f(IFG_\delta$-α-$lcl(A)) \subseteq IFcl(f(A))$, for each intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X,T).

Proof. Assume that f is an intuitionistic fuzzy G_δ-α-locally continuous function and A be an intuitionistic fuzzy set in an intuitionistic fuzzy topological space (X,T). Hence, $f^{-1}(IFcl(f(A)))$ is an intuitionistic fuzzy G_δ-α-locally closed set in an intuitionistic fuzzy topological space (X,T). By Proposition(iv) of (3.1)

$$IFG_\delta - \alpha - lcl(f^{-1}(f(A))) \subseteq f^{-1}(IFclf(A))$$

Since f is an intuitionistic fuzzy injective function, IFG_δ-α-$lcl(A) \subseteq f^{-1}(IFclf(A))$
Taking f on both sides, $f(IFG_\delta$-α-$lcl(A)) \subseteq f(f^{-1}(IFclf(A)))$
Since f is an intuitionistic fuzzy surjective function, $f(IFG_\delta$-α-$lcl(A)) \subseteq IFcl(f(A))$
Conversely, assume that $f(IFG_\delta$-α-$lcl(A)) \subseteq IFcl(f(A))$, for each intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X,T). Let B be an intuitionistic fuzzy closed set in an intuitionistic fuzzy topological space (Y,S). Then $B = IFcl(B)$. Since f is an intuitionistic fuzzy surjective function and by assumption,

$$B = IFcl(B) = IFcl(f(f^{-1}(B))) \supseteq f(IFG_\delta - \alpha - lcl(f^{-1}(B)))$$
$$f^{-1}(B) \supseteq f^{-1}(f(IFG_\delta - \alpha - lcl(f^{-1}(B))))$$

Since f is an intuitionistic fuzzy injective function,

$$f^{-1}(B) \supseteq IFG_\delta\text{-}\alpha\text{-}lcl(f^{-1}(B)) \tag{3.5}$$

But

$$f^{-1}(B) \subseteq IFG_\delta\text{-}\alpha\text{-}lcl(f^{-1}(B)) \tag{3.6}$$

From (3.5) and (3.6) implies that $f^{-1}(B) = IFG_\delta - \alpha - lcl(f^{-1}(B)$. That is, $f^{-1}(B)$ is an intuitionistic fuzzy $G_\delta - \alpha$-locally closed set in an intuitionistic fuzzy topological space (X,T). Thus, f is an intuitionistic fuzzy G_δ-α-locally continuous function. □

Proposition 3.15. *Let (X,T) and (Y,S) be any two intuitionistic fuzzy topological spaces. Let $f : (X,T) \to (Y,S)$ be an intuitionistic fuzzy bijective function. If f is an intuitionistic fuzzy G_δ-α-locally continuous function. Then if $A \in I^Y$ is an intuitionistic fuzzy closed set, then $f^{-1}(A)$=IFG_δ-α-lcl $(f^{-1}(A))$.*

Proof. Let A be an intuitionistic fuzzy closed set in an intuitionistic fuzzy topological space (Y,S). By Proposition(iv)of (3.1)

$$IFG_\delta\text{-}\alpha\text{-}lcl(f^{-1}(A)) \subseteq f^{-1}(IFcl(A)) = f^{-1}(A) \tag{3.7}$$

Since $A = IFcl(A)$. But

$$f^{-1}(A) \subseteq IFG_\delta\text{-}\alpha\text{-}lcl(f^{-1}(A)) \tag{3.8}$$

From (3.7) and (3.8) implies that $f^{-1}(A)$=IFG_δ-α-lcl $(f^{-1}(A)$ □

Proposition 3.16. *Let (X,T), (Y,S) and (Z,R) be any three intuitionistic fuzzy topological spaces. Let $f : (X,T) \to (Y,S)$ be an intuitionistic fuzzy G_δ-α-locally continuous function. If $f(X) \subset Z \subset Y$ then $g : (X,T) \to (Z,R)$ where $R = S/Z$ restricting the range of f is an intuitionistic fuzzy G_δ-α-locally continuous function.*

Proof. Let B be an intuitionistic fuzzy closed set in an intuitionistic fuzzy topological space (Z, R). Then $B = S/Z$, for some intuitionistic fuzzy closed set A of an intuitionistic fuzzy topological space (Y, S). If $f(X) \subset Z \subset Y, f^{-1}(A) = g^{-1}(B)$. Since $f^{-1}(A)$ is an intuitionistic fuzzy G_δ-α-locally closed set in an intuitionistic fuzzy topological space (X, T). Hence, $g^{-1}(B)$ is an intuitionistic fuzzy G_δ-α-locally closed set in an intuitionistic fuzzy topological space (X, T). Therefore, g is an intuitionistic fuzzy G_δ-α-locally continuous function. □

Proposition 3.17. *Let (X, T), (X_1, T_1) and (X_2, T_2) be any three intuitionistic fuzzy topological spaces and $P_i : X_1 \times X_2 \to X_i$ be an intuitionistic fuzzy projection of $X_1 \times X_2$ onto X_i. If $f : X \to X_1 \times X_2$ is an intuitionistic fuzzy G_δ-α-locally continuous function. Then $P_i \circ f : X \to X_i$ is also an intuitionistic fuzzy G_δ-α-locally continuous function.*

Proof. Let A be an intuitionistic fuzzy closed set in an intuitionistic fuzzy topological spaces (X_i, T_i) $(i = 1, 2)$, $(P_i \circ f)^{-1}(A) = f^{-1}(P_i^{-1}(A))$. Since P_i is an intuitionistic fuzzy mapping $P_i^{-1}(A)$ is an intuitionistic fuzzy closed set in an intuitionistic fuzzy topological spaces $X_1 \times X_2$. Hence, $f^{-1}(P_i^{-1}(A))$ is an intuitionistic fuzzy G_δ-α-locally closed set in an intuitionistic fuzzy topological space (X, T). Hence, $P_i \circ f$ is an intuitionistic fuzzy G_δ-α-locally continuous function. □

Proposition 3.18. *Let (X, T) and (Y, S) be any two intuitionistic fuzzy topological spaces. If intuitionistic fuzzy graph function $g : X \to X \times Y$ is an intuitionistic fuzzy G_δ-α-locally continuous function. Then $f : (X, T) \to (Y, S)$ is an intuitionistic fuzzy G_δ-α-locally continuous function.*

Proof. Let g be an intuitionistic fuzzy G_δ-α-locally continuous function and $x_{r,s}$ be any intuitionistic fuzzy point in an intuitionistic fuzzy topological space (X, T).

If $B \in N_\varepsilon^{IFG_\delta\text{-}\alpha\text{-}lq} f(x_{r,s})$ in an intuitionistic fuzzy topological space (Y, S), $X \times B \in N_\varepsilon^{IFG_\delta\text{-}\alpha\text{-}lq} g(x_{r,s})$ in an intuitionistic fuzzy topological space $X \times Y$. Since g is an intuitionistic fuzzy G_δ-α-locally continuous function, there exists $A \in N_\varepsilon(x_{r,s})$ such that $g(A) \subseteq X \times B$. By Definition 3.9, we have $f(A) \subseteq B$. Therefore, f is an intuitionistic fuzzy G_δ-α-locally continuous function. □

Definition 3.19. Let(X, T) and (Y, S) be two intuitionistic fuzzy topological spaces. Let $f : (X, T) \to (Y, S)$ be an intuitionistic fuzzy mapping. Then f is said to be an

(i) intuitionistic fuzzy G_δ-α-locally irresolute function, if for each intuitionistic fuzzy G_δ-α-locally closed set A in an intuitionistic fuzzy topological space (Y, S), $f^{-1}(A)$ is an intuitionistic fuzzy G_δ-α-locally closed set in an intuitionistic fuzzy topological space (X, T).

(ii) intuitionistic fuzzy weakly G_δ-α-locally continuous function, if for each intuitionistic fuzzy G_δ-α-locally closed set A in an intuitionistic fuzzy topological space (Y, S), $f^{-1}(A)$ is an intuitionistic fuzzy closed set in an intuitionistic fuzzy topological space (X, T).

Proposition 3.20. *Let (X, T) and (Y, S) be any two intuitionistic fuzzy topological spaces. Let $f : (X, T) \to (Y, S)$ be an intuitionistic fuzzy mapping. Then the following statements are equivalent*

(i) *f is an intuitionistic fuzzy G_δ-α-locally irresolute function.*

(ii) *for every intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X,T), f $(IFG_\delta$-α-$lcl(A)) \subseteq IFG_\delta$-$\alpha$-$lcl(f(A))$.*

(iii) *for every intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (Y,S), IFG_δ-α-$lcl(f^{-1}(A)) \subseteq f^{-1}$ $(IFG_\delta$-α-$lcl(A))$.*

Proof.

(i)⇒(ii) Let A be an intuitionistic fuzzy set in an intuitionistic fuzzy topological space (X,T). Suppose f is an intuitionistic fuzzy G_δ-α-locally irresolute function. Now, IFG_δ-α-$lcl(f(A))$ is an intuitionistic fuzzy G_δ-α-locally closed set in an intuitionistic fuzzy topological space (Y,S). By hypothesis, f^{-1} $(IFG_\delta$-α-$lcl(f(A)))$ is an intuitionistic fuzzy G_δ-α-locally closed set in an intuitionistic fuzzy topological space (X,T) and hence,

$$A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(IFG_\delta - \alpha - lcl(f(A)))$$

Now, IFG_δ-α-$lcl(A) \subseteq f^{-1}$ $(IFG_\delta$-α-$lcl(f(A)))$
That is, f $(IFG_\delta$-α-$lcl(A)) \subseteq IFG_\delta$-$\alpha$-$lcl(f(A))$

(ii)⇒(iii) Let A be an intuitionistic fuzzy set in an intuitionistic fuzzy topological space (Y,S), then $f^{-1}(A)$ is an intuitionistic fuzzy set in an intuitionistic fuzzy topological space (X,T). By (ii),

$$f(IFG_\delta - \alpha - lcl(f^{-1}(A))) \subseteq IFG_\delta - \alpha - lcl(f(f^{-1}(A)))$$

Since f is an intuitionistic fuzzy bijective function,

$$IFG_\delta - \alpha - lcl(f^{-1}(A)) \subseteq f^{-1}(IFG_\delta - \alpha - lcl(A))$$

(iii)⇒(i) Suppose A is intuitionistic fuzzy IFG_δ-α-locally closed set in an intuitionistic fuzzy topological space (Y,S). Then, IFG_δ-α-$lcl(A) = A$. By hypothesis,

$$IFG_\delta - \alpha - lcl(f^{-1}(A)) \subseteq f^{-1}(IFG_\delta - \alpha - lcl(A))$$
$$IFG_\delta - \alpha - lcl(f^{-1}(A)) \subseteq f^{-1}(A).$$

□

Definition 3.21. Let (X,T) and (Y,S) be any two intuitionistic fuzzy topological spaces. Let $f:(X,T)\to(Y,S)$ be an intuitionistic fuzzy mapping. Then f is said to be an

(i) intuitionistic fuzzy G_δ-α-locally function, if for each intuitionistic fuzzy closed set A in an intuitionistic fuzzy topological space (X,T), $f(A)$ is an intuitionistic fuzzy G_δ-α-locally closed set in an intuitionistic fuzzy topological space (Y,S).

(ii) intuitionistic fuzzy strongly G_δ-α-locally function, if for each intuitionistic fuzzy G_δ-α-locally closed set A in an intuitionistic fuzzy topological space (X,T), $f(A)$ is an intuitionistic fuzzy G_δ-α-locally closed set in an intuitionistic fuzzy topological space (Y,S).

Proposition 3.22. *Let (X,T) and (Y,S) be any two intuitionistic fuzzy topological spaces. Let $f:(X,T)\to(Y,S)$ be an intuitionistic fuzzy mapping. Then the following statements are equivalent*

(i) *f is an intuitionistic fuzzy G_δ-α-locally function.*

(ii) *for each intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (Y,S) and each intuitionistic fuzzy closed set B of an intuitionistic fuzzy topological* *space (X,T) with $f^{-1}(A) \subseteq B$, there is an intuitionistic fuzzy G_δ-α-locally closed set C of an intuitionistic fuzzy topological space (Y,S) with $A \subseteq C$ such that $f^{-1}(C) \subseteq B$.*

(iii) *f^{-1} $(IFG_\delta$-α-$lcl(A)) \subseteq IFcl(f^{-1}(A))$, for each intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (Y,S).*

(iv) *$f(IFint(B)) \subseteq IFG_\delta$-$\alpha$-$lint(f(B))$, for each intuitionistic fuzzy set B of an intuitionistic fuzzy topological space (X,T).*

Proof.

(i)⇒(ii) Suppose f is an intuitionistic fuzzy G_δ-α-locally function. Let A be any intuitionistic fuzzy set A in an intuitionistic fuzzy topological space (Y,S). Let B be any intuitionistic fuzzy closed set in an intuitionistic fuzzy topological space (X,T) such that $f^{-1}(A) \subseteq B$. Let $C = \overline{f(\overline{B})}$. Then C is an intuitionistic fuzzy G_δ-α-locally closed set in an intuitionistic fuzzy topological space (Y,S) and $A \subseteq C$. We have, $f^{-1}(C) = f^{-1}(\overline{f(\overline{B})}) = \overline{f^{-1}(f(\overline{B}))} \subseteq B$. Therefore, $f^{-1}(C) \subseteq B$.

(ii)⇒(i) Let D be an intuitionistic fuzzy open set in an intuitionistic fuzzy topological space (X,T). Put $A = \overline{f(D)}$and $B = \overline{D}$. Thus, $f^{-1}(A) = f^{-1}(\overline{f(D)}) = \overline{f^{-1}(f(D))} \subseteq \overline{D}$. By hypothesis, there exists an intuitionistic fuzzy topological space (Y,S) with $A \subseteq C$ such that $f^{-1}(C) \subseteq B = \overline{D}$. Then, $\overline{f^{-1}(C)} \supseteq D \Rightarrow D \subseteq f^{-1}(\overline{C})$. Hence,

$$f(D) \subseteq f(f^{-1}(\overline{C})) \subseteq \overline{C} \tag{3.9}$$

Also, since $A \subseteq C$, we have $\overline{f(D)} \subseteq C$. This implies

$$f(D) \supseteq \overline{C} \tag{3.10}$$

From (3.9) and (3.10), we get $f(D) = \overline{C}$ is an intuitionistic fuzzy G_δ-α-locally closed set in an intuitionistic fuzzy topological space (Y,S). Hence, f is an intuitionistic fuzzy G_δ-α-locally function.

(ii)⇒(iii) Let A be an intuitionistic fuzzy set in an intuitionistic fuzzy topological space (Y,S). Since $IFcl(f^{-1}(A))$ is an intuitionistic fuzzy closed set in an intuitionistic fuzzy topological space (X,T) with $f^{-1}(A) \subseteq IFcl(f^{-1}(A))$. Then by (ii), there is an intuitionistic fuzzy G_δ-α-locally closed set C in an intuitionistic fuzzy topological space (Y,S) with $A \subseteq C$, $f^{-1}(IFG_\delta\text{-}\alpha\text{-}lcl(A)) \subseteq IFG_\delta\text{-}\alpha\text{-}lcl((C)) \subseteq f^{-1}(C) \subseteq IFcl(f^{-1}(A))$. Therefore, f^{-1} $(IFG_\delta\text{-}\alpha\text{-}lcl(A)) \subseteq IFcl(f^{-1}(A))$.

(iii)⇒(iv) f^{-1} $(IFG_\delta\text{-}\alpha\text{-}lcl(A)) \subseteq IFcl(f^{-1}(A))$, for each intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (Y,S). Putting $A = \overline{f(B)}$,

$$\begin{aligned} f^{-1}(IFG_\delta - \alpha - lcl(\overline{f(B)}) &\subseteq IFcl(f^{-1}(\overline{f(B)})) \\ &\subseteq IFcl(f^{-1}(f(\overline{B}))) \subseteq IFcl(\overline{B}) \\ &\subseteq \overline{IFint(B)} \\ f^{-1}(\overline{IFG_\delta\text{-}\alpha\text{-}lint(f(B))}) &\subseteq \overline{IFint(B)} \end{aligned}$$

Taking complement on both sides,

$$\begin{aligned} \overline{f^{-1}(\overline{IFG_\delta\text{-}\alpha\text{-}lint(f(B))})} &\supseteq \overline{\overline{IFint(B)}} \\ f^{-1}(IFG_\delta - \alpha - lint(f(B))) &\supseteq IFint(B) \end{aligned}$$

Therefore, $f(IFint(B)) \subseteq IFG_\delta\text{-}\alpha\text{-}lint(f(B))$.

(iv)⇒(i). It is obvious. □

Definition 3.23. Let (X,T) and (Y,S) be any two intuitionistic fuzzy topological spaces. Let $f:(X,T)\to(Y,S)$ be an intuitionistic fuzzy mapping. Then f is said to be an intuitionistic fuzzy G_δ-α-locally homeomorphism if f is one to one, onto, intuitionistic fuzzy G_δ-α-locally irresolute function and intuitionistic fuzzy strongly G_δ-α-locally function.

Proposition 3.24. *Let (X,T) and (Y,S) be any two intuitionistic fuzzy topological spaces. If $f:(X,T)\to(Y,S)$ is an intuitionistic fuzzy G_δ-α-locally homeomorphism. Then the following statements are valid.*

(i) *For any intuitionistic fuzzy set A in an intuitionistic fuzzy topological space (X,T),*

$$IFG_\delta-\alpha-lcl(f(A)) = f(IFG_\delta-\alpha-lcl(A))$$

(ii) *For any intuitionistic fuzzy set A in an intuitionistic fuzzy topological space (X,T),*

$$f(\overline{IFG_\delta\text{-}\alpha\text{-}lint(\overline{A})}) = \overline{IFG_\delta\text{-}\alpha\text{-}lint(f(\overline{A}))}$$

(iii) *For any intuitionistic fuzzy set A in an intuitionistic fuzzy topological space (Y,S),*

$$IFG_\delta-\alpha-lcl(f^{-1}(A)) = f^{-1}(IFG_\delta-\alpha-lcl(A))$$

(iv) *For any intuitionistic fuzzy set A in an intuitionistic fuzzy topological space (Y,S),*

$$f^{-1}(\overline{IFG_\delta\text{-}\alpha\text{-}lint(\overline{A})}) = \overline{IFG_\delta\text{-}\alpha\text{-}lint(f^{-1}(\overline{A}))}$$

Proof.

(i) Let A be any intuitionistic fuzzy set in an intuitionistic fuzzy topological space (X,T). Since f is an intuitionistic fuzzy G_δ-α-locally irresolute function. By (ii) and (iii) of Proposition(3.8)

$$f(IFG_\delta\text{-}\alpha\text{-}lcl(A)) \subseteq IFG_\delta\text{-}\alpha\text{-}lcl(f(A)) \tag{3.11}$$

$$IFG_\delta\text{-}\alpha\text{-}lclf(A) \subseteq f(IFG_\delta\text{-}\alpha\text{-}lcl(A)) \tag{3.12}$$

From (3.11) and (3.12) implies that $IFG_\delta\text{-}\alpha\text{-}lclf(A) = f(IFG_\delta\text{-}\alpha\text{-}lcl(A))$

(ii) Let A be any intuitionistic fuzzy set in an intuitionistic fuzzy topological space (X,T). Since f is an intuitionistic fuzzy G_δ-α-locally homeomorphism. Then by above condition (ii), $IFG_\delta\text{-}\alpha\text{-}lclf(A) = f(IFG_\delta\text{-}\alpha\text{-}lcl(A))$. Now,

$$\overline{IFG_\delta\text{-}\alpha\text{-}lint(\overline{f(A)})} = \overline{f(IFG_\delta\text{-}\alpha\text{-}lint(\overline{A})}$$

$$f(\overline{IFG_\delta\text{-}\alpha\text{-}lint(\overline{A})}) = \overline{IFG_\delta\text{-}\alpha\text{-}lint(f(\overline{A}))}$$

(iii) Let A be any intuitionistic fuzzy set in an intuitionistic fuzzy topological space (Y,S). Since f is an intuitionistic fuzzy G_δ-α-locally homeomorphism. Since f is an intuitionistic fuzzy strongly G_δ-α-locally function. Also, f^{-1} is an intuitionistic fuzzy G_δ-α-locally irresolute function. By (ii) and (iii) of Proposition(3.8)

$$IFG_\delta\text{-}\alpha\text{-}lcl(f^{-1}(A)) \subseteq f^{-1}(IFG_\delta\text{-}\alpha\text{-}lcl(A)) \tag{3.13}$$

$$f^{-1}(IFG_\delta\text{-}\alpha\text{-}lcl(A)) \subseteq IFG_\delta\text{-}\alpha\text{-}lcl(f^{-1}(A)) \tag{3.14}$$

From (3.13) and (3.14) implies that $IFG_\delta\text{-}\alpha\text{-}lcl(f^{-1}(A)) = f^{-1}(IFG_\delta\text{-}\alpha\text{-}lcl(A))$

(iv) Let A be any intuitionistic fuzzy set in an intuitionistic fuzzy topological space (Y, S). Since f is an intuitionistic fuzzy G_δ-α-locally homeomorphism. Then by above condition (iii), $IFG_\delta\text{-}\alpha\text{-}lcl(f^{-1}(A)) = f^{-1}(IFG_\delta\text{-}\alpha\text{-}lcl(A))$
Taking complement on both sides,

$$\overline{IFG_\delta\text{-}\alpha\text{-}lint(\overline{f^{-1}(A)})} = \overline{f^{-1}(IFG_\delta\text{-}\alpha\text{-}lint(\overline{A}))}$$

$$f^{-1}(\overline{IFG_\delta\text{-}\alpha\text{-}lint(\overline{A})}) = \overline{IFG_\delta\text{-}\alpha\text{-}lint(f^{-1}(\overline{A}))}$$

□

Proposition 3.25. *Let (X, T), (Y, S) and (Z, R) be any three intuitionistic fuzzy topological spaces. If $f : (X, T) \to (Y, S)$ and $g : (Y, S) \to (Z, R)$ be any two intuitionistic fuzzy mappings. Then the following statements are valid.*

(i) *If f is an intuitionistic fuzzy G_δ-α-locally irresolute function and g is an intuitionistic fuzzy G_δ-α-locally continuous function, then $g \circ f$ is an intuitionistic fuzzy G_δ-α-locally continuous function.*

(ii) *If f is an intuitionistic fuzzy G_δ-α-locally continuous function and g is an intuitionistic fuzzy weakly G_δ-α-locally continuous function, then $g \circ f$ is an intuitionistic fuzzy G_δ-α-locally irresolute function.*

Proof.

(i) Let A be any intuitionistic fuzzy closed set in an intuitionistic fuzzy topological space (Z, R). Since g is an intuitionistic fuzzy G_δ-α-locally continuous function, $g^{-1}(A)$ is an intuitionistic fuzzy G_δ-α-locally closed set in an intuitionistic fuzzy topological space (Y, S). Since f is an intuitionistic fuzzy G_δ-α-locally irresolute function, $f^{-1}(g^{-1}(A))$ is an intuitionistic fuzzy G_δ-α-locally closed set in an intuitionistic fuzzy topological space (X, T). Now, $(g \circ f)^{-1}(A) = f^{-1}(g^{-1}(A))$ is an intuitionistic fuzzy G_δ-α-locally closed set in an intuitionistic fuzzy topological space (X, T). Hence, $g \circ f$ is an intuitionistic fuzzy G_δ-α-locally continuous function.

(ii) Let A be any intuitionistic fuzzy G_δ-α-locally closed set in an intuitionistic fuzzy topological space (Z, R). Since g is an intuitionistic fuzzy weakly G_δ-α-locally continuous function, $g^{-1}(A)$ is an intuitionistic fuzzy closed set in an intuitionistic fuzzy topological space (Y, S). Since f is an intuitionistic fuzzy G_δ-α-locally continuous function, $f^{-1}(g^{-1}(A))$ is an intuitionistic fuzzy G_δ-α-locally closed set in an intuitionistic fuzzy topological space (X, T). Now, $(g \circ f)^{-1}(A) = f^{-1}(g^{-1}(A))$ is an intuitionistic fuzzy G_δ-α-locally closed set in an intuitionistic fuzzy topological space (X, T). Hence, $g \circ f$ is an intuitionistic fuzzy G_δ-α-locally irresolute function. □

Definition 3.26. An intuitionistic fuzzy topological space (X, T) is said to be an intuitionistic fuzzy G_δ-α-local T_2 space if and only if for every intuitionistic fuzzy points $c_{r,s}$ and $d_{m,n}$ in an intuitionistic fuzzy topological space (X, T) and $c \neq d$ there exists an intuitionistic fuzzy G_δ-α-locally open sets $G = \langle x, \mu_G, \gamma_G \rangle$, $H = \langle x, \mu_H, \gamma_H \rangle$ with $\mu_G(c) = 0$, $\gamma_G(c) = 1$, $\mu_H(d) = 1$, $\gamma_H(d) = 0$ and $G \cap H = 0_\sim$.

Proposition 3.27. *Let (X,T) and (Y,S) be any two intuitionistic fuzzy topological spaces. Let $f:(X,T)\to(Y,S)$ be an intuitionistic fuzzy injective mapping and intuitionistic fuzzy G_δ-α-locally continuous function. If (Y,S) is an intuitionistic fuzzy T_2 space, then (X,T) is an intuitionistic fuzzy G_δ-α-local T_2 space.*

Proof. Let $c_{r,s}$ and $d_{m,n}$ be an intuitionistic fuzzy points in an intuitionistic fuzzy topological space (X,T) and $c\neq d$. By intuitionistic fuzzy injective function of f, we have $f(c)\neq f(d)$. Since, (Y,S) is an intuitionistic fuzzy T_2 space, there exists an intuitionistic fuzzy open sets $G=\langle y,\mu_G,\gamma_G\rangle$, $H=\langle y,\mu_H,\gamma_H\rangle$ of S with $\mu_G(f(c))=0$, $\gamma_G(f(c))=1$, $\mu_H(f(d))=1$, $\gamma_H(f(d))=0$ and $G\cap H=0_\sim$. Since f is an intuitionistic fuzzy G_δ-α-locally continuous function. This implies $f^{-1}(G)=\langle x,f^{-1}(\mu_G),f^{-1}(\gamma_G)\rangle$, $f^{-1}(H)=\langle x,f^{-1}(\mu_H),f^{-1}(\gamma_H)\rangle$ are $N_\varepsilon^{IFG_\delta\text{-}\alpha\text{-}lq}(c_{r,s})$ and $N_\varepsilon^{IFG_\delta\text{-}\alpha\text{-}lq}(d_{m,n})$ respectively. That is, $f^{-1}(G)$ and $f^{-1}(H)$ are intuitionistic fuzzy G_δ-α-locally open sets. Now,

$$f^{-1}(\mu_G)(c_{r,s})=\mu_G(f(c))=0$$
$$f^{-1}(\gamma_G)(c_{r,s})=\gamma_G(f(c))=1$$
$$f^{-1}(\mu_H)(d_{m,n})=\mu_H(f(d))=1$$
$$f^{-1}(\gamma_H)(d_{m,n})=\gamma_H(f(d))=0$$

and

$$f^{-1}(G)\cap f^{-1}(H)=f^{-1}(G\cap H)=f^{-1}(0_\sim)=0_\sim.$$

Hence, (X,T) is an intuitionistic fuzzy G_δ-α-local T_2 space. □

Proposition 3.28. *Let (X,T) and (Y,S) be any two intuitionistic fuzzy topological spaces. Let $f:(X,T)\to(Y,S)$ be an intuitionistic fuzzy injective mapping and intuitionistic fuzzy weakly G_δ-α-locally continuous function. If (Y,S) is an intuitionistic fuzzy G_δ-α-local T_2 space, then (X,T) is an intuitionistic fuzzy T_2 space.*

Proof. Let $c_{r,s}$ and $d_{m,n}$ be an intuitionistic fuzzy points in an intuitionistic fuzzy topological space (X,T) and $c\neq d$. By intuitionistic fuzzy injective function of f, we have $f(c)\neq f(d)$. Since, (Y,S) is an intuitionistic fuzzy G_δ-α-local T_2 space, there exists an intuitionistic fuzzy G_δ-α-locally open sets $G=\langle y,\mu_G,\gamma_G\rangle$, $H=\langle y,\mu_H,\gamma_H\rangle$ of S with $\mu_G(f(c))=0$, $\gamma_G(f(c))=1$, $\mu_H(f(d))=1$, $\gamma_H(f(d))=0$ and $G\cap H=0_\sim$. Since f is an intuitionistic fuzzy weakly G_δ-α-locally continuous function. This implies $f^{-1}(G)=\langle x,f^{-1}(\mu_G),f^{-1}(\gamma_G)\rangle$,$f^{-1}(H)=\langle x,f^{-1}(\mu_H),f^{-1}(\gamma_H)\rangle$ are $N_\varepsilon(c_{r,s})$ and $N_\varepsilon(d_{m,n})$ respectively. That is, $f^{-1}(G)$ and $f^{-1}(H)$ are intuitionistic fuzzy open sets. Now,

$$f^{-1}(\mu_G)(c_{r,s})=\mu_G(f(c))=0$$
$$f^{-1}(\gamma_G)(c_{r,s})=\gamma_G(f(c))=1$$
$$f^{-1}(\mu_H)(d_{m,n})=\mu_H(f(d))=1$$
$$f^{-1}(\gamma_H)(d_{m,n})=\gamma_H(f(d))=0$$

and

$$f^{-1}(G)\cap f^{-1}(H)=f^{-1}(G\cap H)=f^{-1}(0_\sim)=0_\sim.$$

Hence, (X,T) is an intuitionistic fuzzy T_2 space. □

Definition 3.29. An intuitionistic fuzzy topological space (X, T) is said to be an intuitionistic fuzzy G_δ-α-local Urysohn space if and only if for every intuitionistic fuzzy points $c_{r,s}$ and $d_{m,n}$ in an intuitionistic fuzzy topological space (X, T) and $c \neq d$ there exists an intuitionistic fuzzy G_δ-α-locally open sets $G = \langle x, \mu_G, \gamma_G\rangle$, $H = \langle x, \mu_H, \gamma_H\rangle$ with $\mu_G(c) = 0$, $\gamma_G(c) = 1$, $\mu_H(d) = 1$, $\gamma_H(d) = 0$ and IFG_δ-α-$lcl(G) \cap IFG_\delta$-α-$lcl(H) = 0_\sim$.

Proposition 3.30. *Every intuitionistic fuzzy G_δ-α-local Urysohn space is an intuitionistic fuzzy G_δ-α-local T_2 space.*

Proof. Let (X, T) be an intuitionistic fuzzy G_δ-α-local Urysohn space. Then for every intuitionistic fuzzy points $c_{r,s}$ and $d_{m,n}$ in an intuitionistic fuzzy topological space (X, T) and $c \neq d$ there exists an intuitionistic fuzzy G_δ-α-locally open sets $G = \langle x, \mu_G, \gamma_G\rangle$, $H = \langle x, \mu_H, \gamma_H\rangle$ with $\mu_G(c) = 0$, $\gamma_G(c) = 1$, $\mu_H(d) = 1$, $\gamma_H(d) = 0$ and IFG_δ-α-$lcl(G) \cap IFG_\delta$-α-$lcl(H) = 0_\sim$. Since $G \subseteq IFG_\delta$-α-$lcl(G)$ and $H \subseteq IFG_\delta$-α-$lcl(H)$. Then $G \cap H \subseteq IFG_\delta$-$\alpha$-$lcl(G) \cap IFG_\delta$-$\alpha$-$lcl(H)$. This implies $G \cap H = 0_\sim$. Hence (X, T) is an intuitionistic fuzzy G_δ-α-local T_2 space. □

4. Interrelation

Definition 4.1. Let(X, T) and (Y, S) be two intuitionistic fuzzy topological spaces. Let $f : (X, T) \to (Y, S)$ be an intuitionistic fuzzy mapping. Then f is said to be an

(i) intuitionistic fuzzy locally continuous function, if for each intuitionistic fuzzy closed set A in an intuitionistic fuzzy topological space (Y, S), $f^{-1}(A)$ is an intuitionistic fuzzy locally closed set in an intuitionistic fuzzy topological space (X, T).

(ii) intuitionistic fuzzy G_δ-locally continuous function, if for each intuitionistic fuzzy closed set A in an intuitionistic fuzzy topological space (Y, S), $f^{-1}(A)$ is an intuitionistic fuzzy G_δ-locally closed set in an intuitionistic fuzzy topological space (X, T).

Proposition 4.2. *Let (X, T) and (Y, S) be two intuitionistic fuzzy topological spaces. Let $f : (X, T) \to (Y, S)$ be an intuitionistic fuzzy locally continuous function. Then f is an intuitionistic fuzzy G_δ-locally continuous function.*

Proof. Let A be an intuitionistic fuzzy closed set in an intuitionistic fuzzy topological space (Y, S). Since f is an intuitionistic fuzzy locally continuous function,$f^{-1}(A)$ is an intuitionistic fuzzy locally closed set in intuitionistic fuzzy topological space (X, T). Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy G_δ-locally closed set, $f^{-1}(A)$ is also an intuitionistic fuzzy G_δ-locally closed set. Hence f is an intuitionistic fuzzy G_δ-locally continuous function. □

Remark 4.3. The converse of the Proposition 4.1 need not true as shown in Example 4.1.

Example 4.4. Let $X = \{a\}$. Consider the intuitionistic fuzzy sets $A_n, n = 0, 1, 2, ...$ as follows. We define the intuitionistic fuzzy sets $A_n = \langle x, \mu_{A_n}, \gamma_{A_n}\rangle, n = 0, 1, 2...$ by $\mu_{A_n}(x) = \frac{n}{10n+1}$ and $\gamma_{A_n}(x) = 1 - \frac{n}{10n+1}$. Then the family $T = \{0_\sim, 1_\sim, A_n : n = 0, 1, ...\}$ is an intuitionistic fuzzy topology on X. Let $Y = \{a\}$ and $F = \langle y, \frac{a}{0.9}, \frac{a}{0.1}\rangle$ be an intuitionistic fuzzy set in Y. Then the family $S = \{0_\sim, 1_\sim, F\}$ is an intuitionistic fuzzy topology on Y. Define a function $f : (X, T) \to (Y, S)$ be an identity function. Now, f is an intuitionistic fuzzy G_δ-locally continuous function. But f is not intuitionistic fuzzy locally continuous function.

Proposition 4.5. *Let (X,T) and (Y,S) be two intuitionistic fuzzy topological spaces. Let $f:(X,T)\to(Y,S)$ be an intuitionistic fuzzy locally continuous function. Then f is an intuitionistic fuzzy G_δ-α-locally continuous function.*

Proof. Let A be an intuitionistic fuzzy closed set in an intuitionistic fuzzy topological space (Y,S). Since f is an intuitionistic fuzzy locally continuous function,$f^{-1}(A)$ is an intuitionistic fuzzy locally closed set in an intuitionistic fuzzy topological space (X,T). Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy G_δ-α-locally closed set, $f^{-1}(A)$ is also an intuitionistic fuzzy G_δ-α-locally closed set. Hence f is an intuitionistic fuzzy G_δ-α-locally continuous function. □

Remark 4.6. The converse of the Proposition 4.2 need not true as shown in Example 4.2.

Proposition 4.7. *Let (X,T) and (Y,S) be two intuitionistic fuzzy topological spaces. Let $f:(X,T)\to(Y,S)$ be an intuitionistic fuzzy G_δ-locally continuous function. Then f is an intuitionistic fuzzy G_δ-α-locally continuous function.*

Proof. Let A be an intuitionistic fuzzy closed set in an intuitionistic fuzzy topological space (Y,S). Since f is an intuitionistic fuzzy G_δ-locally continuous function,$f^{-1}(A)$ is an intuitionistic fuzzy G_δ-locally closed set in an intuitionistic fuzzy topological space (X,T). Since every intuitionistic fuzzy G_δ-locally closed set is an intuitionistic fuzzy G_δ-α-locally closed set, $f^{-1}(A)$ is also an intuitionistic fuzzy G_δ-α-locally closed set. Hence f is an intuitionistic fuzzy G_δ-α-locally continuous function. □

Remark 4.8. The converse of the Proposition 4.3 need not true as shown in Example 4.2.

Example 4.9. Let $X=\{a,b\}$ be a nonempty set. Let $A=\langle x,(\frac{a}{0.3},\frac{b}{0.5}),(\frac{a}{0.2},\frac{b}{0.3})\rangle$ and $B=\langle x,(\frac{a}{0.5},\frac{b}{0.6}),(\frac{a}{0.2},\frac{b}{0.3})\rangle$ be intuitionistic fuzzy sets of X. Then the family $T=\{0_\sim,1_\sim,A,B\}$ is an intuitionistic fuzzy topology on X. Let $Y=\{a,b\}$ be a nonempty set. Let $F=\langle y,(\frac{a}{0.7},\frac{b}{0.5}),(\frac{a}{0.3},\frac{b}{0.2})\rangle$ be an intuitionistic fuzzy set of Y. Then the family $S=\{0_\sim,1_\sim,F\}$ is an intuitionistic fuzzy topology on Y. Define a function $f:(X,T)\to(Y,S)$ as $f(a)=b$ and $f(b)=a$. Now, f is an intuitionistic fuzzy G_δ-α-locally continuous function. But f is not an intuitionistic fuzzy locally continuous function and intuitionistic fuzzy G_δ-locally continuous function.

Definition 4.10. An intuitionistic fuzzy topological space (X,T) is said to be intuitionistic fuzzy G_δ-α-local connected if and only if the only intuitionistic fuzzy sets which are both intuitionistic fuzzy G_δ-α-locally open set and intuitionistic fuzzy G_δ-α-locally closed set are $0_\sim$ and $1_\sim$.

Proposition 4.11. *Let (X,T) and (Y,S) be any two intuitionistic fuzzy topological spaces. If $f:(X,T)\to(Y,S)$ is an intuitionistic fuzzy G_δ-α-locally continuous surjective function and (X,T) is an intuitionistic fuzzy G_δ-α-local connected space then (Y,S) is an intuitionistic fuzzy connected space.*

Proof. Let (X,T) be an intuitionistic fuzzy G_δ-α-local connected space. Suppose that (Y,S) is not an intuitionistic fuzzy connected space. Then there exists a proper intuitionistic fuzzy set A such that A is both intuitionistic fuzzy open set and intuitionistic fuzzy closed set in (Y,S). Since f is an intuitionistic fuzzy G_δ-α-locally continuous surjective function, then $f^{-1}(A)$ is both intuitionistic fuzzy G_δ-α-locally open set and intuitionistic fuzzy G_δ-α-locally closed set in (X,T), which is contradiction. Hence, (Y,S) is an intuitionistic fuzzy connected space. □

Definition 4.12. Let (X,T) be an intuitionistic fuzzy topological space. If a family $\{\langle x,\mu_{G_j},\gamma_{G_j}\rangle : j\in J\}$ of an intuitionistic fuzzy G_δ-α-locally open sets in X satisfies the condition $\bigcup\{\langle x,\mu_{G_j},\gamma_{G_j}\rangle : j\in J\} = 1_\sim$ then it is called as an intuitionistic fuzzy G_δ-α-locally open cover of an intuitionistic fuzzy topological space (X,T).

Definition 4.13. An intuitionistic fuzzy topological space (X,T) is said to be intuitionistic fuzzy G_δ-α-local compact if every intuitionistic fuzzy G_δ-α-locally open cover of $\{A_j : j\in J\}$ of an intuitionistic fuzzy topological space (X,T), there exists a finite subfamily $J_o\subset J$ such that $1_\sim = \bigcup\{A_j : j\in J_o\}$.

Proposition 4.14. *Let (X,T) and (Y,S) be any two intuitionistic fuzzy topological spaces. If $f : (X,T)\to(Y,S)$ is an intuitionistic fuzzy G_δ-α-locally continuous bijective function and (X,T) is an intuitionistic fuzzy G_δ-α-local compact space then (Y,S) is an intuitionistic fuzzy G_δ-α-local compact space.*

Proof. Let $\{A_j : j\in J\}$ be an intuitionistic fuzzy open cover of an intuitionistic fuzzy topological space (Y,S) such that

$$1_\sim = \bigcup_{j\in J} A_j \tag{4.1}$$

Since f is an intuitionistic fuzzy G_δ-α-locally continuous bijective function, $\{f^{-1}(A_j) : j\in J\}$ is an intuitionistic fuzzy G_δ-α-locally open cover of an intuitionistic fuzzy topological space (X,T).
From (3.15),

$$f^{-1}(1_\sim) = f^{-1}(\bigcup_{j\in J} A_j)$$

$$1_\sim = \bigcup_{j\in J} f^{-1}(A_j)$$

Now, $\{f^{-1}(A_j) : j\in J\}$ is an intuitionistic fuzzy G_δ-α-locally open cover of an intuitionistic fuzzy topological space (X,T). Since (X,T) is an intuitionistic fuzzy G_δ-α-local compact space, then there exist a finite subcover $\{f^{-1}(A_j) : j = 1,2,3,.....n\}$ of $\{f^{-1}(A_j) : j\in J\}$ is an intuitionistic fuzzy topological space (X,T). Then,

$$1_\sim = \bigcup_{j=1}^{n} f^{-1}(A_j)$$

Now,

$$f(1_\sim) = f(\bigcup_{j=1}^{n} f^{-1}(A_j))$$

Since f is an intuitionistic fuzzy surjective function,

$$1_\sim = \bigcup_{j=1}^{n} f(f^{-1}(A_j)) = \bigcup_{j=1}^{n} A_j$$

implies that (Y, S) is an intuitionistic fuzzy G_δ-α-local compact space. □

References

[1] K. T. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20 (1986) 87–96.
[2] G. Balasubramanian, Maximal fuzzy topologies, Kybernetika 31 (1995) 459–464.
[3] D. Coker, An introduction to intuitionistic fuzzy topological spaces, Fuzzy Sets and Systems 88(1) (1997) 81–89.
[4] D. Coker and M. Demirci, On intuitionistic fuzzy points, Notes IFS. 1(2) (1995) 79–84
[5] M. Ganster and I. L. Relly, Locally closed sets and LC-continuous functions, Internat. J. Math. Math. Sci. 12 (1989) 417–424.
[6] H. Gurcay and D. Coker, On fuzzy continuity in intuitiionistic fuzzy topological spaces, J. Fuzzy Math. 5 (1997) 365–378.
[7] I. M. Hanafy, Completely continuous function in intuitionistic fuzzy topological space, Czechoslovak Math. J. 53 (2003) 793–803.
[8] L. A. Zadeh, Fuzzy sets, Information and Control 9(1965) 338–353.

R. Narmada Devi (narmadadevi23@gmail.com)
Department of mathemathics, Sri Saradha College for Women, Salem - 16, Tamil Nadu, India.

E. Roja (arudhay@yahoo.co.in)
Department of mathemathics, Sri Saradha College for Women, Salem - 16, Tamil Nadu, India.

M. K. Uma (arudhay@yahoo.co.in)
Department of mathemathics, Sri Saradha College for Women, Salem - 16, Tamil Nadu, India.