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A note on variable transformation

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Abstract. This article proposes to revisit and comment on the process of transformations between probability and possibility which is known as Variable Transformation. The main contribution of this article is to suggest a new transformation procedure rooted in operation of superimposition of sets. It is expected that the new transformation principle which is suggested in this article would be the appropriate one in dealing with probability and possibility.

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1. Introduction

Possibility theory is a mathematical theory dealing with certain types of uncertainties and is considered as an alternative to probability theory. Possibility theory is devoted to the handling of incomplete information. The process of transformation from probability to possibility had received attention in the past. This question is philosophically interesting as a part of debate between probability and fuzzy sets.

The conversion problem between probability and possibility has its roots in possibility - probability consistency principle of Zadeh [13], that he introduced in the paper founding possibility theory. The transformation between probability and possibility has been studied by many researchers. Most of these studies examined principles that must be satisfied for transformations and devised an equation satisfying them. Later on Dubios and Prade further contributed to the development of the possibility theory. In Zadeh's view, possibility distributions were meant to provide a graded semantics to natural statements. The transformation between probabilities to possibility is useful in some practical problems as: constructing a fuzzy membership function from a statistical data Krishnapuram [12], combining probabilities and possibilities in expert systems Klir [11] and reducing complecated complexity Dubois [10]. In other words, the transformation from possibility to probability or conversely is useful in case of decision making when the experts need precise informations to take any decision. A long standing debate took place in literature on the relationship between probability and possibility. In literature, we can see three most commonly used principles linking probability with possibility. These are Zadeh consistency principle, Klir consistency principle and Dubois and Prade consistency principles. It can be found that Zadeh himself was not satisfied with his principle and hence we would not like to deal with this. Klir's principle was also criticized for many reasons. After finding the inadequacy of the two principles to find a possible link between possibility and probability, Dubois and Prade further contributed to the development of another principle. But later on it was found that this did not keep pace with their definition of a normal fuzzy number (see for example Baruah ([5],[6]). All these principles were applied to different fields but they were found not too appropriate for every circumstances. As a consequence of which we can see the existence of many principles relating probability with possibility. Most of these studies examined the principles that must be satisfied for transformations and devised an equation satisfying them in a heuristic way. In this article, we shall discuss one such a principle whiched was named as Variable transformation. Mouchaweb, Bouguelid, Biillaudel, RIERA[8] had proposed a transformation from probability to possibility which they named as Variable Transformation and it was claimed that this transformation is the most specific and that best distinguishes the confused elements. Let us have a look at this transformation in brief.

2. Variable transformation

Mouchaweb, Bouguelid, Biillaudel, RIERA [8] proposed a transformation from probability to possibility which they named as Variable transformation.This transformation is different from those proposed by Zadeh, Klir and Dubois-Prade and it was expressed as follows:

$$\pi_i = (\frac{p_i}{p_1})^{k(1-p_i)}$$

where k is a constant which guarantee the following condition of consistency:

$$\forall w \in X : \pi(w) \geq (w).$$

This condition is a particular case of Dubious- Prade consistency principle but there is a condition that the value of 'k' must belong to the following interval:

$$0 \leq k \leq \frac{log p_n}{(1-p_n).log\frac{p_n}{p_1}}.$$

It was mentioned by them that this above mentioned transformation is different from Klir's transformation in the sense that Klir's transformation has a constant power α which belongs to the open interval $]0,1[$ while the power $k.(1-p_i)$ in variable transformation , is a variable to make it more specific. So it can be said that the authors were in the opinion that the transformation proposed by Klir was not a specific one. In this article, we would like to discuss some of the issues involved in dealing with the procedures.

Now the question arises whether the transformation called VT is acceptable for all circumstances. To see its appropriateness, it is to be interpreted even more general way to understand the underlying principles which motivated the authors to work in that direction. It can be mentioned here that the transformation was the result of modification of Klir's consistency principle on the one hand and Dubios and Prade consistency principle on the other. Thus the so called Variable transformation seemed to be based on two debatable principles from the standpoint of defining the new consistency principle between probability and possibility. The reason behind such a claim may be mentioned in the form that both Klir consistency principle and Dubois- Prade consistency principle were based on some misconceptions. After finding some problems with Klir consistency principle, Dubois and Prade found another transformation linking probability and possibility but later on it was observed that unlike others, these principles also can be criticized in many ways.There is the use of the term measure with possibility which is not reasonable. Again probability and possibility are described over the same space which contradicts their own definition of a normal fuzzy number described with the help of two reference functions. Further, the principles deal with discrete case and nothing was mentioned about continuous cases. There are some other authors, who also found some questionable properties in the aforesaid principlesfor example, Alt and Yovits [1], countered the arguments of Dubois- Prade in the following way: although possibility theory employs weaker rules than probability theory, in manipulating uncertainty, the basic structure of the two theories are not comparable. Hence even though manipulating uncertainty within possibility theory results in a greater loss of information than corresponding uncertainty in probability theory, it is neither necessary nor desirable to lose or gain information solely by transforming uncertainty from one representation to another. Further, the transformation called VT was claimed to produce the most informative results in discrete cases while nothing was mentioned about continuous cases. This is not desirable. We would however like to mention here that our intention is only to focus on the ideas underlying and not on their technical details.

So, while a number of measures have been suggested, we shall find the following one introduced in [5], to be the most useful for our purposes.This consistency principle was named as "The Randomness- Fuzziness Consistency Principles" in which the concept of superimposition of sets introduced by Baruah [4] and also the definition of a normal fuzzy number as proposed by Dubois and Prade played a vital role. This can be viewed as a bridge by which probability and possibility can be connected. Our main intention is to revisit the Variable Transformation from the viewpoints of the suggested principle.

3. Randomness-fuzziness consistency principles:

Baruah ([5], [6], [7]) introduced a framework for reasoning with the link between probability - possibility. The development of this principle focused mainly on the existence of two laws of randomness which are required to define a law of fuzziness. In other words , not one but two laws of fuzziness is required to define a law of randomness on two disjoint spaces which in turn can construct a fuzzy membership function. Fundamental to this approach is the idea that possibility distribution can be viewed as a combination of distributions of which one is a probability distribution and the other is a complementary probability distribution (see [2, 3]). The consistency principle introduced in the manner can be explained mathematically in the following form:

For a normal fuzzy number of the type $N = [\alpha, \beta, \gamma]$ with membership functions

$$\mu_N(x) = \psi_1(x) \text{ if } \alpha \le x \le \gamma, \; = \psi_2(x) \text{ if } \beta \le x \le \gamma, \text{ and } = 0, \text{otherwise}$$

with

$$\psi_1(\alpha) = \psi_2(\gamma) = 0, \; \psi_1(\beta) = \psi_2(\beta) = 1.$$

This transformation is named as "The Randomness- Fuzziness Consistency principles" and it is expected that the shortcomings which are observed in the existing principles will be reduced to a great extent in this procedure. It was thus established that two laws of randomness are needed to define one possibility law. Accordingly, the left reference function of a normal fuzzy number which is nothing but a distribution function, would lead to entropy E_1. In a similar manner, the right reference function of the normal fuzzy number, which is nothing but a complementary distribution function, would lead to another entropyE_2.The pair$[E_1, E_2]$ found can rightly be called fuzzy entropy in the classical sense of defining Shannon's entropy for a discrete law of randomness. Discretizing a law of randomness for a continuous variable should not be of much problem, which in turn can be used to define fuzzy entropy $[E_1, E_2]$, where $E_1$and E_2 are Shannon's entropies for the left reference function and right reference function respectively. This was discussed in more details in Dhar et al.[9]

With the above result, we would like to establish the fact that the spirit of this approach is to our opinion, better founded than the existing ones. If this be the case, then it is obvious that the results of all the transformations which basically depend on the existing link between probability and possibility or conversely would be illogical from our standpoints, the reasons for which are discussed in the previous section. That is why the principles cannot be accepted for further studies and also those who depended on these results without having the in depth thinking would have to reconsider the procedures developed with the existing principles linking possibility with probability. In other words, we would like to say here that the method of linking probability with possibility which is suggested by us is preferable among various other existing transformation procedures because of its logical foundations and appropriate mathematical frameworks. While dealing with a subject like mathematics, it is very important to see whether the things which are in use are constructed within proper mathematical frameworks or not. It is necessary because otherwise we would have to be contended with some results having no logic at all. Hence it is expected that the above mentioned method of transformation would be workable in all respect and it is for this reason this principle of consistency is suggested in this article. From the above, it can be said that the researchers who tried to link probability with possibility had ignored one most important thing that two probability spaces are required to define a possibility space. That is to say that while developing their principles, it was seen that possibility was defined in the same space over which probability was defined. Various other principles which were developed one after another from time to time without having any logical thinking. But one thing can be noticed that none of the researchers, who were dealt with finding a link between probability and possibility, was satisfied with the principles developed by their predecessors. It can be seen from their attempts to find a new consistency principle. As a result of which we can find a myriad of principles in this respect. In this article, we would like to say that there should be one principle instead of many because in mathematics no one should be in a dilemma regarding the choice of any principle. Otherwise, it would certainly lead to a chaotic state. The newcomers in the domain would be overwhelmed by the multitudes of ways to link probability with possibility. Their reaction may be assumed in the following way: either they will accept all of them and will apply them to more or less at random or will accept only one of them and would use it in every context. Both these attitudes are wrong and can be seriously misleading. It is because of this reason, we would like to stress on the fact that in a subject like Mathematics, there should be only one such principle which is found on appropriate mathematical frameworks. In such a case, we would like to stress on adopting the one suggested in this article. In other words, we can say that it has become increasingly clear that there are some aspects which do not allow finding the link effectively. In this regard, the principle suggested in this work, has been recognized as the potential tool for enhancing our ability to deal with the problems outlined.

4. Conclusions

This paper presented a summary of a very important transformation principle linking probability and possibility which is called Variable transformation and in this process , it was argued that this principle was not based on logical arguments as those of other principles existing in the literature. So there arises the need of a principle which can remove the shortcomings which can be found in the existing principles. Consequently, a new principle for finding consistency is suggested.With that proposal, we would like to discard those existing principles along with the so called Variable Transformation. Further, the suggested principle seems to be more informative and logical than others.

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