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Intuitionistic fuzzy contra weakly generalized continuous mappings

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Abstract. The purpose of this paper is to introduce and study the concepts of intuitionistic fuzzy contra weakly generalized continuous mappings in intuitionistic fuzzy topological space. Some of their properties are explored.

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1. Introduction

Fuzzy set (FS) as proposed by Zadeh ([15]) in 1965, is a framework to encounter uncertainty, vagueness and partial truth and it represents a degree of membership for each member of the universe of discourse to a subset of it. After the introduction of fuzzy topology by Chang ([2]) in 1968, there have been several generalizations of notions of fuzzy sets and fuzzy topology. By adding the degree of non-membership to FS, Atanassov ([1]) proposed intuitionistic fuzzy set (IFS) in 1986 which appeals more accurate to uncertainty quantification and provides the opportunity to precisely model the problem, based on the existing knowledge and observations. In 1997, Coker ([3]) introduced the concept of intuitionistic fuzzy topological space. This paper aspires to overtly enunciate the notion of intuitionistic fuzzy contra weakly generalized continuous mappings in intuitionistic fuzzy topological space and study some of their properties. We provide some characterizations of intuitionistic fuzzy contra weakly generalized continuous mappings and establish the relationships with other classes of early defined forms of intuitionistic mappings.

2. Preliminaries

Definition 2.1 ([1])**.** Let X be a non empty fixed set. An *intuitionistic fuzzy set* (IFS in short) A in X is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$ where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

Definition 2.2 ([1])**.** Let A and B be IFSs of the forms

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\} \quad \text{and} \quad B = \{\langle x, \mu_B(x), \nu_B(x)\rangle / x \in X\}.$$

Then

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$,
(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,
(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle / x \in X\}$,
(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle / x \in X\}$,
(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle / x \in X\}$.

For the sake of simplicity, the notation $A = \langle x, \mu_A, \nu_A\rangle$ shall be used instead of $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$. Also for the sake of simplicity, we shall use the notation $A = \langle x, (\mu_A, \mu_B), (\nu_A, \nu_B)\rangle$ instead of $A = \langle x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B)\rangle$.

The intuitionistic fuzzy sets $0_\sim = \{\langle x, 0, 1\rangle / x \in X\}$ and $1_\sim = \{\langle x, 1, 0\rangle / x \in X\}$ are the *empty set* and the *whole set* of X, respectively.

Definition 2.3 ([3])**.** An *intuitionistic fuzzy topology* (IFT in short) on a non empty set X is a family τ of IFSs in X satisfying the following axioms:

(a) $0_\sim, 1_\sim \in \tau$,
(b) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,
(c) $\cup G_i \in \tau$ for any arbitrary family $\{G_i / i \in J\} \subseteq \tau$.

In this case, the pair (X, τ) is called an *intuitionistic fuzzy topological space* (IFTS in short) and any IFS in τ is known as an *intuitionistic fuzzy open set* (IFOS in short) in X.

The complement A^c of an IFOS A in an IFTS (X, τ) is called an *intuitionistic fuzzy closed set* (IFCS in short) in X.

Definition 2.4 ([3])**.** Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A\rangle$ be an IFS in X. Then the *intuitionistic fuzzy interior* and an *intuitionistic fuzzy closure* are defined by

$$int(A) = \cup\{G/G \quad \text{is an IFOS in} \quad X \quad \text{and} \quad G \subseteq A\},$$
$$cl(A) = \cap\{K/K \quad \text{is an IFCS in} \quad X \quad \text{and} \quad A \subseteq K\}.$$

Note that for any IFS A in (X, τ), we have $cl(A^c) = (int(A))^c$ and $int(A^c) = (cl(A))^c$.

Definition 2.5. An IFS $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$ in an IFTS (X, τ) is said to be

(a) *intuitionistic fuzzy semi closed set* ([7]) (IFSCS in short) if $int(cl(A)) \subseteq A$,
(b) *intuitionistic fuzzy α-closed set* ([7]) (IFαCS in short) if $cl(int(cl(A))) \subseteq A$,
(c) *intuitionistic fuzzy pre-closed set* ([7]) (IFPCS in short) if $cl(int(A)) \subseteq A$,
(d) *intuitionistic fuzzy regular closed set* ([7]) (IFRCS in short) if $cl(int(A)) = A$,

(e) *intuitionistic fuzzy generalized closed set* ([14]) (IFGCS in short) if $cl(A) \subseteq U$ whenever $A \subseteq U$, and U is an IFOS,

(f) *intuitionistic fuzzy generalized semi closed set* ([13]) (IFGSCS in short) if $scl(A) \subseteq U$, whenever $A \subseteq U$, and U is an IFOS,

(g) *intuitionistic fuzzy α generalized closed set* ([11])(IFαGCS in short) if $\alpha cl(A) \subseteq U$, whenever $A \subseteq U$, and U is an IFOS.

(h) *intuitionistic fuzzy γ closed set* ([6]) (IF γCS in short) if $int(cl(A)) \cap cl(int(A)) \subseteq A$.

An IFS A is called *intuitionistic fuzzy semi open set, intuitionistic fuzzy α-open set, intuitionistic fuzzy pre-open set, intuitionistic fuzzy regular open set, intuitionistic fuzzy generalized open set, intuitionistic fuzzy generalized semi open set, intuitionistic fuzzy α generalized open set and intuitionistic fuzzy γ open set* (IFSOS, IFαOS, IFPOS, IFROS, IFGOS, IFGSOS, IFαGOS and IFγOS) if the complement A^c is an IFSCS, IFαCS, IFPCS, IFRCS, IFGCS, IFGSCS, IFαGCS and IFγCS respectively.

Definition 2.6 ([8])**.** An IFS $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle /x \in X\}$ in an IFTS (X, τ) is said to be an *intuitionistic fuzzy weakly generalized closed set* (IFWGCS in short) if $cl(int(A)) \subseteq U$, whenever $A \subseteq U$ and U is an IFOS in X.

The family of all IFWGCSs of an IFTS (X, τ) is denoted by IFWGC(X).

Definition 2.7 ([8])**.** An IFS $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle /x \in X\}$ is said to be an *intuitionistic fuzzy weakly generalized open set* (IFWGOS in short) in (X, τ) if the complement A^c is an IFWGCS in X.

The family of all IFWGOSs of an IFTS (X, τ) is denoted by IFWGO(X).

Remark 2.8 ([8])**.** Every IFCS, IFαCS, IFGCS, IFRCS, IFPCS, IFαGCS is an IFWGCS but the converses need not be true in general.

Definition 2.9 ([9])**.** Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A\rangle$ be an IFS in X. Then the *intuitionistic fuzzy weakly generalized interior* and an *intuitionistic fuzzy weakly generalized closure* are defined by

$$wgint(A) = \cup \{G/G \text{ is an IFWGOS in } X \text{ and } G \subseteq A\},$$

$$wgcl(A) = \cap \{K/K \text{ is an IFWGCS in } X \text{ and } A \subseteq K\}.$$

Definition 2.10 ([3])**.** Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). If $B = \{\langle y, \mu_B(y), \nu_B(y)\rangle /y \in Y\}$ is an IFS in Y, then the *pre-image* of B under f denoted by $f^{-1}(B)$, is the IFS in X defined by

$$f^{-1}(B) = \left\{\left\langle x, f^{-1}(\mu_B(x)), f^{-1}(\nu_B(x))\right\rangle /x \in X\right\},$$

where $f^{-1}(\mu_B(x)) = \mu_B(f(x))$. If $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle /x \in X\}$ is an IFS in X, then the *image* of A under f denoted by $f(A)$ is the IFS in Y defined by

$$f(A) = \{\langle y, f(\mu_A(y)), f_-(\nu_A(y))\rangle /y \in Y\}$$

where $f_-(\nu_A) = 1 - f(1 - \nu_A)$.

Definition 2.11. Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be

(a) ([5]) *intuitionistic fuzzy continuous* (IF continuous in short) if $f^{-1}(B)$ is an IFOS in X for every IFOS B in Y,

(b) ([4]) *intuitionistic fuzzy contra continuous* if $f^{-1}(B)$ is an IFCS in X for every IFOS B in Y,

(c) ([4]) *intuitionistic fuzzy contra semi continuous* if $f^{-1}(B)$ is an IFSCS in X for every IFOS B in Y,

(d) ([4]) *intuitionistic fuzzy contra α continuous* if $f^{-1}(B)$ is an IFαCS in X for every IFOS B in Y,

(e) ([4]) *intuitionistic fuzzy contra pre continuous* if $f^{-1}(B)$ is an IFPCS in X for every IFOS B in Y,

(f) ([12]) *intuitionistic fuzzy contra α generalized continuous* if $f^{-1}(B)$ is an IFαGCS in X for every IFOS B in Y,

(g) ([10]) *intuitionistic fuzzy weakly generalized continuous* (IFWG continuous in short) if $f^{-1}(B)$ is an IFWGCS in X for every IFCS B in Y,

(h) ([9]) *intuitionistic fuzzy weakly generalized irresolute* (IFWG irresolute in short) if $f^{-1}(B)$ is an IFWGCS in X for every IFWGCS B in Y.

Definition 2.12 ([8])**.** An IFTS (X,τ) is said to be an *intuitionistic fuzzy $_wT_{1/2}$ space* (IF$_wT_{1/2}$ space) if every IFWGCS in X is an IFCS in X.

Definition 2.13 ([8])**.** An IFTS (X,τ) is said to be an *intuitionistic fuzzy $_{wg}T_q$ space* (IF$_{wg}T_q$ space) if every IFWGCS in X is an IFPCS in X.

3. Intuitinistic fuzzy contra weakly generalized continuous mappings

In this section, we introduce intuitionistic fuzzy contra weakly generalized continuous mappings and study some of their properties.

Definition 3.1. A mapping $f:(X,\tau)\to(Y,\sigma)$ is called an *intuitionistic fuzzy contra weakly generalized continuous* (IF contra WG continuous in short) *mapping* if $f^{-1}(B)$ is an IFWGCS in (X,τ) for every IFOS B of (Y,σ).

Example 3.2. Let $X=\{a,b\}, Y=\{u,v\}$ and $T_1=\langle x,(0.2,0.3),(0.8,0.7)\rangle, T_2=\langle y,(0.2,0.2),(0.8,0.7)\rangle$. Then $\tau=\{0_\sim,T_1,1_\sim\}$ and $\sigma=\{0_\sim,T_2,1_\sim\}$ are IFTs on X and Y respectively. Consider a mapping $f:(X,\tau)\to(Y,\sigma)$ defined as $f(a)=u$ and $f(b)=v$. This mapping f is an intuitionistic fuzzy contra weakly generalized continuous mapping.

Theorem 3.3. *Every intuitionistic fuzzy contra continuous mapping is an intuitionistic fuzzy contra weakly generalized continuous mapping but not conversely.*

Proof. Let $f:(X,\tau)\to(Y,\sigma)$ be an intuitionistic fuzzy contra continuous mapping. Let A be an IFOS in Y. By hypothesis, $f^{-1}(A)$ is an IFCS in X. Since every IFCS is an IFWGCS, $f^{-1}(A)$ is an IFWGCS in X. Hence f is an intuitionistic fuzzy contra weakly generalized continuous mapping. □

Example 3.4. Let $X=\{a,b\}, Y=\{u,v\}$ and $T_1=\langle x,(0.4,0.5),(0.6,0.5)\rangle, T_2=\langle y,(0.2,0.3),(0.8,0.7)\rangle$. Then $\tau=\{0_\sim,T_1,1_\sim\}$ and $\sigma=\{0_\sim,T_2,1_\sim\}$ are IFTs on X and Y respectively. Consider a mapping $f:(X,\tau)\to(Y,\sigma)$ defined as $f(a)=u$ and $f(b)=v$. This f is an intuitionistic fuzzy contra weakly generalized continuous mapping but not an intuitionistic fuzzy contra continuous mapping, since the IFS T_2 is an IFOS in Y but $f^{-1}(T_2) = \langle x, (0.2, 0.3), (0.8, 0.7)\rangle$ is not an IFCS in X.

Theorem 3.5. *Let $f : (X, \tau) \to (Y, \sigma)$ be an intuitionistic fuzzy contra weakly generalized continuous mapping and (X, τ) an $IF_wT_{1/2}$ space. Then f is an intuitionistic fuzzy contra continuous mapping.*

Proof. Let B be an IFOS in Y. By hypothesis, $f^{-1}(B)$ is an IFWGCS in X. Since (X, τ) is an $IF_wT_{1/2}$ space, $f^{-1}(B)$ is an IFCS in X. Hence f is an intuitionistic fuzzy contra continuous mapping. □

Theorem 3.6. *Let $f : (X, \tau) \to (Y, \sigma)$ be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) and (X, τ) an $IF_wT_{1/2}$ space. Then the following statements are equivalent.*

(a) *f is an intuitionistic fuzzy contra weakly generalized continuous mapping,*
(b) *f is an intuitionistic fuzzy contra continuous mapping.*

Proof. Obvious. □

Theorem 3.7. *Every intuitionistic fuzzy contra pre-continuous mapping is an intuitionistic fuzzy contra weakly generalized continuous mapping but not conversely.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be an intuitionistic fuzzy contra pre-continuous mapping. Let A be an IFOS in Y. By hypothesis, $f^{-1}(A)$ is an IFPCS in X. Since every IFPCS is an IFWGCS, $f^{-1}(A)$ is an IFWGCS in X. Hence f is an intuitionistic fuzzy contra weakly generalized continuous mapping. □

Example 3.8. Let $X = \{a, b\}, Y = \{u, v\}$ and $T_1 = \langle x, (0.5, 0.3), (0.5, 0.7)\rangle, T_2 = \langle x, (0.8, 0.8), (0.2, 0.2)\rangle$ and $T_3 = \langle y, (0.9, 0.3), (0.1, 0.6)\rangle$. Then $\tau = \{0_\sim, T_1, T_2, 1_\sim\}$ and $\sigma = \{0_\sim, T_3, 1_\sim\}$ are IFTs on X and Y respectively. Consider a mapping $f : (X, \tau) \to (Y, \sigma)$ defined as $f(a) = u$ and $f(b) = v$. This f is an intuitionistic fuzzy contra weakly generalized continuous mapping but not an intuitionistic fuzzy contra pre-continuous mapping, since the IFS T_3 is an IFOS in Y but $f^{-1}(T_3) = \langle x, (0.9, 0.3), (0.1, 0.6)\rangle$ is not an IFPCS in X.

Theorem 3.9. *Let $f : (X, \tau) \to (Y, \sigma)$ be an intuitionistic fuzzy contra weakly generalized continuous mapping and (X, τ) an $IF_{wg}T_q$ space. Then f is an intuitionistic fuzzy contra pre-continuous mapping.*

Proof. Let B be an IFOS in Y. By hypothesis, $f^{-1}(B)$ is an IFWGCS in X. Since (X, τ) is an $IF_{wg}T_q$ space, $f^{-1}(B)$ is an IFPCS in X. Hence f is an intuitionistic fuzzy contra pre-continuous mapping. □

Theorem 3.10. *Let $f : (X, \tau) \to (Y, \sigma)$ be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) and (X, τ) an $IF_{wg}T_q$ space. Then the following statements are equivalent.*

(a) *f is an intuitionistic fuzzy contra weakly generalized continuous mapping,*
(b) *f is an intuitionistic fuzzy contra pre-continuous mapping.*

Proof. Obvious. □

Theorem 3.11. *Every intuitionistic fuzzy contra α continuous mapping is an intuitionistic fuzzy contra weakly generalized continuous mapping but not conversely.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be an intuitionistic fuzzy contra α continuous mapping. Let A be an IFOS in Y. By hypothesis, $f^{-1}(A)$ is an IFαCS in X. Since every IFαCS is an IFWGCS, $f^{-1}(A)$ is an IFWGCS in X. Hence f is an intuitionistic fuzzy contra weakly generalized continuous mapping. □

Example 3.12. Let $X = \{a, b\}, Y = \{u, v\}$ and $T_1 = \langle x, (0.5, 0.3), (0.5, 0.7)\rangle$, $T_2 = \langle y, (0.4, 0.2), (0.6, 0.8)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Consider a mapping $f : (X, \tau) \to (Y, \sigma)$ defined as $f(a) = u$ and $f(b) = v$. This f is an intuitionistic fuzzy contra weakly generalized continuous mapping but not an intuitionistic fuzzy contra α continuous mapping, since the IFS T_2 is an IFOS in Y but $f^{-1}(T_2) = \langle x, (0.4, 0.2), (0.6, 0.8)\rangle$ is not an IFαCS in X.

Theorem 3.13. *Every intuitionistic fuzzy contra α generalized continuous mapping is an intuitionistic fuzzy contra weakly generalized continuous mapping but not conversely.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be an intuitionistic fuzzy contra α generalized continuous mapping. Let A be an IFOS in Y. By hypothesis, $f^{-1}(A)$ is an IFαGCS in X. Since every IFαGCS is an IFWGCS, $f^{-1}(A)$ is an IFWGCS in X. Hence f is an intuitionistic fuzzy contra weakly generalized continuous mapping. □

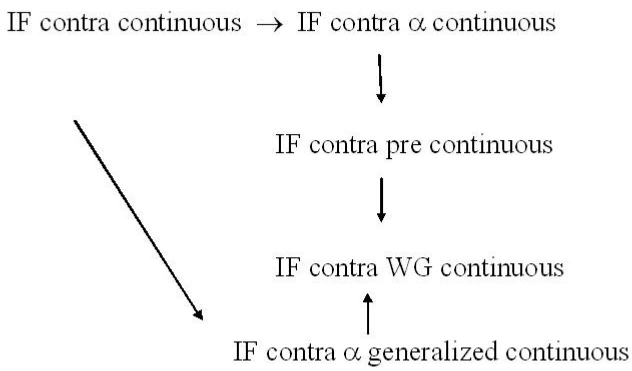
Example 3.14. Let $X = \{a, b\}, Y = \{u, v\}$ and $T_1 = \langle x, (0.5, 0.6), (0.5, 0.4)\rangle$, $T_2 = \langle y, (0.3, 0.5), (0.6, 0.5)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Consider a mapping $f : (X, \tau) \to (Y, \sigma)$ defined as $f(a) = u$ and $f(b) = v$. This f is an intuitionistic fuzzy contra weakly generalized continuous mapping but not an intuitionistic fuzzy contra α generalized continuous mapping, since the IFS T_2 is an IFOS in Y but $f^{-1}(T_2) = \langle x, (0.3, 0.5), (0.6, 0.5)\rangle$ is not an IFαGCS in X.

Remark 3.15. An intuitionistic fuzzy contra semi continuous mapping and an intuitionistic fuzzy contra weakly generalized continuous mapping are independent to each other as seen from the following examples.

Example 3.16. Let $X = \{a, b\}, Y = \{u, v\}$ and $T_1 = \langle x, (0.4, 0.3), (0.6, 0.7)\rangle$, $T_2 = \langle y, (0.4, 0.3), (0.6, 0.7)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Consider a mapping $f : (X, \tau) \to (Y, \sigma)$ defined as $f(a) = u$ and $f(b) = v$. This f is an intuitionistic fuzzy contra semi continuous mapping but not an intuitionistic fuzzy contra weakly generalized continuous mapping , since the IFS T_2 is an IFOS in Y but $f^{-1}(T_2) = \langle x, (0.4, 0.3), (0.6, 0.7)\rangle$ is not an IFWGCS in X.

Example 3.17. Let $X = \{a, b\}, Y = \{u, v\}$ and $T_1 = \langle x, (0.9, 0.7), (0.1, 0.2)\rangle, T_2 = \langle y, (0.7, 0.6), (0.3, 0.4)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Consider a mapping $f : (X, \tau) \to (Y, \sigma)$ defined as $f(a) = u$ and $f(b) = v$. This f is an intuitionistic fuzzy contra weakly generalized continuous mapping but not an intuitionistic fuzzy contra semi continuous mapping, since the IFS T_2 is an IFOS in Y but $f^{-1}(T_2) = \langle x, (0.7, 0.6), (0.3, 0.4)\rangle$ is not an IFSCS in X.

The relation among various types of intuitionistic fuzzy contra continuities are given in the following diagram.



The reverse implications are not true in general in the above diagram. In this diagram "$A \to B$" we mean A implies B but not conversely.

Theorem 3.18. *Let $f : (X, \tau) \to (Y, \sigma)$ be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then the following statements are equivalent.*
(a) *f is an intuitionistic fuzzy contra weakly generalized continuous mapping,*
(b) *$f^{-1}(B)$ is an IFWGOS in X for every IFCS B in Y.*

Proof. (a) ⇒ (b): Let B be an IFCS in Y. Then B^c is an IFOS in Y. By hypothesis, $f^{-1}(B^c) = (f^{-1}(B))^c$ is an IFWGCS in X. Hence $f^{-1}(B)$ is an IFWGOS in X.
(b) ⇒ (a): Let B be an IFOS in Y. Then B^c is an IFCS in Y. By (b), $f^{-1}(B^c) = (f^{-1}(B))^c$ is an IFWGOS in X. Hence $f^{-1}(B)$ is an IFWGCS in X. Therefore f is an intuitionistic fuzzy contra weakly generalized continuous mapping. □

Theorem 3.19. *Let $f : (X, \tau) \to (Y, \sigma)$ be a mapping and let $f^{-1}(A)$ be an IFRCS in X for every IFOS A in Y. Then f is an intuitionistic fuzzy contra weakly generalized continuous mapping.*

Proof. Let A be an IFOS in Y. By hypothesis, $f^{-1}(A)$ is an IFRCS in X. Since every IFRCS is an IFWGCS, $f^{-1}(A)$ is an IFWGCS in X. Hence f is an intuitionistic fuzzy contra weakly generalized continuous mapping. □

Theorem 3.20. *Let $f : (X, \tau) \to (Y, \sigma)$ be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Suppose that one of the following properties hold:*
(a) *$f(wgcl(A)) \subseteq int(f(A))$ for each IFS A in X,*
(b) *$wgcl(f^{-1}(B)) \subseteq f^{-1}(int(B))$ for each IFS B in Y,*
(c) *$f^{-1}(cl(B)) \subseteq wgint(f^{-1}(B))$ for each IFS B in Y.*
Then f is an intuitionistic fuzzy contra weakly generalized continuous mapping.

Proof. (a) ⇒ (b): Let B be an IFS in Y. From the assumption we have, $f(wgcl(f^{-1}(B))) \subseteq int(f(f^{-1}(B))) \subseteq int(B)$. This implies $wgcl(f^{-1}(B)) \subseteq f^{-1}(int(B))$ for each IFS B in Y.
(b) ⇒ (c): It can be proved by using the complement.

Suppose that (c) holds. Let B be an IFCS in Y. By our assumption, we have $f^{-1}(B) \subseteq f^{-1}(cl(B)) \subseteq wgint(f^{-1}(B)) \subseteq f^{-1}(B)$. Hence $wgint(f^{-1}(B)) = f^{-1}(B)$. This implies $f^{-1}(B)$ is an IFWGOS in X. Hence f is an intuitionistic fuzzy contra weakly generalized continuous mapping. □

Theorem 3.21. *Let $f : (X, \tau) \to (Y, \sigma)$ be a bijective mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is an intuitionistic fuzzy contra weakly generalized continuous mapping if $cl(f(A)) \subseteq f(wgint(A))$ for every IFS A in X.*

Proof. Let A be an IFCS in Y. By hypothesis, $cl(f(f^{-1}(A))) \subseteq f(wgint(f^{-1}(A)))$. Since f is onto, $f(f^{-1}(A)) = A$. Therefore $A = cl(A) = cl(f(f^{-1}(A))) \subseteq f(wgint(f^{-1}(A)))$. This implies $f^{-1}(A) \subseteq f^{-1}(f(wgint(f^{-1}(A)))) = wgint(f^{-1}(A)) \subseteq f^{-1}(A)$. Thus $f^{-1}(A)$ is an IFWGOS in X. Hence f is an intuitionistic fuzzy contra weakly generalized continuous mapping. □

Theorem 3.22. *Let $f : (X, \tau) \to (Y, \sigma)$ be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is an intuitionistic fuzzy contra weakly generalized continuous mapping if $f^{-1}(wgcl(B)) \subseteq int(f^{-1}(B))$ for every IFS B in Y.*

Proof. Let B be an IFCS in Y. Since every IFCS is an IFWGCS, we have $wgcl(B) = B$. By hypothesis, $f^{-1}(B) = f^{-1}(wgcl(B)) \subseteq int(f^{-1}(B)) \subseteq f^{-1}(B)$. This implies $f^{-1}(B)$ is an IFOS in X. Hence f is an intuitionistic fuzzy contra continuous mapping. Then by Theorem 3.3, f is an intuitionistic fuzzy contra weakly generalized continuous mapping. □

Theorem 3.23. *An intuitionistic fuzzy continuous mapping $f : (X, \tau) \to (Y, \sigma)$ is an intuitionistic fuzzy contra weakly generalized continuous mapping if IFWGO(X)= IFWGC(X).*

Proof. Let A be an IFOS in Y. By hypothesis, $f^{-1}(A)$ is an IFOS in X. Since every IFOS is an IFWGOS, $f^{-1}(A)$ is an IFWGOS in X. Thus $f^{-1}(A)$ is an IFWGCS in X, by hypothesis. Hence f is an intuitionistic fuzzy contra weakly generalized continuous mapping. □

Theorem 3.24. *Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \delta)$ be any two mappings. Then the following statements hold.*

(a) *Let $f : (X, \tau) \to (Y, \sigma)$ be an intuitionistic fuzzy contra weakly generalized continuous mapping and $g : (Y, \sigma) \to (Z, \delta)$ an intuitionistic fuzzy continuous mapping. Then their composition $gof : (X, \tau) \to (Z, \delta)$ is an intuitionistic fuzzy contra weakly generalized continuous mapping.*

(b) *Let $f : (X, \tau) \to (Y, \sigma)$ be an intuitionistic fuzzy contra weakly generalized continuous mapping and $g : (Y, \sigma) \to (Z, \delta)$ an intuitionistic fuzzy contra continuous mapping. Then their composition $gof : (X, \tau) \to (Z, \delta)$ is an intuitionistic fuzzy weakly generalized continuous mapping.*

(c) *Let $f : (X, \tau) \to (Y, \sigma)$ be an intuitionistic fuzzy weakly generalized irresolute mapping and $g : (Y, \sigma) \to (Z, \delta)$ an intuitionistic fuzzy contra weakly generalized continuous mapping. Then their composition $gof : (X, \tau) \to (Z, \delta)$ is an intuitionistic fuzzy contra weakly generalized continuous mapping.*

Proof. (a) Let A be an IFOS in Z. According to the hypothesis, $g^{-1}(A)$ is an IFOS in Y. Since f is an intuitionistic fuzzy contra weakly generalized continuous mapping, $f^{-1}(g^{-1}(A)) = (gof)^{-1}(A)$ is an IFWGCS in X. Hence gof is an intuitionistic fuzzy contra weakly generalized continuous mapping.

(b) Let A be an IFOS in Z. Hypothetically stating, $g^{-1}(A)$ is an IFCS in Y. Since f is an intuitionistic fuzzy contra weakly generalized continuous mapping, $f^{-1}(g^{-1}(A)) = (gof)^{-1}(A)$ is an IFWGOS in X. Hence gof is an intuitionistic fuzzy weakly generalized continuous mapping.

(c) Let A be an IFOS in Z. To state hypothetically, $g^{-1}(A)$ is an IFWGCS in Y. Since f is an intuitionistic fuzzy weakly generalized irresolute mapping, $f^{-1}(g^{-1}(A)) = (gof)^{-1}(A)$ is an IFWGCS in X. Hence gof is an intuitionistic fuzzy contra weakly generalized continuous mapping. □

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