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Some characterizations of intra-regular semigroups by their generalized fuzzy ideals

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Abstract. In this paper, we give some characterizations of intra-regular semigroups by the properties of their $(\in, \in \vee q_k)$-fuzzy right ideals, $(\in, \in \vee q_k)$-fuzzy left ideals, $(\in, \in \vee q_k)$-fuzzy interior ideals, $(\in, \in \vee q_k)$-fuzzy bi-ideals and $(\in, \in \vee q_k)$-fuzzy generalized bi-ideals.

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1. Introduction

A large number of real word problems in almost all disciplines including engineering, medical science, environmental science, management science, social sciences and artificial intelligence are full of complexities and various types of uncertainties while dealing with them in many occasions. To deal with the complexity of vagueness and uncertainties in an effective way, Zadeh [22] generalized crisp sets, the basis of classical mathematics, by introducing the concept of fuzzy sets. The theory of fuzzy sets has been successfully applied to various fields, especially where classical mathematical approaches are of limited effectiveness. Fuzzy sets are also closely related to other soft computing models such as rough sets [19], vague sets [8] and soft sets [15, 6, 7].

Many authors have applied fuzzy sets to generalized the basic theories of various algebraic structures. The concept of fuzzy substructures of groups was initiated by Rosenfeld [20]. The theory of fuzzy semigroups and fuzzy ideals in semigroups was introduced by Kuroki in [13] and [14]. The theoretical exposition of fuzzy semigroups and their application in fuzzy coding, fuzzy finite state machines and fuzzy languages was considered by Mordeson [16, 17] in a systematic way. Murali [18] proposed the concept of belongingness of a fuzzy point to a fuzzy subset under a natural equivalence on fuzzy subsets. By using the concepts of *belongingness to* (denoted by $\in$) and *quasi-coincidence* (denoted by q) of a fuzzy point with a fuzzy subgroup, Bhakat and Das [1, 2], proposed the concept of (α,β)-fuzzy subgroups, where where $\alpha,\beta \in \{\in, q, \in\vee q, \in\wedge q\}$ and $\alpha \neq \in\wedge q$. In particular, $(\in,\in\vee q)$-fuzzy subgroup is an important and useful generalization of Rosenfeld's fuzzy subgroup. These fuzzy subgroups are further studied in [3, 4]. The concept of $(\in,\in\vee q_k)$-fuzzy subgroups is a viable generalization of Rosenfeld's fuzzy subgroups. Davvaz [5] defined $(\in,\in\vee q_k)$-fuzzy subnearrings and ideals of nearrings. In [9] Jun and Song initiated the study of (α,β)-fuzzy interior ideals of a semigroup which is a natural generalization of fuzzy interior ideals in [10]. Kazanci and Yamak [12] studied $(\in,\in\vee q_k)$-fuzzy bi-ideals of semigroups.

In the present study, we follow the above line of exploration and propose some useful characterizations of intra-regular semigroups by considering the properties of their $(\in,\in\vee q_k)$-fuzzy right ideals, $(\in,\in\vee q_k)$-fuzzy left ideals, $(\in,\in\vee q_k)$-fuzzy interior ideals, $(\in,\in\vee q_k)$-fuzzy bi-ideals and $(\in,\in\vee q_k)$-fuzzy generalized bi-ideals.

2. $(\in,\in\vee q_k)$-fuzzy Ideals in Semigroups

Throughout this paper S will denote a semigroup and k be an arbitrary element of $[0,1)$ unless otherwise specified. A non-empty subset A of S is called a subsemigroup of S if $A^2 \subseteq A$. A non-empty subset J of S is called a left (right) ideal of S if $SJ \subseteq I$ $(JS \subseteq I)$. J is called a two-sided ideal or simply an ideal of S if it is both left and right ideal of S. A non-empty subset B of S is called a generalized bi-ideal of S if $BSB \subseteq B$. A non-empty subset B of S is called a bi-ideal of S if it is both a subsemigroup and a generalized bi-ideal of S. A subsemigroup I of S is called an interior ideal of S if $SIS \subseteq I$.

Definition 2.1. For a fuzzy set f of a semigroup S and $t \in [0,1]$, the crisp set $U(f;t) = \{x \in S \mid f(x) \geq t\}$ is called a level subset of the fuzzy set f.

Definition 2.2. Let f and g be any two fuzzy subsets of S. Then the product $f \circ g$ is a fuzzy subset of S defined by

$$(f \circ g)(a) = \begin{cases} \bigvee\limits_{a=bc} \left(f(b) \wedge g(c)\right), & \text{if there exists } b, c \in S \text{ such that } a = bc, \\ 0, & \text{otherwise.} \end{cases}$$

Definition 2.3. For any $t \in (0,1]$, a fuzzy subset f of S defined as

$$f(y) = \begin{cases} t, & \text{if } y = x, \\ 0, & \text{otherwise,} \end{cases}$$

is called a *fuzzy point* with support x and value t, which is denoted by x_t.

A fuzzy point x_t is said to *belong* to (*resp. quasi-coincident with*) a fuzzy set f, written as $x_t \in f$ (*resp.* $x_t q f$) if $f(x) \geq t$ (*resp.* $f(x) + t > 1$). If $x_t \in f$ or $x_t q f$, then we write $x_t \in \vee q f$. The symbol $\overline{\in\vee q}$ means $\in\vee q$ does not hold. For any two fuzzy subsets f and g of S, $f \leq g$ means that $f(x) \leq g(x)$ for all $x \in S$.

Generalizing the concept of $x_t q f$, Jun [10, 11], defined $x_t q_k f$, where $k \in [0,1)$, as $f(x) + t + k > 1$. By $x_t \in \vee q_k f$ we shall mean that $x_t \in f$ or $x_t q_k f$.

Definition 2.4 ([21])**.** A fuzzy subset of S is called an $(\in, \in \vee q_k)$-fuzzy subsemigroup of S if for all $x, y \in S$ and $t, r \in (0, 1]$ the following condition holds:

$$x_t \in f, y_r \in f \Rightarrow (xy)_{\min\{t,r\}} \in \vee q_k f.$$

Lemma 2.5 ([21])**.** *Let f be a fuzzy subset of S. Then f is an $(\in, \in \vee q_k)$-fuzzy subsemigroup of S if and only if* $f(xy) \geq \min\left\{f(x), f(y), \frac{1-k}{2}\right\}$.

Definition 2.6 ([21])**.** A fuzzy subset f of S is called an $(\in, \in \vee q_k)$-fuzzy left (resp. right) ideal of S if for all $x, y \in S$ and $t \in (0, 1]$ the following condition holds:

$$y_t \in f \Rightarrow (xy)_t \in \vee q_k f \ \ (\text{resp. } x_t \in f \Rightarrow (xy)_t \in \vee q_k f).$$

Lemma 2.7 ([21])**.** *Let f be a fuzzy subset of S. Then f is an $(\in, \in \vee q_k)$-fuzzy left ideal of S if and only if* $f(xy) \geq \min\left\{f(y), \frac{1-k}{2}\right\}$.

Lemma 2.8 ([21])**.** *Let f be a fuzzy subset of S. Then f is an $(\in, \in \vee q_k)$-fuzzy right ideal of S if and only if* $f(xy) \geq \min\left\{f(x), \frac{1-k}{2}\right\}$.

A fuzzy subset f of S is called an $(\in, \in \vee q_k)$-fuzzy ideal of S if it is both an $(\in, \in \vee q_k)$-fuzzy left ideal and an $(\in, \in \vee q_k)$-fuzzy right ideal of S.

Definition 2.9 ([21])**.** A fuzzy subset f of S is called an $(\in, \in \vee q_k)$-fuzzy generalized bi-ideal of S, if for all x, y,$z \in S$ and t, $r \in (0, 1]$, we have

$$x_t \in f, z_r \in f \Rightarrow (xyz)_{\min\{t,r\}} \in \vee q_k f.$$

An $(\in, \in \vee q_k)$-fuzzy generalized bi-ideal of S is called an $(\in, \in \vee q_k)$-fuzzy bi-ideal of S if it is also an $(\in, \in \vee q_k)$-fuzzy subsemigroup of S.

Lemma 2.10 ([21])**.** *LA fuzzy subset f of S is an $(\in, \in \vee q_k)$-fuzzy bi-ideal of S if and only if it satisfies the following conditions.*
(i) $f(xy) \geq \min\left\{f(x), f(y), \frac{1-k}{2}\right\}$ *for all $x, y \in S$ and $k \in [0, 1)$.*
(ii) $f(xyz) \geq \min\left\{f(x), f(z), \frac{1-k}{2}\right\}$*for all $x, y, z \in S$ and $k \in [0, 1)$.*

Lemma 2.11 ([21])**.** *A fuzzy subset f of S is an $(\in, \in \vee q_k)$-fuzzy generalized bi-ideal of S if and only if* $f(xyz) \geq \min\left\{f(x), f(z), \frac{1-k}{2}\right\}$*for all $x, y, z \in S$ and $k \in [0, 1)$.*

Definition 2.12. A fuzzy subsemigroup f of S is called an $(\in, \in \vee q_k)$-fuzzy interior ideal of S if

$$y_t \in f \Rightarrow (xyz)_t \in \vee q_k f$$

for all $x, y, z \in S$ and $t \in (0, 1]$.

Lemma 2.13 ([21])**.** *A fuzzy subset f of S is an $(\in, \in \vee q_k)$-fuzzy interior ideal of S if and only if it satisfies the following conditions.*
(i) $f(xy) \geq \min\left\{f(x), f(y), \frac{1-k}{2}\right\}$ *for all $x, y \in S$ and $k \in [0, 1)$.*
(ii) $f(xyz) \geq \min\left\{f(y), \frac{1-k}{2}\right\}$*for all $x, y, z \in S$ and $k \in [0, 1)$.*

Definition 2.14. Let A be any subset of S. Then the characteristic function $(C_A)_k$ is defined as

$$(C_A)_k(a) = \begin{cases} \frac{1-k}{2}, & \text{if } a \in A, \\ 0, & \text{otherwise.} \end{cases}$$

Example 2.15. Let $S = \{1, 2, 3\}$ be a semigroup with binary operation "$\cdot$", as defined by the following Cayley table:

$\cdot$	1	2	3
1	1	1	1
2	2	2	2
3	3	3	3

Clearly, $(S, \cdot)$ is semigroup and $\{1\}$, $\{2\}$ and $\{3\}$ are left ideals of S. Let δ be a fuzzy subset of S such that

$$\delta(1) = 0.9,\ \delta(2) = 0.6,\ \delta(3) = 0.5.$$

Then it is easy to verify that δ is an $(\in, \in \vee q_k)$-fuzzy ideal of S.

Example 2.16. Let $S = \{1, 2, 3, 4\}$ be a semigroup with binary operation "$\cdot$", as defined by the following Cayley table:

$\cdot$	1	2	3	4
1	1	1	1	1
2	1	2	2	2
3	1	3	3	3
4	1	2	2	2

Let δ be a fuzzy subset of S such that

$$\delta(1) = 0.8,\ \delta(2) = 0.7,\ \delta(3) = 0.5,\ \ \delta(4) = 0.6.$$

Then it is easy to verify that δ is an $(\in, \in \vee q_k)$-fuzzy bi-ideal of S.

Example 2.17. Let $S = \{a, b, c, d\}$ be a semigroup with binary operation "$\cdot$", as defined by the following Cayley table:

$\cdot$	a	b	c	d
a	a	a	a	a
b	a	b	c	c
c	a	c	c	c
d	a	c	c	c

Let δ be a fuzzy subset of S such that

$$\delta(a) = 0.9,\ \delta(b) = 0.7,\ \delta(c) = 0.5, \delta(d) = 0.6.$$

Then it is easy to verify that δ is an $(\in, \in \vee q_k)$-fuzzy interior ideal of S.

Lemma 2.18 ([21])**.** *A fuzzy subset f of S is an $(\in, \in \vee q_k)$-fuzzy generalized bi-ideal of S if and only if $f(xyz) \geq \min\left\{f(x), f(z), \frac{1-k}{2}\right\}$for all $x, y, z \in S$ and $k \in [0, 1)$.*

Lemma 2.19 ([21])**.** *A non-empty subset I of a semigroup S is an interior ideal if and only if $(C_I)_k$ is an $(\in, \in \vee q_k)$-fuzzy interior ideal of S.*

Lemma 2.20 ([21])**.** *A non-empty subset B of a semigroup S is bi-ideal if and only if $(C_B)_k$ is an $(\in, \in \vee q_k)$-fuzzy bi-ideal of S.*

Lemma 2.21 ([21]). *Let f and g be any fuzzy subsets of semigroup S. Then following properties hold.*
(*i*) $(f \wedge_k g) = (f_k \wedge g_k)$.
(*ii*) $(f \circ_k g) = (f_k \circ g_k)$.

Lemma 2.22 ([21]). *Let A and B be any non-empty subsets of a semigroup S. Then the following properties hold.*
(*i*) $(C_A \wedge_k C_B) = (C_{A\cap B})_k$.
(*ii*) $(C_A \circ_k C_B) = (C_{AB})_k$.

3. Characterizations of intra-regular semigroups in terms of $(\in, \in \vee q_k)$-fuzzy ideals

An element a of a semigroup S is called intra-regular if there exist $x, y \in S$ such that $a = xa^2y$. We refer to S as an intra-regular semigroup if every element of S is intra-regular. It is worth noting that in general neither intra regular semigroups are regular semigroups nor regular semigroups are intra-regular semigroups. Nevertheless, both the concepts coincide for commutative semigroups [21].

Theorem 3.1. *For a semigroup S the following are equivalent.*
(*i*) *S is intra-regular.*
(*ii*) *$I \cap B \cap R \subseteq IBR$ for every interior I, bi-ideal B and right R of S.*
(*iii*) *$I[a] \cap B[a] \cap R[a] \subseteq I[a]B[a]R[a]$, for some a in S.*

Proof. $(i) \Rightarrow (ii)$: Let S be intra-regular semigroup, I an interior ideal, B a bi-ideal and R a right ideal of S. Let $a \in I \cap B \cap R$ then $a \in I$, $a \in B$ and $a \in R$. Since S is intra-regular therefore for each $a \in S$ there exist $x, y \in S$ such that

$$\begin{aligned} a &= xaay = x(xaay)(xaay)y = x^2aayxaay^2 \\ &= (x^2xaay)(ayxa)(ay^2) \in (SIS)(BSB)(RS) \\ &\subseteq IBR. \end{aligned}$$

Thus $I \cap B \cap R \subseteq IBR$.

$(ii) \Rightarrow (iii)$ is obvious.

$(iii) \Rightarrow (i)$: As $a \cup SaS$, $a \cup a^2 \cup aSa$ and $a \cup aS$ are interior ideal, bi-ideal and right ideal of S generated by an element a of S respectively. Thus by assumption we have

$$\begin{aligned} &(a \cup SaS) \cap (a \cup a^2 \cup aSa) \cap (a \cup aS) \subseteq (a \cup SaS)(a \cup a^2 \cup aSa)(a \cup aS) \\ &= (a^2 \cup a^3 \cup a^2Sa \cup SaSa \cup SaSa^2 \cup SaSaSa)(a \cup aS) \\ &= a^3 \cup a^4 \cup a^2Sa^2 \cup SaSa^2 \cup SaSa^3 \cup SaSaSa^2 \cup a^3S \\ &\qquad \cup a^4S \cup a^2Sa^2S \quad \cup SaSa^2S \cup SaSa^3S \cup SaSaSa^2S \\ &\subseteq a^3 \cup a^4 \cup Sa^2S \cup a^2Sa^2 \cup Sa^2. \end{aligned}$$

Therefore $a = a^3 = aaa = a^3aa = aa^2aa = xa^2y$ where $x = y = a$ or $a = a^4 = aa^2a = pa^2q$ where $p = q = a$ or $a = ua^2v$ or $a = a^2na^2 = aana^2 = aa^2na^2na^2 = ra^2s$ where $r = a$ and $s = na^2na^2$ or $a = la^2 = laa = lla^2a = ta^2m$ where $t = ll$ and $m = a$ for some $x, y, p, q, r, u, v, l, m, t$ and n in S. Hence S is intra-regular. □

Theorem 3.2. *For a semigroup S, the following conditions are equivalent.*

(i) S is intra-regular.

(ii) $f \circ_k g \circ_k h \geq f \wedge_k g \wedge_k h$ for every $(\in, \in \vee q_k)$-fuzzy interior ideal f, $(\in, \in \vee q_k)$-fuzzy bi-ideal g and $(\in, \in \vee q_k)$-fuzzy right ideal h of a semigroup S.

(iii) $f \circ_k g \circ_k h \geq f \wedge_k g \wedge_k h$ for every $(\in, \in \vee q_k)$-fuzzy interior ideal f, $(\in, \in \vee q_k)$-fuzzy generalized bi-ideal g and $(\in, \in \vee q_k)$-fuzzy right ideal h of a semigroup S.

Proof. $(i) \Rightarrow (iii)$: Let f, g and h be any $(\in, \in \vee q_k)$-fuzzy interior ideal, $(\in, \in \vee q_k)$-fuzzy bi-ideal and $(\in, \in \vee q_k)$-fuzzy right ideal of S respectively. Since S is intra-regular therefore for each $a \in S$ there exist $x, y \in S$ such that

$$\begin{aligned} a &= xaay = x(xaay)(xaay)y = x^2aayxaay^2 \\ &= (x^2xaay)(ayxa)(ay^2) = (x^3aay)(ayxa)(ay^2). \end{aligned}$$

Thus

$$\begin{aligned} (f \circ_k g \circ_k h)(a) &= (f \circ g \circ h)(a) \wedge \frac{1-k}{2} \\ &= \left(\bigvee_{a=pq} \{ f(p) \wedge (g \circ h)(q) \} \right) \wedge \frac{1-k}{2} \\ &\geq f\left(x^3aay\right) \wedge (g \circ h)\left((ayxa)(ay^2)\right) \wedge \frac{1-k}{2} \\ &\geq f(a) \wedge \left(\bigvee_{(ayxa)(ay^2)=bc} \{ g(b) \wedge h(c) \} \right) \wedge \frac{1-k}{2} \\ &\geq f(a) \wedge g(ayxa) \wedge h\left(ay^2\right) \wedge \frac{1-k}{2} \\ &\geq f(a) \wedge g(a) \wedge h(a) \wedge \frac{1-k}{2} \\ &\geq f(a) \wedge g(a) \wedge h(a) \wedge \frac{1-k}{2}. \end{aligned}$$

$(iii) \Rightarrow (ii)$ is obvious.

$(ii) \Rightarrow (iii)$: Let $I[a]$, $B[a]$ and $R[a]$ be interior ideal, bi-ideal and right ideal of S generated by a. Then $(C_{I[a]})_k$, $(C_{B[a]})_k$ and $(C_{R[a]})_k$ be $(\in, \in \vee q_k)$-fuzzy interior ideal, $(\in, \in \vee q_k)$-fuzzy bi-ideal and $(\in, \in \vee q_k)$-fuzzy right ideal of semigroup S. Let $a \in S$ and $b \in I[a] \cap B[a] \cap R[a]$, then $b \in I[a]$, $b \in B[a]$ and $b \in R[a]$. Now

$$\begin{aligned} \frac{1-k}{2} &\leq (C_{I[a] \cap B[a] \cap R[a]})_k(b) = ((C_{I[a]})_k \wedge_k (C_{B[a]})_k \wedge_k (C_{R[a]})_k))(b) \\ &\leq ((C_{I[a]})_k \circ_k (C_{B[a]})_k \circ_k (C_{R[a]})_k)(b) = (C_{I[a]B[a]R[a]})_k(b). \end{aligned}$$

Thus $b \in I[a]B[a]R[a]$. Therefore $I[a] \cap B[a] \cap R[a] \subseteq I[a]B[a]R[a]$. So by Theorem 3.1, S is intra-regular. $\square$

Theorem 3.3. *For a semigroup S the following are equivalent.*

(i) S is intra-regular.

(ii) $L \cap B \cap I \subseteq LBI$ for every left L, bi-ideal B and interior I.

(iii) $L[a] \cap B[a] \cap I[a] \subseteq L[a]B[a]I[a]$, for some a in S.

Proof. $(i) \Rightarrow (ii)$: Let S be intra-regular semigroup, L a left ideal, B a bi-ideal and I an interior ideal of S. Let $a \in L \cap B \cap I$ then $a \in L$, $a \in B$ and $a \in I$. Since S is intra-regular therefore for each $a \in S$ there exist $x, y \in S$ such that

$$a = xaay = (xxa)(ayxa)(xaayyy) \in (SL)(BSB)(SIS).$$

Thus $L \cap B \cap I \subseteq LBI$.

$(ii) \Rightarrow (iii)$ is obvious.

$(iii) \Rightarrow (i)$: As $a \cup Sa$, $a \cup a^2 \cup aSa$ and $a \cup SaS$ are left ideal, bi-ideal and interior ideal of S generated by an element a of S respectively. Thus by assumption we have

$$\begin{aligned}
&(a \cup Sa) \cap (a \cup a^2 \cup aSa) \cap (a \cup a^2 \cup SaS) \\
&\subseteq (a \cup Sa)(a \cup a^2 \cup aSa)(a \cup a^2 \cup SaS) \\
&= (a^2 \cup a^3 \cup a^2Sa \cup Sa^2 \cup Sa^3 \cup Sa^2Sa)(a \cup a^2 \cup SaS) \\
&= a^3 \cup a^4 \cup a^2Sa^2 \cup Sa^3 \cup Sa^4 \cup Sa^2Sa^2 \cup a^4 \cup a^5 \cup a^2Sa^3 \cup Sa^4 \\
&\qquad \cup Sa^5 \cup Sa^2Sa^3 \cup a^2SaS \cup a^3SaS \cup a^2SaSaS \\
&\qquad \cup Sa^2SaS \cup Sa^3SaS \cup Sa^2SaSaS \\
&\subseteq a^3 \cup a^4 \cup a^5 \cup a^2Sa^2 \cup a^2S \cup Sa^2S.
\end{aligned}$$

$a = a^3$ or $a = a^4$or $a = a^5$ or $a = a^2pa^2$ or $a = a^2x$ or $a = ua^2v$ for some x, u, v and p in S. If $a = aaaaa = aa^2a^2$. If $a = a^4$ then $a = aa^2a$ If $a = a^5$ then $a = aa^2a^2$.If $a = a^2pa^2$ then $a = a^2pa^2 = aapa^2 = (aa^2p)a^2(pa^2)$.If $a = a^2x$ then $a = aax = aa^2x^2$ Hence S is intra-regular. □

Theorem 3.4. *For a semigroup S, the following conditions are equivalent.*

(i) S is intra-regular.

(ii) $f \circ_k g \circ_k h \geq f \wedge_k g \wedge_k h$ for every $(\in, \in \vee q_k)$-fuzzy left ideal f, $(\in, \in \vee q_k)$-fuzzy bi-ideal g and $(\in, \in \vee q_k)$-fuzzy interior ideal h of a semigroup S.

(iii) $f \circ_k g \circ_k h \geq f \wedge_k g \wedge_k h$ for every $(\in, \in \vee q_k)$-fuzzy left ideal f, $(\in, \in \vee q_k)$-fuzzy generalized bi-ideal g and $(\in, \in \vee q_k)$-fuzzy interior ideal h of a semigroup S.

Proof. $(i) \Rightarrow (iii)$: Let f, g and h be any $(\in, \in \vee q_k)$-fuzzy left ideal, $(\in, \in \vee q_k)$-fuzzy bi-ideal and $(\in, \in \vee q_k)$-fuzzy interior ideal of S. Since S is intra-regular therefore for each $a \in S$ there exist $x, y \in S$ such that

$$a = xaay = (xxa)(ayxa)(xaayyy).$$

Thus

$$\begin{aligned}
&(f \circ_k g \circ_k h)(a) = (f \circ g \circ h)(a) \wedge \tfrac{1-k}{2} \\
&= \left(\bigvee_{a=pq} \{ f(p) \wedge (g \circ h)(q) \} \right) \wedge \tfrac{1-k}{2} \\
&\geq f(xxa) \wedge (g \circ h)((ayxa)(xaayyy)) \wedge \tfrac{1-k}{2} \\
&\geq f(a) \wedge \left(\bigvee_{(ayxa)(xaayyy)=bc} \{ g(b) \wedge h(c) \} \right) \wedge \tfrac{1-k}{2} \\
&\geq f(a) \wedge g(ayxa) \wedge h(xaayyy) \wedge \tfrac{1-k}{2}
\end{aligned}$$

$\geq f(a) \wedge g(a) \wedge h(a) \wedge \frac{1-k}{2}$

$\geq f(a) \wedge g(a) \wedge h(a) \wedge \frac{1-k}{2}$.

$(iii) \Rightarrow (ii)$ is obvious.

$(ii) \Rightarrow (i)$: Let $L[a]$, $B[a]$ and $I[a]$ be left ideal, bi-ideal and interior ideal of S generated by a respectively. Then $(C_{L[a]})_k$, $(C_{B[a]})_k$ and $(C_{I[a]})_k$ be $(\in, \in \vee q_k)$-fuzzy left ideal, $(\in, \in \vee q_k)$-fuzzy bi-ideal and $(\in, \in \vee q_k)$-fuzzy interior ideal of semigroup S. Let $a \in S$ and $b \in L[a] \cap B[a] \cap I[a]$. Then $b \in L[a]$, $b \in B[a]$ and $b \in I[a]$. Now

$$\begin{aligned} \frac{1-k}{2} &\leq (C_{L[a]\cap B[a]\cap I[a]})_k(b) = ((C_{L[a]})_k \wedge_k (C_{B[a]})_k \wedge_k (C_{I[a]})_k))(b) \\ &\leq ((C_{L[a]})_k \circ_k (C_{B[a]})_k \circ_k (C_{I[a]})_k)(b) = (C_{L[a]B[a]I[a]})_k(b). \end{aligned}$$

Thus $b \in L[a]B[a]I[a]$. Therefore $L[a] \cap B[a] \cap I[a] \subseteq L[a]B[a]I[a]$. Hence by Theorem 3.3, S is intra-regular. □

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