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Fuzzy soft topological spaces

Tugbahan Simsekler, Saziye Yuksel

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Abstract. In this paper we define fuzzy soft topology over a fuzzy soft set with fixed parameter set. By this way we define fuzzy soft open sets and fuzzy soft closed sets. We give the definition of belongness of a fuzzy point to a fuzzy soft set and hence we define fuzzy soft interior and fuzzy soft closure point. Also we present the fuzzy soft neighborhood and fuzzy soft Q-neighborhood of a fuzzy point. This paper is a begining point for fuzzy soft topology over a fuzzy soft set with fixed parameter set.

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Corresponding Author: Saziye Yuksel (syuksel@selcuk.edu.tr)

1. Introduction

We can not solve the problems by using mathematical tools generally in the social life since in mathematics the concepts are precise, not subjective. Some theories are developed to eliminate this lack for vagueness such as theory of fuzzy sets [14], theory of rough sets [10], theory of intuitionistic fuzzy sets [4], theory of interval mathematics [5].

Molodtsov [9] introduced soft set theory as a different method for vagueness in 1999. He applied soft set theory to several directions, such as game theory, Riemann integration, Perron integration, smoothness of functions. Maji et. al. [7] defined new notions of soft set theory. Shabir and Naz [12] defined soft topology by using soft sets and presented the basic properties in their paper.

Fuzzy soft set which is a combination of fuzzy and soft sets were first introduced by Maji et.al. [8] in 2001. Many researchers improved this study and gave new results ([1], [3]). Aygunoglu and Aygun [6] applied fuzzy soft sets on group theory. Tanay and Kandemir [13] defined fuzzy soft topology on a fuzzy soft set over an initial universe. They introduced new concepts like fuzzy soft base, fuzzy soft neighborhood system, fuzzy soft subspace topology and they presented basic properties. Roy and Samanta [11] defined fuzzy soft topology over the initial universe and they introduced base and subbase for this space also they gave some characterizations.

In our paper we introduce fuzzy soft topology, which is a generalization of soft topology, over a fuzzy soft set on (U, E) with a fixed set of parameter. By this way we define the notions of fuzzy soft open sets, fuzzy soft closed sets, fuzzy soft neighborhood of a point, fuzzy soft Q-neighborhood of a point, fuzzy soft interior and fuzzy soft closure point. We presented the basic notions of theory.

2. Preliminaries

Throughout this paper U denotes initial universe, E denotes the set of all possible parameters for U , $P(U)$ denotes the power set of U and I^U denotes the set of all fuzzy subset of U and "(U, E)" denotes the universal set U and the parameter set E.

Definition 2.1. [14] A fuzzy set A in U is a set of ordered pairs:

$$A = \{(x, \mu_A(x)) : x \in U\}$$

where $\mu_A(x) : U \to [0, 1] = I$ is a mapping and $\mu_A(x)$ (or $A(x)$) states the grade of belongness of x in A. The family of all fuzzy sets in U is denoted by I^U.

Definition 2.2. [14] Let A, B be two fuzzy sets of I^U

1. A is contained in B if and only if $\mu_A(x) \leq \mu_B(x)$ for every $x \in U$.
2. The union of A and B is a fuzzy set C, denoted by $A \vee B = C$, whose membership function $\mu_C(x) = \max\{\mu_A(x), \mu_B(x)\}$ for every $x \in U$
3. The intersection of A and B is a fuzzy set C, denoted by $A \wedge B = C$ whose membership function $\mu_C(x) = \min\{\mu_A(x), \mu_B(x)\}$ for every $x \in U$.
4. The complement of A is a fuzzy set, denoted by A^c,whose membership function $\mu_{A^c}(x) = 1 - \mu_A(x)$ for every $x \in U$.

Definition 2.3. [9] Let $A \subseteq E$. A pair (F, A) is called soft set over U where F is a mapping given by $F : A \to P(U)$.

Aktas and Cagman [2] showed that every fuzzy set is a soft set. That is fuzzy sets are a special class of soft sets.

Definition 2.4. [11] Let $A \subseteq E$. (f_A, E) is defined to be a fuzzy soft set on (U, E) if f_A:$E \to I^U$ is a mapping defined by $f_A(e) = \mu^e_{f_A}$ where $\mu^e_{f_A} = \bar{O}$ if $e \in E - A$ and $\mu^e_{f_A} \neq \bar{O}$ if $e \in A$ where $\bar{O}(u) = 0$ for each $u \in U$.

Definition 2.5. [11] The complement of a fuzzy soft set (f_A, E) on (U, E) is a fuzzy soft set (f^c_A, E) which is denoted by $(f_A, E)^c$ and $f^c_A : E \to I^U$ is defined by $\mu^e_{f^c_A} = 1 - \mu^e_{f_A}$ if $e \in A$ and $\mu^e_{f^c_A} = \bar{I}$ if $e \in E \backslash A$, where $\bar{I}(u) = 1$ for each $u \in U$.

Example 2.6. Tugba and Gurcan are going to marry and they want to hire a wedding room. The fuzzy soft set (f_A, E) describes the "capacity of the wedding room". Let $U = \{a, b, c, d, e\}$ be the wedding rooms under consideration, $E = \{big = e_1, central = e_2, cheap = e_3, expensive = e_4, elegant = e_5, quality = e_6, good\ serving = e_7\}$ be the parameter set and $A = \{e_2, e_5, e_6\}$ be a subset of E.

$(f_A, E) = \{e_2 = \{a_{0.3}, b_{0.5}, c_{0.9}, d_{0.8}, e_{0.6}\}, e_5 = \{a_{0.8}, b_{0.6}, c_{0.2}, d_{0.1}, e_{0.5}\}, e_6 = \{a_{0.7}, b_{0.5}, c_{0.3}, d_{0.2}, e_{0.4}\}\}$ is a fuzzy soft set on (U, E). The complement of (f_A, E) is,

$(f_A, E)^c = \{e_1 = \{a_1, b_1, c_1, d_1, e_1\}, e_2 = \{a_{0.7}, b_{0.5}, c_{0.1}, d_{0.2}, e_{0.4}\}, e_3 = \{a_1, b_1, c_1, d_1, e_1\}, e_4 = \{a_1, b_1, c_1, d_1, e_1\}, e_5 = \{a_{0.2}, b_{0.4}, c_{0.8}, d_{0.9}, e_{0.5}\}, e_6 = \{a_{0.3}, b_{0.5}, c_{0.7}, d_{0.8}, e_{0.6}\}, e_7 = \{a_1, b_1, c_1, d_1, e_1\}\}$.

Definition 2.7. [11] The fuzzy soft set (f_Φ, E) on (U, E) is defined to be null fuzzy soft set and is denoted by Φ. $f_\Phi(e) = \bar{O}$ for every $e \in E$.

Definition 2.8. [11] The fuzzy soft set (f_E, E) on (U, E) is defined to be absolute fuzzy soft set and is denoted by $E^\sim$. $f_E(e) = \bar{I}$ for every $e \in E$.

Clearly $\Phi^c = E^\sim$ and $E^{\sim c} = \Phi$.

Definition 2.9. [11] Let (f_A, E) and (g_B, E) be two fuzzy soft sets on (U, E). (f_A, E) is defined to be fuzzy soft subset of (g_B, E) if $\mu^e_{f_A} \subseteq \mu^e_{g_B}$ for all $e \in E$, i.e., $\mu^e_{f_A}(u) \leq \mu^e_{g_B}(u)$ for all $u \in U$ and for all $e \in E$ and is denoted by $(f_A, E) \sqsubseteq (g_B, E)$.

Definition 2.10. [11] Let (f_A, E) and (g_B, E) be two fuzzy soft sets on (U, E) . The union of these two fuzzy soft sets is a fuzzy soft set (h_C, E), defined by $h_C(e) = \mu^e_{h_C} = \mu^e_{f_A} \cup \mu^e_{g_B}$ for all $e \in E$ where $C = A \cup B$ and is denoted by $(h_C, E) = (f_A, E) \sqcup (g_B, E)$.

Definition 2.11. [11] Let (f_A, E) and (g_B, E) be two fuzzy soft sets on (U, E) . The intersection of these two fuzzy soft sets is a fuzzy soft set (h_C, E), defined by $h_C(e) = \mu^e_{h_C} = \mu^e_{f_A} \cap \mu^e_{g_B}$ for all $e \in E$ where $C = A \cap B$ and is denoted by $(h_C, E) = (f_A, E) \sqcap (g_B, E)$.

Theorem 2.12. [1] *Let* $((f_A, E)$ *and* $((g_A, E)$ *be two fuzzy soft sets on* (U, E)*. Then the following holds:*

(1) $(f_A, E)^c \sqcap (g_A, E)^c = [(f_A, E) \sqcup (g_A, E)]^c$.
(2) $(f_A, E) \sqcap (g_A, E)]^c = (f_A, E)^c \sqcup (g_A, E)^c$.

Theorem 2.13. [1] *Let* (f_{i_A}, E) *be a family of fuzzy soft sets on* (U, E)*. Then the following holds:*

(1) $\sqcap_i (f_{i_A}, E)^c = (\sqcup_i (f_{i_A}, E))^c$.
(2) $\sqcup_i (f_{i_A}, E)^c = (\sqcap_i (f_{i_A}, E))^c$.

3. Fuzzy soft topological spaces

Tanay and Kandemir [13] defined fuzzy soft topology on a fuzzy soft set over an initial universe. In this definition the parameter sets of fuzzy soft sets generate the fuzzy soft topology are not same they can be chosen arbitrary. In our paper we fix the parameter set of fuzzy soft sets which construct the fuzzy soft topology and give the definition as following.

Definition 3.1. Let (f_A, E) be a fuzzy soft set on (U, E) and τ_f be the collection of fuzzy soft subsets of (f_A, E), then τ_f is said to be a fuzzy soft topology on (f_A, E) if the following conditions hold:

(1) $\Phi, (f_A, E) \in \tau_f$,
(2) If $(f_{i_A}, E) \in \tau_f$, then $\sqcup_i (f_{i_A}, E) \in \tau_f$,
(3) If $(g_A, E), (h_A, E) \in \tau_f$, then $(g_A, E) \sqcap (h_A, E) \in \tau_f$.

The triplet (f_A, E, τ_f) is called a fuzzy soft topology over (f_A, E).

Definition 3.2. Let (f_A, E, τ_f) be a fuzzy soft topological space over (f_A, E). A fuzzy soft subset of (f_A, E) is called fuzzy soft closed if its complement is a member of τ_f.

Theorem 3.3. *Let (f_A, E, τ_f) be a fuzzy soft topological space over (f_A, E). Then,*
(1) *$E^{\sim}, (f_A, E)^c$ are fuzzy soft closed sets,*
(2) *The arbitrary intersection of fuzzy soft closed sets are fuzzy soft closed,*
(3) *The union of two fuzzy soft closed sets is a fuzzy soft closed set.*

Proof. (1) It is clear.
(2) It is clear by Theorem 13.
(3) It is clear by Theorem 12.

□

Example 3.4. Let $U = \{a, b, c, d, e\}$, $E = \{e_1, e_2, e_3, e_4\}$, $A = \{e_1, e_2, e_3\}$ and
$(f_A, E) = \{e_1 = \{a_{0.3}, b_{0.4}, c_{0.5}, d_{0.6}, e_{0.7}\}, e_2 = \{a_{0.8}, b_{0.5}, c_{0.9}, d_{0.5}, e_1\}, e_3 = \{a_{0.6}, b_{0.7}, c_{0.4}, d_{0.5}, e_{0.8}\}\}$,
$(f_{1_A}, E) = \{e_1 = \{a_{0.2}, b_{0.4}, c_{0.1}, d_{0.3}, e_{0.5}\}, e_2 = \{a_{0.7}, b_{0.4}, c_{0.8}, d_{0.3}, e_{0.9}\}, e_3 = \{a_{0.5}, b_{0.6}, c_{0.1}, d_{0.3}, e_{0.8}\}\}$,
$(f_{2_A}, E) = \{e_1 = \{a_{0.3}, b_{0.3}, c_{0.2}, d_{0.5}, e_{0.6}\}, e_2 = \{a_{0.8}, b_{0.3}, c_{0.7}, d_{0.4}, e_{0.8}\}, e_3 = \{a_{0.4}, b_{0.7}, c_{0.2}, d_{0.2}, e_{0.6}\}\}$,
$(f_{3_A}, E) = \{e_1 = \{a_{0.2}, b_{0.3}, c_{0.1}, d_{0.3}, e_{0.5}\}, e_2 = \{a_{0.7}, b_{0.3}, c_{0.7}, d_{0.3}, e_{0.8}\}, e_3 = \{a_{0.4}, b_{0.6}, c_{0.1}, d_{0.2}, e_{0.6}\}\}$,
$(f_{4_A}, E) = \{e_1 = \{a_{0.3}, b_{0.4}, c_{0.2}, d_{0.5}, e_{0.6}\}, e_2 = \{a_{0.8}, b_{0.4}, c_{0.8}, d_{0.4}, e_{0.9}\}, e_3 = \{a_{0.5}, b_{0.7}, c_{0.2}, d_{0.3}, e_{0.8}\}\}$,
$(f_{5_A}, E) = \{e_1 = \{a_{0.1}, b_{0.2}, c_0, d_{0.2}, e_{0.4}\}, e_2 = \{a_{0.5}, b_{0.1}, c_{0.4}, d_{0.1}, e_0\}, e_3 = \{a_{0.2}, b_{0.4}, c_{0.1}, d_{0.1}, e_{0.4}\}\}$.

Then $\tau_f = \{\Phi, (f_A, E), (f_{1_A}, E), (f_{2_A}, E), (f_{3_A}, E), (f_{4_A}, E), (f_{5_A}, E)\}$ is a fuzzy soft topology on (f_A, E).

By the complement we obtain the family fuzzy soft closed sets:

$$\tau_f^c = \{E^{\sim}, (f_A, E)^c, (f_{1_A}, E)^c, (f_{2_A}, E)^c, (f_{3_A}, E)^c, (f_{4_A}, E)^c, (f_{5_A}, E)^c\}$$

where $(f_A, E)^c = \{e_1 = \{a_{0.7}, b_{0.6}, c_{0.5}, d_{0.4}, e_{0.3}\}, e_2 = \{a_{0.2}, b_{0.5}, c_{0.1}, d_{0.5}, e_0\}$,
$e_3 = \{a_{0.4}, b_{0.3}, c_{0.6}, d_{0.5}, e_{0.2}\}, e_4 = \{a_1, b_1, c_1, d_1, e_1\}\}$,
$(f_{1_A}, E)^c = \{e_1 = \{a_{0.8}, b_{0.6}, c_{0.9}, d_{0.7}, e_{0.5}\}, e_2 = \{a_{0.3}, b_{0.6}, c_{0.2}, d_{0.7}, e_{0.1}\}$,
$e_3 = \{a_{0.5}, b_{0.4}, c_{0.9}, d_{0.7}, e_{0.2}\}, e_4 = \{a_1, b_1, c_1, d_1, e_1\}\}$,
$(f_{2_A}, E)^c = \{e_1 = \{a_{0.7}, b_{0.7}, c_{0.8}, d_{0.5}, e_{0.4}\}, e_2 = \{a_{0.2}, b_{0.7}, c_{0.3}, d_{0.6}, e_{0.2}\}$,
$e_3 = \{a_{0.6}, b_{0.3}, c_{0.8}, d_{0.8}, e_{0.4}\}, e_4 = \{a_1, b_1, c_1, d_1, e_1\}\}$,
$(f_{3_A}, E)^c = \{e_1 = \{a_{0.8}, b_{0.7}, c_{0.9}, d_{0.7}, e_{0.5}\}, e_2 = \{a_{0.3}, b_{0.7}, c_{0.3}, d_{0.7}, e_{0.2}\}$,
$e_3 = \{a_{0.6}, b_{0.4}, c_{0.9}, d_{0.8}, e_{0.4}\}, e_4 = \{a_1, b_1, c_1, d_1, e_1\}\}$,
$(f_{4_A}, E)^c = \{e_1 = \{a_{0.7}, b_{0.6}, c_{0.8}, d_{0.5}, e_{0.4}\}, e_2 = \{a_{0.2}, b_{0.6}, c_{0.2}, d_{0.6}, e_{0.1}\}$,
$e_3 = \{a_{0.5}, b_{0.3}, c_{0.8}, d_{0.7}, e_{0.2}\}, e_4 = \{a_1, b_1, c_1, d_1, e_1\}\}$,
$(f_{5_A}, E)^c = \{e_1 = \{a_{0.9}, b_{0.8}, c_1, d_{0.8}, e_{0.6}\}, e_2 = \{a_{0.5}, b_{0.9}, c_{0.6}, d_{0.9}, e_1\}$,

$e_3 = \{a_{0.8}, b_{0.6}, c_{0.9}, d_{0.9}, e_{0.6}\}$, $e_4 = \{a_1, b_1, c_1, d_1, e_1\}\}$.

Definition 3.5. Let (f_A, E) be a fuzzy soft set on (U, E) and P_x^λ $(x \in U, \lambda \in (0,1])$ be a fuzzy point in I^U. If $\lambda \leq \mu_{f_A}^e(x)$, for every $e \in A$, then P_x^λ belongs to (f_A, E) and it is denoted by $P_x^\lambda \in^\sim (f_A, E)$.

Definition 3.6. Let P_x^λ be a fuzzy point in I^U. Then (P_x^λ, E) is a fuzzy soft set on (U, E) where $P_x^\lambda(e) = \mu_{P_x^\lambda}^e$, $\mu_{P_x^\lambda}^e(u) = \lambda$, if $u = x$ and $\mu_{P_x^\lambda}^e(u) = 0$ if $u \neq x$ for every $e \in E$ and every $u \in U$.

Lemma 3.7. *Let (f_A, E) be a fuzzy soft set on (U, E) and P_x^λ be a fuzzy point in I^U.*

(1) *$P_x^\lambda \in^\sim (f_A, E)$ if and only if $(P_x^\lambda, A) \sqsubseteq (f_A, E)$.*
(2) *If $(P_x^\lambda, E) \sqcap (f_A, E) = \Phi$, then $P_x^\lambda \notin^\sim (f_A, E)$.*

Definition 3.8. Let (f_A, E, τ_f) be a fuzzy soft topological space over (f_A, E), (g_A, E) be a fuzzy soft subset of (f_A, E) and P_x^λ be a fuzzy point in I^U. (g_A, E) is called a fuzzy soft neighborhood of P_x^λ if there exists a fuzzy soft open set (h_A, E) such that $P_x^\lambda \in^\sim (h_A, E) \sqsubseteq (g_A, E)$.

Example 3.9. We consider the fuzzy soft topology in Example 3.4. Let $P_c^{0.2}$ be a fuzzy point. Then $(f_A, E), (f_{2_A}, E), (f_{4_A}, E)$ are fuzzy soft neighborhoods of $P_c^{0.2}$.

Definition 3.10. Let (f_A, E, τ_f) be a fuzzy soft topological space over (f_A, E), (g_A, E) and (h_A, E) be two fuzzy soft subsets of (f_A, E). (h_A, E) is called a fuzzy soft neighborhood of (g_A, E) if there exists a fuzzy soft open set (s_A, E) such that

$$(g_A, E) \sqsubseteq (s_A, E) \sqsubseteq (h_A, E).$$

Theorem 3.11. *Let (f_A, E, τ_f) be a fuzzy soft topological space over (f_A, E) . Then the following holds:*

(1) *Every $P_x^\lambda \in^\sim (f_A, E)$ has a fuzzy soft neighborhood.*
(2) *If (g_A, E) and (h_A, E) are fuzzy soft neighborhoods of P_x^λ, then $(g_A, E) \sqcap (h_A, E)$ is a fuzzy soft neighborhood of P_x^λ.*
(3) *If (g_A, E) is a fuzzy soft neighborhood of P_x^λ and $(g_A, E) \sqsubseteq (h_A, E)$, then (h_A, E) is a fuzzy soft neighborhood of P_x^λ.*

Proof. (1) (f_A, E) is a fuzzy soft open set and $P_x^\lambda \in^\sim (f_A, E) \sqsubseteq (f_A, E)$, hence the proof is clear.

(2) Let (g_A, E) and (h_A, E) be fuzzy soft neighborhoods of P_x^λ. Then there exist fuzzy soft open sets (g_{1_A}, E) and (h_{1_A}, E) such that $P_x^\lambda \in^\sim (g_{1_A}, E) \sqsubseteq (g_A, E)$ and $P_x^\lambda \in^\sim (h_{1_A}, E) \sqsubseteq (h_A, E)$. Hence $P_x^\lambda \in^\sim (g_{1_A}, E) \sqcap (h_{1_A}, E) \sqsubseteq (g_A, E) \sqcap (h_A, E)$ where $(g_{1_A}, E) \sqcap (h_{1_A}, E)$ is a fuzzy soft open set. Thus $(g_A, E) \sqcap (h_A, E)$ is a fuzzy soft neighborhood of P_x^λ.

(3) Let (g_A, E) be a fuzzy soft neighborhood of P_x^λ and $(g_A, E) \sqsubseteq (h_A, E)$. Then there exists a fuzzy soft open set (g_{1_A}, E) such that $P_x^\lambda \in^\sim (g_{1_A}, E) \sqsubseteq (g_A, E) \sqsubseteq (h_A, E)$. Hence (h_A, E) is a fuzzy soft neighborhood of P_x^λ. □

Definition 3.12. Let (f_A, E, τ_f) be a fuzzy soft topological space over (f_A, E), (g_A, E) be a fuzzy soft subset of (f_A, E) and $P_x^\lambda \in^\sim (f_A, E)$. P_x^λ is called a fuzzy soft interior point of (g_A, E) if there exists a fuzzy soft open set (h_A, E) such that $P_x^\lambda \in^\sim (h_A, E) \sqsubseteq (g_A, E)$.

Definition 3.13. The all fuzzy soft interior points of (g_A, E) is called the fuzzy soft interior of (g_A, E) and is denoted by $(g_A, E)^\circ$.

$(g_A, E)^\circ = \{(P_x^\lambda, A) : P_x^\lambda$ is fuzzy soft interior point of $(g_A, E)\}$

Theorem 3.14. *Let (f_A, E, τ_f) be a fuzzy soft topological space over (f_A, E) and (g_A, E) be a fuzzy soft subset of (f_A, E).*

(1) $(g_A, E)^\circ$ *is the union of all fuzzy soft open sets contained in* (g_A, E).
(2) $(g_A, E)^\circ \sqsubseteq (g_A, E)$.
(3) $(g_A, E)^\circ$ *is a fuzzy soft open set.*
(4) $(g_A, E)^\circ$ *is the biggest fuzzy soft open sets contained in* (g_A, E).
(5) (g_A, E) *is a fuzzy soft open set if and only if* $(g_A, E) = (g_A, E)^\circ$.

Proof. (1) We will show that $(g_A, E)^\circ = \sqcup\{(h_A, E) : (h_A, E) \sqsubseteq (g_A, E)$ and (h_A, E) is fuzzy soft open$\}$. Let $P_x^\lambda \in^\sim (g_A, E)^\circ$. Then there exists a fuzzy soft open set (h_A, E) such that $P_x^\lambda \in^\sim (h_A, E) \sqsubseteq (g_A, E)$. Thus $P_x^\lambda \in^\sim \sqcup(h_A, E)$. Conversely let $P_x^\lambda \in^\sim \sqcup\{(h_A, E){:}(h_A, E) \sqsubseteq (g_A, E)$ and (h_A, E) is fuzzy soft open$\}$. By the fuzzy soft interior point definition $P_x^\lambda \in^\sim (g_A, E)^\circ$.
(2) The proof is obvious by (1).
(3) Since the union of fuzzy soft open sets is fuzzy soft open the proof is clear.
(4) It is clear.
(5) Let (g_A, E) be a fuzzy soft open set. Since $(g_A, E)^\circ$ is the biggest fuzzy soft open set contained in (g_A, E), $(g_A, E) = (g_A, E)^\circ$. Conversely, suppose that $(g_A, E) = (g_A, E)^\circ$. Since $(g_A, E)^\circ$ is a fuzzy soft open, (g_A, E) is a fuzzy soft open set.

□

Theorem 3.15. *Let (f_A, E, τ_f) be a fuzzy soft topological space over (f_A, E) and (g_A, E), (h_A, E) be two fuzzy soft subsets of (f_A, E).*

(1) $\Phi^\circ = \Phi, (f_A, E)^\circ = (f_A, E)$.
(2) $((g_A, E)^\circ)^\circ = (g_A, E)^\circ$.
(3) *If* $(g_A, E) \sqsubseteq (h_A, E)$ *then* $(g_A, E)^\circ \sqsubseteq (h_A, E)^\circ$.
(4) $(g_A, E)^\circ \sqcap (h_A, E)^\circ = [(g_A, E) \sqcap (h_A, E)]^\circ$.
(5) $(g_A, E)^\circ \sqcup (h_A, E)^\circ \sqsubseteq [(g_A, E) \sqcup (h_A, E)]^\circ$.

Proof. (1) The proof is obvious.
(2) Let $(g_A, E)^\circ = (s_A, E)$. Since (s_A, E) is a fuzzy soft open set $(s_A, E)^\circ = (s_A, E)$, so $((g_A, E)^\circ)^\circ = (g_A, E)^\circ$.
(3) Let $(g_A, E) \sqsubseteq (h_A, E)$. $(g_A, E)^\circ \sqsubseteq (g_A, E)$ and hence $(g_A, E)^\circ \sqsubseteq (h_A, E)$ also $(h_A, E)^\circ$ is the biggest fuzzy soft open set contained in (h_A, E) and $(g_A, E)^\circ \sqsubseteq (h_A, E)^\circ$.
(4) $(g_A, E)^\circ \sqsubseteq (g_A, E)$ and $(h_A, E)^\circ \sqsubseteq (h_A, E)$. Hence $(g_A, E)^\circ \sqcap (h_A, E)^\circ \sqsubseteq (g_A, E) \sqcap (h_A, E)$. Since the biggest fuzzy soft open set contained in $(g_A, E) \sqcap (h_A, E)$ is $[(g_A, E) \sqcap (h_A, E)]^\circ$, $(g_A, E)^\circ \sqcap (h_A, E)^\circ \sqsubseteq [(g_A, E) \sqcap (h_A, E)]^\circ$.
Conversely, $[(g_A, E) \sqcap (h_A, E)]^\circ \sqsubseteq (g_A, E)^\circ$ and $[(g_A, E) \sqcap (h_A, E)]^\circ \sqsubseteq (h_A, E)^\circ$. Hence $[(g_A, E) \sqcap (h_A, E)]^\circ \sqsubseteq (g_A, E)^\circ \sqcap (h_A, E)^\circ$.

(5) $(g_A, E)^\circ \sqsubseteq (g_A, E)$ and $(h_A, E)^\circ \sqsubseteq (h_A, E)$. Then $(g_A, E)^\circ \sqcup (h_A, E)^\circ \sqsubseteq (g_A, E) \sqcup (h_A, E)$. The biggest fuzzy soft open set contained in $(g_A, E) \sqcup (h_A, E)$ is $[(g_A, E) \sqcup (h_A, E)]^\circ$ and so $(g_A, E)^\circ \sqcup (h_A, E)^\circ \sqsubseteq [(g_A, E) \sqcup (h_A, E)]^\circ$.

□

The converse inclusion for (5) does not hold generally as shown in the following example:

Example 3.16. We consider the fuzzy soft topology given in Example 3.4. Let $(g_A, E) = \{e_1 = \{a_{0.2}, b_{0.3}, c_{0.5}, d_{0.5}, e_{0.5}\}, e_2 = \{a_{0.5}, b_{0.2}, c_{0.8}, d_{0.2}, e_{0.3}\}, e_3 = \{a_{0.4}, b_{0.5}, c_{0.4}, d_{0.2}, e_{0.5}\}\}$ and $(h_A, E) = \{e_1 = \{a_{0.3}, b_{0.4}, c_{0.4}, d_{0.3}, e_{0.6}\}, e_2 = \{a_{0.8}, b_{0.4}, c_{0.7}, d_{0.4}, e_{0.9}\}, e_3 = \{a_{0.5}, b_{0.7}, c_{0.3}, d_{0.4}, e_{0.9}\}\}$.

Then $(g_A, E)^\circ = (f_{5_A}, E)$ and $(h_A, E)^\circ = (f_{3_A}, E) \sqcup (f_{5_A}, E) = (f_{3_A}, E)$ and hence $(g_A, E)^\circ \sqcup (h_A, E)^\circ = (f_{3_A}, E)$.

$(g_A, E) \sqcup (h_A, E) = \{e_1 = \{a_{0.3}, b_{0.4}, c_{0.5}, d_{0.5}, e_{0.6}\}, e_2 = \{a_{0.8}, b_{0.4}, c_{0.8}, d_{0.4}, e_{0.9}\}, e_3 = \{a_{0.5}, b_{0.7}, c_{0.4}, d_{0.4}, e_{0.9}\}$ and $[(g_A, E) \sqcup (h_A, E)]^\circ = (f_{1_A}, E) \sqcup (f_{2_A}, E) \sqcup (f_{3_A}, E) \sqcup (f_{5_A}, E) = (f_{4_A}, E)$. However (f_{4_A}, E) is not a fuzzy soft subset of (f_{3_A}, E).

Definition 3.17. Let (f_A, E, τ_f) be a fuzzy soft topological space over (f_A, E) and (g_A, E) be a fuzzy soft subset of (f_A, E). The intersection of all fuzzy soft closed sets containing (g_A, E) is called fuzzy soft closure of (g_A, E).

$(g_A, E)^- = \sqcap\{(h_A, E) : (g_A, E) \sqsubseteq (h_A, E)$ and (h_A, E) is fuzzy soft closed$\}$

Theorem 3.18. *Let (f_A, E, τ_f) be a fuzzy soft topological space over (f_A, E) and (g_A, E), (s_A, E) be two fuzzy soft subsets of (f_A, E). Then,*

(1) *$(g_A, E)^-$ is a fuzzy soft closed set.*
(2) $(g_A, E) \sqsubseteq (g_A, E)^-$.
(3) *$(g_A, E)^-$ is the smallest fuzzy soft closed set containing (g_A, E).*
(4) *If $(g_A, E) \sqsubseteq (s_A, E)$ then $(g_A, E)^- \sqsubseteq (s_A, E)^-$.*
(5) *(g_A, E) is fuzzy soft closed if and only if $(g_A, E) = (g_A, E)^-$.*
(6) $((g_A, E)^-)^- = (g_A, E)^-$.
(7) $[(g_A, E) \sqcup (s_A, E)]^- = (g_A, E)^- \sqcup (s_A, E)^-$.
(8) $[(g_A, E) \sqcap (s_A, E)]^- \sqsubseteq (g_A, E)^- \sqcap (s_A, E)^-$.

Proof. (1) Obvious.

(2) Obvious.

(3) Obvious.

(4) Let $(g_A, E) \sqsubseteq (s_A, E)$. Since $(s_A, E) \sqsubseteq (s_A, E)^-$, $(g_A, E) \sqsubseteq (s_A, E)^-$. The smallest fuzzy soft closed set containing (g_A, E) is $(g_A, E)^-$, hence $(g_A, E)^- \sqsubseteq (s_A, E)^-$.

(5) Obvious.

(6) Let $(g_A, E)^- = (k_A, E)$. Since (k_A, E) is fuzzy soft closed $(k_A, E)^- = (k_A, E)$ and so $((g_A, E)^-)^- = (g_A, E)^-$.

(7) $(g_A, E) \sqsubseteq (g_A, E) \sqcup (s_A, E)$ and $(s_A, E) \sqsubseteq (g_A, E) \sqcup (s_A, E)$. By (4), $(g_A, E)^- \sqsubseteq [(g_A, E) \sqcup (s_A, E)]^-$ and $(s_A, E)^- \sqsubseteq [(g_A, E) \sqcup (s_A, E)]^-$. Hence $(g_A, E)^- \sqcup (s_A, E)^- \sqsubseteq [(g_A, E) \sqcup (s_A, E)]^-$.

$(g_A, E) \sqcup (s_A, E) \sqsubseteq (g_A, E)^- \sqcup (s_A, E)^-$. $(g_A, E)^- \sqcup (s_A, E)^-$ is a fuzzy soft closed set. Since $[(g_A, E) \sqcup (s_A, E)]^-$ is the smallest fuzzy soft closed set containing $(g_A, E) \sqcup (s_A, E)$, we have $[(g_A, E) \sqcup (s_A, E)]^- \sqsubseteq (g_A, E)^- \sqcup (s_A, E)^-$. Thus $[(g_A, E) \sqcup (s_A, E)]^- = (g_A, E)^- \sqcup (s_A, E)^-$.

(8) $(g_A, E) \sqsubseteq (g_A, E)^-$ and $(s_A, E) \sqsubseteq (s_A, E)^-$. Thus $(g_A, E) \sqcap (s_A, E) \sqsubseteq (g_A, E)^- \sqcap (s_A, E)^-$ and $(g_A, E)^- \sqcap (s_A, E)^-$ is a fuzzy soft closed set since the intersection of two fuzzy soft closed sets is fuzzy soft closed. The smallest fuzzy soft closed set containing $(g_A, E) \sqcap (s_A, E)$ is $[(g_A, E) \sqcap (s_A, E)]^-$. Hence $[(g_A, E) \sqcap (s_A, E)]^- \sqsubseteq (g_A, E)^- \sqcap (s_A, E)^-$.

□

The converse inclusion of (8) is not true generally as shown in the following example.

Example 3.19. We consider the fuzzy soft topology given in Example 3.4. Let $(g_A, E) = \{e_1 = \{a_{0.2}, b_{0.3}, c_{0.5}, d_{0.5}, e_{0.5}\}, e_2 = \{a_{0.5}, b_{0.2}, c_{0.8}, d_{0.2}, e_{0.3}\}, e_3 = \{a_{0.4}, b_{0.5}, c_{0.4}, d_{0.2}, e_{0.5}\}\}$ and $(s_A, E) = \{e_1 = \{a_{0.3}, b_{0.4}, c_{0.4}, d_{0.3}, e_{0.6}\}, e_2 = \{a_{0.8}, b_{0.4}, c_{0.6}, d_{0.4}, e_{0.9}\}, e_3 = \{a_{0.5}, b_{0.7}, c_{0.3}, d_{0.4}, e_{0.9}\}\}$. Then $(g_A, E)^- = E^\sim$ and $(s_A, E)^- = E^\sim$. Thus $(g_A, E)^- \sqcup (s_A, E)^- = E^\sim$.

$(g_A, E) \sqcap (s_A, E) = \{e_1 = \{a_{0.2}, b_{0.3}, c_{0.4}, d_{0.3}, e_{0.5}\}, e_2 = \{a_{0.5}, b_{0.2}, c_{0.6}, d_{0.2}, e_{0.3}\}, e_3 = \{a_{0.4}, b_{0.5}, c_{0.3}, d_{0.2}, e_{0.5}\}\}$ and $[(g_A, E) \sqcap (s_A, E)]^- = (f_{5_A}, E)^c$. Also $E^\sim$ is not a fuzzy soft subset of $(f_{5_A}, E)^c$.

Definition 3.20. Let P_x^λ be a fuzzy point in I^U and (f_A, E) be a fuzzy soft set on (U, E). P_x^λ is said to be quasi-coincident with (f_A, E), denoted by $P_x^\lambda q(f_A, E)$ if $\lambda + \mu_{f_A}^e(x) > 1$ for each $e \in A$.

Definition 3.21. Let (f_A, E) and (g_A, E) be two fuzzy soft sets on (U, E). (f_A, E) is said to be quasi-coincident with (g_A, E), denoted by $(f_A, E)q(g_A, E)$, if there exists $u \in U$ such that $\mu_{f_A}^e(u) + \mu_{g_A}^e(u) > 1$, for every $e \in A$. If this is truewe can say that (f_A, E) and (g_A, E) is quasi-coincident at u.

Definition 3.22. Let (f_A, E, τ_f) be a fuzzy soft topological space over (f_A, E) and P_x^λ be a fuzzy point in I^U. A fuzzy soft subset (v_A, E) of (f_A, E) is called a Q-fuzzy soft neighborhood of P_x^λ if there exists a fuzzy soft open set (w_A, E) such that $P_x^\lambda q(w_A, E) \sqsubseteq (v_A, E)$.

Example 3.23. We consider the fuzzy soft topology in Example 3.4. Let $\mathrm{P}_b^{0.8}$ be a fuzzy point. Then (f_A, E), (f_{1_A}, E), (f_{2_A}, E), (f_{3_A}, E) and (f_{4_A}, E) are fuzzy soft open Q-neighborhoods of $\mathrm{P}_b^{0.8}$

Theorem 3.24. *Let (f_A, E, τ_f) be a fuzzy soft topological space over (f_A, E). The following holds:*

(1) *If (g_A, E) is a Q-fuzzy soft neighborhood of P_x^λ, then P_x^λ is quasi-coincident with (g_A, E).*
(2) *If (g_A, E) and (h_A, E) are Q-fuzzy soft neighborhoods of P_x^λ, then $(g_A, E) \sqcap (h_A, E)$ is a Q- fuzzy soft neighborhood of P_x^λ.*
(3) *If (g_A, E) is a Q- fuzzy soft neighborhood of P_x^λ and $(g_A, E) \sqsubseteq (h_A, E)$, then (h_A, E) is a Q-fuzzy soft neighborhood of P_x^λ.*

Proposition 3.25. *Let (g_A, E) and (h_A, E) be two fuzzy soft sets on (U, E). $(g_A, E) \sqsubseteq (h_A, E)$ if and only if (g_A, E) is not quasi-coincident with $(h_A, E)^c$.*

Proof.

$$(g_A, E) \sqsubseteq (h_A, E) \Leftrightarrow \mu^e_{g_A}(u) \leq \mu^e_{h_A}(u)$$

for every $e \in E$ and every $u \in U$.

$$\Leftrightarrow \mu^e_{g_A}(u) + 1 - \mu^e_{h_A}(u) \leq 1$$

$$\Leftrightarrow \mu^e_{g_A}(u) + \mu^e_{h^c_A}(u) \leq 1.$$

Thus (g_A, E) is not quasi-coincident with $(h_A, E)^c$. □

Theorem 3.26. *Let (f_A, E, τ_f) be a fuzzy soft topological space over (f_A, E), P^λ_x be a fuzzy point in I^U and (g_A, E) be a fuzzy soft subset of (f_A, E). $P^\lambda_x \in^\sim (g_A, E)^-$ if and only if each Q-fuzzy soft neighborhood of P^λ_x is quasi-coincident with (g_A, E) at x.*

Proof. $P^\lambda_x \in^\sim (g_A, E)^-$ if and only if for every fuzzy soft closed set (v_A, E) containing (g_A, E), $P^\lambda_x \in^\sim (v_A, E)$ i.e., $\lambda \leq \mu^e_{v_A}(x)$, for every $e \in A$. By complement we obtain $P^\lambda_x \in^\sim (g_A, E)^-$ if and only if for every fuzzy soft open set $(k_A, E) \sqsubseteq (g_A, E)^c$, $1 - \lambda \geq \mu^e_{k_A}(x)$ for every $e \in A$. That is, for every fuzzy soft open set (k_A, E) satisfying $1 - \lambda < \mu^e_{k_A}(x)$, (k_A, E) is not contained in $(g_A, E)^c$. By proposition 3.25., (k_A, E) is not contained in $(g_A, E)^c$ if and only if (k_A, E) is quasi-coincident with $((g_A, E)^c)^c = (g_A, E)$. □

4. Conclusion

Fuzzy soft sets are very popular subject for researchers. This hybrid model which is more general than fuzzy and soft sets can be applied several directions easily. In this paper we construct a topology over a fuzzy soft set with fixed set parameter and define the basic concepts. Our next study is to introduce fuzzy soft seperation axioms. Also one can try to define fuzzy soft compactness, fuzzy soft connectedness and so on.

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Tugbahan Simsekler (tugbahan@selcuk.edu.tr)
Department of mathematics, Faculty science, University of Selcuk, Postal code 42100, Konya, Turkey

Saziye Yuksel (syuksel@selcuk.edu.tr)
Department of mathemathics, Faculty science, University of Selcuk, Postal code 42100, Konya, Turkey