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On $(\in, \in \vee q)$-intuitionistic fuzzy h-ideals of hemirings

Mohsen Asghari-Larimi, Young Bae Jun

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Abstract. The notion of intuitionistic fuzzy sets was introduced by Atanassov as a generalization of the notion of fuzzy sets. Using the notion of "belongingness $(\in)$" and "quasi-coincidence (q)" of fuzzy points in fuzzy sets, we introduce the concepts of $(\in, \in \vee q)$-intuitionistic fuzzy ideal, $(\in, \in \vee q)$-intuitionistic fuzzy k-ideal and $(\in, \in \vee q)$-intuitionistic fuzzy h-ideal of hemirings, and some interesting properties are investigated.

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Corresponding Author: Mohsen Asghari-Larimi (asghari2004@yahoo.com)

1. Introduction

Given a set H, a fuzzy subset of H (or a fuzzy set in H) is, by definition, an arbitrary mapping $\mu : H \longrightarrow [0, 1]$ where [0,1] is the closed interval in reals whose endpoints are 0 and 1. This important concept of a fuzzy set has been introduced by Zadeh in [19]. Since then, many papers on fuzzy sets appeared showing the importance of the concept and its applications (see, for example, [2, 6]).

After the introduction of fuzzy sets by Zadeh, there have been a number of generalizations of this fundamental concept. The notion of intuitionistic fuzzy sets introduced by Atanassov [3, 6] is one among them. An intuitionistic fuzzy set gives both a membership degree and a non-membership degree. The membership and non-membership values induce an indeterminacy index, which models the hesitancy of deciding the degree to which an object satisfies a particular property. As the basis for the study of intuitionistic fuzzy set theory, many operations and relations over intuitionistic fuzzy sets were introduced [4, 5]. Many concepts in fuzzy set theory were also extended to intuitionistic fuzzy set theory, such as intuitionistic fuzzy relations, intuitionistic L-fuzzy sets, intuitionistic fuzzy implications, intuitionistic fuzzy grade of hypergroups, intuitionistic fuzzy logics, and the degree of similarity between intuitionistic fuzzy sets, etc., [10].

In [7] Biswas applied the concept of intuitionistic fuzzy sets to the theory of groups and studied intuitionistic fuzzy subgroups of a group.

The idea of quasi-coincidence of a fuzzy point with a fuzzy set, which is mentioned in [16], played a vital role to generate some different types of fuzzy subgroups. Bhakat and Das [8] gave the concepts of (α, β)-fuzzy subgroups by using the notion of $(\in)$ and (q) between a fuzzy point and a fuzzy subgroup, where α, β are any two of $\{\in, q, \in \vee q, \in \wedge q\}$ with $\alpha \neq \in \wedge q$, and introduced the concept of an $(\in, \in \vee q)$-fuzzy subgroup. In [9] $(\in, \in \vee q)$- fuzzy subrings and ideals defined. In [14] Jun and Song initiated the study of (α, β)-fuzzy interior ideals of a semigroup. In [17] Shabir et al. studied characterizations of regular semigroups by (α, β)-fuzzy ideals. In [18] Yuan et al. redefined (α, β)-intuitionistic fuzzy subgroups. In [15] Kazanci and Yamak studied $(\in, \in \vee q)$-fuzzy bi-ideals of a semigroup. Generalizing the concept of the quasi-coincident of a fuzzy point with a fuzzy subset. Dudek et al. [11] introduced the concept of $(\in, \in \vee q)$-fuzzy h-ideal (k-ideal) of a hemiring. In [13] Jun et al. studied $(\in, \in \vee q_k)$-fuzzy ideals of hemirings. In [1] Abdullah et al. studied (α, β)-intuitionistic fuzzy ideals in hemirings. In [12] Jun studied (α, β)-fuzzy ideals of hemirings. This paper continues this line of research.

The paper is organized as follows: in Section 2 some fundamental definitions on fuzzy sets and intuitionistic fuzzy sets are explored; in Section 3, we define $(\in, \in \vee q)$-intuitionistic fuzzy ideals of hemirings, $(\in, \in \vee q)$-intuitionistic fuzzy k-ideal, and $(\in, \in \vee q)$-intuitionistic fuzzy h-ideal of a hemiring. Finally, in Section 4, we produce some relations between $(\in, \in \vee q)$-intuitionistic fuzzy ideals with $(\in, \in \vee q)$-fuzzy ideals and with anti $(\in, \in \vee q)$-fuzzy ideals, and then establish some useful theorems.

2. Preliminaries

A *semiring* is an algebraic system $(R, +, \cdot)$ consisting of a non-empty set R together with two binary operations called addition "$+$" and multiplication "$\cdot$", here $x \cdot y$ will be denoted by juxtaposition for all $x, y \in R$, such that $(R, +)$ and $(R, \cdot)$ are semigroups connected by the following distributive laws: $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$ for all $a, b, c \in R$. An element $0 \in R$ is called a *zero* of R if $a + 0 = 0 + a = a$ and $a0 = 0a = a$ for all $a \in R$. A semiring with zero and a commutative addition is called a *hemiring*. A nonempty subset X of R is called a *subhemiring* of R if $X \cdot X \subseteq X$ and $X + X \subseteq X$. A non-empty subset I of a semiring R is said to be a *left* (resp. *right*) *ideal* of R if it is closed under the addition and $RI \subseteq I$ (resp. $IR \subseteq I$). A left ideal which is also a right ideal is called an *ideal*. A left (resp. right) ideal I of a hemiring R is called a *left* (resp. *right*) *k-ideal* of R if for any $a, b \in I$ and $x \in R$ whenever $x + a = b$ then $x \in I$. A left (resp. right) ideal I of a hemiring R is called a *left* (resp. *right*) *h-ideal* of R if for any $a, b \in I$ and all $x, y \in R$ whenever $x + a + y = b + y$ then $x \in I$.

The concept of a fuzzy set in a non-empty set was introduced by Zadeh [19] in 1965. Let X be a non-empty set. A mapping $\mu : X \longrightarrow [0; 1]$ is called a *fuzzy set* in X. The *complement* of μ, denoted by μ^c, is the fuzzy set in X given by $\mu^c(x) = 1 - \mu(x)$ for all $x \in X$.

For any $t \in [0,1]$ and fuzzy set μ of X, the set

$$U(\mu,t) = \{x \in X | \mu(x) \geq t\} \ \ (\text{respectively}, \ L(\mu,t) = \{x \in X | \mu(x) \leq t\}),$$

is called an *upper* (respectively, *lower*) *t-level cut* of μ.

Definition 2.1. An *intuitionistic fuzzy set* (IFS for short) A in a non-empty set X is an object having the form

$$A = \{(x, \mu_A(x), \lambda_A(x)) | x \in X\},$$

where the functions $\mu_A : X \longrightarrow [0;1]$ and $\lambda_A : X \longrightarrow [0;1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\lambda_A(x)$) of each element $x \in X$ with respect to the set A, respectively, and $0 \leq \mu_A(x)+\lambda_A(x) \leq 1$ for all $x \in X$ (see [3, 4]). For the sake of simplicity, we shall use the symbol $A = (\mu_A, \lambda_A)$ for the IFS $A = \{(x, \mu_A(x), \lambda_A(x)) | x \in X\}$. Denote by $IFS(X)$ the set of all intuitionistic fuzzy sets in X.

Definition 2.2 ([3]). Let $A = (\mu_A, \lambda_A)$ and $B = (\mu_B, \lambda_B)$ be intuitionistic fuzzy sets in X. Then

(1) $A \subseteq B$ iff $\mu_A(x) \leq \mu_B(x)$ and $\lambda_A(x) \geq \lambda_B(x)$ for all $x \in X$,
(2) $A = B$ iff $A \subseteq B$ and $B \subseteq A$,
(3) $A^c = \{(x, \lambda_A(x), \mu_A(x)) | x \in X\}$,
(4) $A \cap B = \{(x, \min\{\mu_A(x), \mu_B(x)\}, \max\{\lambda_A(x), \lambda_B(x)\}) | x \in X\}$,
(5) $A \cup B = \{(x, \max\{\mu_A(x), \mu_B(x)\}, \min\{\lambda_A(x), \lambda_B(x)\}) | x \in X\}$,
(6) $\Box A = \{(x, \mu_A(x), \mu^c_A(x)) | x \in X\}$,
(7) $\Diamond A = \{(x, \lambda^c_A(x), \lambda_A(x)) | x \in X\}$.

Definition 2.3 ([16]). Let $Y \subseteq X$ and $t \in [0,1]$. We define $t_Y \in F(X)$ as follows:

$$t_Y(x) = \begin{cases} t & \text{if } x \in Y \\ 0 & \text{if } x \in X \backslash Y. \end{cases}$$

In particular, if Y is a singleton, say x, then $t_{\{x\}}$ is called a *fuzzy point with support* x and *value* t and is denoted by x_t.

Definition 2.4 ([16]). Let μ be a fuzzy subset of X and x_t be a fuzzy point.

(1) If $\mu(x) \geq t$, then we say x_t belongs to μ, and write $x_t \in \mu$.
(2) If $\mu(x) + t > 1$, then we say x_t is quasi-coincident with μ, and write $x_t q \mu$.
(3) $x_t \in \vee q\mu \Longleftrightarrow x_t \in \mu$ or $x_t q\mu$.
(4) $x_t \in \wedge q\mu \Longleftrightarrow x_t \in \mu$ and $x_t q\mu$.

In what follows, unless otherwise specified, α and β will denote any one of $\in, q, \in \vee q$ or $\in \wedge q$ with $\alpha \neq \in \wedge q$. To say that $x_t \overline{\alpha} \mu$ means that $x_t \alpha \mu$ does not hold. We defined

$$U(\alpha\mu, t) = \{x \in X |\ x_t \alpha \mu\},$$

where $\alpha \in \{\in, q, \in \vee q\}$.

Definition 2.5 ([11]). A fuzzy subset μ of R is said to be an $(\in, \in \vee q)$-*fuzzy left (*resp. *right) ideal* of a hemiring R if

$$x \in U(\in \mu, t),\ y \in U(\in \mu, r) \Longrightarrow x + y \in U(\in \vee q\mu, t \wedge r),$$
$$x \in U(\in \mu, t) \Longrightarrow yx \in U(\in \vee q\mu, t)(\text{resp. } xy \in U(\in \vee q\mu, t)),$$

for all $x, y \in R$ and $t, r \in (0,1]$. A fuzzy subset which is an $(\in, \in \vee q)$-fuzzy left and right ideal is called an *$(\in, \in \vee q)$-fuzzy ideal.*
An $(\in, \in \vee q)$-fuzzy ideal μ of a hemiring R satisfying the following condition:

$$x + a = b,\ a \in U(\in \mu, t),\ b \in U(\in \mu, r) \Longrightarrow x \in U(\in \vee q\mu, t \wedge r),$$

for all $a, b, x \in R$ and $t, r \in (0,1]$ is called an *$(\in, \in \vee q)$-fuzzy k-ideal.*
An $(\in, \in \vee q)$-fuzzy ideal μ of a hemiring R satisfying the following condition:

$$x + a + y = b + y,\ a \in U(\in \mu, t),\ b \in U(\in \mu, r) \Longrightarrow x \in U(\in \vee q\mu, t \wedge r),$$

for all $a, b, x, y \in R$ and $t, r \in (0,1]$ is called an *$(\in, \in \vee q)$-fuzzy h-ideal.*

Lemma 2.6 ([11])**.** *A fuzzy subset μ of a hemiring R is an $(\in, \in \vee q)$-fuzzy h-ideal (resp. k-ideal) of R if and only if it satisfies:*

(a) $\mu(x + y) \geq \min\{\mu(x), \mu(y), 0.5\}$,
(b) $\mu(yx) \geq \min\{\mu(x), 0.5\}$,
(c) $\mu(xy) \geq \min\{\mu(x), 0.5\}$,
(d) $x + a + y = b + y \Longrightarrow \mu(x) \geq \min\{\mu(a), \mu(b), 0.5\}$,
(resp. (e) $x + a = b \Longrightarrow \mu(x) \geq \min\{\mu(a), \mu(b), 0.5\}$*),*

for all $a, b, x, y \in R$.

3. $(\in, \in \vee q)$-Intuitionistic fuzzy ideals of hemirings

In what follows, let R denote a hemiring and $t \in (0,1]$.

Definition 3.1. Let μ be a fuzzy set in X. We define

$L(\in \mu, t) = \{x \in X |\ \mu(x) \leq t\}$,
$L(q\mu, t) = \{x \in X |\ \mu(x) + t \leq 1\})$,
$L(\in \vee q\mu, t) = \{x \in X |\ \mu(x) + t \leq 1 \text{ or } \mu(x) \leq t\}$.

Then the set $L(\alpha\mu, t)$ is called a *lower t-level cut* of $\alpha\mu$, where $\alpha \in \{\in, q, \in \vee q\}$.

It is clear that $L(\in \mu, t) = L(\mu, t)$.

Corollary 3.2 ()**.** *Let μ be a fuzzy set in X. Then for all $t \in (0,1]$ we have*

(1) $U(\in \vee q\mu, t) = U(\in \mu, t) \bigcup U(q\mu, t)$,
(2) $L(\in \vee q\mu, t) = L(\in \mu, t) \bigcup L(q\mu, t)$.

Corollary 3.3 ([11])**.** *For any fuzzy subset λ of X and $t \in (0,1]$, we consider two subsets:*

$$Q(\lambda, t) = \{x \in X |\ x_t q \lambda\} \text{ and } [\lambda]_t = \{x \in X |\ x_t \in \vee q \lambda\}.$$

Then $[\lambda]_t = U(\lambda, t) \bigcup Q(\lambda, t)$.

Theorem 3.4 ()**.** *Let μ be a fuzzy set in X. Then we have*

(1) *If $t \in (0, 0.5]$, then $U(\in \vee q\mu, t) = U(\in \mu, t)$,*
(2) *If $t \in (0.5, 1]$, then $U(\in \vee q\mu, t) = U(q\mu, t)$.*

Proof. (1) If $t \in (0, 0.5]$, then $1 - t \in [0.5, 1)$. Thus $t \leq 1 - t$. By Corollary 3.2, it is clear that $U(\in \mu, t) \subseteq U(\in \vee q\mu, t)$. Let $x \notin U(\in \mu, t)$. Then $\mu(x) < t$ and so $\mu(x) < 1 - t$. This shows that $x \notin U(q\mu, t)$, and hence $x \notin (U(\in \mu, t) \bigcup U(q\mu, t))$. Thus $U(\in \mu, t) \supseteq U(\in \vee q\mu, t)$. Therefore $U(\in \mu, t) = U(\in \vee q\mu, t)$.

(2) If $t \in (0.5, 1]$, then $1 - t \in [0, 0.5)$. Thus $1 - t < t$. By Theorem 3.2, we have $U(q\mu, t) \subseteq U(\in \vee q\mu, t)$. Let $x \notin U(q\mu, t)$, then $\mu(x) + t \leq 1$ and so

$\mu(x) \le 1-t < t$. This shows that $x \notin U(\in \mu, t)$, and thus $x \notin (U(\in \mu, t) \bigcup U(q\mu, t))$. Hence $U(q\mu, t) \supseteq U(\in \vee q\mu, t)$. Therefore $U(q\mu, t) = U(\in \vee q\mu, t)$. □

Corollary 3.5 ([12]). *Every fuzzy subset λ of X satisfies the following assertion:*

$$t \in (0, 0.5] \Longrightarrow [\lambda]_t = U(\lambda, t).$$

Theorem 3.6. *Let μ be a fuzzy set in X. Then we have*

(1) *If $t \in (0, 0.5]$, then $L(\in \vee q\mu, t) = L(\in \mu, t)$,*
(2) *If $t \in [0.5, 1]$, then $L(\in \vee q\mu, t) = L(q\mu, t)$.*

Proof. The proof is similar to that of Theorem 3.4. □

Definition 3.7. Let $A = (\mu_A, \lambda_A) \in IFS(R)$. Then $A = (\mu_A, \lambda_A)$ is called an (α, β)-*intuitionistic fuzzy left* (resp. *right*) *ideal* of hemiring R if

(1) $x \in U(\alpha\mu_A, t),\ y \in U(\alpha\mu_A, r) \Longrightarrow x + y \in U(\beta\mu_A, t \wedge r)$,
(2) $x \in U(\alpha\mu_A, t) \Longrightarrow yx \in U(\beta\mu_A, t)$(resp. $xy \in U(\beta\mu_A, t)$),
(3) $x \in L(\alpha\lambda_A, t),\ y \in L(\alpha\lambda_A, r) \Longrightarrow x + y \in L(\beta\lambda_A, t \vee r)$,
(4) $x \in L(\alpha\lambda_A, t) \Longrightarrow yx \in L(\beta\lambda_A, t)$(resp. $xy \in L(\beta\lambda_A, t)$),

for all $x, y \in R$ and $t, r \in (0, 1]$. A fuzzy subset which is an (α, β)-intuitionistic fuzzy left and right ideal is called an (α, β)-*intuitionistic fuzzy ideal.*

A fuzzy subset μ (resp. λ) of R is said to be an (resp. *anti*) (α, β)-*fuzzy ideal* of hemiring R if it satisfies the conditions (1) and (2) (resp. (3) and (4)) of Definition 3.7.

Definition 3.8. An (α, β)-intuitionistic fuzzy ideal $A = (\mu_A, \lambda_A)$ of a hemiring R satisfying the following condition:

(1) $x + a = b,\ a \in U(\alpha\mu_A, t),\ b \in U(\alpha\mu_A, r) \Longrightarrow x \in U(\beta\mu_A, t \wedge r)$,
(2) $x + a = b,\ a \in L(\alpha\lambda_A, t),\ b \in L(\alpha\lambda_A, r) \Longrightarrow x \in L(\beta\lambda_A, t \vee r)$,

for all $a, b, x \in R$ and $t, r \in (0, 1]$ is called an (α, β)-*intuitionistic fuzzy k-ideal.*

A fuzzy subset μ (resp. λ) of R is said to be an (resp. *anti*) (α, β)-*fuzzy k-ideal* of hemiring R if it satisfies the condition (1) (resp. (2)) of Definition 3.8.

Definition 3.9. An (α, β)-intuitionistic fuzzy ideal $A = (\mu_A, \lambda_A)$ of a hemiring R satisfying the following condition:

(1) $x + a + y = b + y,\ a \in U(\alpha\mu_A, t),\ b \in U(\alpha\mu_A, r) \Longrightarrow x \in U(\beta\mu_A, t \wedge r)$,
(2) $x + a + y = b + y,\ a \in L(\alpha\lambda_A, t),\ b \in L(\alpha\lambda_A, r) \Longrightarrow x \in L(\beta\lambda_A, t \vee r)$,

for all $a, b, x, y \in R$ and $t, r \in (0, 1]$ is called an (α, β)-*intuitionistic fuzzy h-ideal.*

A fuzzy subset μ (resp. λ) of R is said to be an (resp. *anti*) (α, β)-*fuzzy h-ideal* of hemiring R if it satisfies the condition (1) (resp. (2)) of Definition 3.9.

Theorem 3.10. *Let λ be a fuzzy subset of a hemiring R and $t, r \in (0, 1]$. Then:*

(1) $(a1)$ $x \in L(\in \lambda, t),\ y \in L(\in \lambda, r) \Longrightarrow x + y \in L(\in \vee q\lambda, t \vee r)$ *and*
$(a2)$ $\lambda(x + y) \le \max\{\lambda(x), \lambda(y), 0.5\}$ *for all $x, y \in R$ are equivalent.*
(2) $(b1)$ $x \in L(\in \lambda, t) \Longrightarrow yx \in L(\in \vee q\lambda, t)$ *and*
$(b2)$ $\lambda(yx) \le \max\{\lambda(x), 0.5\}$ *for all $x, y \in R$ are equivalent.*
(3) $(c1)$ $x \in L(\in \lambda, t) \Longrightarrow xy \in L(\in \vee q\lambda, t)$ *and*
$(c2)$ $\lambda(xy) \le \max\{\lambda(x), 0.5\}$ *for all $x, y \in R$ are equivalent.*

(4) $(d1)$ $x+a+y=b+y,\ a\in L(\in\lambda,t),\ b\in L(\in\lambda,r)\Longrightarrow x\in L(\in\vee q\lambda,t\vee r)$ *and*
$(d2)$ $x+a+y=b+y\Longrightarrow\lambda(x)\leq\max\{\lambda(a),\lambda(b),0.5\}$ *for all* $a,b,x,y\in R$ *are equivalent.*

(5) $(e1)$ $x+a=b,\ a\in L(\in\lambda,t),\ b\in L(\in\lambda,r)\Longrightarrow x\in L(\in\vee q\lambda,t\vee r)$ *and* $(e2)$ $x+a=b\Longrightarrow\lambda(x)\leq\max\{\lambda(a),\lambda(b),0.5\}$ *for all* $a,b,x\in R$ *are equivalent.*

Proof. $(a1)\Longrightarrow(a2)$. Assume that there exist $x,y\in R$ such that

$$\lambda(x+y)>\max\{\lambda(x),\lambda(y),0.5\}.$$

Choose $t\in(0,1]$ such that $\lambda(x+y)>t\geq\max\{\lambda(x),\lambda(y),0.5\}$. Then $x\in L(\in\lambda,t)$ and $y\in L(\in\lambda,t)$. But $\lambda(x+y)>t$, so $x+y\overline{\in}L(\in\lambda,t)$ and $\lambda(x+y)+t>2t\geq 1$. Then we have

$$x+y\overline{\in}L(\in\vee q\lambda,t)=L(\in\vee q\lambda,t\vee t),$$

which is a contradiction. Thus $\lambda(x+y)\leq\max\{\lambda(x),\lambda(y),0.5\}$. Hence (a2) holds.

$(a2)\Longrightarrow(a1)$. Let

$$\lambda(x+y)\leq\max\{\lambda(x),\lambda(y),0.5\}.$$

Assume that $t,r\in(0,1]$ such that $x\in L(\in\lambda,t)$ and $y\in L(\in\lambda,r)$. Then $\lambda(x)\leq t$ and $\lambda(y)\leq r$. Hence

$$\lambda(x+y)\leq\max\{\lambda(x),\lambda(y),0.5\}\leq\max\{t,r,0.5\}.$$

If $\max\{t,r\}\leq 0.5$, then $\lambda(x+y)\leq 0.5$, and so $\lambda(x+y)+\max\{t,r\}\leq 0.5+0.5=1$, which implies $x+y\in L(q\lambda,t\vee r)$. If $\max\{t,r\}>0.5$, then $\lambda(x+y)\leq\max\{t,r\}$, which implies that $x+y\in L(\in\lambda,t\vee r)$. Hence (a1) holds.

$(b1)\Longrightarrow(b2)$. Assume that there exist $x,y\in R$ such that $\lambda(yx)>\max\{\lambda(x),0.5\}$. Choose $t\in(0,1]$ such that $\lambda(yx)>t\geq\max\{\lambda(x),0.5\}$. Then $x\in L(\in\lambda,t)$ but $\lambda(yx)>t$, so $yx\overline{\in}L(\in\lambda,t)$ and $\lambda(yx)+t>2t\geq 1$. Then we obtain $yx\overline{\in}L(\in\vee q\lambda,t)$, which is a contradiction. Thus $\lambda(yx)\leq\max\{\lambda(x),0.5\}$. Hence (b2) holds.

$(b2)\Longrightarrow(b1)$. Let $\lambda(yx)\leq\max\{\lambda(x),0.5\}$. Assume that $t\in(0,1]$ such that $x\in L(\in\lambda,t)$. Then $\lambda(x)\leq t$. Hence $\lambda(yx)\leq\max\{\lambda(x),0.5\}\leq\max\{t,0.5\}$. If $t\leq 0.5$, then $\lambda(yx)\leq 0.5$, and so $\lambda(yx)+t\leq 0.5+0.5=1$, which implies that $yx\in L(q\lambda,t)$. If $t>0.5$, then $\lambda(yx)\leq t$, which implies that $yx\in L(\in\lambda,t)$. Hence (b1) holds.

$(d1)\Longrightarrow(d2)$. Suppose that there exist $a,b,x,y\in R$ such that $x+a+y=b+y$. Assume that $\lambda(x)>\max\{\lambda(a),\lambda(b),0.5\}$. Choose $t\in(0,1]$ such that $\lambda(x)>t\geq\max\{\lambda(a),\lambda(b),0.5\}$. Then $a,b\in L(\in\lambda,t)$. But $x\overline{\in}L(\in\lambda,t)$ and $\lambda(x)+t>2t\geq 1$, so $x\overline{\in}L(q\lambda,t)$. Then we obtain $x\overline{\in}L(\in\vee q\lambda,t)$, which is a contradiction. Thus $\lambda(x)\leq\max\{\lambda(a),\lambda(b),0.5\}$. Hence (d2) holds.

$(d2)\Longrightarrow(d1)$. Let $a,b,x,y\in R,\ t,r\in(0,1]$, $x+a+y=b+y$ and $a\in L(\in\lambda,t),\ b\in L(\in\lambda,r)$. If $\max\{\lambda(a),\lambda(b),0.5\}=\lambda(a)$, then

$$\lambda(x)\leq\max\{\lambda(a),\lambda(b),0.5\}=\lambda(a)\leq t\leq\max\{t,r\}.$$

Thus $x \in L(\in \lambda, t \vee r)$, implying that $x \in L(\in \vee q\lambda, t \vee r)$.
Similarly, if $\max\{\lambda(a), \lambda(b), 0.5\} = \lambda(b)$, then $x \in L(\in \vee q\lambda, t \vee r)$.
Let $\max\{\lambda(a), \lambda(b), 0.5\} = 0.5$. If $\max\{t, r\} \geq 0.5$, then

$$\lambda(x) \leq \max\{\lambda(a), \lambda(b), 0.5\} = 0.5 \leq \max\{t, r\},$$

which implies $x \in L(\in \lambda, t \vee r)$ and so $x \in L(\in \vee q\lambda, t \vee r)$. If $\max\{t, r\} < 0.5$, then $0.5 < 1 - \max\{t, r\} < 1$. Thus $\lambda(x) \leq 0.5 \leq 1 - \max\{t, r\}$, which implies that $x \in L(q\lambda, t \vee r)$ and so $x \in L(\in \vee q\lambda, t \vee r)$. Hence (d1) holds. □

Corollary 3.11. *A fuzzy subset λ of a hemiring R is an anti $(\in, \in \vee q)$-fuzzy h-ideal of R if and only if it satisfies:*

(1) $\forall x, y \in R,\ \lambda(x + y) \leq \max\{\lambda(x), \lambda(y), 0.5\}$,
(2) $\forall x, y \in R,\ \lambda(yx) \leq \max\{\lambda(x), 0.5\}$,
(3) $\forall x, y \in R,\ \lambda(xy) \leq \max\{\lambda(x), 0.5\}$,
(4) $\forall a, b, x, y \in R,\ x + a + y = b + y \Longrightarrow \lambda(x) \leq \max\{\lambda(a), \lambda(b), 0.5\}$.

Corollary 3.12. *A fuzzy subset λ of a hemiring R is an anti $(\in, \in \vee q)$-fuzzy k-ideal of R if and only if it satisfies:*

(1) $\forall x, y \in R,\ \lambda(x + y) \leq \max\{\lambda(x), \lambda(y), 0.5\}$,
(2) $\forall x, y \in R,\ \lambda(yx) \leq \max\{\lambda(x), 0.5\}$,
(3) $\forall x, y \in R,\ \lambda(xy) \leq \max\{\lambda(x), 0.5\}$,
(4) $\forall a, b, x \in R,\ x + a = b \Longrightarrow \lambda(x) \leq \max\{\lambda(a), \lambda(b), 0.5\}$.

Corollary 3.13. *A fuzzy subset λ of a hemiring R is an anti $(\in, \in \vee q)$-fuzzy ideal of R if and only if it satisfies:*

(1) $\forall x, y \in R,\ \lambda(x + y) \leq \max\{\lambda(x), \lambda(y), 0.5\}$,
(2) $\forall x, y \in R,\ \lambda(yx) \leq \max\{\lambda(x), 0.5\}$,
(3) $\forall x, y \in R,\ \lambda(xy) \leq \max\{\lambda(x), 0.5\}$.

Example 3.14. Let $R = \{0, 1, 2, 3, 4\}$ and let the operations be given by the following tables holds:

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

and

.	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

Let μ and λ be two fuzzy subset of R defined by

$$\mu(x) = \begin{cases} 1 & \text{if } x \in \{0, 1\} \\ \frac{x-1}{x} & \text{if } x \in \{2, 3, 4\} \end{cases}, \quad \lambda(x) = \begin{cases} 0 & \text{if } x \in \{0, 1\} \\ \frac{1}{x} & \text{if } x \in \{2, 3, 4\} \end{cases}$$

Then $(R, +, .)$ is a hemiring and $A = (\mu_A,\ \lambda_A)$ is an $(\in, \in \vee q)$-intuitionistic fuzzy h-ideal (resp. k-ideal) of R.

Theorem 3.15. *Let λ be an anti $(\in, \in \vee q)$-fuzzy h-ideal of R. Then we have*

(1) *If $t \in [0.5, 1]$, then $L(\in \lambda, t) \neq \varnothing$ is a h-ideal of R.*
(2) *If $t \in (0, 0.5]$, then $L(q\lambda, t) \neq \varnothing$ is a h-ideal of R.*

Proof. (1) Let λ be an anti $(\in, \in \vee q)$-fuzzy h-ideal of R, and let $t \in [0.5, 1]$ be such that $L(\in \lambda, t) \neq \varnothing$. Let $x, y \in L(\in \lambda, t)$ be such that $x + y \overline{\in} L(\in \lambda, t)$. Then $\lambda(x) \leq t$ and $\lambda(y) \leq t$, but $\lambda(x + y) > t$. Since λ is an anti $(\in, \in \vee q)$-fuzzy h-ideal of R. By Corollary 3.11(1), we get

$$t < \lambda(x + y) \leq \max\{\lambda(x), \lambda(y), 0.5\}.$$

If $\max\{\lambda(x), \lambda(y), 0.5\} = \lambda(x)$, then $x \overline{\in} L(\in \lambda, t)$, which is a contradiction. Similarly, if $\max\{\lambda(x), \lambda(y), 0.5\} = \lambda(y)$, then $y \overline{\in} L(\in \lambda, t)$, which is a contradiction. If $\max\{\lambda(x), \lambda(y), 0.5\} = 0.5$, then

$$0.5 \leq t < \lambda(x + y) \leq \max\{\lambda(x), \lambda(y), 0.5\} = 0.5,$$

which is a contradiction. Thus $x + y \in L(\in \lambda, t)$.

If $x \in L(\in \lambda, t)$ and $y \in R$ be such that $yx \overline{\in} L(\in \lambda, t)$, then $\lambda(x) \leq t$, but $\lambda(yx) > t$. Since λ is an anti $(\in, \in \vee q)$-fuzzy h-ideal of R. By Corollary 3.11(2), we get

$$t < \lambda(yx) \leq \max\{\lambda(x), 0.5\},$$

If $\max\{\lambda(x), 0.5\} = \lambda(x)$, then $x \overline{\in} L(\in \lambda, t)$, which is a contradiction.
If $\max\{\lambda(x), 0.5\} = 0.5$, then

$$0.5 \leq t < \lambda(yx) \leq \max\{\lambda(x), 0.5\} = 0.5,$$

which is a contradiction. Thus $yx \in L(\in \lambda, t)$. Similarly, let $x \in L(\in \lambda, t)$ and $y \in R$. Then $xy \in L(\in \lambda, t)$.

Now, let $a, b \in L(\in \lambda, t)$, $x, y \in R$ and $x + a + y = b + y$ be such that $x \overline{\in} L(\in \lambda, t)$. Then $\lambda(a) \leq t$ and $\lambda(b) \leq t$, but $\lambda(x) > t$. Since λ is an anti $(\in, \in \vee q)$-fuzzy h-ideal of R. By Corollary 3.11(4), we get

$$t < \lambda(x) \leq \max\{\lambda(a), \lambda(b), 0.5\},$$

If $\max\{\lambda(a), \lambda(b), 0.5\} = \lambda(a)$, then $a \overline{\in} L(\in \lambda, t)$, which is a contradiction. Similarly, if $\max\{\lambda(a), \lambda(b), 0.5\} = \lambda(b)$, then $b \overline{\in} L(\in \lambda, t)$, which is a contradiction. If $\max\{\lambda(a), \lambda(b), 0.5\} = 0.5$, then $0.5 \leq t < \lambda(x) \leq \max\{\lambda(a), \lambda(b), 0.5\} = 0.5$, which is a contradiction. Thus $x \in L(\in \lambda, t)$.

(2) Let λ be an anti $(\in, \in \vee q)$-fuzzy h-ideal of R, and let $t \in (0, 0.5]$ such that $L(q\lambda, t) \neq \varnothing$. Let $x, y \in L(q\lambda, t)$ be such that $x + y \overline{\in} L(q\lambda, t)$. Then $\lambda(x) + t \leq 1$ and $\lambda(y) + t \leq 1$ but $\lambda(x + y) + t > 1$. Since λ is an anti $(\in, \in \vee q)$-fuzzy h-ideal of R. By Corollary 3.11(1), we get

$$1 - t < \lambda(x + y) \leq \max\{\lambda(x), \lambda(y), 0.5\}.$$

If $\max\{\lambda(x), \lambda(y), 0.5\} = \lambda(x)$, then $x \overline{\in} L(q\lambda, t)$, which is a contradiction. Similarly, if $\max\{\lambda(x), \lambda(y), 0.5\} = \lambda(y)$, then $y \overline{\in} L(q\lambda, t)$, which is a contradiction. Let $\max\{\lambda(x), \lambda(y), 0.5\} = 0.5$. Since $t \in (0, 0.5]$, then $1 - t \in [0.5, 1)$ and so

$$0.5 \leq 1 - t < \lambda(x + y) \leq \max\{\lambda(x), \lambda(y), 0.5\} = 0.5,$$

which is a contradiction. Thus $x + y \in L(q\lambda, t)$.

Let $x \in L(q\lambda, t)$ and $y \in R$ be such that $yx \overline{\in} L(q\lambda, t)$ Then $\lambda(x) + t \leq 1$, but $\lambda(yx) + t > 1$. Since λ is an anti $(\in, \in \vee q)$-fuzzy h-ideal of R. By Corollary 3.11(2), we get

$$1 - t < \lambda(yx) \leq \max\{\lambda(x), 0.5\},$$

If $\max\{\lambda(x), 0.5\} = \lambda(x)$, then $\lambda(x) > 1-t$ and so $x\overline{\in}L(q\lambda, t)$, which is a contradiction. If $\max\{\lambda(x), 0.5\} = 0.5$, then

$$0.5 \le 1-t < \lambda(yx) \le \max\{\lambda(x), 0.5\} = 0.5,$$

which is a contradiction. Thus $yx\in L(q\lambda, t)$. Similarly, let $x \in L(q\lambda, t)$ and $y \in R$. Then $xy\in L(q\lambda, t)$.

Now, let $a, b \in L(q\lambda, t)$, $x, y \in R$ and $x + a + y = b + y$ be such that $x\overline{\in}L(q\lambda, t)$. Then $\lambda(a)+t \le 1$ and $\lambda(b)+t \le 1$, but $\lambda(x)+t > 1$. Since λ is an anti $(\in, \in \vee q)$-fuzzy h-ideal of R. By Corollary 3.11(4), we get

$$1-t < \lambda(x) \le \max\{\lambda(a), \lambda(b), 0.5\},$$

If $\max\{\lambda(a), \lambda(b), 0.5\} = \lambda(a)$, then $a\overline{\in}L(q\lambda, t)$, which is a contradiction.
Similarly, if $\max\{\lambda(a), \lambda(b), 0.5\} = \lambda(b)$, then $b\overline{\in}L(q\lambda, t)$, which is a contradiction. If $\max\{\lambda(a), \lambda(b), 0.5\} = 0.5$, then $0.5 \le 1-t < \lambda(x) \le \max\{\lambda(a), \lambda(b), 0.5\} = 0.5$, which is a contradiction. Thus $x\in L(q\lambda, t)$. □

Corollary 3.16. *Let λ be an anti $(\in, \in \vee q)$-fuzzy k-ideal of R. Then we have*

(1) *If $t \in [0.5, 1]$, then $L(\in \lambda, t) \neq \varnothing$ is a K-ideal of R.*
(2) *If $t \in (0, 0.5]$, then $L(q\lambda, t) \neq \varnothing$ is a K-ideal of R.*

Corollary 3.17. *Let λ be an anti $(\in, \in \vee q)$-fuzzy ideal of R. Then we have*

(1) *If $t \in [0.5, 1]$, then $L(\in \lambda, t) \neq \varnothing$ is an ideal of R.*
(2) *If $t \in (0, 0.5]$, then $L(q\lambda, t) \neq \varnothing$ is an ideal of R.*

Theorem 3.18. *Let A be a h-ideal of R, and let λ and μ be fuzzy subset of R defined by*

$$\mu_A(x) = \begin{cases} \ge 0.5 & \text{if } x \in A \\ 0 & o.w. \end{cases}, \quad \lambda_A(x) = \begin{cases} \le 0.5 & \text{if } x \in A \\ 1 & o.w. \end{cases}$$

Then

(1) *$A = (\mu_A, \lambda_A)$ is an $(\in, \in \vee q)$-intuitionistic fuzzy h-ideal of R.*
(2) *$A = (\mu_A, \lambda_A)$ is an $(q, \in \vee q)$-intuitionistic fuzzy h-ideal of R.*

Proof. (1) If $t, r \in (0, 1]$, then $A = (\mu_A, \lambda_A)$ must satisfies the following conditions,
$(a1)$ $x \in L(\in \lambda, t),\ y \in L(\in \lambda, r) \Longrightarrow x + y \in L(\in \vee q\lambda, t \vee r)$,
$(a2)$ $x \in U(\in \mu, t),\ y \in U(\in \mu, r) \Longrightarrow x + y \in U(\in \vee q\mu, t \vee r)$,
$(b1)$ $x \in L(\in \lambda, t) \Longrightarrow yx \in L(\in \vee q\lambda, t)$,
$(b2)$ $x \in U(\in \mu, t) \Longrightarrow yx \in U(\in \vee q\mu, t)$,
$(c1)$ $x \in L(\in \lambda, t) \Longrightarrow xy \in L(\in \vee q\lambda, t)$,
$(c2)$ $x \in U(\in \mu, t) \Longrightarrow xy \in U(\in \vee q\mu, t)$,
$(d1)$ $x + a + y = b + y,\ a \in L(\in \lambda, t),\ b \in L(\in \lambda, r) \Longrightarrow x \in L(\in \vee q\lambda, t \vee r)$,
$(d2)$ $x + a + y = b + y,\ a \in U(\in \mu, t),\ b \in U(\in \mu, r) \Longrightarrow x \in U(\in \vee q\mu, t \vee r)$
for all $a, b, x, y \in R$.

(a1) Let $x, y \in R$ and $t, r \in (0, 1]$ be such that $x \in L(\in \lambda_A, t),\ y \in L(\in \lambda_A, r)$. Then $\lambda_A(x) \le t$ and $\lambda_A(y) \le r$. Let $\max\{t, r\} = 1$. Hence $\lambda_A(x) = 1$ or $\lambda_A(y) = 1$. Then $\lambda(x+y) \le 1 = \max\{\lambda(x), \lambda(y), 0.5\}$. By Theorem 3.10(1), we have $x + y \in L(\in \vee q\lambda_A, t \vee r)$. If $\max\{t, r\} \neq 1$, then $\lambda_A(x) \le 0.5$ and

$\lambda_{_A}(y) \leq 0.5$. Thus $x, y \in A$. Since A is a h-ideal of R, we have $x + y \in A$. This implies

$$\lambda_{_A}(x+y) \leq 0.5 = \max\{\lambda(x), \lambda(y), 0.5\}.$$

Therefore $x + y \in L(\in \vee q\lambda_{_A}, t \vee r)$.

(a2) Let $x, y \in R$ and $t, r \in (0,1]$ be such that $x \in U(\in \mu_{_A}, t)$, $y \in U(\in \mu_{_A}, r)$. Then $\mu_{_A}(x) \geq t > 0$ and $\mu_{_A}(y) \geq r > 0$. Thus $\mu_{_A}(x) \geq 0.5$ and $\mu_{_A}(y) \geq 0.5$, and so $x, y \in A$. Since A is a h-ideal of R, we have $x + y \in A$. Thus $\mu_{_A}(x+y) \geq 0.5$. If $\max\{t, r\} \leq 0.5$, then $\mu_{_A}(x+y) \geq \max\{t, r\}$, and so $x + y \in U(\in \mu_{_A}, t \vee r)$. If $\max\{t, r\} > 0.5$, then $\mu_{_A}(x+y) + \max\{t, r\} > 0.5+0.5 = 1$, and so $x+y \in U(q\mu_{_A}, t\vee r)$. Therefore $x+y \in U(\in \vee q\mu_{_A}, t\vee r)$.

(b1) Let $x, y \in R$ and $t \in (0,1]$ be such that $x \in L(\in \lambda_{_A}, t)$. Then $\lambda_{_A}(x) \leq t$. If $\lambda_{_A}(x) = 1$. Since $\lambda(yx) \leq 1 = \max\{\lambda(x), 0.5\}$. By Theorem 3.10(2), we have $yx \in L(\in \vee q\lambda_{_A}, t)$. If $\lambda_{_A}(x) \neq 1$, then $\lambda_{_A}(x) \leq 0.5$, thus $x \in A$. Since A is a h-ideal of R, we have $yx \in A$. Thus $\lambda_{_A}(yx) \leq 0.5 = \max\{\lambda(x), 0.5\}$. Therefore $yx \in L(\in \vee q\lambda_{_A}, t)$.

(b2) Let $x, y \in R$ and $t \in (0,1]$ be such that $x \in U(\in \mu_{_A}, t)$. Then $\mu_{_A}(x) \geq t > 0$. Thus $\mu_{_A}(x) \geq 0.5$, and so $x \in A$. Since A is a h-ideal of R, we have $yx \in A$. Thus $\mu_{_A}(yx) \geq 0.5$. If $t \leq 0.5$, then $\mu_{_A}(yx) \geq t$, and so $yx \in U(\in \mu_{_A}, t)$. If $t > 0.5$, then $\mu_{_A}(yx) + t > 0.5 + 0.5 = 1$, and so $yx \in U(q\mu_{_A}, t)$. Therefore $yx \in U(\in \vee q\mu_{_A}, t)$.

Similarly we can prove $(c1)$ and $(c2)$.

(d1) Let $a, b, x, y \in R$, $x + a + y = b + y$ and $t, r \in (0,1]$ be such that $a \in L(\in \lambda_{_A}, t)$, $b \in L(\in \lambda_{_A}, r)$. Then $\lambda_{_A}(a) \leq t$ and $\lambda_{_A}(b) \leq r$. Let $\max\{t, r\} = 1$. Then $\lambda_{_A}(a) = 1$ or $\lambda_{_A}(b) = 1$. Hence $\lambda(x) \leq 1 = \max\{\lambda(a), \lambda(b), 0.5\}$. By Theorem 3.10(4), we have $x \in L(\in \vee q\lambda_{_A}, t \vee r)$. Let $\max\{t, r\} \neq 1$. Then $\lambda_{_A}(a) \leq 0.5$ and $\lambda_{_A}(b) \leq 0.5$. Thus $a, b \in A$. Since A is a h-ideal of R, we have $x \in A$. Hence $\lambda_{_A}(x) \leq 0.5$. This implies

$$\lambda_{_A}(x) \leq 0.5 = \max\{\lambda(a), \lambda(b), 0.5\}.$$

Therefore $x \in L(\in \vee q\lambda_{_A}, t \vee r)$.

(d2) Let $a, b, x, y \in R$, $x + a + y = b + y$ and $t, r \in (0,1]$ be such that $a \in U(\in \mu_{_A}, t)$, $b \in U(\in \mu_{_A}, r)$. Then $\mu_{_A}(a) \geq t > 0$ and $\mu_{_A}(b) \geq r > 0$. Thus $\mu_{_A}(a) \geq 0.5$ and $\mu_{_A}(b) \geq 0.5$, and so $a, b \in A$. Since A is a h-ideal of R, we have $x \in A$. Thus $\mu_{_A}(x) \geq 0.5$. If $\max\{t, r\} \leq 0.5$, then $\mu_{_A}(x) \geq \max\{t, r\}$, and so $x \in U(\in \mu_{_A}, t \vee r)$. If $\max\{t, r\} > 0.5$, then $\mu_{_A}(x) + \max\{t, r\} > 0.5 + 0.5 = 1$, and so $x \in U(q\mu_{_A}, t \vee r)$. Therefore $x \in U(\in \vee q\mu_{_A}, t \vee r)$.

□

Theorem 3.19. *Let A be a k-ideal of R, and let λ and μ be fuzzy subset of R defined by*

$$\mu_A(x) = \begin{cases} \geq 0.5 & \text{if } x \in A \\ 0 & o.w. \end{cases}, \quad \lambda_A(x) = \begin{cases} \leq 0.5 & \text{if } x \in A \\ 1 & o.w. \end{cases}$$

Then

(1) $A = (\mu_{_A}, \lambda_{_A})$ *is an* $(\in, \in \vee q)$*-intuitionistic fuzzy k-ideal of R.*

(2) $A = (\mu_{_A}, \lambda_{_A})$ *is an a* $(q, \in \vee q)$*-intuitionistic fuzzy k-ideal of R.*

Proof. The proof is similar to that of Theorem 3.18. □

Corollary 3.20. *Let A be an ideal of R, and let λ and μ be fuzzy subset of R defined by*

$$\mu_A(x) = \begin{cases} \geq 0.5 & \text{if } x \in A \\ 0 & o.w. \end{cases}, \quad \lambda_A(x) = \begin{cases} \leq 0.5 & \text{if } x \in A \\ 1 & o.w. \end{cases}$$

Then

(1) *$A = (\mu_A, \lambda_A)$ is an $(\in, \in \vee q)$-intuitionistic fuzzy ideal of R.*
(2) *$A = (\mu_A, \lambda_A)$ is an a $(q, \in \vee q)$-intuitionistic fuzzy ideal of R.*

4. $(\in, \in \vee q)$-Intuitionistic fuzzy ideals with (anti) $(\in, \in \vee q)$-Fuzzy ideals

In this section, let R be a hemiring. It is clear that, $A = (\mu_A,\ \lambda_A)$ is an $(\in, \in \vee q)$-intuitionistic fuzzy ideal of R if and only if μ_A is an $(\in, \in \vee q)$-fuzzy ideal and λ_A is an anti $(\in, \in \vee q)$-fuzzy ideal of R. But, we introduce some relations between $(\in, \in \vee q)$-intuitionistic fuzzy ideals with $(\in, \in \vee q)$-fuzzy ideals and with anti $(\in, \in \vee q)$-fuzzy ideals.

Theorem 4.1. *Let R be a hemiring. Then, $\square A = (\mu_A,\ \mu_A^c)$ is an $(\in, \in \vee q)$-intuitionistic fuzzy h-ideal of R if and only if μ_A is an $(\in, \in \vee q)$-fuzzy h-ideal of R.*

Proof. Let μ_A be an $(\in, \in \vee q)$-fuzzy h-ideal of R. By Corollary 3.11, it is sufficient to show that μ_A^c satisfies the conditions:

(1) $\forall x, y \in R,\ \mu_A^c(x + y) \leq \max\{\mu_A^c(x), \mu_A^c(y), 0.5\}$,
(2) $\forall x, y \in R,\ \mu_A^c(yx) \leq \max\{\mu_A^c(x), 0.5\}$,
(3) $\forall x, y \in R,\ \mu_A^c(xy) \leq \max\{\mu_A^c(x), 0.5\}$,
(4) $\forall a, b, x, y \in R,\ x + a + y = b + y \Longrightarrow \mu_A^c(x) \leq \max\{\mu_A^c(a), \mu_A^c(b), 0.5\}$.

Since μ_A is an $(\in, \in \vee q)$-fuzzy h-ideal of R. Then

Case(1) For $x, y \in R$, we have $\mu_A(x + y) \geq \min\{\mu_A(x), \mu_A(y), 0.5\}$, and so

$$1 - \mu_A^c(x + y) \geq \min\{1 - \mu_A^c(x), 1 - \mu_A^c(y), 0.5\}.$$

Which implies

$$\mu_A^c(x + y) \leq 1 - \min\{1 - \mu_A^c(x), 1 - \mu_A^c(y), 0.5\}.$$

Therefore

$$\mu_A^c(x + y) \leq \max\{\mu_A^c(x), \mu_A^c(y), 0.5\}.$$

Case(2) For $x, y \in R$, we have $\mu_A(yx) \geq \min\{\mu_A(x), 0.5\}$, and so

$$1 - \mu_A^c(yx) \geq \min\{1 - \mu_A^c(x), 0.5\}.$$

Which implies

$$\mu_A^c(yx) \leq 1 - \min\{1 - \mu_A^c(x), 0.5\}.$$

Therefore

$$\mu_A^c(yx) \leq \max\{\mu_A^c(x), 0.5\}.$$

Case(3) Similarly, for $x, y \in R$, we have $\mu_A(xy) \geq \min\{\mu_A(x), 0.5\}$, and so $1 - \mu_A^c(xy) \geq \min\{1 - \mu_A^c(x), 0.5\}$. Which implies $\mu_A^c(xy) \leq 1 - \min\{1 - \mu_A^c(x), 0.5\}$. Therefore $\mu_A^c(xy) \leq \max\{\mu_A^c(x), 0.5\}$.

Case(4) For $a, b, x, y \in R$, $x+a+y = b+y$, we have $\mu_A(x) \geq \min\{\mu_A(a), \mu_A(b), 0.5\}$, and so

$$1 - \mu_A^c(x) \geq \min\{1 - \mu_A^c(a), 1 - \mu_A^c(b), 0.5\}.$$

Which implies

$$\mu_A^c(x) \leq 1 - \min\{1 - \mu_A^c(a), 1 - \mu_A^c(b), 0.5\}.$$

Therefore

$$\mu_A^c(x) \leq \max\{\mu_A^c(a), \mu_A^c(b), 0.5\}.$$

This completes the proof. □

Corollary 4.2. *Let R be a hemiring. Then, $\Diamond A = (\lambda_A^c, \lambda_A)$ is an $(\in, \in \vee q)$-intuitionistic fuzzy h-ideal of R if and only if λ_A is an anti $(\in, \in \vee q)$-fuzzy h-ideal of R.*

Theorem 4.3. *Let R be a hemiring. Then, $\Box A = (\mu_A,\ \mu_A^c)$ is an $(\in, \in \vee q)$-intuitionistic fuzzy k-ideal of R if and only if μ_A is an $(\in, \in \vee q)$-fuzzy k-ideal of R.*

Proof. The proof is similar to that of Theorem 4.1. □

Corollary 4.4. *Let R be a hemiring. Then, $\Diamond A = (\lambda_A^c, \lambda_A)$ is an $(\in, \in \vee q)$-intuitionistic fuzzy k-ideal of R if and only if λ_A is an anti $(\in, \in \vee q)$-fuzzy k-ideal of R.*

Theorem 4.5. *Let R be a hemiring. Then, $\Box A = (\mu_A,\ \mu_A^c)$ is an $(\in, \in \vee q)$-intuitionistic fuzzy ideal of R if and only if μ_A is an $(\in, \in \vee q)$-fuzzy ideal of R.*

Proof. The proof is similar to that of Theorem 4.1. □

Corollary 4.6. *Let R be a hemiring. Then, $\Diamond A = (\lambda_A^c, \lambda_A)$ is an $(\in, \in \vee q)$-intuitionistic fuzzy h-ideal of R if and only if λ_A is an anti $(\in, \in \vee q)$-fuzzy h-ideal of R.*

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Mohsen Asghari-Larimi (asghari2004@yahoo.com)
Department of Mathematics, Golestan University, Postal code 49138-15759, Gorgan, Iran.

Young Bae Jun (skywine@gmail.com)
Department of Mathematics Education, Gyeongsang National University, Chinju 660-701, Korea.