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On IF-rough oscillatory region and it's application in decision making

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Abstract. The aim of this paper is to introduce a new concept of IF-rough oscillatory region and to show its application in the field of decision making. It is a generalization of the previous concept fuzzy-rough oscillatory region.

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1. Introduction

The concept of Rough Set, introduced by Z. Pawlak in 1980[8], is a powerful Mathematical tool to deal with incompleteness. L.A. Zadeh introduced the concept of Fuzzy Set theory in 1965[11].Later on K.T Atanassov[1] introduced the concept of IF set in 1983.Though the name introduced by Atanassov was Intuitionistic fuzzy set but later on a controversy arose in connection with intuitionistic fuzzy logic. So to avoid this point through out this paper we are using IF set instead of intuitionistic fuzzy set. A.Mukherjee and S.Halder introduced the concept of Fuzzy Oscillatory Region in 2007[7].S.Halder introduced IF Oscillatory region in 2008[3].In this paper, the concept of IF-Rough Oscillatory Region is introduced. With the help of this concept we can find the pattern of any attribute of an object and can draw decision about an unknown object.

In section 2, some important required information is cited.

In section 3, the concept of IF-Rough Oscillatory Region is defined and its properties are studied.

In section 4, the application of the above defined concept is shown with an example of practical field from the paper[12].Lastly,the conclusion is drawn.

This paper is a generalization of the previous paper of the authors[4].since IF set is used in this paper so non-membership function is also used and so its result are far better from the previous concept.

2. Preliminaries

In this section, some of the important required concepts necessary to go further through this paper is shown.

2.1 Rough Set[8] :

Let U be a finite non-empty set, called universe and R be an equivalence relation on U,called indiscernibility relation. By R(x) we mean that the set of all y such that xRy,i.e. R(x)=$[x]_R$ is containing the element x.

Let X be a subset of U. We want to characterize the set X with respect to R. The Lower approximation of a set X w.r.t R is the set of all objects,which surely belong to X i.e. $R_*(X)=\{x{:}R(x)\subseteq X\}$. The Upper approximation of X w.r.t R is the set of all objects, which are partially belonging to X i.e. $R^*(x)=\{x{:}R(x)\cap X\neq\phi\}$.Fuzzy set is defined by employing the fuzzy membership function, where rough set is defined by approximations. The difference of the upper and the lower approximation is a boundary region.Any Rough Set has a non-empty boundary region where as any crisp set has an empty boundary region. The lower approximation is called interior and the upper approximation is called closure of the set.With this interior we may form a topological space.

2.2 IF-Set[1]:

Definition 2.2.1: Let X denotes a Universal set.Then the membership function μ_A by which a fuzzy set A is usually defined has the form $\mu_A{:}X\rightarrow[0,1]$.

Definition 2.2.2:

Let E be a fixed Universe.An IF set A in E is an object having the form

$A=\{\langle x,\mu_A(x),\gamma_A(x)\rangle{:}x\in E\}$, where the functions $\mu_A{:}E\rightarrow[0,1]$ and $\gamma_A{:}E\rightarrow[0,1]$define the degree of membership and the degree of non-membership respectively of the element $x\in E$ to the set A, and $\forall x\in E$, $0\leq \mu_A(x)+\gamma_A(x)\leq 1$.

Fuzzy set can be viewed as IF sets but not conversely. Basically, IF sets based models may be adequate in situations when we face human testimonies, opinions, etc. involving answers of the type: Yes, No, Does not apply.

2.3 On Fuzzy-rough Oscillatory region[4]:

The lower approx set forms the topology. The elements of this topology are open sets i.e. the elements of lower approx. are open sets. And the elements of upper approximations are closed sets. With this topology we can now introduce some operators as follows:

Definition 2.3.1: The operator Λ , Int, Cl and V : $I^x \rightarrow I^x$ is defined as

(i) $\Lambda_{a_j}(x) = \inf\{ \mu_{a_j}(x_i) : \mu_{a_j}(x_i) \geq \mu_{a_j}(x), x_i \in G$,G is an open set , $j=1,2,\cdots,n\}$
$=\hat{I}$,if no such open set exists.

(ii) $\mathrm{Int}_{a_j}(x)=\sup\{\mu_{a_j}(x_i) : \mu_{a_j}(x_i) \leq \mu_{a_j}(x), x_i \in G$,G is an open set , $j=1,2,\cdots,n\}$
$=\phi$,if no such open set exists.

(iii)$Cl_{a_j}(x)$ = inf { $\mu_{a_j}(x_i)$: $\mu_{a_j}(x_i) \geq \mu_{a_j}(x)$, $x_i \in G$,G is an closed set , j=1,2,$\cdots$,n}

= Î,if no such closed set exists.

(iv)$V_{a_j}(x)$ = sup{$\mu_{a_j}(x_i)$: $\mu_{a_j}(x_i) \leq \mu_{a_j}(x)$, $x_i \in G$,G is an closed set , j=1,2,$\cdots$,n}

=ϕ,if no such closed set exists.

where $\mu_{a_j}(x_i)$is the membership value of any particular attribute a_j of any object x_i and $\mu_{a_j}(x)$ is the membership value of unknown object for a particular attribute a_j

Definition 2.3.2: An operator O^o: $I^X \to I^X$ such that $O^o a_j(x) = \Lambda_{a_j}(x)$- $Int_{a_j}(x)$.

This operator is said to be fuzzy-rough open oscillatory operator and an operator O^c:$I^X \to I^X$ such that $O^c a_j(x) = Cl_{a_j}(x)$- $V_{a_j}(x)$.

This operator is said to be fuzzy rough closed oscillatory operator.

and We have, $h_{a_j}(x)$ = inf { $\Lambda_{a_j}(x)$,$Cl_{a_j}(x)$} - sup{ $Int_{a_j}(x)$, $V_{a_j}(x)$}

From the above relation following cases may arise:

Case I: $h_{a_j}(x) = \Lambda_{a_j}(x)$ - $Int_{a_j}(x)$;

Case II:$h_{a_j}(x) = \Lambda_{a_j}(x)$ - $V_{a_j}(x)$;

Case III: $h_{a_j}(x) = Cl_{a_j}(x)$ - $Int_{a_j}(x)$;

Case IV: $h_{a_j}(x) = Cl_{a_j}(x)$ - $V_{a_j}(x)$;

Case V: $h_{a_j}(x)$ = Î - $V_{a_j}(x)$;

Case VI: $h_{a_j}(x)$ = Î -$Int_{a_j}(x)$;

Case VII: $h_{a_j}(x) = \Lambda_{a_j}(x)$ - ϕ;

Case VIII: $h_{a_j}(x) = Cl_{a_j}(x)$ -ϕ ;

Case IX: $ha_j(x)$ = Î - ϕ;

So while drawing any decision by the help of height of oscillation at first we have to check the membership values of the height of oscillation and then the structure. Finally we can draw a conclusion about an unknown object or about the pattern of that unknown object with the help of value of height of oscillation as follows: To make conclusion from height of oscillation we face various cases. We distribute these cases in three parts: (1) Stable, (2) Unstable;(3) Oscillating.

3. On IF-rough Oscillatory Region:

In this section,the concept of IF-Rough Oscillatory region is introduced .It is an extended part of the previous section 2.3. Let us consider a data set containing set of objects X = {x_i: i = 1,2,$\cdots$,n} with the set of attribute A = {a_j: j = 1,2,$\cdots$,n}. The data set may contain linguistic attributes which may be intuitionistic in nature. We find the value for each object x_i with linguistic attribute a_i between [0,1].Here each object is denoted as {$\langle x_i,\mu_{a_j}(x_i), \gamma_{a_j}(x_i)\rangle$: i =1,2,$\cdots$,m and j =1,2,$\cdots$,n}, where $\mu_{a_j}(x_i)$ is the degree of belong ness and γ_{a_j} is the degree of non-belong ness of an object x_i corresponding to the attribute a_j. The lower approximation set forms the topology. i.e. lower approximations are open sets and upper approximations are closed sets. With this topology we can now introduce some operators as follows:

Definition 3.1: An operator Λ from $I^X \to I^X$is defined as

Λ_{a_j}(x)=supγ{infμ{$\langle$ μ_{a_j}(x$_i$), γ_{a_j}(x$_i$)$\rangle$:x$_i$ is an open set with μ_{a_j}(x$_i$)$\geq$ μ_{a_j}(x) and γ_{a_j}(x$_i$)$\leq$ γ_{a_j}(x),i =1,2,$\cdots$,m and j =1,2,$\cdots$,n}}

=Î, if no such open set exist

Where μ_{a_j}(x$_i$) is the degree of belong ness of x$_i$ corresponding to the attribute a$_j$ and γ_{a_j}(x$_i$) is the degree of non belong ness of x$_i$ corresponding to the attribute a$_j$, μ_{a_j}(x) is the degree of belong ness of an unknown object x corresponding to the attribute a$_j$ and γ_{a_j}(x) is the degree of non belong ness of an unknown object x corresponding to the attribute a$_j$.

Example 3.2: Let us consider a data set as follows:

	a_1	a_2	a_3	a_4	d
x_1	<0.7,0.3>	<0.8,0.1>	<0.6,0.3>	<0.6,0.4>	1
x_2	<0.5,0.4>	<0.7,0.3>	<0.6,0.4>	<0.8,0.2>	1
x_3	<0.4,0.4>	<0.3,0.1>	<0.5,0.3>	<0.6,0.4>	1
x_4	<0.4,0.4>	<0.3,0.1>	<0.5,0.3>	<0.6,0.4>	0
x_5	<0.6,0.2>	<0.5,0.3>	<0.7,0.2>	<0.7,0.3>	1

where decision attribute is taken 1 for positive decision and 0 for negative decision. Here X = {x$_1$, x$_2$, x$_3$, x$_4$, x$_5$}, the set of objects d is any decision. Then L.A (W) ={x$_1$, x$_2$, x$_5$} and U.A (W) ={x$_1$, x$_2$, x$_3$, x$_4$, x$_5$}, where W ={ x$_1$, x$_2$, x$_3$,x$_5$}.

Let P be any object with attribute values a$_1$ <0.5,0.3>, a$_2$ <0.1,0.6>, a$_3$ <0.8,0.2>, a$_4$ <0.5,0.3>. Therefore Λ_{a_1}(p) =supγ{infμ{<0.6,0.2>,<0.7,0.3>}}=<0.6,0.2>, Λ_{a_2}(p) = <0.5,0.3>, Λ_{a_3}(p) = Î and Λ_{a_4}(p) = <0.7,0.3>.

Definition 3.3: An operator Λ^c from $I^X \rightarrow I^X$is defined as

$\Lambda^c_{a_j}$(x)=infμ{supγ{$\langle$ γ_{a_j}(x$_i$), μ_{a_j}(x$_i$)$\rangle$:x$_i$ is an open set with γ_{a_j}(x$_i$)$\leq$ γ_{a_j}(x) and μ_{a_j}(x$_i$)$\geq$ μ_{a_j}(x),i =1,2,$\cdots$,m and j =1,2,$\cdots$,n}}.

=ϕ,if no such open set exists[ϕ implies undefined set].

Each symbol is similar as Definition 3.1.

Example 3.4: From example 3.2 we get, $\Lambda^c_{a_1}$(p)= infμ{supγ{<0.3,0.7>,<0.2,0.6>}}
=<0.3,0.7>, $\Lambda^c_{a_2}$(p)= infμ{supγ{<0.1,0.8>,<0.3,0.7>,<0.3,0.5>}}
=infμ{<0.3,0.7>,<0.3,0.5>}=<0.3,0.5> $\Lambda^c_{a_3}$(p)=ϕ $\Lambda^c_{a_4}$(p)
= infμ{supγ{<0.2,0.8>,<0.3,0.7>}}=<0.3,0.7>.

Definition 3.5: An operator Int from $I^X \rightarrow I^X$is defined as

Int$_{a_j}$(x)=infγ{supμ{$\langle$ μ_{a_j}(x$_i$), γ_{a_j}(x$_i$)$\rangle$:x$_i$ is an open set with μ_{a_j}(x$_i$)$\leq$ μ_{a_j}(x) and γ_{a_j}(x$_i$)$\geq$ γ_{a_j}(x),i =1,2,$\cdots$,m and j =1,2,$\cdots$,n}}.

=ϕ,if no such open set exists.

Each symbol is similar as Definition 3.1.

Example 3.6: From example 3.2 we get, Int$_{a_1}$(p)= <0.5,0.4>,Int$_{a_2}$(p)=ϕ ,Int$_{a_3}$(p)= <0.7,0.2>, Int$_{a_4}$(p)=ϕ.

Definition 3.7: An operator Int^c from $I^X \rightarrow I^X$is defined as

Int$^c_{a_j}$(x)=supμ{infγ{$\langle$ μ_{a_j}(x$_i$), γ_{a_j}(x$_i$)$\rangle$:x$_i$ is an open set with μ_{a_j}(x$_i$)$\geq$ μ_{a_j}(x) and γ_{a_j}(x$_i$)$\leq$ γ_{a_j}(x),i =1,2,$\cdots$,m and j =1,2,$\cdots$,n}}.

=Î,if no such open set exists.

Each symbol is similar as Definition 3.1.

Example 3.8: From example 3.2 we get, $\text{Int}^c_{a_1}$(p)= <0.4,0.5>,$\text{Int}^c_{a_2}$(p)=Î ,$\text{Int}^c_{a_2}$(p)= <0.2,0.7>, $\text{Int}^c_{a_2}$(p)=Î.

Definition 3.9: An operator Cl from $I^X \to I^X$ is defined as

Cl_{a_j}(x)=supγ{infμ{$\langle$ $\mu_{a_j}(x_i)$, $\gamma_{a_j}(x_i)\rangle$:x_i is a closed set with $\mu_{a_j}(x_i) \geq \mu_{a_j}(x)$ and $\gamma_{a_j}(x_i) \leq \gamma_{a_j}(x)$,i =1,2,$\cdots$,m and j =1,2,$\cdots$,n}}.

=î,if no such closed set exists.

Each symbol is similar as Definition 3.1.

Example 3.10: From example 3.2 we get, Cl_{a_1}(p)= <0.6,0.2>,Cl_{a_2}(p)=<0.3,0.1> ,Cl_{a_3}(p)= Î, Cl_{a_4}(p)=<0.7,0.3>.

Definition 3.11: An operator Cl^c from $I^X \to I^X$ is defined as

$\text{Cl}^c_{a_j}$(x)=infμ{supγ{$\langle$ $\gamma_{a_j}(x_i)$, $\mu_{a_j}(x_i)\rangle$:x_i is an closed set with $\gamma_{a_j}(x_i) \leq \gamma_{a_j}(x)$ and $\mu_{a_j}(x_i) \geq \mu_{a_j}(x)$,i =1,2,$\cdots$,m and j =1,2,$\cdots$,n}}.

=ϕ,if no such closed set exists.

Each symbol is similar as Definition 3.1.

Example 3.12: From example 3.2 we get, $\text{Cl}^c_{a_1}$(p)= <0.3,0.7>,$\text{Cl}^c_{a_2}$(p)=<0.3,0.5> ,$\text{Cl}^c_{a_3}$(p)= ϕ, $\text{Cl}^c_{a_4}$(p)=<0.3,0.7>.

Definition 3.13: An operator V from $I^X \to I^X$ is defined as

V_{a_j}(x)=infγ{supμ{$\langle$ $\mu_a(x_i)$, $\gamma_{a_j}(x_i)\rangle$:x_i is an closed set with $\mu_{a_j}(x_i) \leq \mu_{a_j}(x)$ and $\gamma_{a_j}(x_i) \geq \gamma_{a_j}(x)$,i =1,2,$\cdots$,m and j =1,2,$\cdots$,n}}.

=ϕ,if no such closed set exists.

Each symbol is similar as Definition 3.1.

Example 3.14: From example 3.2 we get, V_{a_1}(p)= <0.5,0.4>,V_{a_2}(p)=ϕ ,V_{a_3}(p)= <0.7,0.2>, V_{a_4}(p)=ϕ.

Definition 3.15: An operator V^c from $I^X \to I^X$ is defined as

$V^c_{a_j}$(x)=supμ{infγ{$\langle$ $\mu_{a_j}(x_i)$, $\gamma_{a_j}(x_i)\rangle$:x_i is an closed set with $\mu_{a_j}(x_i) \geq \mu_{a_j}(x)$ and $\gamma_{a_j}(x_i) \leq \gamma_{a_j}(x)$,i =1,2,$\cdots$,m and j =1,2,$\cdots$,n}}.

=Î,if no such closed set exists.

Each symbol is similar as Definition 3.1.

Example 3.16: From example 3.2 we get, $V^c_{a_1}$(p)= <0.4,0.5>,$V^c_{a_2}$(p)=Î ,$V^c_{a_3}$(p)= <0.2,0.7>, $V^c_{a_4}$(p)=Î.

With the help of above defined operators we can define two oscillatory operators as follows:

Definition 3.17: An operator O^o: $I^X \to I^X$ is said to be an IF-rough open oscillatory operator of an object x with attribute a_j if $O^o a_j(x) = \Lambda_{a_j}(x)$ - $\text{Int}_{a_j}(x)$ and an operator O^{oc}: $I^X \to I^X$ is said to be an IF-rough compliment open oscillatory operator of an object x with attribute a_j if $O^{oc} a_j(x) = \text{Int}^c_{a_j}(x)$ - $\Lambda^c_{a_j}(x)$. The IF-rough open oscillating operator implies the membership value of the length of oscillation of each attribute of a particular object in the lower approximation region and the IF-rough compliment open oscillatory operator implies the non-membership value of the length of oscillation of each attribute of a particular object in the lower approximation region.

Example 3.18: From example 3.2 we get,$O^o a_1(p) = \Lambda_{a_1}(p)$- $Int_{a_1}(p)$ = <0.6,0.2> - <0.5,0.4> = <0.1,0.2>, $O^o a_2(p)$ = <0.5,0.3> -ϕ , $O^o a_3(p)$ = $\hat{I}$ - <0.7,0.2>, $O^o a_4(p)$ = <0.7,0.3> -ϕ and $O^{oc} a_1(x)$ =<0.4,0.5>-<0.3,0.7>=<0.1,0.2>, $O^{oc} a_2(x)$ =$\hat{I}$-<0.3,0.5>, $O^{oc} a_3(x)$ =<0.2,0.7>-ϕ, $O^{oc} a_4(x)$ =$\hat{I}$-<0.3,0.7>.

The graph is given below:

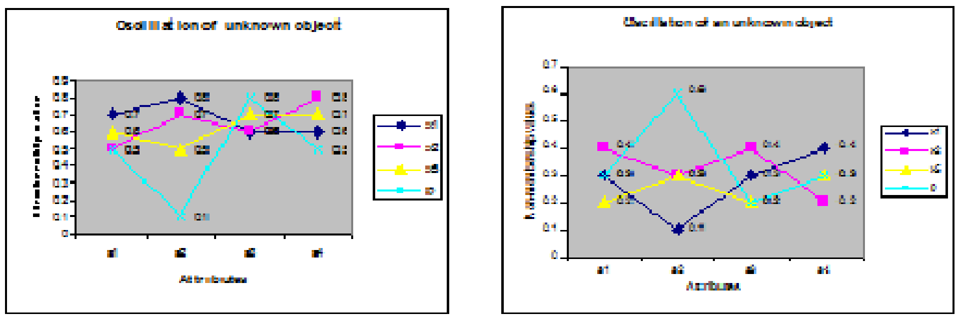


Figure 1: The line chart of the pattern of IF-rough oscillation.

Remark 3.19: From the above graph it is clear that ($\Lambda_{a_1}(p)$,$Int_{a_1}(p)$) is the range within which an attribute a_1 of an object p may oscillate And $O_{a_1}(p)$ is the length of oscillation of the object p for the attribute a_1 within the region of lower approximation .Here the oscillation of the object p w.r.to the attribute a_2 is very high Now if the sum of membership grade and non-membership grade of any oscillating operator $O^o_{a_j}(x)$ is less than or equal to 0.5 than by the help of this operator some decision making problems can be solved But there may arise cases as $O^o_{a_j}(x) = \hat{I}$ - $Int_{a_j}(x)$ or $O^o_{a_j}(x) = \Lambda_{a_j}(x)$ - ϕ . Or $O^o_{a_j}(x) = \hat{I}$ - ϕ . Then this operator doesn't helps by any way in decision making problems. Now let us introduce some theorems to show how the IF rough open oscillating operator may be used in decision making problems

Theorem 3.20: For any object x , if $O^o a_j(x) = 0$, the object is stable or oscillating.

Proof. To prove the theorem two cases may happen:

(a) $O^o a_j(x)$ = <0,1> and (b) $O^o a_j(x)$ = <0,0>

(a) Let $O^o a_j(x)$ = <0,1>

which implies $\Lambda_{a_j}(x)$- $Int_{a_j}(x) = < 0, 1 >$

which implies $\Lambda_{a_j}(x)$ = <0,0> and $Int_{a_j}(x) = < 0, 1 >$,Otherwise if $\Lambda_{a_j}(x)$ = <a,b> and $Int_{a_j}(x)$ = <c,d>,where 0<a,b,c,d<1, then $\Lambda_{a_j}(x)$- $Int_{a_j}(x)$ = <a-c,d-b> $\neq$ <0,1> , which is possible if and only if a-c = 0 which implies a = c.......(1)

and d-b =1 which implies d=1+b,which is possible iff b=0 and hence d =1. Now if d =1,obviously c = 0 and hence a = 0 from (1). i.e <0,0> and <0,1> are the only open sets. And only possible object of such pattern must be with membership value 0 and non-membership value 1 i.e x = <0,1> i.e it is converging towards a fixed point and the object x = <0,1> is an open set and hence obviously stable i.e the object is in positive or in negative region.

(b) $O^o a_j(x)$ = <0,0>

which implies Λ_{a_j}(x)- Int$_{a_j}$(x) = <0,0>
which implies Λ_{a_j}(x)= Int$_{a_j}$(x) = <a,b>, where a,b $\in$ [0,1]
which implies <a,b> is the only open set and the only possible value of x must be <a,b>, where a,b $\in$ [0,1].
which implies a + b<1 [from the definition of IF-set] ....(2)
From the above expression (2) four cases may arise:
(i) b = 0, a<1, then <a,0> is the only open set and x must be <a,0>, a (0,1].Then the object is either in the positive or in the negative region i.e. the object is stable.
(ii) a = 0, b<1, then <0,b> is the only open set and x must be <0,b>, b (0,1]. Then the object is either in the positive or in the negative region i.e. the object is stable.
(iii) a$\neq$ 0 and b$\neq$ 0, then the object is also oscillating because the decision of the object depends on the value of a and b.
(iv) a = 0, b = 0, This case is an absurd case.

Remark 3.21: For any object x , if O^{oc}a$_j$(x) = 0, the object is stable or oscillating.

Theorem 3.22: For any object x, if O^{o}a$_j$(x) = <1,0>, then the object is in lower approximation or in outside region.

Proof. Let O^{o}a$_j$(x) = <1,0>
which implies Λ_{a_j}(x)- Int$_{a_j}$(x) = $<1,0>$
which implies Λ_{a_j}(x) = <1,0> and Int$_{a_j}$(x) = $<0,0>$,Otherwise if Λ_{a_j}(x) = <a,b> and Int$_{a_j}$(x) = <c,d>,where 0<a,b,c,d<1, then Λ_{a_j}(x)- Int$_{a_j}$(x) = <a-c,d-b>$\neq$ <1,0> , which is possible if and only if d-b = 0 which implies d = b.......(3)
and a-c =1 which implies a=1+c>1,which is possible iff c=0 and hence a =1. Now if a =1,obviously b = 0 and hence d = 0 from (3). i.e <1,0> and <0,0> are the only open sets. And only possible object of such pattern must be with membership value 1 and non-membership value 0 i.e x = <1,0> i.e it is converging towards a fixed point and the object x = <1,0> is an open set and hence obviously stable i.e the object is in positive or in negative region.

Remark 3.23: For any object x, if O^{oc}a$_j$(x) = <1,0>, then the object is in lower approximation or in outside region.
We now define another oscillating operator as follows:

Definition 3.24: An operator O^{cl}: I^X $\rightarrow$ I^X of an object x such that O^{cl}a$_j$(x) = Cl$_{a_j}$(x) - V$_{a_j}$(x), is called closed IF-rough oscillatory operator and an operator O^{clc}:I^X $\rightarrow$ I^X is said to be an If-rough compliment closed oscillatory operator of an object x with attribute a$_j$ if O^{clc}a$_j$(x) = V$^c_{a_j}$(x) - Cl$_{a_j}$(x).

Example 3.25: From example 3.2 we get, O^{cl}a$_1$(p) = <0.6,0.2>-<0.5,0.4>
= <0.1,0.2>, O^{cl}a$_2$(p) = <0.3, 0.1> -ϕ , O^{cl}a$_3$(p) = Î- <0.7,0.2>, O^{cl}a$_4$(p)
= <0.7,0.3> -ϕ and
O^{clc}a$_1$(p) =<0.1,0.2> ,O^{clc}a$_2$(p) =Î-<0.3,0.5> ,O^{clc}a$_3$(p)
=<0.2,0.7>-ϕ ,O^{clc}a$_4$(p) =Î-<0.3,0.7> .

Theorem 3.26: For any object x, O^{cl}a$_j$(x) = 0, the object is in boundary region.

Proof. To prove the theorem two cases may happen:

(a) $O^{cl}a_j(x) = <0,1>$ and (b) $O^{cl}a_j(x) = <0,0>$.

(a) Let $O^{cl}a_j(x) = <0,1>$ which implies $Cl_{a_j}(x) - V_{a_j}(x) = <0,1>$,

which implies $Cl_{a_j}(x) = <0,0>$ and $V_{a_j}(x) = <0,1>$, Otherwise if $Cl_{a_j}(x) = <a,b>$ and $V_{a_j}(x) = <c,d>$,where $0<a,b,c,d<1$, then $Cl_{a_j}(x)- V_{a_j}(x) = <a-c,d-b> \neq <0,1>$, which is possible if and only if a-c = 0 which implies a = c.....(4)

and d-b =1 which implies d=1+b>1,which is possible iff b=0 and hence d =1. Now if d =1,obviously c = 0 and hence a = 0 from (4). i.e $<0,0>$ and $<0,1>$ are the only closed sets. And only possible object of such pattern must be with membership value 0 and non-membership value 1 i.e $x = <0,1>$ i.e it is converging towards a fixed point and the object $x = <0,1>$ is an closed set and hence obviously the object is boundary region.

(b) Let $O^{cl}a_j(x) = <0,0>$ which implies $Cl_{a_j}(x) - V_{a_j}(x) = <0,0>$.

which implies $Cl_{a_j}(x) = V_{a_j}(x) = <a,b>$,where $a,b \in [0,1]$. which implies $<a,b>$ is the only open set and the only possible value of x must be $<a,b>$, where $a,b \in[0,1]$.

which implies $a + b<1$ [from the definition of IF-set] ......(5)

From the above expression (5) four cases may arise:

(i) $b = 0$, $a<1$, then $<a,0>$ is closed set and x must be $<a,0>$, $a\in (0,1]$. Then the object is in the boundary region or in the lower approximation.

(ii) $a = 0$, $b<1$, then ¡0,b¿ is the only closed set and x must be $<0,b>$, $b\in (0,1]$. Then the object is in the boundary region or in the lower approximation.

(iii) $a\neq 0$ and $b\neq 0$, then the object is completely in the boundary region tending to go towards lower approximation or outside region depending on the value of a and b.

(iv) $a = 0$, $b = 0$, This case is an absurd case.

Remark 3.27: For any object x, $O^{clc}a_j(x) = 0$, the object is in boundary region.

Theorem 3.28: For any object x, $O^{cl}a_j(x) = <1,0>$, the object is in boundary region.

Proof. Let $O^{cl}a_j(x) = <1,0>$ which implies $Cl_{a_j}(x) - V_{a_j}(x) = <1,0>$,

which implies $Cl_{a_j}(x) = <1,0>$ and $V_{a_j}(x) = <0,0>$, Otherwise if $Cl_{a_j}(x) = <a,b>$ and $V_{a_j}(x) = <c,d>$,where $0<a,b,c,d<1$, then $Cl_{a_j}(x)- V_{a_j}(x) = <a-c,d-b> \neq <1,0>$, which is possible if and only if d-b = 0 which implies d =b.....(6)

and a-c =1 which implies a=1+c>1,which is possible iff c=0 and hence a =1.

Now if a =1,obviously b = 0 and hence d = 0 from (6).

i.e $<1,0>$ and $<0,0>$ are the only closed sets. And only possible object of such pattern must be with membership value 1 and non-membership value 0 i.e $x = <0,1>$ i.e it is converging towards a fixed point and the object $x = <1,0>$ is an closed set and hence obviously the object is boundary region.

Remark 3.29: For any object x, $O^{clc}a_j(x) = <1,0>$, the object is in boundary region.

Theorem 3.30: For any object x, if $O^{o}a_j(x) = 0$, then $O^{cl}a_j(x) = 0$

Proof. Let $O^{o}a_j(x) = 0$, Then two cases arise:

(a) $O^{o}a_j(x)= <0,0>$ and (b) $O^{o}a_j(x) = <0,1>$.

(a) Let $O^o a_j(x)$ = <0,0> which implies $\Lambda_{a_j}(x)$- $Int_{a_j}(x)$ = <0,0>
which implies $\Lambda_{a_j}(x)$ = <a,b> and $Int_{a_j}(x)$ = <a,b>, where a,b $\in$ [0,1]
which implies <a,b> is the only open set and the only possible value of x is <a,b>.
which implies $Cl_{a_j}(x)$ = <a,b> and $V_{a_j}(x)$ = <a,b>, where x = <a,b>.
which implies $Cl_{a_j}(x)$- $V_{a_j}(x)$ = <0,0>
which implies $O^{cl}a_j(x) = 0$. Hence proved.
(b) Let $O^o a_j(x)$ = <0,1>
which implies $\Lambda_{a_j}(x)$- $Int_{a_j}(x)$ = <0,1>
which implies $\Lambda_{a_j}(x)$ =<a,0> and $Int_{a_j}(x)$ = <a,1>, where a$\in$ [0,1]
which implies <a,0> and <a,1> are the only open sets and the only possible value of x is <a,1>
which implies a +1 <1[from the definition of IF-set] It is possible if and only if a = 0, so x must be <0,1> and the only open sets are <0,0> and <0,1>.
which implies $Cl_{a_j}(x)$ = <0,0> and $V_{a_j}(x)$ = <0,1>, where x = <0,1>.
which implies $Cl_{a_j}(x)$- $V_{a_j}(x)$= <0,1>.
which implies $O^{cl}a_j(x)$= 0.Hence proved.

Theorem 3.31: If $O^o a_j(x)$=<0,0>,then $O^{oc}a_j(x)$=<0,0>.

Proof. Let $O^o a_j(x)$ = <0,0>
which implies $\Lambda_{a_j}(x)$- $Int_{a_j}(x)$ = <0,0>
which implies $\Lambda_{a_j}(x)$ = <a,b> and $Int_{a_j}(x)$ = <a,b>, where a,b $\in$[0,1]
which implies <a,b> is the only open set and the only possible value of x is <a,b>
which implies $\Lambda^c_{a_j}(x)$ = <b,a> and $Int^c_{a_j}(x)$ = <b,a>
which implies $Int^c_{a_j}(x)$ -$\Lambda^c_{a_j}(x)$ = <0,0>
which implies $O^{oc}a_j(x)$=<0,0>.

Theorem 3.32: $O^{cl}a_j(x)$=<0,0>,then $O^{clc}a_j(x)$=<0,0>

Proof. Let $O^{cl}a_j(x)$=<0,0> which implies $Cl_{a_j}(x)$- $V_{a_j}(x)$ = <0,0>
which implies $Cl_{a_j}(x)$ = <a,b> and $V_{a_j}(x)$ = <a,b>, where a,b$\in$ [0,1]
which implies <a,b> is the closed set and the only possible value of x is <a,b>
which implies $Cl^c_{a_j}(x)$ = <b,a> and $V^c_{a_j}(x)$ = <b,a>
which implies $Cl^c_{a_j}(x)$ -$V^c_{a_j}(x)$ = <0,0>
which implies $O^{clc}a_j(x)$=<0,0>.

Remark 3.33: Next we define the height of oscillation as follows:
$h_{a_j}(x)$=supγ{infμ {$\Lambda_{a_j}(x)$, $Cl_{a_j}(x)$}-infγ{sup μ{$Int_{a_j}(x)$,$V_{a_j}(x)$}}......(A)
and $h^c_{a_j}(x)$= infμ {supγ{$Int^c_{a_j}(x)$,$V^c_{a_j}(x)$}-sup μ{ infγ{$\Lambda^c_{a_j}(x)$, $Cl^c_{a_j}(x)$}}......(B)
From example 3.2, $h_{a_1}(p)$ = <0.6,0.2>-<0.5,0.4>=<0.1,0.2>, $h^c_{a_1}(x)$=<0.1,0.2>, $h_{a_2}(p)$ =<0.3,0.1>-ϕ , $h^c_{a_2}(x)$= Î-<0.3,0.7>, $h_{a_3}(p)$=Î-<0.7,0.2>, $h^c_{a_3}(x)$=<0.2,0.7>-ϕ , $h_{a_4}(p)$=<0.6,0.2>-ϕ , $h^c_{a_4}(x)$=Î-<0.3,0.7>. From this expression 3.33(A), we may have various cases. Let us first study the 4 cases of $h_{a_j}(x)$, the part of $h^c_{a_j}(x)$ is inside these cases.

Case I : Let if possible $h_{a_j}(x)$ = $\Lambda_{a_j}(x)$- $Int_{a_j}(x)$ .Then the following sub cases may arise:

Sub case (A): $h_{a_j}(x) = <0,0>$,
Sub case (B): $h_{a_j}(x) = \hat{I}$- $Int_{a_j}(x)$,
Sub case (C): $h_{a_j}(x) = \Lambda_{a_j}(x)$-$\phi$,
Sub case (D): $0_\sim < h_{a_j}(x) < 1_\sim$,
Sub case (E): $h_{a_j}(x) = \hat{I}$-ϕ.
Let us study the above five sub cases:

Sub case (A): If $h_{a_j}(x) = <0,0>$,the decision may be drawn as in theorem 3.20.

Sub case (B): $h_{a_j}(x) = \hat{I}$- $Int_{a_j}(x)$.

In this sub case the attributes may lie in lower approximation or outside region or in boundary region. So we need to check the numerical value of the difference between the membership value of the attribute a_j and $Int_{a_j}(x)$ of the object x.

Let the difference be $d = < \mu_{a_j}(x_i)$, $\gamma_{a_j}(x_i)>$- $Int_{a_j}(x)$.

Three cases may arise:

(i) If $\mu(d) \geq 0.5$, the attribute lie in outside region.

(ii) If $0 < \mu(d) < 0.5$, the attribute lie in boundary region, tending towards lower approximation.

(iii) If $\mu(d) = 0$, the attribute is in lower approximation.

Example 3.33.1: Let us consider the information table given below:

	a_1	a_2	a_3	d
x_1	<0.7,0.2>	<0.6,0.3>	<0.8,0.1>	1
x_2	<0.6,0.2>	<0.5,0.1>	<0.7,0.3>	1
x_3	<0.6,0.3>	<0.5,0.1>	<0.7,0.2>	1
x_4	<0.6,0.3>	<0.5,0.1>	<0.7,0.2>	0

Here open set=$\{x_1,x_2\}$ and closed set=$\{x_1,x_2,x_3,x_4\}$ Let p_1 be any object with attribute values:a_1 <0.8,0.1>, a_2 <0.7,0.0>, a_3 <1.0,0.0>

$h_{a_1}(p1) = \sup\gamma\{\inf\mu\ \{\Lambda_{a_1}(p_1), Cl_{a_1}(p_1)\}$-$\inf\gamma\{\sup\ \mu\{Int_{a_1}(p_1),V_{a_1}(p_1)\}\}$

$=\hat{i}$-$\inf\gamma\{\sup\ \mu\{Int_{a_1}(p_1),V_{a_1}(p_1)\}\}[\Lambda_{a_1}(p_1)= Cl_{a_1}(p_1)=\hat{I}]$

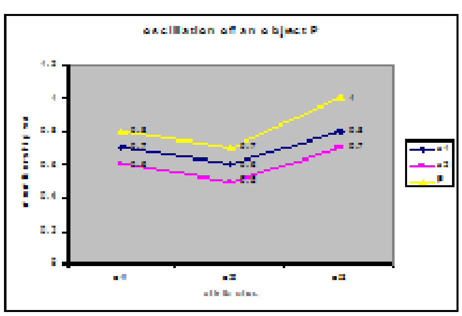
$= \hat{I}$- $\inf\gamma\{\sup\ \mu\{<0.7,0.2>,<0.7,0.2>\} = \hat{I}$- <0.7,0.2>,[Pattern $\hat{I}$- $Int_{a_j}(x)$]

Here d = <0.8,0.1>-<0.7,0.2> = <0.1, 0.1>

$\mu(d)=0.1<0.5$, the attribute is in boundary region, tending towards lower approx.

Similarly the attributes a_2, a_3 lie in the boundary region, tending towards lower approximation.

The figure is given below:



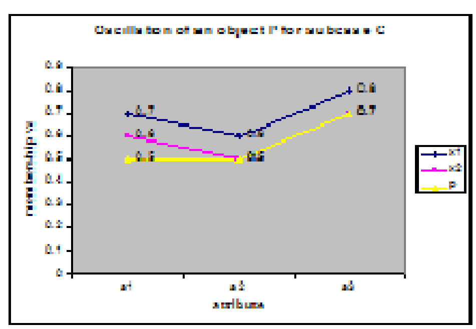


Figure 2: The line chart of the pattern of IF-rough oscillation of Case-I, Subcase-B.

Sub case (C): $h_{a_j}(x) = \Lambda_{a_j}(x)$-$\phi$.

We need to check the numerical value of the difference between the membership value of the attribute a_j and $\Lambda_{a_j}(x)$ of the object x.

Let the difference be d = $< \mu_{a_j}(x)$, $\gamma_{a_j}(x)$>- $Int_{a_j}(x)$.

Three cases may arise:

(i) If μ(d)$\geq$ 0.5, the attribute lie in outside region.

(ii) If 0< μ(d) < 0.5 , the attribute lie in boundary region, tending towards lower approximation.

(iii) If μ(d) = 0, the attribute is in lower approximation.

Example 3.33.2: Let us consider the information table :

	a_1	a_2	a_3	d
x_1	<0.6,0.2>	<0.6,0.2>	<0.8,0.1>	1
x_2	<0.8,0.1>	<0.7,0.2>	<0.9,0.1>	1
x_3	<0.8,0.1>	<0.7,0.2>	<0.9,0.1>	1
x_4	<0.8,0.1>	<0.7,0.2>	<0.9,0.1>	0

Here open set={x_1} and closed set={x_1,x_2,x_3,x_4}

Let p_2 be any object with attribute values:a_1 <0.5,0.3>, a_2 <0.5,0.3>, a_3 <0.7,0.1>

$h_{a_1}(p_2)$ = supγ{infμ {$\Lambda_{a_1}(p_2)$, $Cl_{a_1}(p_2)$}-infγ{sup μ{$Int_{a_1}(p_2)$,$V_{a_1}(p_2)$}}

=supγ{infμ {<0.6,0.2>,<0.6,0.2>}-ϕ[Since $Int_{a_1}(p_2)$

=$V_{a_1}(p_2)$=ϕ]

= <0.6,0.2>-ϕ,[Pattern $\Lambda_{a_j}(x)$-ϕ]

Here d = <0.6,0.2>-<0.5,0.3> = <0.1, 0.1>

μ(d)=0.1<0.5, the attribute is in boundary region, tending towards lower approx.

Similarly the attributes a_2, a_3 lie in the boundary region, tending towards lower approximation.

The figure is given below:

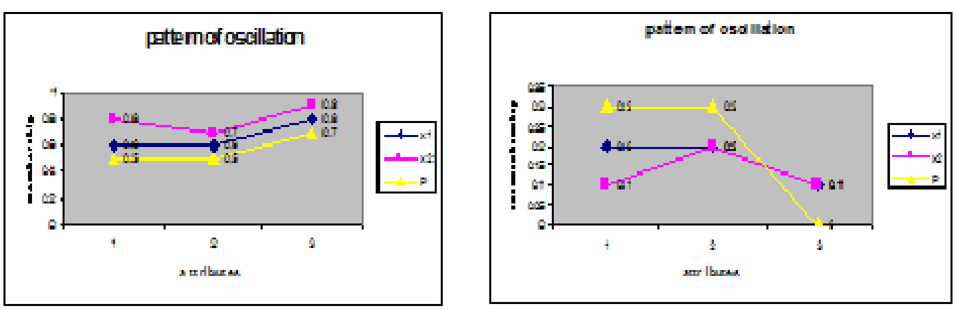


Figure 3: The line chart of the pattern of IF-rough oscillation of Case-I,Subcase-C.

Sub case (D): $0_\sim$ <$h_{a_j}(x)$<$1_\sim$.

In this case let s = μ(h_{a_j}(x))+ γ(h_{a_j}(x)).

If s $\leq$ 0.5 ,the attribute is in boundary region tending toward lower approximation, since the height of oscillation is very small.

If s > 0.5, then three cases may arise:

(i) If μ(h_{a_j}(x))>0.5, the attribute is tending towards boundary region.

(ii) If μ(h_{a_j}(x)) <0.5, the attribute is tending to go lower approximation.

(iii) If μ(h_{a_j}(x)) =0, the attribute is in lower approximation.

Example 3.33.3: Let the information table be:

	a_1	a_2	a_3	d
x_1	<0.7,0.3>	<0.8,0.1>	<0.6,0.3>	1
x_2	<0.5,0.4>	<0.7,0.3>	<0.6,0.4>	1
x_3	<0.4,0.4>	<0.3,0.1>	<0.5,0.3>	1
x_4	<0.4,0.4>	<0.3,0.1>	<0.5,0.3>	0

Here open set ={x_1 , x_2} and closed set ={ x_1 , x_2 ,x_3 ,x_4}.

Let p_3 be any object with attribute values:a_1 <0.7,0.3>, a_2 <0.8,0.2>, a_3 <0.6,0.3>

$h_{a_1}(p_3)$ = supγ{infμ {$\Lambda_{a_1}(p_3)$, $Cl_{a_1}(p_3)$}-infγ{sup μ{$Int_{a_1}(p_3)$,$V_{a_1}(p_3)$}}

=supγ{infμ {<0.7,0.3>,<0.7,0.3>}-infγ{sup μ{<0.7,0.3>,<0.7,0.3>}}

= <0.7,0.3>-<0.7,0.3>[Pattern Λ_{a_j}(x)-Int_{a_j}(x)]=<0.0,0.0>

The attribute is in lower approximation.

Similarly the attributes a_2, a_3 lie in the lower approximation.

The figure is given below:

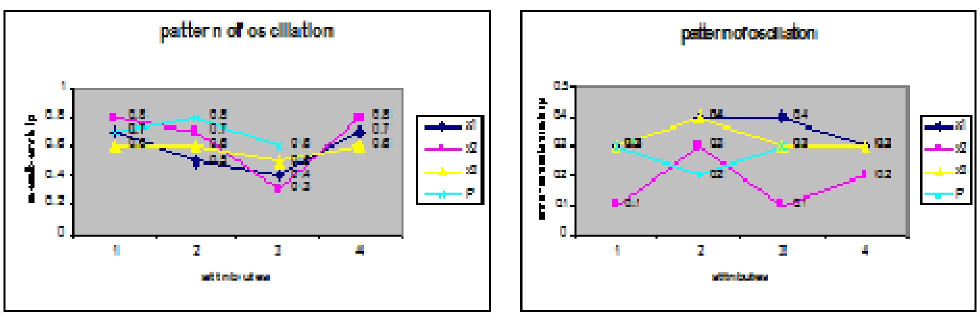


Figure 4: The line chart of the pattern of IF-rough oscillation of Case-I,Subcase-D.

Sub case (E): h_{a_j}(x) = Î-ϕ,

Example 3.33.4: Let the information table be:

	a_1	a_2	a_3	d
x_1	<0.7,0.2>	<0.6,0.2>	<0.8,0.1>	1
x_2	<0.6,0.2>	<0.5,0.1>	<0.7,0.3>	1
x_3	<0.6,0.3>	<0.5,0.1>	<0.7,0.2>	1
x_4	<0.6,0.3>	<0.5,0.1>	<0.7,0.2>	0

Here open set ={x_1 , x_2} and closed set ={ x_1 , x_2 ,x_3 ,x_4}.

Let p_4 be any object with attribute values: a_1 <0.8,0.1>, a_2 <0.7,0.3>, a_3 <1.0,0.0>. h_{a_2}(p)=Î-ϕ. The figure is given below:

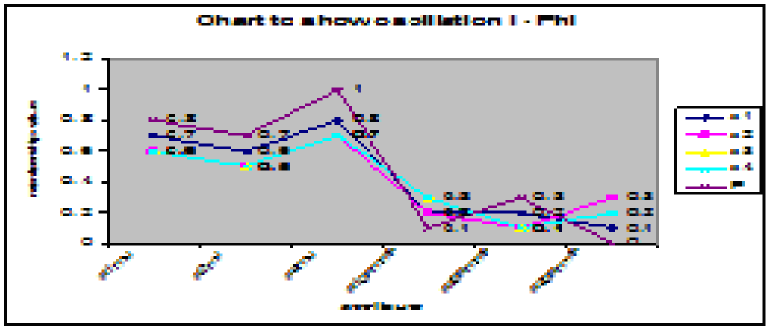


Figure 5: The line chart of the pattern of IF-rough oscillation of Case-I,Subcase-E.

This is an unstable case. We cannot make any decision. So we need to make decision with the help of $h^c_{a_j}$(x).From the expression of remark 3.33(B), following four cases may arise:

Case-I′: $h^c_{a_j}(x) = Int^c_{a_j}(x) - \Lambda^c_{a_j}(x)$.

Case-II′: $h^c_{a_j}(x) = Int^c_{a_j}(x) - Cl^c_{a_j}(x)$.

Case-III′: $h^c_{a_j}(x) = V^c_{a_j}(x) - \Lambda^c_{a_j}(x)$.

Case-IV′: $h^c_{a_j}(x) = V^c_{a_j}(x) - Cl^c_{a_j}(x)$.

Let us study the above cases as follows:

Case-I′: $h^c_{a_j}(x) = Int^c_{a_j}(x) - \Lambda^c_{a_j}(x)$.

There also following cases may arise:

Sub case (A): $h^c_{a_j}(x)$ =<0,0>.

Sub case (B): $h^c_{a_j}(x)$ = Î- $\Lambda^c_{a_j}(x)$

Sub case (C): $h^c_{a_j}(x) = Int^c_{a_j}(x) - \phi$

Sub case (D): $0_\sim < h^c_{a_j}(x) < 1_\sim$

Sub case (E): $h^c_{a_j}(x)$ = Î-ϕ

Let us study the above cases as follows:

Sub case (A): If $h^c_{a_j}(x)$ =<0,0>, the decision may be drawn same as remark 3.21.

Sub case (B): $h^c_{a_j}(x)$= Î- $\Lambda^c_{a_j}(x)$

We need to check the numerical value of the difference between the membership value of the attribute a_j and $\Lambda^c_{a_j}(x)$ of the object x.

Let the difference be d = < $\gamma^c_{a_j}(x_i)$, $\mu^c_{a_j}(x_i)$> -$\Lambda^c_{a_j}(x)$.

Three cases may arise:

(i) If $\gamma(d) \geq 0.5$,the attribute tends to go outside region.

(ii) If 0< $\gamma(d) < 0.5$, lie in boundary region, tending towards lower approximation.

(iii) If γ (d) =0, the attribute is in lower approximation.

Example 3.33.5: Let the information table be:

	a_1	d
x_1	<0.6,0.2>	1
x_2	<0.8,0.1>	1
x_3	<0.8,0.1>	1
x_4	<0.8,0.1>	0

Here open set $=\{x_1\}$ and closed set $=\{x_1, x_2, x_3, x_4\}$.

Let p_5 be any object with attribute values: a_1 <0.5,0.3>.

$h^c_{a_1}(p_5)=\inf\mu\{\sup\gamma\{Int^c_{a_1}(p_5), V^c_{a_1}(p_5)\}$-sup $\mu\{\inf\gamma\{\Lambda^c_{a_1}(p_5), Cl^c_{a_1}(p_5)\}$

$=\hat{I}$-sup $\mu\{\inf\gamma\{<0.2,0.6>,<0.2,0.6>\}\}$[Since,$Int^c_{a_1}(p_5)=V^c_{a_1}(p5)=\hat{I}$]

$=\hat{I}$-<0.6,0.2>, [Pattern $\hat{I}$- $\Lambda^c_{a_j}(x)$]

d=<0.3,0.5>-<0.2,0.6>=<0.1,0.1>

$\gamma(d) < 0.5$, therefore the attribute lie in boundary region, tending towards lower approximation.

Sub case (C): $h^c_{a_j}(x) = Int^c_{a_j}(x) - \phi$

In this sub case the attributes may lie in lower approximation or outside region or in boundary region. So we need to check the numerical value of the difference between the membership value of the attribute a_j and $Int^c_{a_j}(x)$ of the object x.

Let the difference be $d = Int^c_{a_j}(x) - < \gamma_{a_j}(x_i), \mu_{a_j}(x_i) >$.

Three cases may arise:

(i) If γ (d)$\geq$ 0.5, the attribute lie in outside region.

(ii) If 0< γ (d) <0.5 , the attribute lie in boundary region, tending towards lower approximation.

(iii) If γ (d) =0, the attribute is in lower approximation.

Example 3.33.6: Let the information table be:

	a_1	a_2	a_3	d
x_1	<0.7,0.2>	<0.6,0.3>	<0.8,0.1>	1
x_2	<0.6,0.2>	<0.5,0.1>	<0.7,0.3>	1
x_3	<0.6,0.3>	<0.5,0.1>	<0.7,0.2>	1
x_4	<0.6,0.3>	<0.5,0.1>	<0.7,0.2>	0

Here open set $=\{x_1, x_2\}$ and closed set $=\{x_1, x_2, x_3, x_4\}$. Let p_6 be any object with attribute values: a_1 <0.8,0.1>, a_2 <0.7,0.0>, a_3 <1.0,0.0>.

$h^c_{a_1}(p_6)=\inf\mu\{\sup\gamma\{Int^c_{a_1}(p_6), V^c_{a_1}(p_6)\}$-sup $\mu\{\inf\gamma\{\Lambda^c_{a_1}(p6), Cl^c_{a_1}(p_6)\}$

$=\inf\mu\{\sup\gamma\{<0.2,0.7>,<0.2,0.7>\}\}-\phi[\Lambda^c_{a_1}(p_6)= Cl^c_{a_1}(p_6)=\phi]$

=<0.2,0.7>-ϕ,[Pattern $Int^c_{a_j}(x)\phi$]

d=<0.2,0.7>-<0.1,0.8>=<0.1,0.1>

$\gamma(d)=0.1 < 0.5$, therefore the attribute lie in boundary region, tending towards lower approximation.Similarly,the other attributes lie in the boundary region, tending towards lower approximation.

Sub case (D): $0_\sim < h^c_{a_j}(x) < 1_\sim$

In this case let $s = \mu(h^c_{a_j}(x)) + \gamma(h^c_{a_j}(x))$.

If $s \leq 0.5$, the attribute is in boundary region tending toward lower approx., since the height of oscillation is very small.

If $s > 0.5$, then three cases may arise:

(i) If $\gamma(h^c_{a_j}(x)) \geq 0.5$, the attribute is in boundary region, tending to go outside.

(ii) If $\gamma(\mathrm{h}^c_{a_j}(\mathrm{x})) < 0.5$, the attribute is in boundary, tending to go lower approximation.

(iii) If $\gamma(\mathrm{h}^c_{a_j}(\mathrm{x})) = 0$, the attribute is in lower approximation.

Sub case (E): $\mathrm{h}^c_{a_j}(\mathrm{x})$ = î-ϕ, This is an unstable case. We cannot make any decision of this object.

Case-II′: $\mathrm{h}^c_{a_j}(\mathrm{x}) = \mathrm{Int}^c_{a_j}(\mathrm{x})$ - $\mathrm{Cl}^c_{a_j}(\mathrm{x})$.

Here also following cases may arise:

Sub case (A): $\mathrm{h}^c_{a_j}(\mathrm{x})$ =<0,0>,the decision may be drawn same as remark 3.21.

Sub case (B): $\mathrm{h}^c_{a_j}(\mathrm{x})$ = Î- $\mathrm{Cl}^c_{a_j}(\mathrm{x})$

Sub case (C): $\mathrm{h}^c_{a_j}(\mathrm{x}) = \mathrm{Int}^c_{a_j}(\mathrm{x})$ -ϕ , this case is same as Case-I′,Subcase(C).

Sub case (D): $0_\sim < \mathrm{h}^c_{a_j}(\mathrm{x}) < 1_\sim$

Sub case (E): $\mathrm{h}^c_{a_j}(\mathrm{x})$ = Î-ϕ , this is an unstable case.

Let us study the remaining two cases as follows:

Sub case (B): $\mathrm{h}^c_{a_j}(\mathrm{x})$ = Î- $\mathrm{Cl}^c_{a_j}(\mathrm{x})$

We need to check the numerical value of the difference between the membership value of the attribute a_j and $\mathrm{Cl}^c_{a_j}(\mathrm{x})$ of the object.

Let the difference be d =< $\gamma_{a_j}(\mathrm{x}_i)$,$\mu_{a_j}(\mathrm{x}_i)$>- $\mathrm{Cl}^c_{a_j}(\mathrm{x})$.

Three cases may arise:

(i) If $\gamma(\mathrm{d}) \geq 0.5$, the attribute is in outside region.

(ii) If γ(d) ¡ 0.5 ,the attribute is tends towards the boundary.

(iii) If $\gamma(\mathrm{d}) = 0$, the attribute is in boundary region.

Sub case (D): $0_\sim < \mathrm{h}^c_{a_j}(\mathrm{x}) < 1_\sim$.

Let s = $\mu(\mathrm{h}^c_{a_j}(\mathrm{x}))+ \gamma(\mathrm{h}^c_{a_j}(\mathrm{x}))$.

If s $\geq$ 0.5 ,the attribute is in boundary region, tending towards lower approx.

If s > 0.5, then three cases may arise:

(i) If $\gamma(\mathrm{h}^c_{a_j}(\mathrm{x})) < 0.5$, the attribute tends to go towards the lower approximation.

(ii) If $\gamma(\mathrm{h}^c_{a_j}(\mathrm{x})) > 0.5$, the attribute is tending to go towards outside region.

(iii) If $\gamma(\mathrm{h}^c_{a_j}(\mathrm{x})) = 0$, the attribute is in boundary, tending towards lower approximation .

Case-III′: $\mathrm{h}^c_{a_j}(\mathrm{x}) = \mathrm{V}^c_{a_j}(\mathrm{x})$ - $\Lambda^c_{a_j}(\mathrm{x})$

Here also following cases may arise:

Sub case (A): $\mathrm{h}^c_{a_j}(\mathrm{x})$ =<0,0>,the decision may be drawn same as remark 3.21.

Sub case (B): $\mathrm{h}^c_{a_j}(\mathrm{x})$ = Î- $\Lambda^c_{a_j}(\mathrm{x})$,this case is same as Case-I′,Subcase(B).

Sub case (C): $\mathrm{h}^c_{a_j}(\mathrm{x}) = \mathrm{V}^c_{a_j}(\mathrm{x})$ -ϕ

Sub case (D): $0_\sim < \mathrm{h}^c_{a_j}(\mathrm{x}) < 1_\sim$

Sub case (E): $\mathrm{h}^c_{a_j}(\mathrm{x})$ = Î- ϕ , this is an unstable case.

Let us study the remaining two cases as follows:

Sub case (C): $\mathrm{h}^c_{a_j}(\mathrm{x}) = \mathrm{V}^c_{a_j}(\mathrm{x})$ -ϕ

We need to check the numerical value of the difference between the membership value of the attribute a_j and $\mathrm{V}^c_{a_j}(\mathrm{x})$ of the object x. Let the difference be d = $\mathrm{V}^c_{a_j}(\mathrm{x})$- < $\gamma_{a_j}(\mathrm{x}_i)$,$\mu_{a_j}(\mathrm{x}_i)$>.

Three cases may arise: (i) If $\gamma(\mathrm{d}) \leq 0.5$, the attribute is tending towards the boundary.

(ii) If $\gamma(\mathrm{d}) > 0.5$, the attribute is in outside region.

(iii) If γ(d) =0, the attribute is in boundary.

Example 3.33.7: Let the information table be:

	a_1	a_2	a_3	d
x_1	<0.6,0.1>	<0.6,0.2>	<0.7,0.2>	1
x_2	<0.7,0.2>	<0.6,0.3>	<0.65,0.1>	1
x_3	<0.75,0.2>	<0.7,0.2>	<0.75,0.3>	1
x_4	<0.75,0.2>	<0.7,0.2>	<0.75,0.3>	0

Here open set $=\{x_1 , x_2\}$ and closed set $=\{ x_1 , x_2 ,x_3 ,x_4\}$. Let p_7 be any object with attribute values: a_1 <0.9,0.0>, a_2 <0.8,0.1>, a_3 <0.9,0.05>.

$h^c_{a_1}(p_7)=\inf\mu\ \{\sup\gamma\{Int^c_{a_1}(p_7),V^c_{a_1}(p_7)\}\text{-}\sup\ \mu\{\ \inf\gamma\{\Lambda^c_{a_1}(p_7),\ Cl^c_{a_1}(p_7)\}\}$

$=\inf\mu\{\sup\gamma\{<0.1,0.6>,<0.1,0.6>\}\}\text{-}\phi[\Lambda^c_{a_1}(p_7)=Cl^c_{a_1}(p_7)=\phi]$

$=<0.1,0.6>\text{-}\phi$,[Pattern $V^c_{a_j}(x)\phi$]

$d=<0.1,0.6>\text{-}<0.0,0.9>=<0.1,0.3>$

$\gamma(d)=0.1 < 0.5$, therefore the attribute lie in boundary region.Similarly,the other attributes lie in the boundary region.

The figure is given below:

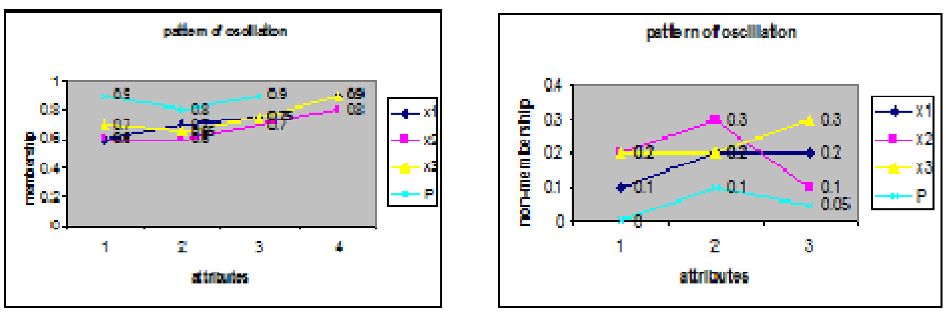


Figure 6: The line chart of the pattern of IF-rough oscillation of Case-III′,Subcase-C.

Sub case (D): $0_\sim < h^c_{a_j}(x) < 1_\sim$.

We need to check the numerical value of $s=\mu(h^c_{a_j}(x))+\gamma(h^c_{a_j}(x))$.

If $s \leq 0.5$,the attribute is in boundary region, tending towards lower approximation.

If d> 0.5 ,then three cases may arise:

(i) If $\gamma\ (h^c_{a_j}(x)) < 0.5$, the attribute is in boundary, tending towards lower approximation.

(ii) If $\gamma\ (h^c_{a_j}(x)) \geq 0.5$, the attribute is in boundary.

(iii) If $\gamma\ (h^c_{a_j}(x)) =0$, the attribute is in lower approximation.

Example 3.33.8: Let the information table be:

	a_1	a_2	a_3	d
x_1	<0.5,0.2>	<0.6,0.4>	<0.8,0.2>	1
x_2	<0.7,0.3>	<0.8,0.2>	<1.0,0.0>	1
x_3	<0.65,0.35>	<0.75,0.25>	<0.95,0.05>	1
x_4	<0.65,0.35>	<0.75,0.25>	<0.95,0.05>	0

Here open set $=\{x_1, x_2\}$ and closed set $=\{x_1, x_2, x_3, x_4\}$. Let p_8 be any object with attribute values: a_1 <0.67,0.3>, a_2 <0.77,0.2>, a_3 <0.97,0.0>.

$h^c_{a_1}(p_8)=\inf\mu\ \{\sup\gamma\{Int^c_{a_1}(p_8), V^c_{a_1}(p_8)\}\text{-}\sup\ \mu\{\inf\gamma\{\Lambda^c_{a_1}(p_8), Cl^c_{a_1}(p_8)\}\}$

$=\inf\mu\ \{\sup\gamma\{\phi,<0.35,0.65>\}\}\text{-}\sup\ \mu\{\inf\gamma\{<0.3,0.7>,<0.3,0.7>\}\}$

$=<0.35,0.65>-<0.3,0.7>=<0.05,0.05>$

s=0.1< 0.5,the attribute lie in boundary region, tending towards the lower approximation. Similarly,the other attributes lie in the lower approximation.

The figure is given below:

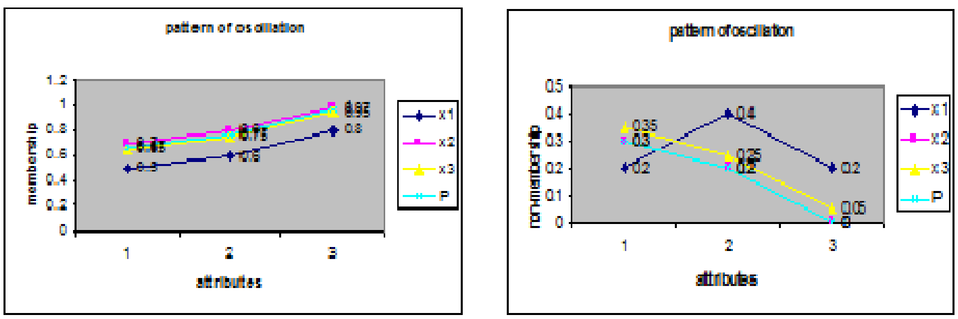


Figure 7: The line chart of the pattern of IF-rough oscillation of Case-III′,Subcase-D.

Case-IV′: $h^c_{a_j}(x) = V^c_{a_j}(x) - Cl^c_{a_j}(x)$

Here also following cases may arise:

Sub case (A): $h^c_{a_j}(x) = <0,0>$,this case is same as Case-II′,Subcase(B).

Sub case (B): $h^c_{a_j}(x) = \hat{I} - Cl^c_{a_j}(x)$,this case is same as Case-III′,Subcase(C).

Sub case (C): $h^c_{a_j}(x) = V^c_{a_j}(x) - \phi$,same as above.

Sub case (D): $0_\sim < h^c_{a_j}(x) < 1_\sim$.

Sub case (E): $h^c_{a_j}(x) = \hat{I}-\phi$, this an unstable case. Let us study the remaining one cases as follows:

Sub case (D): $0_\sim < h^c_{a_j}(x) < 1_\sim$.

In this case the attribute is in boundary region.

We need to check the numerical value of $s = \mu(h^c_{a_j}(x)) + \gamma(h^c_{a_j}(x))$.

If $s \leq 0.5$,the attribute is in boundary region, tending towards lower approximation.

If $s > 0.5$,then three cases may arise:

(i) If $\gamma(h^c_{a_j}(x)) < 0.5$, the attribute is in boundary, tending towards lower approximation.

(ii) If $\gamma\ (h^c_{a_j}(x)) \geq 0.5$, the attribute is in boundary region.

(iii) If γ ($h^c_{a_j}$(x)) =0, the attribute is in lower approximation.

Example 3.33.9: Let the information table be:

	a_1	a_2	a_3	d
x_1	<0.7,0.1>	<0.8,0.2>	<0.6,0.3>	1
x_2	<0.8,0.2>	<0.9,0.1>	<0.7,0.3>	1
x_3	<0.75,0.2>	<0.85,0.1>	<0.65,0.3>	1
x_4	<0.75,0.2>	<0.85,0.1>	<0.65,0.3>	0

Here open set ={x_1 , x_2} and closed set ={ x_1 , x_2 ,x_3 ,x_4}. Let p_9 be any object with attribute values: a_1 <0.75,0.2>, a_2 <0.85,0.1>, a_3 <0.65,0.3>.

$h^c_{a_1}(p_9)$= infμ{supγ{$\text{Int}^c_{a_1}(p_9)$,$V^c_{a_1}(p_9)$}-sup μ{ infγ{$\Lambda^c_{a_1}(p_9)$, $\text{Cl}^c_{a_1}(p_9)$}}

=infμ{supγ{ϕ,<0.2,0.75>}}-sup μ{ infγ{<0.2,0.8>,<0.2,0.75>}}

=<0.2,0.75>-<0.2,0.8>=<0.0,0.05>

s=0.05< 0.5,the attribute lie in boundary region, tending towards the lower approximation. Similarly,the other attributes lie in the lower approximation.

The figure is given below:

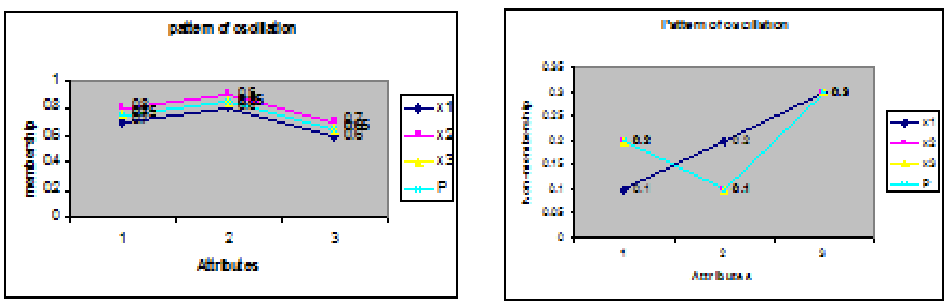


Figure 8: The line chart of the pattern of IF-rough oscillation of Case-IV′,Subcase-D.

Case II :

Let if possible h_{a_j}(x) = Λ_{a_j}(x)- V_{a_j}(x) .

Then the following sub cases may arise:

Sub case (A): h_{a_j}(x) = <0,0>

Sub case (B): h_{a_j}(x) = Î- V_{a_j}(x)

Sub case (C): h_{a_j}(x) = Λ_{a_j}(x)-ϕ

Sub case (D): $0_\sim$ <h_{a_j}(x) <$1_\sim$.

Sub case (E): h_{a_j}(x) = Î-ϕ

Let us study the above five sub cases:

Sub case (A): If h_{a_j}(x) = <0,0>,the decision may be drawn as in theorem 3.20.

Sub case (B): h_{a_j}(x) = Î- V_{a_j}(x).

In this sub case the attributes may lie in outside region or in boundary region. So we need to check the numerical value of the difference between the membership value of the attribute a_j and V_{a_j}(x) of the object x.

Let the difference be d = < μ_{a_j}(x$_i$) ,γ_{a_j}(x$_i$)>-V$_{a_j}$(x).

Three cases may arise:

(i) If μ(d)$\geq$ 0.5, the attribute lie in the outside region.

(ii) If 0< μ(d)< 0.5 , the attribute lie in the boundary region, tending towards the lower approximation.

(iii) If μ(d) = 0, the attribute is in the lower approximation.

Example 3.33.10: Let the information table be:

	a_1	a_2	a_3	d
x_1	<0.6,0.1>	<0.6,0.2>	<0.7,0.2>	1
x_2	<0.7,0.2>	<0.6,0.3>	<0.65,0.1>	1
x_3	<0.75,0.2>	<0.7,0.2>	<0.75,0.3>	1
x_4	<0.75,0.2>	<0.7,0.2>	<0.75,0.3>	0

Here open set ={x_1 , x_2} and closed set ={ x_1 , x_2 ,x_3 ,x_4}. Let p_{10} be any object with attribute values: a_1 <0.9,0.0>, a_2 <0.8,0.0>, a_3 <0.9,0.05>.

$h_{a1}(p_{10})$= supγ{infμ {$\Lambda_{a1}(p_{10})$,$Cl_{a1}(p_{10})$}}-infγ{sup μ{$Int_{a1}(p_{10})$,$V_{a1}(p_{10})$}}

= Î-infγ{supμ {<0.7,0.2>,<0.75,0.2>}}[Since $\Lambda_{a1}(p_{10})$=$Cl_{a1}(p_{10})$=Î]

=Î-<0.75,0.2>[Pattern Î-V]

d=<0.9,0.0>-<0.75,0.2>=<0.15,0.1>

μ(d)=0.15< 0.5,the attribute lie in boundary region, tending towards the lower approximation. Similarly,the other attributes lie in the lower approximation.

The figure is given below:

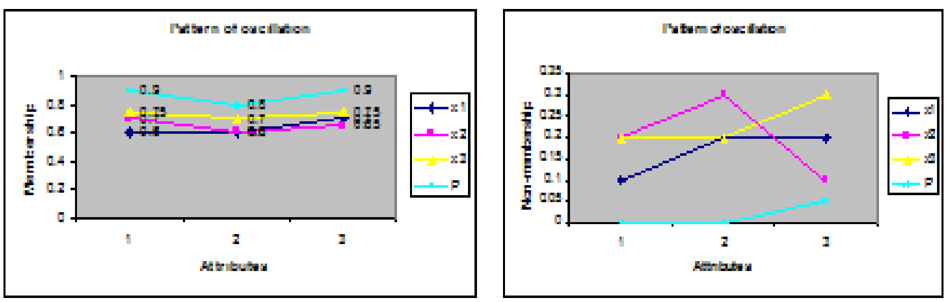


Figure 9: The line chart of the pattern of IF-rough oscillation of Case-II,Subcase-B.

Sub case (C): h_{a_j}(x) = Λ_{a_j}(x)-ϕ,same as caseI.

Sub case (D): 0 <h_{a_j}(x) <1 In this case let s = μ(h_{a_j}(x))+ γ(h_{a_j}(x)).

If s $\leq$ 0.5 ,the attribute is in boundary region tending toward lower approx., since the height of oscillation is very small.

If s > 0.5, then three cases may arise:

(i)If μ(h_{a_j}(x))$\geq$ 0.5, the attribute is in the boundary region, tending towards the outside region.

(ii) If μ(h_{a_j}(x)) < 0.5, the attribute is tending to go lower approximation.

(iii) If μ(h_{a_j}(x)) =0, the attribute is in the lower approximation.

Example 3.33.11: Let the information table be:

	a_1	a_2	a_3	d
x_1	<0.5,0.4>	<0.6,0.4>	<0.8,0.2>	1
x_2	<0.7,0.3>	<0.8,0.2>	<1.0,0.0>	1
x_3	<0.65,0.35>	<0.75,0.25>	<0.95,0.05>	1
x_4	<0.65,0.35>	<0.75,0.25>	<0.95,0.05>	0

Here open set $=\{x_1, x_2\}$ and closed set $=\{x_1, x_2, x_3, x_4\}$. Let p_{11} be any object with attribute values: a_1 <0.67,0.3>, a_2 <0.77,0.2>, a_3 <0.97,0.0>.

$h_{a1}(p_{11})= \sup\gamma\{\inf\mu\ \{\Lambda_{a1}(p_{11}),Cl_{a1}(p_{11})\}\}-\inf\gamma\{\sup\ \mu\{Int_{a1}(p_{11}),V_{a1}(p_{11})\}\}$

$= \sup\gamma\{\inf\mu\ \{<0.7,0.3>,<0.7,0.3>\}\}-\inf\gamma\{\sup\mu\ \{<0.5,0.4>,<0.65,0.35>\}\}$

$=<0.7,0.3>-<0.65,0.35>$[Pattern $\Lambda_{a_j}(x)$-$V_{a_j}(x)$]

$=<0.05,0.05>$

s=0.15<0.5,the attribute lie in the boundary region tending towards the lower approximation, since the height of oscillation is very small.Similarly,the other attributes lie in the lower approximation.

The figure is given below:

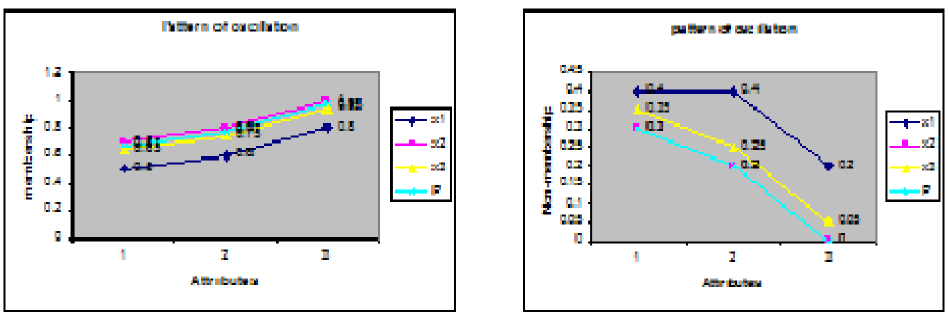


Figure 10: The line chart of the pattern of IF-rough oscillation of Case-II,Subcase-D.

Sub case (E): $h_{a_j}(x)$ = Î- ϕ, this case is same as case I.

Case III :

Let if possible $h_{a_j}(x) = Cl_{a_j}(x)$-$Int_{a_j}(x)$.

Then the following sub cases may arise:

Sub case (A): $h_{a_j}(x)$ = <0,0>.

Sub case (B): $h_{a_j}(x)$ = Î- $Int_{a_j}(x)$.

Sub case (C): $h_{a_j}(x) = Cl_{a_j}(x)$-ϕ

Sub case (D): $0_\sim < h_{a_j}(x) < 1_\sim$.

Sub case (E): $h_{a_j}(x)$= Î-ϕ

Let us study the above five sub cases:

Sub case (A): If $h_{a_j}(x)$ = <0,0>,the decision may be drawn as in theorem 3.20.

Sub case (B): $h_{a_j}(x)$= Î- $Int_{a_j}(x)$, this case is same as case I.

Sub case (C): $h_{a_j}(x) = Cl_{a_j}(x)$-ϕ

We need to check the numerical value of the difference between the membership value of the attribute a_j and Cl_{a_j}(x) of the object x.

Let the difference be d = Cl_{a_j}(x) - <$\mu_{a_j}(x_i)$, $\gamma_{a_j}(x_i)$¿.

Three cases may arise:

(i) If μ(d) $\geq$ 0.5 ,the attribute tends to go outside region.

(ii) If 0< μ(d) < 0.5 , lie in the boundary region, tending towards the lower approximation.

(iii) If μ(d) =0, the attribute is in the lower approximation.

Example 3.33.12 Let the information table be:

	a_1	a_2	d
x_1	<0.5,0.1>	<0.7,0.2>	1
x_2	<0.6,0.1>	<0.65,0.2>	1
x_3	<0.4,0.2>	<0.6,0.1>	1
x_4	<0.4,0.2>	<0.6,0.1>	0

Here open set ={x_1 , x_2} and closed set ={ x_1 , x_2 ,x_3 ,x_4}. Let p_{12} be any object with attribute values:a_1 <0.1,0.2>, a_2 <0.2,0.3>.

$h_{a1}(p_{12})$ = supγ{infμ {$\Lambda_{a1}(p_{12})$, $Cl_{a1}(p_{12})$}-infγ{sup μ{$Int_{a1}(p_{12})$,$V_{a1}(p_{12})$}}

=supγ{infμ {<0.5,0.1>,<0.4,0.2>}-ϕ[Since,$Int_{a1}(p_{12})$=$V_{a1}(p_{12})$=ϕ]

=<0.4,0.2>-ϕ[Pattern Cl_{a_j}(x)-ϕ]

d=<0.4,0.2>-<0.1,0.2>=<0.3,0.0>.

μ(d)=0.3<0.5,i.e. the attribute lie in the boundary region, tending towards the lower approximation.

Similarly the attributes $_{a2}$, $_{a3}$.

Sub case (D): $0_\sim$ <h_{a_j}(x) <$1_\sim$.

In this case let s = μ(h_{a_j}(x))+ γ(h_{a_j}(x)).

If s $\leq$ 0.5 ,the attribute is in boundary region tending toward lower approx., since the height of oscillation is very small.

If s > 0.5, then three cases may arise:

(i) If μ(h_{a_j}(x))$\geq$ 0.5, the attribute is in the boundary region, tending towards the lower approximation.

(ii) If μ(h_{a_j}(x))< 0.5, the attribute is tending to go lower approximation.

(iii) If μ(h_{a_j}(x)) =0, the attribute is in the lower approximation.

Example 3.33.13: Let the information table be:

	a_1	a_2	a_3	d
x_1	<0.8,0.1>	<0.9,0.1>	<0.7,0.2>	1
x_2	<0.55,0.2>	<0.65,0.2>	<0.45,0.1>	1
x_3	<0.7,0.2>	<0.8,0.1>	<0.6,0.3>	1
x_4	<0.7,0.2>	<0.8,0.1>	<0.6,0.3>	0

Here open set ={x_1 , x_2} and closed set ={ x_1 , x_2 ,x_3 ,x_4}. Let p_{13} be any object with attribute values: a_1 <0.6,0.2>, a_2 <0.7,0.2>, a_3 <0.5,0.3>.

$h_{a1}(p_{13})$= supγ{infμ {$\Lambda_{a1}(p_{13})$,$Cl_{a1}(p_{13})$}}-infγ{sup μ{$Int_{a1}(p_{13})$,$V_{a1}(p_{13})$}}

= supγ{infμ {<0.8,0.2>,<0.7,0.2>}}-infγ{supμ {<0.55,0.2>,<0.55,0.2>}}=<0.7,0.2>-<0.55,0.2>=<0.15,0.0>

s=0.15<0.5,the attribute lie in the boundary region tending towards the lower approximation, since the height of oscillation is very small.Similarly,the other attributes lie in the lower approximation.

The figure is given below:

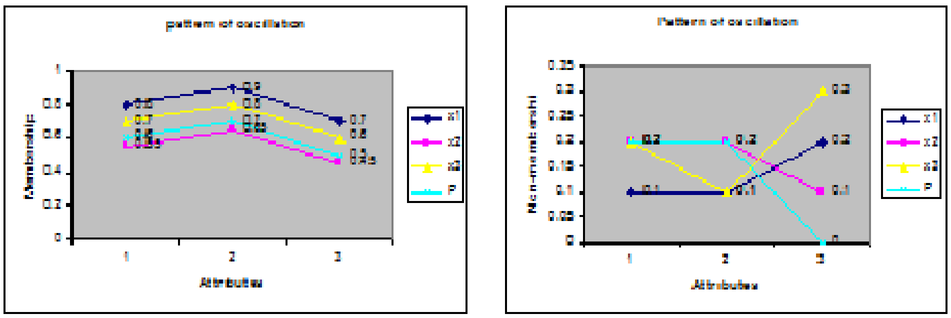


Figure 11: The line chart of the pattern of IF-rough oscillation of Case-III,Subcase-D.

Sub case (E): h_{a_j}(x) = Î-ϕ , this case is same as caseI.

Case IV : Let if possible h_{a_j}(x) = Cl_{a_j}(x)- V_{a_j}(x) .

Then the following sub cases may arise:

Sub case (A): h_{a_j}(x) = <0,0>

Sub case (B): h_{a_j}(x) = Î- V_{a_j}(x)

Sub case (C):h_{a_j}(x) = Cl_{a_j}(x)-ϕ

Sub case (D): $0_\sim$ <h_{a_j}(x)<$1_\sim$.

Sub case (E): h_{a_j}(x) = Î-ϕ ,this case is same as caseI.

Let us study the above five sub cases:

Sub case (A): If h_{a_j}(x) = <0,0>,the decision may be drawn as in theorem 3.20.

Sub case (B): h_{a_j}(x) = Î- V_{a_j}(x),this case is same as case II,subcase(B).

Sub case (C): h_{a_j}(x) = Cl_{a_j}(x)-ϕ,this case is same as CaseIII,subcase(C).

Sub case (D): $0_\sim$ <h_{a_j}(x)<$1_\sim$.

In this case let s = μ(h_{a_j}(x))+ γ(h_{a_j}(x)).

If s $\leq$ 0.5 ,the attribute is in boundary region tending toward lower approximation, since the height of oscillation is very small.

If s > 0.5, then three cases may arise:

(i) If μ(h_{a_j}(x))> 0.5, the attribute is in the boundary region, tending towards the lower approximation.

(ii) If μ(h_{a_j}(x)) < 0.5, the attribute is tending to go towards the lower approximation.

(iii) Ifμ(h_{a_j}(x)) =0, the attribute is in the lower approximation.

Example 3.33.14: Let the information table be:

	a_1	a_2	a_3	d
x_1	<0.7,0.1>	<0.8,0.2>	<0.6,0.3>	1
x_2	<0.8,0.2>	<0.9,0.1>	<0.7,0.3>	1
x_3	<0.75,0.2>	<0.85,0.1>	<0.65,0.3>	1
x_4	<0.75,0.2>	<0.85,0.1>	<0.65,0.3>	0

Here open set $=\{x_1 , x_2\}$ and closed set $=\{ x_1 , x_2 ,x_3 ,x_4\}$. Let p_{14} be any object with attribute values: a_1 <0.75,0.2>, a_2 <0.85,0.1>, a_3<0.65,0.3>.

$h_{a1}(p_{14})= \sup\gamma\{\inf\mu \{\Lambda_{a1}(p_{14}),Cl_{a1}(p_{14})\}\}-\inf\gamma\{\sup \mu\{Int_{a1}(p_{14}),V_{a1}(p_{14})\}\}$

$= \sup\gamma\{\inf\mu \{<0.8,0.2>,<0.75,0.2>\}\}-\inf\gamma\{\sup\mu \{\phi,<0.75,0.2>\}\}=<0.75,0.2>-<0.75,0.2>=<0.0,0.0>$

Therefore the attribute lie in the lower approximation. Similarly,the other attributes lie in the lower approximation.

The figure is given below:

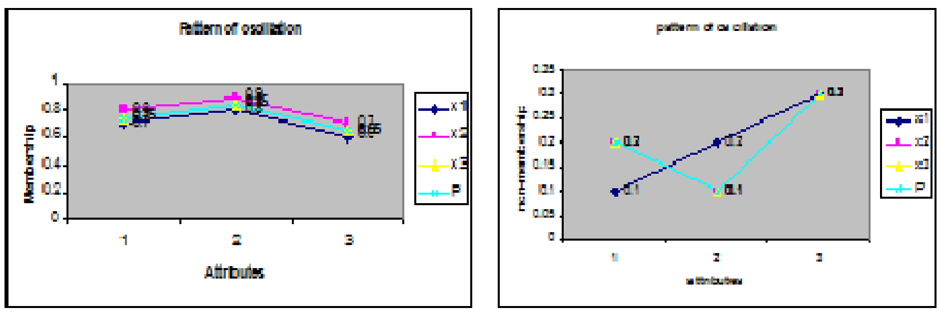


Figure 12: The line chart of the pattern of IF-rough oscillation of Case-IV,Subcase-D.

Sub case (E): $h_{a_j}(x) = \hat{I}- \phi$, this is an an unstable case.

The above cases are also distributed in three parts as stable, unstable and oscillating.

The above cases are oscillating.

4 Application of Intuitionistic fuzzy- rough oscillatory region:

While collecting data sometimes we face human testimonies, opinions etc. involving answers of the type: Yes, No and don't know. In such cases IF membership value is required. Such as some attributes e.g. humidity, wind etc. cannot be explained only by the membership value. The attributes can take membership value and the non-membership value between [0,1]. Since IF-set is a generalizing form of fuzzy set, we developed our concept to IF-rough oscillatory region. And the application of IF-rough oscillatory region is given below. The main focus of this section is to find the height of oscillation. With the help of height of oscillation we can draw decision about an unknown object.

We take the initial table from paper [12] about the rice chilo suppressalis in Lujiang area in china from 1980 to 1998 as follows:

Table 3.1:
Initial information table be:

Year	Month	Day	Temp	Humidity	Rain	Wind	sun	Pressure	Pest density
1980	6	1	26	86	28	2.5	5	752	197
	6	2	25	84	2	1.5	2	756	24
	6	3	28	86	0	3.0	5	760	32
	6	4	27	88	16	2.5	0	757	86
	6	5	28	84	1	1.0	0	748	357
	...	...	...	...	...	...	...	...	...

We can find that, in the initial information table 3.1 there are seven attributes besides the decision attribute 'Pest density'. In general, the agricultural technicians record these attributes by their experience, so there are some attributes that are dispensable to 'Pest Density'. After removing the redundant attributes by rough set theory, we get the case base as table 3.2.

Table 3.2:
Case base generated form initial information table by rough sets:

Year	Month	Day	Temp	Humidity	Wind	Pest density
1980	7	1	29	86	2.5	197
	7	2	25	84	1.5	24
	7	3	24	86	3.0	32
	7	4	23	88	1.5	86
	7	5	23	84	1.0	357
	...	...	...	...	...	...

In our application, all attributes in the pest case base are numerical, and the new fuzzy case base can be described (by equation 1)as table 3.3.

To convert the crisp variable or linguistic variable to fuzzy membership value many researchers had introduced many concept. Out of these concepts introduced by Wang Zhenyu, Hang Xiaoshu and Xiong Fanlum[12] is as follows:

Here we describe, in brief, the fuzzy set theory which is used for transforming the case based reasoning in fuzzy description. By the fuzzy representation, different from the vague or incorrect representation, the cases will be feasible and hence it is possible to match and to retrieve the solved cases. As the data in the pest case based reasoning are mainly numerical, they use the corresponding fuzzy strategies.

A n-dimensional pattern case based reasoning $A_i=[A_{i1},A_{i2},...,A_{in}]$ is represented as a 3n-dimensional vector

$A_i=[\mu_{low(Ai1)}(A_i),...,\mu_{high(Ain)}(A_i)]$ $=[{y_1}^{(0)},{y_2}^{(0)},...,{y_{3n}}^{(0)}]$. Where the μ values indicate the membership functions of the corresponding linguistic p-sets low, medium, high along each feature axis. According to that, the attribute of the pest case base set can be represented as

A=[Temperature, Humidity, Rain, Wind, Pressure, Pest density] .After the transforming using p-sets ,they got such attributes set as expression (1).Here they use the p-fuzzy sets in one -dimensional form,with range[0,1],represented as

$\pi(A_j;c;\lambda)=2(1-|A_j-c|/\lambda)^2$,for $\lambda/2\leq|A_j-c|\leq\lambda$.
$=1-2(\ |A_j-c|/\lambda)^2$,for $0\leq|A_j-c|\leq\lambda/2$
=0,otherwise...............(1)

Where $\lambda(\geq 0)$ is the radius of the π-function with c as the central point.

According to the domain knowledge, we set the parameters as follows:

C_{TL}=20, C_{TH}=26, T=10; C_{HL}=75,C_{HH}=90, H=15;C_{WL}=2,C_{WH}=6, W=4; C_{PL}=80,C_{PH}=350, P=80.

Table 3.3:

Year	Month	Day	Temp (Low,High)	Humidity (Low,High)	Wind (Low,High)	Pest (Low,High)
1980	7	1	<0.02,0.98>	<0.14,0.85>	<0.97,0.030>	<0,0>
	7	2	<0.50,0.50>	<0.32,0.68>	<0.97,0.0>	<0.18,0>
	7	3	<0.68,0.32>	<0.14,0.85>	<0.875,0.125>	<0.32,0>
	7	4	<0.82,0.18>	<0.035,0.965>	<0.97,0.0>	<0.99,0.0>
	7	5	<0.82,0.18>	<0.32,0.68>	<0.875,0>	<0.0,0.98>

Let the values of high (temp, humidity and wind) attributes values denote the membership degree and low (temp, humidity and wind) denote the non membership values of the attributes temp, humidity and wind respectively. We get the new information table as follows:

Table 3.4:

Day(d)	Temp(T)	Humidity(H)	Wind(W)	Pest density
d_1	<0.98,0.02>	<0.85,0.14>	<0.03,0.97>	<0.0,0.0>
d_2	<0.5,0.5>	<0.68,0.32>	<0.0,0.97>	<0,0.18>
d_3	<0.32,0.68>	<0.85,0.14>	<0.13,0.86>	<0.0,0.32>
d_4	<0.18,0.82>	<0.96,0.04>	<0.0, 0.97>	<0.0,0.99>
d_5	<0.18,0.82>	<0.68,0.32>	<0.0,0.88>	<0.98,0.0>

Here we see that it is not possible to find lower and upper approximation taking all the attributes together. So first we find the lower and upper approximation taking single attribute separately and then draw conclusion. First consider the attribute temperature. The lower and upper approximation for temperature is given by

L.A {Pest density 0} = { d_1 ,d_2 , d_3}

U.A {Pest density 0} = { d_1 ,d_2 , d_3 ,d_4 ,d_5}

Let us draw the decision of the day having temperature: T = <0.6,0.3>.

Then h_T(d)= <0.98,0.02>-<0.5,0.5>, [Pattern Λ-Int]

= <0.48,0.3>.

Hence by caseI,subcase(D) of remark 3.33 ,the attribute temperature is tending towards the boundary region.

The lower and upper approximation for humidity is given by

L.A {Pest density 0} = { d_1 ,d_3 , d_4}

U.A {Pest density 0} = { d_1 ,d_2 , d_3 ,d_4 ,d_5}

Let we want to draw decision of the day having humidity:H= <0.7,0.2>

Then h_H(d)= infμ {Λ_H(d), Cl_H(d)}-supμ{Int_H(d),V_H(d)}

= infμ{<0.85,0.14>,<0.85,0.14>}-supμ{ϕ ,<0.68,0.32>}

= <0.85,0.14>-<0.68,0.32>, [Pattern Λ-V]

= <0.17,0.18>,s=0.35<0.5

Hence by case II,subcase(D) of remark 3.33,the attribute humidity is in the lower approximation.

Now, the lower and upper approximation for wind is given by
L.A {Pest density 0} = { d_1 ,d_3}
U.A {Pest density 0} = {d_1 ,d_2 , d_3 ,d_4}
Let we want to draw decision of the day having wind: d: wind = <0.3,0.4>.
Then h_W(d)= inf $\mu\{\Lambda_w(d)$, $Cl_W(d)\}$-sup$\mu\{Int_W(d), V_W(d)\}$
= Î-supμ<0.13,0.86>,<0.13,0.86> = Î-<0.13,0.86>,[Pattern Î -Int]
Let d =<0.3,0.4>-<0.13,0.86> = <0.17,0.46>
s = 0.17+0.46=0.63>0.5 Hence by case I,subcase(B) of the remark 3.33, the attribute wind is in the boundary region , tending towards the lower approximation.

Therefore we conclude that among the three attributes lies on boundary and one in lower approximation.

5 Conclusion:

The concept studied in this paper helps us to draw conclusion about the pattern of an unknown object from a large no of data, when the actual answer is distributed in three sections - yes, no, unknown. So IF membership value is required here. This method helps us to draw conclusion about unknown objects with missing data which is very common in practical field. An example with Pest density is cited in this paper which indicates how this method is helpful in practical life.

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