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On the topological structure of intuitionistic fuzzy soft sets

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Abstract. The aim of this paper is to construct topologies on intuitionistic fuzzy soft sets. The concept of intuitionistic fuzzy soft topologies is introduced and their basic properties are given.

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1. Introduction

To solve complicated problems in economics, engineering, environmental science and social science, methods in classical mathematics are not always successful because of various types of uncertainties present in these problems. There are several theories: probability theory, fuzzy set theory [12], rough set theory [9] and the interval mathematics which we can consider as mathematical tools for dealing with uncertainties. But all these theories have their own difficulties. For example, probability theory can deal only with stochastically stable phenomena (see [8]). To overcome these kinds of difficulties, Molodtsov [8] proposed a completely new approach, which is called soft set theory, for modeling uncertainty.

In soft set theory, we observe that in most cases the parameters are vague words or sentences involve vague words. Considering this point, Maji et al. [7] proposed the concept of intuitionistic fuzzy soft sets by combining soft sets with intuitionistic fuzzy sets. Gunduz et al. [6] introduced the concept of intuitionistic fuzzy soft module and simplified the sigh of intuitionistic fuzzy soft sets.

Chang [3] proposed fuzzy topological spaces. D.Coker [4] defined intuitionistic fuzzy topologies. Shabir et al. [10] introduce soft topological spaces. Tanay et al. [11] discussed fuzzy soft topological spaces.

In this paper, we define intuitionistic fuzzy soft topologies and give their basic properties.

2. Preliminaries

Throughout this paper, U denotes initial universe, E denotes the set of all possible parameters and I denotes $[0,1]$. We only consider the case where U and E are both nonempty finite sets. "Intuitionistic fuzzy" is briefly "IF".

2.1. On lattices.

Definition 2.1 ([5]). Let $(L,\leq)$ be a poset.

(1) L is called a lattice, if $a\vee b\in L,\ a\wedge b\in L$ for any $a,b\in L$.

(2) L is called a complete lattice, if $\vee\ S\in L,\ \wedge\ S\in L$ for any $S\subseteq L$.

(3) L is called distributive, if $a\vee(b\wedge c)=(a\vee b)\wedge(a\vee c)$, $a\wedge(b\vee c)=(a\wedge b)\vee(a\wedge c)$ for any $a,b,c\in L$.

(4) L is called a complete distributive lattice (resp. a distributive lattice), if L is a complete lattice (resp. a lattice) and distributive.

Obviously, every complete lattice (resp. a complete distributive lattice) is a lattice (resp. a distributive lattice).

Definition 2.2 ([5]). Let L be a lattice with top element 1_L and bottom element 0_L and let $a,b\in L$. b is called a complement element of a, if $a\vee b=1_L, a\wedge b=0_L$.

If $a\in L$ has complement element, then it is unique. We denote the complement element of a by a'.

Let $a,b\in R$. We define

$$a\vee b=\max\{a,b\}\ \ and\ \ a\wedge b=\min\{a,b\}.$$

Let A be any bounded subset of R. We define

$$\bigvee A=sup\ A\ \ and\ \ \bigwedge A=inf\ A.$$

We denote $J=\{(a,b)\in I\times I: a+b\leq 1\}$.

Definition 2.3. Let $(a,b),(c,d)\in I\times I$. We define

(1) $(a,b)=(c,d)\iff a=c,\ b=d$.

(2) $(a,b)\sqcup(c,d)=(a\vee c,b\wedge d)$, $(a,b)\sqcap(c,d)=(a\wedge c,b\vee d)$.

(3) $(a,b)'=(b,a)$.

Moreover, for $\{(a_\alpha,b_\alpha):\alpha\in\Gamma\}\subseteq I\times I$,

$\bigsqcup\limits_{\alpha\in\Gamma}(a_\alpha,b_\alpha)=(\bigvee\limits_{\alpha\in\Gamma}a_\alpha,\bigwedge\limits_{\alpha\in\Gamma}b_\alpha)$ and $\prod\limits_{\alpha\in\Gamma}(a_\alpha,b_\alpha)=(\bigwedge\limits_{\alpha\in\Gamma}a_\alpha,\bigvee\limits_{\alpha\in\Gamma}b_\alpha))$.

Definition 2.4. Let $(a,b),(c,d)\in J$ and let $S\ \subseteq J\times J$. $(a,b)S(c,d)$, if a $\leq c$ and $b\geq d$. We denote S by $\leq$.

Obviously, $(a,b)=(c,d)\iff (a,b)\leq(c,d)$ and $(c,d)\leq(a,b)$.

Remark 2.5. (1) $(J,\leq)$ is a poset with $0_J=(0,1)$ and $1_J=(1,0)$.

(2) $(a,b)''=(a,b)$.

(3) $((a,b)\sqcup(c,d))\sqcup(e,f)=(a,b)\sqcup((c,d)\sqcup(e,f))$,
$((a,b)\sqcap(c,d))\sqcap(e,f)=(a,b)\sqcap((c,d)\sqcap(e,f))$.

(4) $(a,b)\sqcup(c,d)=(c,d)\sqcup(a,b)$, $(a,b)\sqcap(c,d)=(c,d)\sqcap(a,b)$.
(5) $((a,b)\sqcup(c,d))\sqcap(e,f)=((a,b)\sqcap(e,f))\sqcup((c,d)\sqcap(e,f))$.
$((a,b)\sqcap(c,d))\sqcup(e,f)=((a,b)\sqcup(e,f))\sqcap((c,d)\sqcup(e,f))$.
(6) $(\bigsqcup\limits_{\alpha\in\Gamma}(a_\alpha,b_\alpha))'=\bigsqcap\limits_{\alpha\in\Gamma}(a_\alpha,b_\alpha)'$, $(\bigsqcap\limits_{\alpha\in\Gamma}(a_\alpha,b_\alpha))'=\bigsqcup\limits_{\alpha\in\Gamma}(a_\alpha,b_\alpha)'$.

Proposition 2.6. $(J,\leq,\sqcap,\sqcup)$ *is a complete distributive lattice.*

Proof. By Remark 2.5, $(J,\leq)$ is a poset.

For any $\{(a_\alpha,b_\alpha):\alpha\in\Gamma\}\subseteq J$, $\bigsqcup\limits_{\alpha\in\Gamma}(a_\alpha,b_\alpha)=(\bigvee\limits_{\alpha\in\Gamma}a_\alpha,\bigwedge\limits_{\alpha\in\Gamma}b_\alpha)$. Put

$$\bigvee_{\alpha\in\Gamma}a_\alpha=a,\ \bigwedge_{\alpha\in\Gamma}b_\alpha=b.$$

For any $\varepsilon>0$, there exists $\alpha_\varepsilon\in\Gamma$ such that $a_{\alpha_\varepsilon}>a-\varepsilon$. Then $a+b-\varepsilon<a_{\alpha_\varepsilon}+b\leq a_{\alpha_\varepsilon}+b_{\alpha_\varepsilon}\leq 1$. Thus $a+b\leq 1$. This implies $\bigsqcup\limits_{\alpha\in\Gamma}(a_\alpha,b_\alpha)\in J$.

Similarly, we can prove $\bigsqcap\limits_{\alpha\in\Gamma}(a_\alpha,b_\alpha)\in J$. Hence J is a complete lattice. By Remark 2.5, J is a distributive. Therefore, J is a complete distributive lattice. □

2.2. **IF sets and IF topologies.** In this paper, $\mathcal{F}(U)$ denotes the family of all fuzzy sets in U.

Definition 2.7 ([1, 2])**.** An IF set A in U is an object having the from

$$A=\{<x,\mu_A(x),\nu_A(x)>:x\in U\},$$

where $\mu_A,\nu_A\in\mathcal{F}(U)$ satisfying $0\leq\mu_A(x)+\nu_A(x)\leq 1$ for each $x\in U$, and $\mu_A(x),\nu_A(x)$ are used to define the degree of membership and the degree of non-membership of the element x to A, respectively.

For the sake of simplicity, we redefine Definition 2.7 and give Definition 2.8.

Definition 2.8. A is called an IF set in U, if $A=(\mu_A,\nu_A)$ and for each $x\in U$, $A(x)=(\mu_A(x),\nu_A(x))\in J$, where $\mu_A,\nu_A\in\mathcal{F}(U)$ are used to define the degree of membership and the degree of non-membership of the element x to A, respectively.

In this paper, $\mathcal{IF}(U)$ denotes the family of all IF sets in U.

Let $A,B\in\mathcal{IF}(U)$ and let $(J,\leq,\sqcap,\sqcup)$ be a complete distributive lattice. Then some IF relations and IF operations are defined as follows:
(1) $A=B\Longleftrightarrow A(x)=B(x)$ for each $x\in U$.
(2) $A\subseteq B\Longleftrightarrow A(x)\leq B(x)$ for each $x\in U$.
(3) $(A\cup B)(x)=A(x)\sqcup B(x)$ for each $x\in U$.
(4) $(A\cap B)(x)=A(x)\sqcap B(x)$ for each $\in U$.
(5) $A'(x)=A(x)'$ for each $x\in U$.
Moreover,

$$(\bigcup_{\alpha\in\Gamma}A_\alpha)(x)=\bigsqcup_{\alpha\in\Gamma}A_\alpha(x)\ for\ each\ x\in U$$

and

$$(\bigcap_{\alpha\in\Gamma}A_\alpha)(x)=\bigsqcap_{\alpha\in\Gamma}A_\alpha(x)\ for\ each\ x\in U,$$

where $\{A_\alpha:\alpha\in\Gamma\}\subseteq\mathcal{IF}(U)$.

Obviously, $A=B\iff A\subseteq B$ and $B\subseteq A$.

Example 2.9. Let $U = \{x_1, x_2, x_3, x_4, x_5\}$. We defined

$A(x_1) = (0.2, 0.8), A(x_2) = (0.6, 0.3), A(x_3) = (0, 1), A(x_4) = (1, 0), A(x_5) = (0.4, 0.4).$

Then $A \in \mathcal{IF}(U)$. We denote it by

$$A = \frac{(0.2, 0.8)}{x_1} + \frac{(0.6, 0.3)}{x_2} + \frac{(0, 1)}{x_3} + \frac{(1, 0)}{x_4} + \frac{(0.4, 0.4)}{x_5}.$$

We have

$$A' = \frac{(0.8, 0.2)}{x_1} + \frac{(0.3, 0.6)}{x_2} + \frac{(1, 0)}{x_3} + \frac{(0, 1)}{x_4} + \frac{(0.4, 0.4)}{x_5}.$$

Let

$$B = \frac{(0.2, 0.5)}{x_1} + \frac{(0.2, 0.8)}{x_2} + \frac{(0.8, 0.1)}{x_3} + \frac{(0.3, 0.6)}{x_4} + \frac{(0, 1)}{x_5}.$$

Then

$$A \cap B = \frac{(0.2, 0.8)}{x_1} + \frac{(0.2, 0.8)}{x_2} + \frac{(0, 1)}{x_3} + \frac{(0.3, 0.6)}{x_4} + \frac{(0, 1)}{x_5},$$

$$A \cup B = \frac{(0.2, 0.5)}{x_1} + \frac{(0.6, 0.3)}{x_2} + \frac{(0.8, 0.1)}{x_3} + \frac{(1, 0)}{x_4} + \frac{(0.4, 0.4)}{x_5}.$$

Definition 2.10 ([13])**.** Let $A \in \mathcal{IF}(U)$.

(1) A is called an IF empty set, if $A(x) = (0, 1)$ for each $x \in U$. We denote it by $\widetilde{0}$.

(2) A is called an IF universe set, if $A(x) = (1, 0)$ for each $x \in U$. We denote it by $\widetilde{1}$.

Definition 2.11 ([4])**.** Let $\tau \subseteq \mathcal{IF}(U)$. Then τ is called an IF topology on U, if

(i) $\widetilde{0}, \widetilde{1} \in \tau$,

(ii) $A, B \in \tau$ implies $A \cap B \in \tau$,

(iii) $\{A_\alpha : \alpha \in \Gamma\} \subseteq \tau$ implies $\bigcup\{A_\alpha : \alpha \in \Gamma\} \in \tau$.

The pair (U, τ) is called an IF topological space. Every member of τ is called an IF open set in U. A is called an IF closed set in U if $A' \in \tau$.

Example 2.12. Let A and B are the IF in Example 2.9. Then

$$\tau = \{\widetilde{0}, \widetilde{1}, A, B, A \cap B, A \cup B\}$$

is an IF topology on U.

2.3. **IF soft sets.**

Definition 2.13 ([8])**.** Let $A \subseteq E$. A pair (f, A) is called a fuzzy soft set over U, if f is a mapping given by $f : A \to \mathcal{F}(U)$. We denote (f, A) by f_A.

In other words, a fuzzy soft set f_A over U is a parameterized family of fuzzy sets in the universe U.

Definition 2.14 ([6, 7])**.** Let $A \subseteq E$. A pair (f, A) is called an IF soft set over U, where f is a mapping given by $f : A \to \mathcal{IF}(U)$. We denote (f, A) (resp. $\mu_{f(e)}$, $\nu_{f(e)}$) by f_A (resp. f_e, f^e).

In other words, an IF soft set f_A over U is a parameterized family of IF sets in the universe U, and $\mu_{f(e)} = f_e \in \mathcal{F}(U)$, $\nu_{f(e)} = f^e \in \mathcal{F}(U)$, $f(e) = (f_e, f^e) \in I\mathcal{F}(U)$ and $f(e)(x) = (f_e(x), f^e(x)) \in J$ for any $e \in A$ and $x \in U$.

Let $A \subseteq E$. Denote

$$\mathcal{IFS}(U)_A = \{f_A : f_A \text{ is an IF soft set over } U\},$$

$$\mathcal{IFS}(U) = \{f_A : f_A \text{ is an IF soft set over } U \text{ and } A \subseteq E\}.$$

Obviously,

$$\mathcal{IFS}(U) = \bigcup_{A \subseteq E} \mathcal{IFS}(U)_A.$$

Example 2.15. Let $U = \{x_1, x_2, x_3, x_4, x_5\}$ be a universe consisting of five houses as possible alternatives, and let $A = \{e_1, e_2, e_3, e_4\} \subseteq E$ be a set of parameters considered by the decision makers, where e_1, e_2, e_3 and e_4 represent the parameters "beautiful", "modern", "cheap" and "in the green surroundings", respectively.

Now, we consider a soft set f_A, which describes the "attractiveness of the houses" that Mr.X is going to buy. In this case, to define the soft set f_A means to point out beautiful houses, modern houses and so on. Consider the mapping f given by "houses(.)", where (.) is to be filled in by one of the parameters $e_i \in A$. For instance, $f(e_1)$ means "houses(beautiful)", and its functional value is the set consisting of all the beautiful houses in U. Let f_A be an IF soft set over U, defined as follows

$$\begin{aligned}
f(e_1) &= \frac{(0.2,0.8)}{x_1} + \frac{(0.6,0.3)}{x_2} + \frac{(0.2,0.6)}{x_3} + \frac{(0.4,0.5)}{x_4} + \frac{(0.4,0.4)}{x_5}.\\
f(e_2) &= \frac{(0.2,0.5)}{x_1} + \frac{(0.9,0.1)}{x_2} + \frac{(0.1,0.8)}{x_3} + \frac{(0.3,0.6)}{x_4} + \frac{(0.5,0.3)}{x_5}.\\
f(e_3) &= \frac{(0,1)}{x_1} + \frac{(0,1)}{x_2} + \frac{(0.5,0.4)}{x_3} + \frac{(0,1)}{x_4} + \frac{(0,1)}{x_5}.\\
f(e_4) &= \frac{(0,1)}{x_1} + \frac{(0.5,0.5)}{x_2} + \frac{(0,1)}{x_3} + \frac{(0.5,0.4)}{x_4} + \frac{(0,1)}{x_5}.
\end{aligned}$$

Then the IF soft set f_A is described by the following Table 1.

TABLE 1. Tabular representation of the fuzzy soft set f_A

	x_1	x_2	x_3	x_4	x_5
e_1	(0.2,0.8)	(0.6,0.3)	(0.2,0.6)	(0.4,0.5)	(0.4,0.4)
e_2	(0.2,0.5)	(0.9,0.1)	(0.1,0.8)	(0.3,0.6)	(0.5,0.3)
e_3	(0,1)	(0,1)	(0.5,0.4)	(0,1)	(0,1)
e_4	(0,1)	(0.5,0.5)	(0,1)	(0.5,0.4)	(0,1)

Definition 2.16 ([7]). Let $A, B \in \mathcal{IFS}(U)$.

(1) f_A is called IF soft subset of g_B, if $A \subseteq B$ and $f(e) \subseteq g(e)$ for any $e \in A$. We write $f_A \mathrel{\widetilde{\subset}} g_B$.

(2) f_A and g_B are called IF soft equal, if $f_A \mathrel{\widetilde{\subset}} g_B$ and $g_A \mathrel{\widetilde{\subset}} f_B$. We write $f_A = g_B$.

Obviously, $f_A = g_B$ if and only if $A = B$ and $f(e) = g(e)$ for any $e \in A$.

Definition 2.17 ([7]). Let $A, B \in \mathcal{IFS}(U)$.

(1) The intersection of f_A and g_B is the IF soft set h_C, where $C = A \cap B$, and $h(e) = f(e) \cap g(e)$ for any $e \in C$. We write $f_A \widetilde{\cap} g_B = h_C$.

(2) The union f_A and g_B is the IF soft set h_C, where $C = A \cup B$, and for any $e \in C$,

$$h(e) = \begin{cases} f(e), & e \in A - B \\ g(e), & e \in B - A \\ f(e) \cup g(e) & e \in A \cap B \end{cases}$$

We write $f_A \widetilde{\cup} g_B = h_C$.

3. IF soft topologies

In this section, we introduce the concept of IF soft topologies.

Definition 3.1. The relative complement of an IF soft set f_E is denoted by f'_E and is defined by

$$(f_E)' = f'_E,$$

where

$$f' : E \to \mathcal{IF}(U)$$

is a mapping given by $f'(\varepsilon) = (f(\varepsilon))'$ for each $\varepsilon \in E$.

Proposition 3.2. *Let $f_E, g_E \in \mathcal{IFS}(U)_E$. Then*

$$(f_E \widetilde{\cap} g_E)' = f'_E \widetilde{\cup} g'_E,\ (f_E \widetilde{\cup} g_E)' = f'_E \widetilde{\cap} g'_E.$$

Proof. Let $f_E \widetilde{\cap} g_E = h_E$, where

$$h'(e) = (h(e))' = (f(e) \cap g(e))' = (f(e))' \cup (g(e))' = f'(e) \cup g'(e)$$

for any $e \in E$. Then $(f_E \widetilde{\cap} g_E)' = h'_E$ and $f'_E \widetilde{\cap} g'_E = h'_E$. Thus

$$(f_E \widetilde{\cap} g_E)' = f'_E \widetilde{\cup} g'_E.$$

Similarly, we can prove $(f_E \widetilde{\cup} g_E)' = f'_E \widetilde{\cap} g'_E$. □

Definition 3.3. Let $f_E \in \mathcal{IFS}(U)_E$.

(1) f_E is called absolute IF soft over U, if f(e)=$\widetilde{1}$ for any $e \in E$. We denoted it by U_E.

(2) f_E is called relative null IF soft over U, if f(e)=$\widetilde{0}$ for any $e \in E$. We denoted it by $\varnothing_E$.

Obviously, $\varnothing_E$=U'_E and U_E=$\varnothing'_E$.

Definition 3.4. Let $\tau \subseteq \mathcal{IFS}(U)_E$ and $\tau' = \{f_E : f'_E \in \tau\}$. Then τ is called an IF soft topology on U if the following conditions are satisfied:

(i) $U_E, \varnothing_E \in \tau$,

(ii) $f_E, g_E \in \tau$ implies $f_E \widetilde{\cap} g_E \in \tau$,

(iii) $\{(f_\alpha)_E : \alpha \in \Gamma\} \subseteq \tau$ implies $\widetilde{\bigcup} \{(f_\alpha)_E : \alpha \in \Gamma\} \in \tau$.

The pair (U, τ, E) is called an IF soft topological space over U. Every member of τ is called an IF soft open set in U. f_E is called an IF soft closed set in U if $f_E \in \tau'$.

Example 3.5. Let $U = \{x_1, x_2\}$ and $E = \{e_1, e_2\}$. Let $f_E, g_E, h_E, l_E \in \mathcal{IFS}(U)_E$ where

$$f(e_1) = \frac{(0.2, 0.8)}{x_1} + \frac{(0.6, 0.3)}{x_2},\ f(e_2) = \frac{(0.2, 0.5)}{x_1} + \frac{(0.9, 0.1)}{x_2};$$
$$g(e_1) = \frac{(0.1, 0.8)}{x_1} + \frac{(0.6, 0.1)}{x_2},\ g(e_2) = \frac{(0.2, 0.8)}{x_1} + \frac{(0.8, 0.1)}{x_2};$$
$$h(e_1) = \frac{(0.2, 0.8)}{x_1} + \frac{(0.6, 0.1)}{x_2},\ h(e_2) = \frac{(0.2, 0.5)}{x_1} + \frac{(0.9, 0.1)}{x_2};$$
$$l(e_1) = \frac{(0.1, 0.8)}{x_1} + \frac{(0.6, 0.3)}{x_2},\ l(e_2) = \frac{(0.2, 0.8)}{x_1} + \frac{(0.8, 0.1)}{x_2}.$$

We have

$$f'(e_1) = \frac{(0.8, 0.2)}{x_1} + \frac{(0.3, 0.6)}{x_2},\ f'(e_2) = \frac{(0.5, 0.2)}{x_1} + \frac{(0.1, 0.9)}{x_2}.$$

Then $h_E = f_E \,\widetilde{\cup}\, g_E$ and $l_E = f_E \,\widetilde{\cap}\, g_E$. Thus $\tau = \{f_E, g_E, h_E, l_E, \varnothing_E, U_E\}$ is an IF soft topology on U.

Proposition 3.6. *Let (U, τ_1, E) and (U, τ_2, E) be two IF soft topologies over U. Denote $\tau_1 \cap \tau_2 = \{f_E : f_E \in \tau_1$ and $f_E \in \tau_2\}$. Then $\tau_1 \cap \tau_2$ is an IF soft topology on U.*

Proof. Obviously $\varnothing_E, U_E \in \tau_1 \cap \tau_2$. Let $f_E, g_E \in \tau_1 \cap \tau_2$. Then $f_E, g_E \in \tau_1$ and $f_E, g_E \in \tau_2$. Note that τ_1 and τ_2 are two IF soft topologies on U. Then $f_E \cap g_E \in \tau_1$ and $f_E \cap g_E \in \tau_2$. Hence $f_E \cap g_E \in \tau_1 \cap \tau_2$. Let $\{(f_\alpha)_E : \alpha \in \Gamma\} \subseteq \tau_1 \cap \tau_2$. Then $(f_\alpha)_E \in \tau_1$ and $(f_\alpha)_E \in \tau_2$ for any $\alpha \in \Gamma$. Since τ_1 and τ_2 are two IF soft topologies on U, $\widetilde{\bigcup} \{(f_\alpha)_E : \alpha \in \Gamma\} \in \tau_1$ and $\widetilde{\bigcup} \{(f_\alpha)_E : \alpha \in \Gamma\} \in \tau_2$. Thus $\widetilde{\bigcup} \{(f_\alpha)_E : \alpha \in \Gamma\} \in \tau_1 \cap \tau_2$. □

Let τ_1 and τ_2 be two IF soft topologies on U. Denote
$\tau_1 \vee \tau_2 = \{f_E \,\widetilde{\cup}\, g_E : f_E \in \tau_1$ *and* $g_E \in \tau_2\}$,
$\tau_1 \wedge \tau_2 = \{f_E \,\widetilde{\cap}\, g_E : f_E \in \tau_1$ *and* $g_E \in \tau_2\}$.

Example 3.7. Let f_E and g_E be two IF soft sets in Example 3.5. Then

$$\tau_1 = \{\varnothing_E, U_E, f_E\},\ \tau_2 = \{\varnothing_E, U_E, g_E\} \text{ and } \tau_1 \cap \tau_2 = \{\varnothing_E, U_E\}$$

are three IF soft topologies on U. But

$$\tau_1 \cup \tau_2 = \{\varnothing_E, U_E, f_E, g_E\},\ \tau_1 \vee \tau_2 = \{\varnothing_E, U_E, f_E, g_E, f_E \,\widetilde{\cup}\, g_E\}$$

and

$$\tau_1 \wedge \tau_2 = \{\varnothing_E, U_E, f_E, g_E, f_E \,\widetilde{\cap}\, g_E\}$$

are not IF soft topologies on U.

Theorem 3.8. *Let (U, τ, E) be an IF soft topologies over U. for any $e \in E$,*

$$\tau(e) = \{f(e) : f_E \in \tau\}$$

is an IF topology on U.

Proof. Let $e \in E$. (1) By $\varnothing_E$, $U_E \in \tau$, $\tilde{0} = \varnothing(e)$ and $\tilde{1} = U(e)$, we have $\tilde{0}, \tilde{1} \in \tau(e)$.

(2) Let $V, W \in \tau(e)$. Then there exist $f_E, g_E \in \tau$ such that $V = f(e)$ and $W = g(e)$. By τ be an IF soft topologies on U, $f_E \widetilde{\cap} g_E \in \tau$. Put $h_E = f_E \widetilde{\cap} g_E$. Then $h_E \in \tau$. Note that $V \cap W = f(e) \cap g(e) = h(e)$ and $\tau(e) = \{f(e) : f_E \in \tau\}$. Then $V \cap W \in \tau(e)$.

(3) Let $\{V_\alpha : \alpha \in \Gamma\} \subseteq \tau(e)$. Then for every $\alpha \in \tau$, there exist $(f_\alpha)_E \in \tau$ such that $V_\alpha = f_\alpha(e)$. By τ be an IF soft topologies on U, $\widetilde{\bigcup} \{(f_\alpha)_E : \alpha \in \Gamma\} \in \tau$. Put $f_E = \widetilde{\bigcup} \{(f_\alpha)_E : \alpha \in \Gamma\}$. Then $f_E \in \tau$. Note that $\bigcup\limits_{\alpha \in \Gamma} V_\alpha = \bigcup \{f_\alpha(e) : \alpha \in \Gamma\} = f(e)$ and $\tau(e) = \{f(e) : f_E \in \tau\}$. Then $\bigcup\limits_{\alpha \in \Gamma} V_\alpha \in \tau(e)$.

Therefore $\tau(e) = \{f(e) : f_E \in \tau\}$ is an IF topology on U. □

Definition 3.9. Let (U, τ, E) be an IF soft topological space and $\mathscr{B} \subseteq \tau$. $\mathscr{B}$ is a basis for τ, if for each $g_E \in \tau$, there exists $\mathscr{B}' \subseteq \mathscr{B}$ such that $g_E = \widetilde{\bigcup} \mathscr{B}'$.

Example 3.10. Let τ be the IF soft topology in Example 3.5. Then

$$\mathscr{B} = \{\varnothing_E, f_E, g_E, l_E, U_E\}$$

is a basis for τ.

Theorem 3.11. *Let $\mathscr{B}$ is a basis for IF soft topology τ. Denote $\mathscr{B}_e = \{f(e) : f_E \in \mathscr{B}\}$ and $\tau(e) = \{f(e) : f_E \in \tau\}$ for any $e \in E$. Then $\mathscr{B}_e$ is a basis for IF topology $\tau(e)$.*

Proof. Let $e \in E$. For any $V \in \tau(e)$, $V = g(e)$ for some $g_E \in \tau$. Note that $\mathscr{B}$ is a basis for τ. Then there exists $\mathscr{B}' \subseteq \mathscr{B}$ such that $g_E = \widetilde{\bigcup} \mathscr{B}'$. So $V = \bigcup \mathscr{B}'_e$, where $\mathscr{B}'_e = \{f(e) : f_E \in \mathscr{B}'\} \subseteq \mathscr{B}_e$. Thus $\mathscr{B}_e$ is basis for IF topology $\tau(e)$ for any $e \in E$. □

4. Some properties of IF soft topologies

In this section, we give some properties of IF soft topologies.

Definition 4.1. Let (U, τ, E) be an IF soft topological space and let $f_E \in \mathcal{IFS}(U)_E$. Then interior and closure of f_E denoted respectively by $int(f_E)$ and $cl(f_E)$, are defined as follows:

$$int(f_E) = \widetilde{\cup} \{g_E \in \tau : g_E \widetilde{\subset} f_E\},$$

$$cl(f_E) = \widetilde{\cap} \{g_E \in \tau' : f_E \widetilde{\subset} g_E\}.$$

Theorem 4.2. *Let (U, τ, E) be an IF soft topological space over U. Then the following properties hold.*

(1) *U_E and $\varnothing_E$ are IF soft closed sets over U.*

(2) *The intersection of any number of IF soft closed sets is an IF soft closed set over U.*

(3) *The union of any two IF soft closed sets is an IF soft closed set over U.*

Proof. This follows from Proposition 3.2. □

Theorem 4.3. *Let (U, τ, E) be an IF soft topological space over U and let $f_E \in \mathcal{IFS}(U)_E$. Then the following properties hold.*

(1) $int(f_E) \mathrel{\widetilde{\subset}} f_E$.
(2) $f_E \mathrel{\widetilde{\subset}} g_E \Longrightarrow int(f_E) \mathrel{\widetilde{\subset}} int(g_E)$.
(3) $int(f_E) \in \tau$.
(4) f_E *is an IF soft open set* $\Longleftrightarrow int(f_E) = f_E$.
(5) $int(int(f_E)) = int(f_E)$.
(6) $int(\varnothing_E) = \varnothing_E$, $int(U_E) = U_E$.

Proof. (1) and (2) are obvious.

(3) Obviously, $\widetilde{\cup}\, \{g_E \in \tau : g_E \mathrel{\widetilde{\subset}} f_E\} \in \tau$. Note that $int(f_E) = \widetilde{\cup}\, \{g_E \in \tau : g_E \mathrel{\widetilde{\subset}} f_E\}$, Then $int(f_E) \in \tau$.

(4) Necessity. By (1), $int(f_E) \mathrel{\widetilde{\subset}} f_E$.

Since $f_E \in \tau$ and $f_E \mathrel{\widetilde{\subset}} f_E$, then $f_E \mathrel{\widetilde{\subset}} \widetilde{\bigcup}\, \{g_E \in \tau : g_E \mathrel{\widetilde{\subset}} f_E\}$. By $int(f_E) = \widetilde{\cup}\, \{g_E \in \tau : g_E \mathrel{\widetilde{\subset}} f_E\}$, $f_E \mathrel{\widetilde{\subset}} int(f_E)$. Thus $int(f_E) = f_E$.

Sufficiency. This holds by (3).

(5) and (6) hold by (3) and (4). □

Theorem 4.4. *Let (U, τ, E) be an IF soft topological space over U and let $f_E \in \mathcal{IFS}(U)_E$. Then the following properties hold.*

(1) $f_E \mathrel{\widetilde{\subset}} cl(f_E)$.
(2) $f_E \mathrel{\widetilde{\subset}} g_E \Longrightarrow cl(f_E) \mathrel{\widetilde{\subset}} cl(g_E)$.
(3) $(cl(f_E))' \in \tau$.
(4) f_E *is an IF soft closed set* $\Longleftrightarrow cl(f_E) = f_E$.
(5) $cl(cl(f_E)) = cl(f_E)$.
(6) $cl(\varnothing_E) = \varnothing_E$, $cl(U_E) = U_E$.

Proof. (1) and (2) are obvious.

(3) By Theorem 4.3 (3),

$$int(f'_E) \in \tau.$$

Since $(cl(f_E))' = (\widetilde{\bigcap}\, \{g_E \in \tau' : f_E \mathrel{\widetilde{\subset}} g_E\})' = \widetilde{\bigcup}\, \{g_E \in \tau : g_E \mathrel{\widetilde{\subset}} f'_E\} = int(f'_E)$, then $(cl(f_E))' \in \tau$.

(4) Necessity. By Theorem 4.4 (1),

$$f_E \mathrel{\widetilde{\subset}} cl(f_E).$$

Since $f_E \in \tau'$ and $f_E \mathrel{\widetilde{\subset}} f_E$, $cl(f_E) = \widetilde{\bigcap}\, \{g_E \in \tau' : f_E \mathrel{\widetilde{\subset}} g_E\} \mathrel{\widetilde{\subset}} \{f_E \in \tau' : f_E \mathrel{\widetilde{\subset}} f_E\}$. Then $cl(f_E) \mathrel{\widetilde{\subset}} f_E$. Thus $f_E = cl(f_E)$.

Sufficiency. This holds by (3).

(5) and (6) hold by (3) and (4). □

Theorem 4.5. *Let (U, τ, E) be an IF soft topological space and let $f_E, g_E \in \mathcal{IFS}(U)_E$.*

(1) $int(f_E) \mathbin{\widetilde{\cap}} int(g_E) = int(f_E \mathbin{\widetilde{\cap}} g_E)$.
(2) $int(f_E) \mathbin{\widetilde{\cup}} int(g_E) \mathrel{\widetilde{\subset}} int(f_E \mathbin{\widetilde{\cup}} g_E)$.
(3) $cl(f_E) \mathbin{\widetilde{\cup}} cl(g_E) = cl(f_E \mathbin{\widetilde{\cup}} g_E)$.
(4) $cl(f_E \mathbin{\widetilde{\cap}} g_E) \mathrel{\widetilde{\subset}} cl(f_E) \mathbin{\widetilde{\cap}} cl(g_E)$.
(5) $(int(f_E))' = cl(f'_E)$.
(6) $(cl(f_E))' = int(f'_E)$.

Proof. (1) Since $f(e) \tilde{\cap} g(e) \tilde{\subseteq} f(e)$ for any $e \in E$, we have $f_E \tilde{\cap} g_E \tilde{\subseteq} f_E$. By Theorem 4.3(2),

$$int(f_E \tilde{\cap} g_E) \tilde{\subseteq} int(f_E).$$

Similarly, $int(f_E \tilde{\cap} g_E) \tilde{\subseteq} int(g_E)$.

Then $int(f_E \tilde{\cap} g_E) \tilde{\subseteq} int(f_E) \tilde{\cap} int(g_E)$.

By Theorem 4.3(1),

$$int(f_E) \tilde{\subseteq} f_E \text{ and } int(g_E) \tilde{\subseteq} g_E.$$

Then $int(f_E) \tilde{\cap} int(g_E) \tilde{\subseteq} f_E \tilde{\cap} g_E$. So $int(f_E) \tilde{\cap} int(g_E) \tilde{\subseteq} int(f_E \tilde{\cap} g_E)$.

Hence, $int(f_E) \tilde{\cap} int(g_E) = int(f_E \tilde{\cap} g_E)$.

Similarly, we can prove (2), (3) and (4).

(5) holds by Proposition 3.2.

$$(int(f_E))' = (\widetilde{\bigcup} \{h_E \in \tau : h_E \tilde{\subseteq} f_E\})' = \widetilde{\bigcap} \{h_E \in \tau' : f'_E \tilde{\subseteq} h_E\} = cl(f'_E).$$

(6) The proof is similar to (5). □

Example 4.6. Let $U=\{x_1, x_2\}$ and $E = \{e_1, e_2\}$.

$$f(e_1) = \frac{(0.2, 0.8)}{x_1} + \frac{(0.6, 0.3)}{x_2}, f(e_2) = \frac{(0.2, 0.5)}{x_1} + \frac{(0.9, 0.1)}{x_2}.$$

Obviously, $\tau = \{f_E, \varnothing_E, U_E\}$ is an IF soft topology on U.

h_E, l_E are defined as follows:

$$g(e_1) = \frac{(0.1, 0.8)}{x_1} + \frac{(0.6, 0.3)}{x_2}, g(e_2) = \frac{(0.2, 0.5)}{x_1} + \frac{(0.9, 0.1)}{x_2};$$

$$h(e_1) = \frac{(0.2, 0.8)}{x_1} + \frac{(0.6, 0.3)}{x_2}, h(e_2) = \frac{(0.2, 0.5)}{x_1} + \frac{(0.7, 0.1)}{x_2}.$$

(1) Obviously, $int(g_E) = \varnothing_E = int(h_E)$ and $g_E \tilde{\cup} h_E = f_E$. Then

$$int(g_E) \tilde{\cup} int(h_E) = \varnothing_E \tilde{\cup} \varnothing_E = \varnothing_E$$

and

$$int(g_E \tilde{\cup} h_E) = int(f_E) = f_E.$$

Thus

$$int(g_E) \tilde{\cup} int(h_E) \neq int(g_E \tilde{\cup} h_E).$$

(2) By Theorem 4.5(5),

$$cl(g'_E) = (int(g_E))' = \varnothing'_E = U_E.$$

Similarly, $cl(h'_E) = U_E$. Then

$$cl(g'_E) \tilde{\cap} cl(h'_E) = U_E \tilde{\cap} U_E = U_E.$$

Similarly,

$$cl(g'_E \tilde{\cap} h'_E) = cl((g_E \tilde{\cup} h_E)') = (int(g_E \tilde{\cup} h_E))' = f'_E.$$

Thus

$$cl(g'_E \tilde{\cap} h'_E) \neq cl(g'_E) \tilde{\cap} cl(h'_E).$$

5. Conclusions

In this paper, we introduced intuitionistic fuzzy soft topologies. The interior and closure of an intuitionistic fuzzy soft set, and bases for an intuitionistic fuzzy soft topology are introduced. Some basic properties of intuitionistic fuzzy soft topologies are given. In future work, we will research intuitionistic fuzzy soft subspace, intuitionistic fuzzy soft points and neighborhoods of a intuitionistic fuzzy soft point and bring them into the structure of intuitionistic fuzzy soft topological spaces.

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