Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 5, No. 1, (January 2013), pp. 213–228
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr

Non-probabilistic sensitivity and uncertainty analysis of atmospheric dispersion

Rituparna Chutia, Supahi Mahanta, D. Datta

Received 4 June 2012; Accepted 19 July 2012

Abstract. Sensitivity and uncertainty analysis have been carried out on Gaussian atmospheric dispersion model. Here sensitivity and uncertainty refers to non-probabilistic analysis. Fuzzy set theory is vastly applied to quantify non-probabilistic sensitivity and uncertainty analysis. Transformation method for optimization of interval at α-cut level in fuzzy arithmetic has been applied for entire process of calculation. Wind speed, discharge velocity, ambient temperature and gas exit temperature are considered as uncertain parameters, due to their imprecise measured value, here uncertain reflex fuzzy number. The model is run for the stability categories unstable, neutral and stable. Hartley-like measure to estimate the most sensitive parameter among the uncertain parameters has been considered. Fuzziness measure for the estimate of uncertainty in ground level concentration due to the sensitive parameters is used. Finally, uncertainty due to most sensitive parameters wind speed and discharge velocity on the pollutant concentration is compared for the stability categories unstable, neutral and stable. Computer codes are generated, for the model, in Matlab.

2010 AMS Classification: 03E72, 28E10, 49Q12, 68T37

Keywords: Transformation method, Fuzziness, Hartley-like measure, Gaussian plume Model, Fuzzy set theory.

Corresponding Author: Rituparna Chutia (Rituparnachutia7@rediffmail.com)

1. Introduction

Atmospheric diffusion model is a mathematical expression relating emission of material into atmosphere to downwind ambient concentration of the material. The main aim of the modelling process is to estimate the concentration of a pollutant at a particular receptor point by calculation from the basic information about the source of pollutant and meteorological conditions. Concentration of air pollutant at a given place is a function of a number of variables, such as rate of emission, distance of the receptor from the source and atmospheric conditions. The most important atmospheric conditions are wind speed, wind direction and vertical temperature structure of local atmosphere. Air pollution dispersion models are subject to scientific uncertainty, but the way this is handled for air quality management policy is different depending on the of modelling and impact under consideration [30].

Information about dispersion model parameters can be gained through measurement, calibration, expert judgement etc. However, value of these parameters may be subject to uncertainty due to lack of measurement point and over-calibration or inaccurate expert judgement. Inherent uncertainty of the input parameter is one of the main causes of uncertainty in model output. Parameter uncertainty is present because not always a single value of a parameter can completely characterize a modelling domain [7]. Traditionally, the available information is interpreted in a probabilistic sense and probability theory has been used to describe this information. Probability theory has certain input requirements and whenever these requirements are met, probability theory will provide powerful results. However, it is clear that not all uncertainties in data or model parameters are random; other source of imprecision that may lead to uncertainty are scare or imprecise data, measurement error or data obtained from expert judgement or subjective interpretation of available information. These kinds of uncertainties cannot be treated solely by probability theory. Thus, usefulness and applicability of other mathematical tools, such as fuzzy set theory or possibility theory should be explored. In the cases when not much data are available, or design values can be only estimated by an expert, the fuzzy set theory is useful as it assigns each value a degree of credibility [29]. The use of fuzzy numbers are proposed as a suitable tools for handling atmospheric dispersion criteria and tackling decisions made under uncertainty. Fuzzy analysis based on fuzzy set introduced by Zadeh [29] is widely used to represent such uncertain knowledge.

Sensitivity analysis aims to quantify relative importance of input variables in determining value of an assigned output variable. Several sensitivity analysis methods exist, including one-at-a-time (OAT) method, fractional experiments, differential analysis, Fourier Amplitude Sensitivity Test (FAST), Sobol's, Monte Carlo Analysis and Response Surface Method etc. [26]. Several sensitivity analysis of various dispersion software tools have been carried out using OAT method ([6], [19], [21]). It can be used as an aid to identifying the important uncertainties for the purpose of prioritizing additional data collection or research [12]. A Sensitivity study examines the way a particular model responds to variations in values of input variables or internal parameters [10]. Parameter sensitivity refers to the case where the output function values are largely effected by variations in the values of one or more parameters. Traditional and most used method of sensitivity analysis is the derivative method which provides local sensitivity analysis. Local sensitivity analysis is based on derivatives of evaluated at particular point and indicates how output will change if the base-line input values are perturbed slightly. But due to the implicit correlation of the parameters under investigation, we need a global sensitivity analysis. Global sensitivity analysis method is based on stepwise regression and this rigorous computational procedure is shortened by carrying out the correlation matrix method. Many global sensitivity analysis techniques are now available, such as Fourier Amplitude Sensitivity Test (FAST) [11], regression-based methods [14] and Sobol's method [9]. A survey of sampling-based methods has been presented in [20]. However, all these require specific probability distribution of all the parameters of interest. In practice, probability distributions of the parameters are not always possible due to their lack of measurement and spare behaviour in their prediction. Accordingly, imprecise method of sensitivity analysis is searched. The Hartley-like measure is the method of sensitivity analysis with imprecise parameter, because non-specificity is quantified using the Hartley-like measure. Parameter sensitivity analysis by Hartley-like measure is a method from imprecise probability theory. When the information about the parameters consists of a central value and a coefficient or range of variation then the Hartley-like measure method of sensitivity analysis is employed [23]. The basic strategy for arriving at a sensitivity analysis or assessment is by successively fixing the input parameters and drawing the effect on the variability of the output. Minimum value to Hartley-like measure of the output with respect to fixing a particular parameter to the most likely value, for a particular point of observation, is considered to be the most sensitive parameter.

A large number of mathematical methods have been developed for handling uncertainties. Probabilistic and statistical methods available are usually preferred for analysis of aleatory uncertainty. Some of the existing methods of aleatory uncertainty analysis are Monte Carlo Simulation [28], First-order second-moment analysis [18], stochastic response surface methods [15], Polynomial Chaos expansion Method [8] and Reliability analysis based approaches [5]. On the other hand, several competing approaches have been suggested when both aleatory and epistemic uncertainties are present. The most important methods of handling epistemic uncertainties consist of interval analysis [1], possibility theory [2] and fuzzy set theory [29]. Uncertainty analysis aims at quantifying amount of fuzziness exits in fuzzy output of a system due to fuzzy inputs. Fuzziness measure in the fuzzy theory is a measure of uncertainty analysis. This measure gives us the amount of fuzziness associated in the output model due to the fuzziness in the input parameters.

In this article, an attempt has been made for sensitivity and uncertainty analysis of atmospheric dispersion under a fuzzy environment. Literature review admits that there exist lots of method for sensitivity and uncertainty analysis under probabilistic environment. However, under fuzzy environment few literature are available. Here, an attempt has been made to fill that gap in atmospheric dispersion. Sensitivity analysis is being carried out using the Hartley-like measure and the sensitive parameters are drawn out. Uncertainty of the final response due to the sensitive parameters are expressed in terms of the Fuzziness measure.

2. Non-probabilistic sensitivity and uncertainty analysis

2.1. **Fuzzy set theory.** The notion of fuzzy set was introduced by Zadeh [29], since then its application has been evident in different field of study. The notion of fuzzy number arises from experiences of everyday life when any phenomena which can be quantified are not characterised in terms of absolutely precise numbers. Fuzzy numbers are convex and normalised fuzzy sets which are defined on the set of real numbers. Membership function of fuzzy number assigns degree of 1 to the most probable value, also called the mean value and lower degrees to other numbers which reflect their proximity to the most probable value according to the used membership function. Thus, the membership function decreases from 1 to 0 on both sides of the most probable value. α-level set or the α-cut of a fuzzy number is an interval defined for a specific value of the membership function.

2.2. **Transformation method.** The Transformation method introduced by Hanss [13] uses a fuzzy α-cut technique, which is based on interval arithmetic. Given a problem with n independent parameters. These parameters are uncertain in the sense that they are fuzzy numbers. Let these fuzzy numbers be represented as $\tilde{x}_1, \tilde{x}_n, \cdots, \tilde{x}_n$, and the function output $q = f(\tilde{x}_1, \tilde{x}_n, \cdots, \tilde{x}_n)$ is also a fuzzy number. Using the α-cuts technique, each input parameters is decomposed into a set $P_i, i = 1, 2, \cdots, n$ of $m+1$ intervals $X_i^{(j)}, j = 1, 2, \cdots, m$ where

$$P_i = \{X_I^{(0)}, X_I^{(1)}, \cdots, X_I^{(m)}\}$$

with

$$X_i^{(j)} = [a_i^{(j)}, b_i^{(j)}], a_i^{(j)} < b_i^{(j)}, i = 1, 2, \cdots, n, j = 1, 2, \cdots m$$

where, $a_i^{(j)}$, $b_i^{(j)}$ denote the lower and upper bound of the interval at the membership level μ_j. Instead of applying standard interval arithmetic, intervals are now transformed into arrays $\hat{X}_i^{(j)}$ of the following form:

$$\hat{X}_i^{(j)} = \left(\overbrace{\alpha_i^{(j)}, \beta_i^{(j)}, \alpha_i^{(j)}, \beta_i^{(j)} \cdots, \alpha_i^{(j)}, \beta_i^{(j)}}^{2^{i-1} pairs} \right)$$

with

$$\alpha_i^{(j)} = \left(\overbrace{a_i^{(j)}, \cdots, a_i^{(j)}}^{2^{i-1} pairs} \right), \beta_i^{(j)} = \left(\overbrace{b_i^{(j)}, \cdots, b_i^{(j)}}^{2^{i-1} pairs} \right),$$

The evaluation of function f is now carried out by evaluating the expression separately at each of the positions of the arrays using the conventional arithmetic. Results obtained is deterministic in decomposed and transformed form which can be re-transformed to get fuzzy valued result using a correction procedure known as recursive approximation. The fuzzy valued result $q^{(j)}$ of the function f can be achieved in its decomposed form

$$Z^{(j)} = [a^{(j)}, b^{(j)}], j = 1, 2, \cdots, m$$

By re-transforming the arrays $\hat{Z}^j$ by a correction procedure using the recursive formulae

$$a^{(j)} = \min_k (a^{(j+1)}, {}^k \hat{z}^{(j)}), b^{(j)} = \max_k (b^{(j+1)}, {}^k \hat{z}^{(j)}), j = 1, 2, \cdots, m$$

and

$$a^{(m)} = \min_k ({}^k \hat{z}^{(j)}) = \max_k ({}^k \hat{z}^{(j)}) = b^{(m)}$$

where, ${}^{k}\hat{z}^{(j)}$ is the k^{th} element of the array $\hat{Z}^{(j)}$ given by

$$^{k}\hat{z}^{(j)} = f(^{k}\hat{x}_1^{(j)}, ^{k}\hat{x}_2^{(j)}, \cdots, ^{k}\hat{x}_n^{(j)})$$

and ${}^{k}\hat{x}_i^{(j)}$ is denotes the k^{th} elements of the array $\hat{X}_i^{(j)}$

2.3. **The Hartley-like measure.** Hartley measure is well established measure of uncertainty in the classical set theory ([3], [22]). This type of measure quantifies the most fundamental type of uncertainty, one expressed in terms of a finite set of possible alternatives. The type of uncertainty quantified by the Hartley measure is well captured by the term non-specificity. This theory was generalised to fuzzy set by Higashi and Klir ([16],[24]). The generalised measure H for any non-empty fuzzy set A defined on a finite universal set X has the form

$$H(A) = \frac{1}{h(A)} \int_0^{h(a)} log_2|A_\alpha| d\alpha$$

where A_α denotes the cardinality of the α-cuts of the fuzzy set A and $h(A)$ height of A. For fuzzy intervals or numbers on the real line, the Hartley-like measure is defined as

$$HL(A) = \int_0^1 log_2(1 + \lambda(A_\alpha)) d\alpha$$

where $\lambda(A_\alpha)$ is the Lebesgue measure of A_α [16]. Mathematically, for a fuzzy number $A = [a_L, a_m, a_R]$ given by the membership function

$$\mu_A(x) = \begin{cases} \frac{x-a_L}{a_m-a_L}, & \text{if } a_L \le x \le a_m \\ \frac{x-a_R}{a_m-a_R}, & \text{if } a_m \le x \le a_R \\ 0, & \text{otherwise} \end{cases} \tag{2.1}$$

the Hartley-like measure is given by the expression below, which is valid for any type of triangular fuzzy number.

$$HL(A) = \frac{1}{(a_R - a_L)ln(2)}([1 + (a_R - a_L)]ln[1 + (a_R - a_L)] - (a_R - a_L))$$

2.4. **Fuzziness measure.** Probabilistic uncertainty is quantified in terms of coefficient of variation defined as the ratio of the standard deviation to the mean. However we are dealing with non-probabilistic uncertainty quantification, hence we will measure the uncertainty in terms of fuzziness measure. Fuzziness of a fuzzy set [3] is defined as the sum of the lack of distinction of the set and its complement. The lack of distinction is measured by

$$1 - |2A(x) - 1|$$

The measure of fuzziness $f(A)$ is then obtained by adding all these measurements

$$f(A) = \sum_{x \epsilon A} (1 - |2A(x) - 1|)$$

For a symmetric triangular fuzzy number, $A = [a_L, a_m, a_R]$ given by the membership function can be written as follows

$$f(A) = \int_{a_L}^{a_m} \left(1 - |2\frac{x - a_L}{a_m - a_L} - 1|\right) dx + \int_{a_R}^{a_m} \left(1 - |2\frac{x - a_R}{a_m - a_R} - 1|\right) dx$$

3. The Gaussian Model

Gaussian type models are widely used in atmospheric dispersion modelling, in particular for regulatory purposes. The Gaussian plume function can be used to predict the ground level pollutant concentration under different stability conditions given a pollution source. The ground level concentration directly downwind is of interest, since pollution concentration will be highest along that axis. Accordingly the value of pollutant concentration is calculated as [17]

$$\chi(x, 0, 0) = \frac{Q}{\pi u \sigma_y \sigma_z} exp(-\frac{H^2}{2\sigma_z^2})$$

where, Q is the emission rate (g/sec), σ_y, σ_z are the dispersion coefficients in cross-wind and vertical direction (m), U is the wind speed (m/sec), H is the effective stack height (m) and x is the downwind distance (m).

The dispersion coefficients σ_y and σ_z depend on the atmospheric stability class and increases with the downwind distance from the pollution source. They are the fundamental of all Gaussian based air pollution models. They can be determined very roughly by reading off a graph, but they can more accurately be determined by the mathematical expression by Eimutis and Konicek [4], which are as

$$\sigma_y = A_y(s)x^{0.9071} \quad \text{and} \quad \sigma_z = A_z(s)x^{q(s)} + R(s)$$

where A_y, A_z, q and R are parameters which depend upon the stability s and distance x.

The effective stack height is the sum of the physical stack height $h(m)$ and the plume rise $\triangle h(m)$, which is given by Moses and Carson equation [25] as follows:

$$\triangle h = Ad\frac{w_0}{u} + \frac{B\sqrt{Q_H}}{u}$$

where, $Q_H = r\pi w_0 C_p(T_s - T_a)$ heat release rate (Cal/sec) and d is the stack diameter (m), r is the stack radius (m), w_0 is the discharge velocity (m/sec), C_p is the specific heat at constant pressure $(Cal/kg-K)$, T_a is the ambient temperature (K) and T_s is the gas exit temperature (K). The parameters A and B depends upon the stability classes and are given by $A = 3.47$, $B = 0.333$ for unstable, $A = 0.35$, $B = 0.17$ for neutral and $A = -1.04$, $B = 0.17$ for stable stability classes.

4. Analyzing technique

The basic strategy for arriving at a sensitivity analysis or assessment is by successively fixing the input parameters and drawing the effect on the variability of the output. The minimum value to Hartley-like measure of the output with respect to fixing a particular parameter to the most likely value, for a particular point of observation, leads to finding the most sensitive parameter.

Evaluation of the output along the downwind distances is carried out by the transformation method, and fuzziness measure for uncertainty analysis on the pollutant concentration due to a parameter is evaluated. This is done by keeping one of the parameters as fuzzy and the other parameters fixed at their most likely values. The fuzzy bound of the output due to the concerned parameter and the respective fuzziness measure are evaluated to analyse uncertainty. The wider the bounds of the output, the more the fuzziness measure, and hence uncertainty would increase when fuzziness increases. Finally the whole process is done for different stability classes and the results thus found can be compared.

Tables 1 and 2 below depict uncertain and crisp inputs to the model. The concentration is evaluated for downwind distance starting from $x = 1$ to 3000 m away along the horizontal direction. These crisp input data are taken from the analysis of [27], also the uncertain parameters are modelled by taking the mean to be the most likely value and considering some degree of variability to get triangular fuzzy numbers. However, membership function of these parameters can be modelled from the actual site data by some methods of construction of the membership function.

Table 1. Triangular fuzzy numbers representing uncertain parameters

Parameters	Lower Bound	Middle Value	Upper Bound
Wind Speed $u(m/sec)$	2.27	3.57	4.6
Discharge Velocity $w_0(m/sec)$	7.2	11.8	16.4
Ambient Temperature $T_a(K)$	273	276	279
Gas Exit Temperature $T_s(K)$	315	355	395

Table 2. Crisp Input Values

Parameters	Values
Specific heat (C_p)	$0.24(Cal/kg-K)$
Physical stack height (h)	$55(m)$
Emission rate (Q)	$1(g/sec)$
Stack diameter (d)	$4(m)$

5. Results and discussions

The main aim of sensitivity analysis is to estimate the change of output of a model with respect to the changes in the model inputs. Sensitivity analysis is the study of how uncertainty in the output of a model can be appropriated to different sources of uncertainty in the input. It will in turn instruct users as to the relative importance of the inputs in determining the output. Sensitivity analysis provides useful risk insights, but alternative approaches are also needed to understand which of the parameters show up as important and why they show up as important. Hartley-like measure of the output concentration has been evaluated for various downwind distances starting from $x = 1$ to 3000 m. Hartley-like measure of the output pollution concentration under different uncertain parameters were obtained for the stability classes A (unstable), D (neutral) and F (stable) which are shown in figs. 1, 2 and 3, which are obtained by successively fixing the uncertain parameters to the most likely values. It has been found that the Hartley-like measure under all stability categories is the least for wind speed, followed by discharge velocity, and that the other uncertain parameters - ambient temperature and gas exit temperature are higher compared to them. Thus wind speed is the most sensitive parameter followed by discharge velocity compared to other uncertain parameters. However under the stability classes A and D in figs. 1 and 2, the Hartley-like measure is lower in case of pollutant concentration, fixing the discharge velocity around 500 m and 900 m away from the pollution source respectively. This is because around the stack, discharge velocity has a significant role in pollution concentration under unstable and neutral stability classes and as we move away from the stack, concentration is governed by wind speed, which means that discharge velocity has no more roles to play in concentration away from the pollution source. Just around the pollution source, under considered fuzzy variability, the uncertain parameters remain insensitive around 100 m, 400 m and 500 m away from the pollution source under the stability classes A, D and F respectively. These analyses show that the fuzzy variability of the input parameters is insensitive around the stack, but gradually their sensitivity starts playing a role.

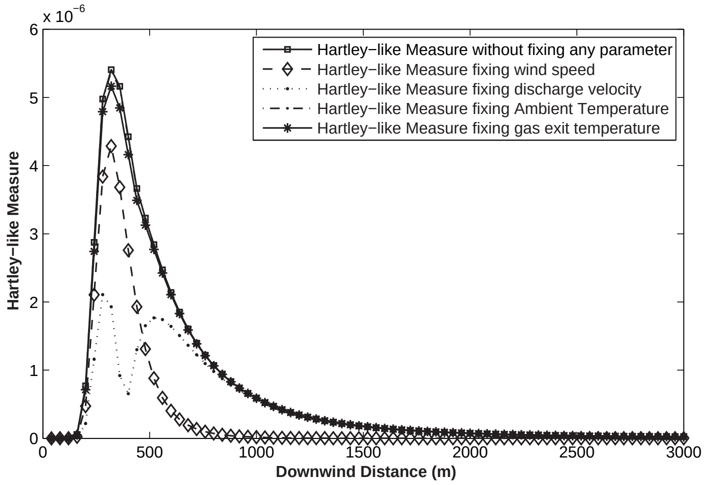

Figure 1. Hartley-like measure of pollutant concentration along the downwind distance under stability class A.

Fuzzy bounds of output pollution concentration due to the parameters wind speed and discharge velocity have been depicted in figs. 4, 5, 6 and figs. 7, 8, 9 respectively under stability classes A, D and F. The fuzziness measures due to wind speed and discharge velocity are shown in figs. 10, 11 and 12. For the unstable stability class A, fuzzy bounds of pollutant concentration due to discharge velocity at downwind distance 320 m are $[2.145E-6, 4.138E-6, 7.211E-6](mg/m^3)$ (fig. 7) and the

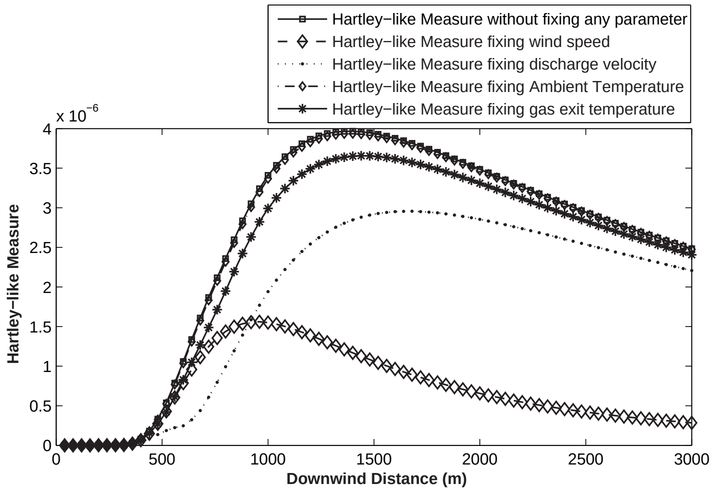

FIGURE 2. Hartley-like measure of pollutant concentration along the downwind distance under stability class D.

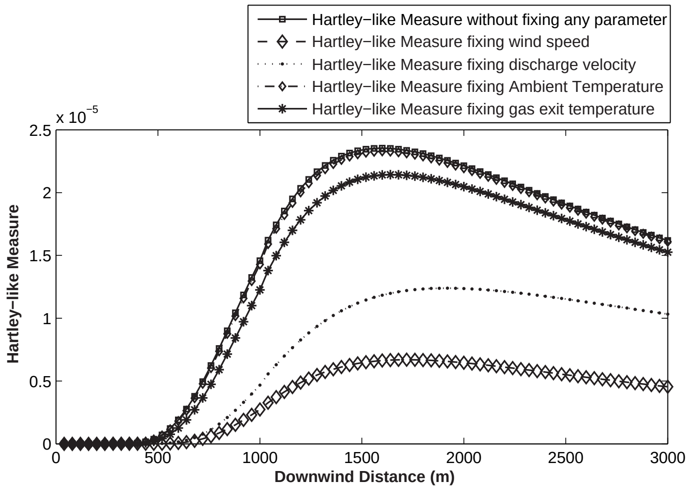

FIGURE 3. Hartley-like measure of pollutant concentration along the downwind distance under stability class F.

respective fuzziness measure is $2.511E-6$ (fig. 10) is the highest. The respective fuzzy bounds due to wind speed are $[2.461E-6, 4.138E-6, 4.689E-6](mg/m^3)$ (fig. 4) and the fuzziness measure is $1.139E-6$ (fig. 10). Gradually the fuzzy bounds as well as the fuzziness measure of pollutant concentration due to discharge velocity decrease when distance increases. Thus the effect of fuzzy variability of discharge velocity on concentration gets lowered as we move away from the source.

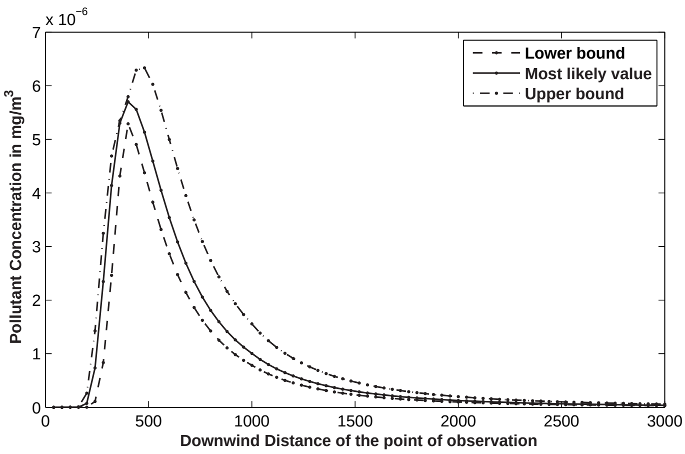

Figure 4. Bounds of pollutant concentration taking only wind speed as uncertain under stability class A.

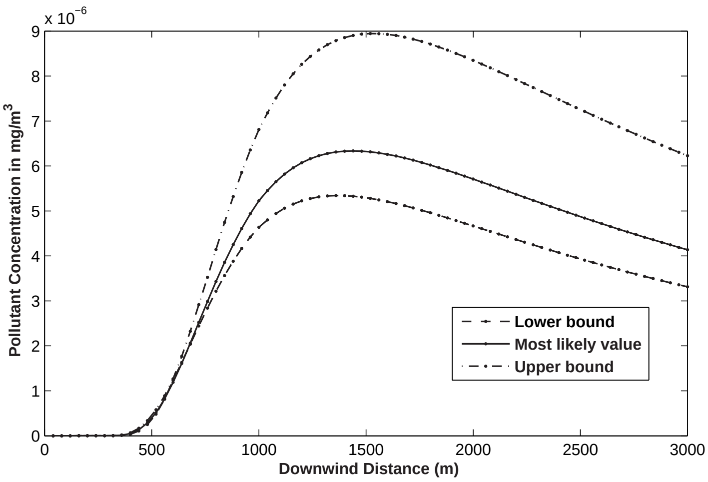

Figure 5. Bounds of pollutant concentration taking only wind speed as uncertain under stability class D.

This is so because discharge velocity at a distance far from the pollution source should be negligible. Fuzzy bound of concentration due to wind speed at 280 m is $[8.338E-6, 2.346E-6, 3.248E-6](mg/m^3)$ which is highest with the highest fuzziness measure $(= 1.197E-6)$ with respect to this concerned bound. Gradually bounds as well as the fuzziness measure decrease, which is evident from figs. 4 and 10. At this downwind distance, fuzzy bounds due to discharge velocity are $[8.584E-6, 2.346E-6, 5.485E-6](mg/m^3)$ and the fuzziness measure is $2.322E-6$.

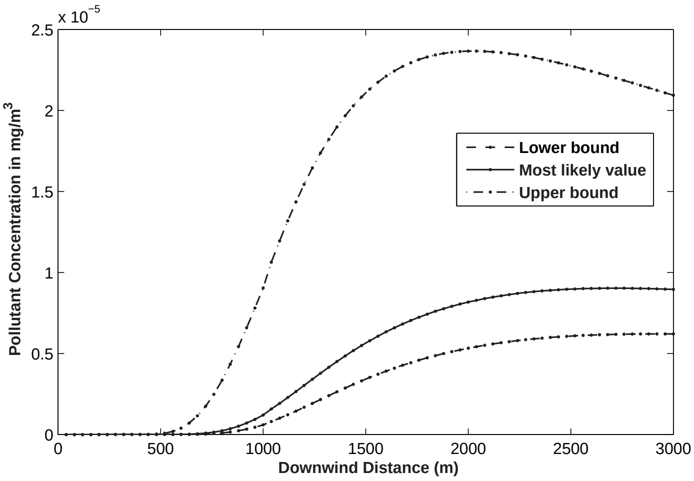

FIGURE 6. Bounds of pollutant concentration taking only wind speed as uncertain under stability class F.

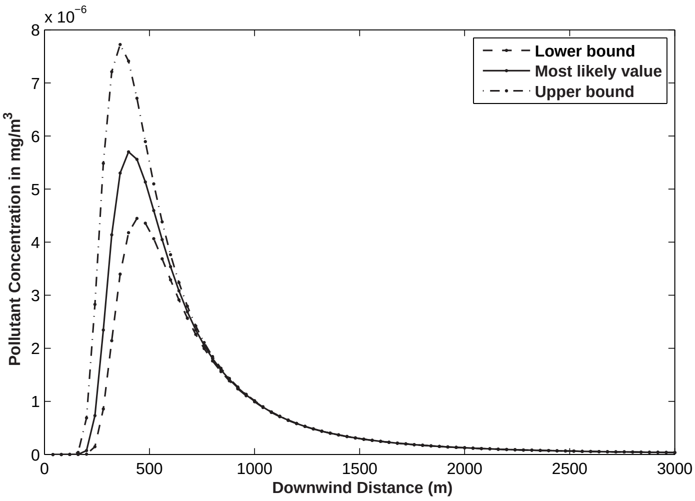

FIGURE 7. Bounds of pollutant concentration taking only discharge velocity as uncertain under stability class A.

This means, the measurement of wind speed at a distance far from the pollution source is more accurate than that at the vicinity. Fig. 10 shows also that the fuzziness measure of discharge velocity is more, compared to wind speed at distance between 100 m to 500 m. Gradually the fuzziness measure for discharge velocity decreases more rapidly as we cross downwind distance 500 m. This is because at

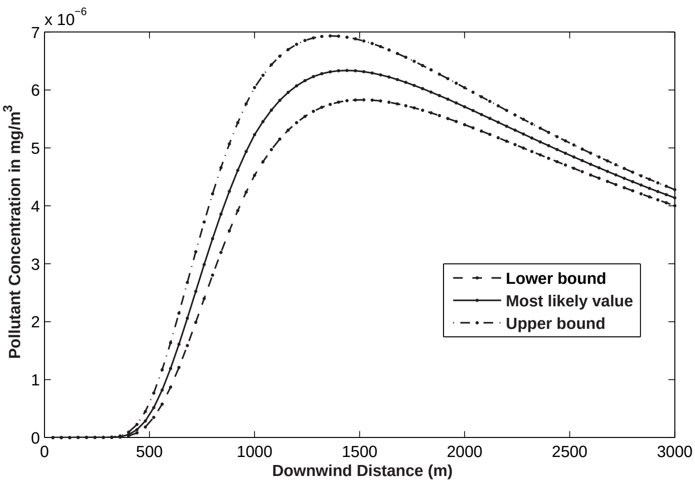

FIGURE 8. Bounds of pollutant concentration taking only discharge velocity as uncertain under stability class D.

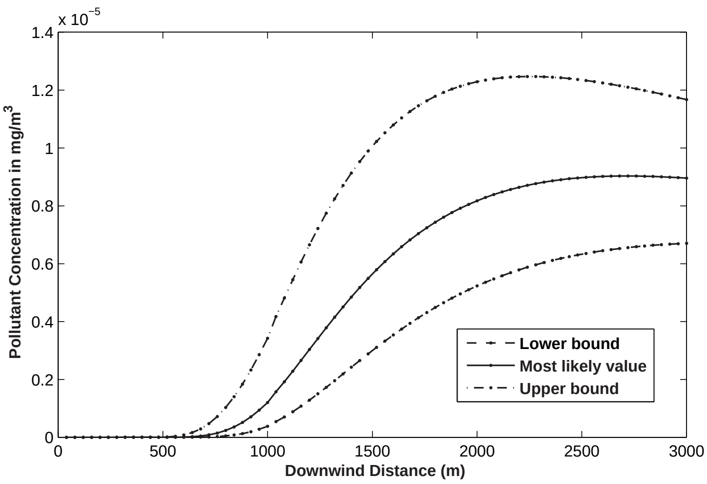

FIGURE 9. Bounds of pollutant concentration taking only discharge velocity as uncertain under stability class F.

the vicinity of the stack, plume is dominated by discharge velocity and as we move away from the source, plume is driven by wind speed.

Pollutant concentration calculations under the neutral stability class D were performed for downwind distances, $x = 1$ to 3000 m. Minimum, most likely and maximum concentration due to the parameters wind speed and discharge velocity are shown in figs. 5 and 8. Fuzzy bounds of concentration due to wind speed are very

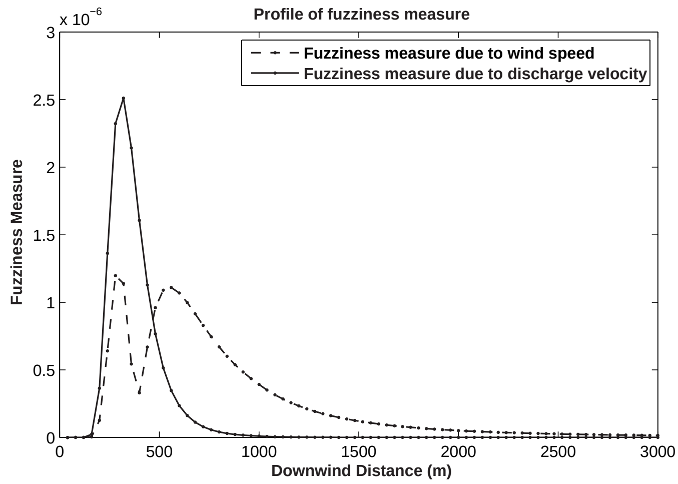

FIGURE 10. Fuzziness measure of pollutant concentration due to wind speed and discharge velocity under stability class A.

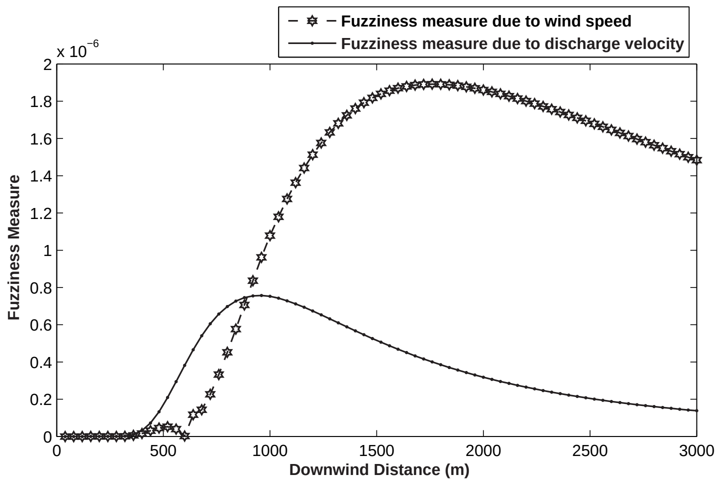

FIGURE 11. Fuzziness measure of pollutant concentration due to wind speed and discharge velocity under stability class D.

narrow around the stack and widen up away from the source. Similar behaviour is seen in the case of discharge velocity. However, bounds become narrow at a far end. Thus the fuzziness measure with respect to wind speed increases along the downwind direction and it decreases with respect to discharge velocity. With reference to wind speed as the fuzzy bounds become wider, uncertainty measured by fuzziness measure increases. This can be seen from fig. 11.

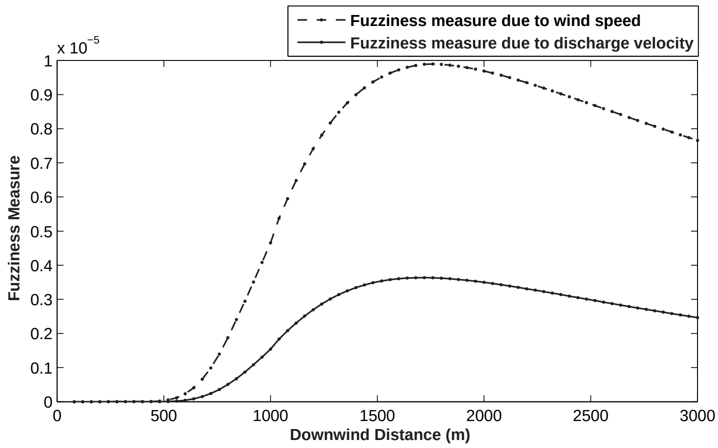

FIGURE 12. Fuzziness measure of pollutant concentration due to wind speed and discharge velocity under stability class F.

Fuzzy bounds of concentration and fuzziness measure under the stability class F were found out for the same downwind distances as that were done for the unstable and the neutral stability classes. Concentration bounds, (figs. 6 and 9), due to wind speed and discharge velocity are almost negligible around 600 m away from the pollution source and so is the fuzziness measure shown in fig. 12. But as we move away from the pollution source, bounds start increasing and so does the fuzziness measure. However the fuzziness measure of concentration always remains higher in the case of wind speed. This is because the most sensitive parameter in this case is wind speed throughout the downwind distances.

We have thus seen that the uncertainty due to wind speed is the maximum. Some uncertainty could be attributed to discharge velocity at the vicinity of the pollution source. The plume is driven by the discharge velocity at the vicinity of the stack but this effect is dissipated by wind speed as the plume move away from the source.

6. Conclusions

This paper demonstrates the applicability of the fuzzy set theory for accounting non-probabilistic uncertainty and sensitivity analysis of atmospheric dispersion. Sensitivity and uncertainty analysis to different physical phenomenon are often done probabilistically; here an attempt has been made to carry out the sensitivity and uncertainty analysis non-probabilistically using the theory of fuzzy sets. The sensitivity analysis by the Hartley-like measure yields that the parameter wind speed is the most sensitive parameter concerning to the pollutant concentration followed by the discharge velocity. And the uncertainty analysis by fuzziness measure to the sensitive parameter revels that uncertainty due to the most sensitive parameter wind speed is more as compared to the discharge velocity, however this uncertainty can be minimized by gaining more knowledge to the concerned parameter. Non-probabilistic uncertainty and sensitivity analysis can be further carried out using trapezoidal and Gaussian membership function. Since these type of numbers are more informative and knowledge base future research in this line can be encouraged.

Acknowledgements. This work has been funded under a BRNS research project sponsored by Department of Atomic Energy, Govt. of India.

References

[1] M. Ayyub Bilal and George J. Klir, Uncertainty Modelling and Analysis in Engineering and the Sciences, chapman and Hall/CRC Press, Boca Ratan, 2006.

[2] R. Bubbico, and B. Mazzarotta, Accidental release of toxic chemicals: Influence of the main input parameters on consequence calculation. Journal Hazardous Materials 151(2-3) (2008) 394–406.

[3] R. N. Colvile, N. K. Woodfield, D.J. Carruthers, B.E.A. Fisher, A. Rickard, S. Neville and A. Hughes, Uncertainty in Dispersion Modelling and Urban Air Quality Mapping, Journal of Environmental Science Policy 5 (2002) 202–220.

[4] A. C. Cullen and H. C. Frey, Probabilistic Techniques in Exposure Assessment, Plenum Press, New York, 1999.

[5] DNV, UDM verification manual. DNV Software, UK. 2006.

[6] X. Du and W. Chen, A Most Probable Point based Method for Uncertainty Analysis, Journal of Design and manufacturing Automation 4(1) (2001) 47–66.

[7] D. Dubois and H. Parde, Possibility theory: An approach to computerized processing of uncertainty, New York, Plenum Press, 1988.

[8] E. C. Eimutis and M. G. Konicek, Derivations of continuous functions for the lateral and vertical atmospheric dispersion coefficients, Atmospheric Environment 16 (1972) 859-863.

[9] M. Hagashi and G. J. Klir, Measure of Uncertainty and Information based on Possibility Distribution, International Journal of General Systems 9(1) (1983) 43–58.

[10] R. V. L. Hartley, Transmission of Information, The Bell System Technical Journal 7 (1928) 535–563.

[11] M. Hanss, The transformation method for the simulation and analysis of systems with uncertain parameters, Fuzzy Sets and Systems 130 (2002) 277–289.

[12] J. C. Helton, Uncertainty and Sensitivity analysis techniques for use in performance assessment for radioactive waste disposal, Reliability Engineering and System Safety 42(2-3) (1993) 327–367.

[13] J. C. Helton, F. J. Davis and J. D. Johnson, A comparison of uncertainty and sensitivity analysis results obtained with random and Latin hypercube sampling, Reliability Engineering System Safety 89(3) (2005) 305–330.

[14] J. C. Helton, J. D. Johnson, C. J. Sallaberry and C. B. Storlie, Survey of sampling-based methods for uncertainty and sensitivity analysis. Reliability Engineering System Safety 91(10-11) (2006) 1175–1209.

[15] J. A. Hoybe, Model Error Propagation and Date Collection Design, An Application in Water Quality Modeling, Water, Air and Soil Pollution 103(1-4) (1998) 101–119.

[16] S. S. Isukapalli and P. G. Georgopoulos, Stochastic Response Surface Methods (SRSMs) for Uncertainty Propagation: Application to environmental and Biological Systems, Risk Analysis 18(3) (1998) 351–363.

[17] G. J. Klir, M. J. Wiermann, Uncertainty based Information. Elements of generalised Information Theory, Physica-Verlg, Heidelberg, 1998.

[18] S. Mahadevan and P. Raghothamachar, Adaptive simulation for system reliability analysis of large structures, Computers and Structures 77 (2000) 725–734.

[19] S. K. Nair, D. B. Chambers, S. H. Park and F. O. Hoffman, Review of Models Used for Determining Consequences of UF6 Release: Model Evaluation Report, Vol. 2, Prepared for

the Office of Nuclear Material Safety and Safeguards, U.S. Nuclear Regulatory Commission, Washington, District of Columbia 20555-0001, NUREG/CR-6481, 1997.
[20] M. Oberguggenberger, J. King and B. Schmelzer, Imprecise probability methods for sensitivity analysis in engineering, 5th International Symposium on Imprecise Probability: Theories and Applications, Prague, Czech Republic, 2007.
[21] M. Saeedi, H. Fakhraee and M. Rezaei Sadrabadi, A Fuzzy Modified Gaussian Air Pollution Dispersion Model, Research Journal of Environmental Sciences 2(3) (2008) 156–169.
[22] A. Saltelli, M. Ratto, S. Tarantola and F. Campolongo, Sensitivity analysis for chemical models. Chemical Reviews 105(7) (2005) 2811–2826.
[23] A. Sengupta and T. K. Pal, Theory and Methodology: On comparing interval numbers, European Journal of Operational Research 127 (2000) 28–43.
[24] K. Shankar Rao, Uncertainty Analysis in Atmospheric Dispersion Modelling, Pure and Applied Geophysics 162 (2005) 1893–1917.
[25] V. V. Shirvaikar and V. J. Daoo, Air Pollution Meteorology, Bhabha Atomic Research Centre, Mumbai, India, 2002.
[26] I. M. Sobol's, Sensitivity estimates for nonlinear mathematical models, Mathematical Modelling Computational Experiments 1(4) (1993) 407–414.
[27] F. W. Thomas, S. G. Carpenter and W. C. Colbaugh, Plume rise estimates for electric generating stations, Journal of Air Pollution and Control Association 20 (1970) 170–177.
[28] Cheng Wang, Parametric Uncertainty Analysis for Complex Engineering Systems, Ph. D. Thesis, MIT, 53-113, 1999.
[29] A. Yegnan, D. G. Williamson and A. J. Graettinger, Uncertainty analysis in air dispersion modelling, Environmental Modelling and Software 17 (2002) 639–649.
[30] L. A. Zadeh, Fuzzy sets, Information and Control 8 (1965) 338–353.

Rituparna Chutia (Rituparnachutia7@rediffmail.com)
Department of Mathematics, Gauhati University, Guwahati-781014, India

Supahi Mahanta (supahi_mahanat@rediffmail.com)
Department of Statistics, Gauhati University, Guwahati-781014, India

D. Datta (ddatta@barc.gov.in)
Health Physics Division, BARC, Mumbai-400085, India