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On cubic Γ-hyperideals in left almost Γ-semihypergroups

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Abstract. In this paper, the concept of cubic sub LA-Γ-semihypergroup is introduced and some results on cubic Γ-hyperideals and cubic bi-Γ-hyperideals in left almost Γ-semihypergroups are provided.

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1. Introduction

Hyperstructure theory was born in 1934 when Marty [9] defined hypergroups, began to analyze their properties and applied them to groups, rational algebraic functions. Now they are widely studied from theoretical point of view and for their applications to many subjects of pure and applied properties. In 1986, Sen and Saha [11] introduced the concept of Γ-semigroup as a generalization of semigroup and ternary semigroup. Many classical notions and results of the theory of semigroups have been extended and generalized to Γ-semigroups. Recently, Yaqoob and Aslam [13] introduced the notion of LA-Γ-semihypergroups as a generalization of commutative semigroups, commutative semihypergroups and of commutative Γ-semigroups. They proved some results in this respect and presented many examples of LA-Γ-semihypergroups. Yaqoob et al. [14, 15] applied rough set theory and soft set theory to LA-Γ-semihypergroups.

In 1965, Zadeh [17] introduced the notion of a fuzzy subset of a non-empty set X, as a function from X to $[0, 1]$. After the introduction of the concept of fuzzy sets by Zadeh, several researchers conducted the researches on the generalization of the notion of fuzzy set with huge applications in computer, logics, automata and many branches of pure and applied mathematics. Rosenfeld [10] defined the concept of fuzzy group. Since then many papers have been published in the field of fuzzy algebra. Recently, fuzzy set theory has been well developed in the context of hyperalgebraic structure theory. The recent books [2, 12] contains a wealth of applications. Davvaz and Leoreanu-Fotea [3] studied the structure of fuzzy Γ-hyperideals in Γ-semihypergroups. Aslam et al. [1] introduced the notion of rough M-hypersystems and fuzzy M-hypersystems in Γ-semihypergroups.

Jun et al. [4] introduced the notion of cubic sub-algebras/ideals in BCK/BCI-algebras, and then they investigated several properties. They discussed relationship between a cubic subalgebra and a cubic ideal. Also, they provided characterizations of a cubic subalgebra/ideal, and considered a method to make a new cubic subalgebra from old one. Also see [5, 6, 7, 8].

In this paper, the concept of cubic sub LA-Γ-semihypergroup is introduced and some results on cubic Γ-hyperideals and cubic bi-Γ-hyperideals in left almost Γ-semihypergroups are provided.

2. Preliminaries and basic definitions

In this section, we recall certain definitions and results needed for our purpose.

Let S be a non-empty set and $\mathcal{P}^*(S)$ be the set of all non-empty subsets of S. The map $\circ : S \times S \to \mathcal{P}^*(S)$ is called hyperoperation or join operation on the set S. A couple $(S, \circ)$ is called a hypergroupoid. Let A and B be two non-empty subsets of S, then we denote

$$A \circ B = \bigcup_{a \in A, b \in B} a \circ b, \qquad a \circ A = \{a\} \circ A \text{ and } a \circ B = \{a\} \circ B.$$

Definition 2.1. [11] Let $S = \{a, b, c, ...\}$ and $\Gamma = \{\alpha, \beta, \gamma, ...\}$ be two sets. Then S is called a Γ-semigroup if there exists a mapping $S \times \Gamma \times S \longrightarrow S$ written as $(a, \gamma, b) \longrightarrow a\gamma b$ satisfying the following identity $(a\alpha b)\beta c = a\alpha(b\beta c)$ for all $a, b, c \in S$ and for all $\alpha, \beta \in \Gamma$. Let K be a non-empty subset of S. Then K is called a sub Γ-semigroup of S, if $a\gamma b \in K$ for all $a, b \in K$ and $\gamma \in \Gamma$.

Definition 2.2. [13] Let H and Γ be two non-empty sets. Then H is called a left almost Γ-semihypergroup (abbreviated as an LA-Γ-semihypergroup) if every $\gamma \in \Gamma$ is a hyperoperation on H, i.e, $x\gamma y \subseteq H$ for every $x, y \in H$, and for every $\gamma, \beta \in \Gamma$ and $x, y, z \in H$ we have $(x\gamma y)\beta z = (z\gamma y)\beta x$.

The law $(x\gamma y)\beta z = (z\gamma y)\beta x$ is called left invertive law. Through out the paper H will denote an LA-Γ-semihypergroup unless otherwise specified. Let A and B be two non-empty subsets of an LA-Γ-semihypergroup H. Then we define

$$A\gamma B = \bigcup \{a\gamma b \mid a \in A,\ b \in B \text{ and } \gamma \in \Gamma\}.$$

Also

$$A\Gamma B = \bigcup_{\gamma \in \Gamma} A\gamma B = \bigcup \{a\gamma b \mid a \in A,\ b \in B \text{ and } \gamma \in \Gamma\}.$$

Example 2.3. Let $H = \{x, y, z, t\}$ and $\Gamma = \{\beta, \gamma\}$ be the sets of binary hyperoperations defined below:

β	x	y	z	t
x	x	x	x	x
y	x	$\{z,t\}$	z	$\{z,t\}$
z	x	z	z	z
t	x	$\{y,t\}$	z	$\{y,t\}$

γ	x	y	z	t
x	x	x	x	x
y	x	$\{y,z,t\}$	z	$\{y,z,t\}$
z	x	z	z	z
t	x	$\{y,t\}$	z	$\{y,t\}$

Clearly H is not a Γ-semihypergroup because $\{y, z, t\} = (t\beta t)\gamma y \neq t\beta(t\gamma y) = \{y, t\}$. Thus H is an LA-Γ-semihypergroup because it satisfies left invertive law.

Every LA-Γ-semihypergroup satisfies the law $(a\alpha b)\beta(c\gamma d) = (a\alpha c)\beta(b\gamma d)$ for all $a, b, c, d \in H$ and $\alpha, \beta, \gamma \in \Gamma$. This law is known as Γ-hypermedial law. (cf. [13]).

Definition 2.4. [13] Let K be a non-empty subset of H. Then K is called a sub LA-Γ-semihypergroup of H if $a\gamma b \subseteq K$ for all $a, b \in K$ and $\gamma \in \Gamma$.

Definition 2.5. [13] A non-empty subset A of an LA-Γ-semihypergroup H is a right (left) Γ-hyperideal of H if $A\Gamma H \subseteq A$ ($H\Gamma A \subseteq A$), and is a Γ-hyperideal of H if it is both a right and a left Γ-hyperideal.

Definition 2.6. [13] A non-empty subset B of an LA-Γ-semihypergroup H is called bi-Γ-hyperideal of H if $B\Gamma B \subseteq B$ and $(B\Gamma H)\Gamma B \subseteq B$.

A bi-Γ-hyperideal B of an LA-Γ-semihypergroup H is proper if $B \neq H$.

Definition 2.7. [13] A non-empty subset B of an LA-Γ-semihypergroup H is called $(1, 2)$-Γ-hyperideal of H if $B\Gamma B \subseteq B$ and $(B\Gamma H)\Gamma B^2 \subseteq B$.

Definition 2.8. [16] A subset M of an LA-Γ-semihypergroup H is called M-hypersystem if for all $a, b \in M$, there exist $x \in H$ and $\gamma, \beta \in \Gamma$, such that $a\gamma(x\beta b) \subseteq M$.

Definition 2.9. [16] A subset N of an LA-Γ-semihypergroup H is called N-hypersystem if for all $a \in N$, there exist $x \in H$ and $\gamma, \beta \in \Gamma$, such that $a\gamma(x\beta a) \subseteq N$.

Now we will recall the concept of interval valued fuzzy sets.

An interval number is $\widetilde{a} = [a^-, a^+]$, where $0 \leqslant a^- \leqslant a^+ \leqslant 1$. Let $D[0, 1]$ denote the family of all closed subintervals of $[0, 1]$, i.e.,

$$D[0,1] = \{\widetilde{a} = [a^-, a^+] : a^- \leqslant a^+, \text{ for } a^-, a^+ \in I\}.$$

We define the operations "$\succeq$", "$\preceq$", "$=$", "rmin" and "rmax" in case of two elements in $D[0, 1]$. We consider two elements $\widetilde{a} = [a^-, a^+]$ and $\widetilde{b} = [b^-, b^+]$ in $D[0, 1]$. Then

(1) $\widetilde{a} \succeq \widetilde{b}$ if and only if $a^- \geq b^-$ and $a^+ \geq b^+$,
(2) $\widetilde{a} \preceq \widetilde{b}$ if and only if $a^- \leq b^-$ and $a^+ \leq b^+$,
(3) $\widetilde{a} = \widetilde{b}$ if and only if $a^- = b^-$ and $a^+ = b^+$,
(4) $\text{rmin}\{\widetilde{a}, \widetilde{b}\} = [\min\{a^-, b^-\}, \min\{a^+, b^+\}]$,
(5) $\text{rmax}\{\widetilde{a}, \widetilde{b}\} = [\max\{a^-, b^-\}, \max\{a^+, b^+\}]$.

It is obvious that $(D[0,1], \preceq, \vee, \wedge)$ is a complete lattice with $\widetilde{0} = [0,0]$ as its least element and $\widetilde{1} = [1,1]$ as its greatest element. Let $\widetilde{a_i} \in D[0,1]$ where $i \in \Lambda$. We define

$$\underset{i\in\Lambda}{\mathrm{rinf}}\widetilde{a}_i = \left[\inf_{i\in\Lambda} a_i^-,\ \inf_{i\in\Lambda} a_i^+\right] \quad \text{and} \quad \underset{i\in\Lambda}{\mathrm{rsup}}\widetilde{a}_i = \left[\sup_{i\in\Lambda} a_i^-,\ \sup_{i\in\Lambda} a_i^+\right].$$

An interval valued fuzzy set (briefly, IVF-set) $\widetilde{\mu}_A$ on X is defined as

$$\widetilde{\mu}_A = \left\{\left\langle x, [\mu_A^-(x), \mu_A^+(x)]\right\rangle : x \in X\right\},$$

where $\mu_A^-(x) \leqslant \mu_A^+(x)$, for all $x \in X$. Then the ordinary fuzzy sets $\mu_A^- : X \to [0,1]$ and $\mu_A^+ : X \to [0,1]$ are called a lower fuzzy set and an upper fuzzy set of $\widetilde{\mu}$, respectively. Let $\widetilde{\mu}_A(x) = [\mu_A^-(x), \mu_A^+(x)]$, then

$$A = \{\langle x, \widetilde{\mu}_A(x)\rangle : x \in X\},$$

where $\widetilde{\mu}_A : X \longrightarrow D[0,1]$.

3. Cubic Γ-hyperideals in left almost Γ-semihypergroup

Jun et al. [4], introduced the concept of cubic sets defined on a non-empty set X as objects having the form:

$$\Xi = \{\langle x, \widetilde{\mu}_\Xi(x), \lambda_\Xi(x)\rangle : x \in X\},$$

which is briefly denoted by $\Xi = \langle \widetilde{\mu}_\Xi, \lambda_\Xi\rangle$, where the functions $\widetilde{\mu}_\Xi : X \longrightarrow D[0,1]$ and $\lambda_\Xi : X \to [0,1]$.

Definition 3.1. Let $\Xi = \langle \widetilde{\mu}_\Xi, \lambda_\Xi\rangle$ and $\mathcal{F} = \langle \widetilde{\mu}_\mathcal{F}, \lambda_\mathcal{F}\rangle$ be two cubic sets in an LA-Γ-semihypergroup H, then

$$\Xi \cap \mathcal{F} = \{\langle x, \mathrm{rmin}\{\widetilde{\mu}_\Xi(x), \widetilde{\mu}_\mathcal{F}(x)\}, \max\{\lambda_\Xi(x), \lambda_\mathcal{F}(x)\}\rangle : x \in H\},$$

and

$$\Xi *_\Gamma \mathcal{F} = \{\langle x, \widetilde{\mu}_{\Xi *_\Gamma \mathcal{F}}(x), \lambda_{\Xi *_\Gamma \mathcal{F}}(x)\rangle : x \in H\},$$

where

$$\widetilde{\mu}_{\Xi *_\Gamma \mathcal{F}}(x) = \begin{cases} \underset{x\in y\gamma z}{\mathrm{rsup}}\{\mathrm{rmin}\{\widetilde{\mu}_\Xi(y), \widetilde{\mu}_\mathcal{F}(z)\}\} & \text{if } x \in y\gamma z, \forall \gamma \in \Gamma \\ [0,0] & \text{otherwise} \end{cases}$$

$$\lambda_{\Xi *_\Gamma \mathcal{F}}(x) = \begin{cases} \inf_{x\in y\gamma z}\{\max\{\lambda_\Xi(y), \lambda_\mathcal{F}(z)\}\} & \text{if } x \in y\gamma z, \forall \gamma \in \Gamma \\ 1 & \text{otherwise.} \end{cases}$$

Denote $\mathcal{C}(H)$ by family of all cubic sets in H.

Proposition 3.2. *Let H be an LA-Γ-semihypergroup, then the set $(\mathcal{C}(H), *_\Gamma)$ is an LA-Γ-semihypergroup.*

Proof. Clearly $\mathcal{C}(H)$ is closed. Let $\Xi_1 = \langle \widetilde{\mu}_{\Xi_1}, \lambda_{\Xi_1}\rangle$, $\Xi_2 = \langle \widetilde{\mu}_{\Xi_2}, \lambda_{\Xi_2}\rangle$ and $\Xi_3 = \langle \widetilde{\mu}_{\Xi_3}, \lambda_{\Xi_3}\rangle$ be in $\mathcal{C}(H)$. Let x be any element of H such that $x \notin y\gamma z$ for some $y, z \in H$ and $\gamma \in \Gamma$. Then for $\gamma, \beta \in \Gamma$, we have

$$((\widetilde{\mu}_{\Xi_1} *_\Gamma \widetilde{\mu}_{\Xi_2}) *_\Gamma \widetilde{\mu}_{\Xi_3})(x) = [0,0] = ((\widetilde{\mu}_{\Xi_3} *_\Gamma \widetilde{\mu}_{\Xi_2}) *_\Gamma \widetilde{\mu}_{\Xi_1})(x).$$

And

$$((\lambda_{\Xi_1} *_\Gamma \lambda_{\Xi_2}) *_\Gamma \lambda_{\Xi_3})(x) = 1 = ((\lambda_{\Xi_3} *_\Gamma \lambda_{\Xi_2}) *_\Gamma \lambda_{\Xi_1})(x).$$

Let x be any element of H such that $x \in y\gamma z$ for some $y, z \in H$. Then for $\gamma, \beta \in \Gamma$ we have

$$\begin{aligned}
& ((\widetilde{\mu}_{\Xi_1} *_\Gamma \widetilde{\mu}_{\Xi_2}) *_\Gamma \widetilde{\mu}_{\Xi_3})(x) = \operatorname*{rsup}_{x \in y\beta z} \{\mathrm{rmin}\{(\widetilde{\mu}_{\Xi_1} *_\Gamma \widetilde{\mu}_{\Xi_2})(y), \widetilde{\mu}_{\Xi_3}(z)\}\} \\
= & \operatorname*{rsup}_{x \in y\beta z} \left\{\mathrm{rmin}\left\{\operatorname*{rsup}_{y \in p\gamma q} \{\mathrm{rmin}\{\widetilde{\mu}_{\Xi_1}(p), \widetilde{\mu}_{\Xi_2}(q)\}\}, \widetilde{\mu}_{\Xi_3}(z)\right\}\right\} \\
= & \operatorname*{rsup}_{x \in y\beta z} \operatorname*{rsup}_{y \in p\gamma q} \{\mathrm{rmin}\{\widetilde{\mu}_{\Xi_1}(p), \widetilde{\mu}_{\Xi_2}(q), \widetilde{\mu}_{\Xi_3}(z)\}\} \\
= & \operatorname*{rsup}_{x \in (p\gamma q)\beta z} \{\mathrm{rmin}\{\widetilde{\mu}_{\Xi_1}(p), \widetilde{\mu}_{\Xi_2}(q), \widetilde{\mu}_{\Xi_3}(z)\}\} \\
= & \operatorname*{rsup}_{x \in (z\gamma q)\beta p} \{\mathrm{rmin}\{\widetilde{\mu}_{\Xi_3}(z), \widetilde{\mu}_{\Xi_2}(q), \widetilde{\mu}_{\Xi_1}(p)\}\} \\
= & \operatorname*{rsup}_{x \in m\beta p} \operatorname*{rsup}_{m \in z\gamma q} \{\mathrm{rmin}\{\widetilde{\mu}_{\Xi_3}(z), \widetilde{\mu}_{\Xi_2}(q), \widetilde{\mu}_{\Xi_1}(p)\}\} \\
= & \operatorname*{rsup}_{x \in m\beta p} \left\{\mathrm{rmin}\left\{\operatorname*{rsup}_{m \in z\gamma q} \{\mathrm{rmin}\{\widetilde{\mu}_{\Xi_3}(z), \widetilde{\mu}_{\Xi_2}(q)\}\}, \widetilde{\mu}_{\Xi_1}(p)\right\}\right\} \\
= & \operatorname*{rsup}_{x \in m\beta p} \{\mathrm{rmin}\{(\widetilde{\mu}_{\Xi_3} *_\Gamma \widetilde{\mu}_{\Xi_2})(m), \widetilde{\mu}_{\Xi_1}(p)\}\} \\
= & ((\widetilde{\mu}_{\Xi_3} *_\Gamma \widetilde{\mu}_{\Xi_2}) *_\Gamma \widetilde{\mu}_{\Xi_1})(x).
\end{aligned}$$

And

$$\begin{aligned}
& ((\lambda_{\Xi_1} *_\Gamma \lambda_{\Xi_2}) *_\Gamma \lambda_{\Xi_3})(x) = \inf_{x \in y\beta z} \{\max\{(\lambda_{\Xi_1} *_\Gamma \lambda_{\Xi_2})(y), \lambda_{\Xi_3}(z)\}\} \\
= & \inf_{x \in y\beta z} \left\{\max\left\{\operatorname*{rsup}_{y \in p\gamma q} \{\max\{\lambda_{\Xi_1}(p), \lambda_{\Xi_2}(q)\}\}, \lambda_{\Xi_3}(z)\right\}\right\} \\
= & \inf_{x \in y\beta z} \inf_{y \in p\gamma q} \{\max\{\lambda_{\Xi_1}(p), \lambda_{\Xi_2}(q), \lambda_{\Xi_3}(z)\}\} \\
= & \inf_{x \in (p\gamma q)\beta z} \{\max\{\lambda_{\Xi_1}(p), \lambda_{\Xi_2}(q), \lambda_{\Xi_3}(z)\}\} \\
= & \inf_{x \in (z\gamma q)\beta p} \{\max\{\lambda_{\Xi_3}(z), \lambda_{\Xi_2}(q), \lambda_{\Xi_1}(p)\}\} \\
= & \inf_{x \in m\beta p} \inf_{m \in z\gamma q} \{\max\{\lambda_{\Xi_3}(z), \lambda_{\Xi_2}(q), \lambda_{\Xi_1}(p)\}\} \\
= & \inf_{x \in m\beta p} \left\{\max\left\{\operatorname*{rsup}_{m \in z\gamma q} \{\max\{\lambda_{\Xi_3}(z), \lambda_{\Xi_2}(q)\}\}, \lambda_{\Xi_1}(p)\right\}\right\} \\
= & \inf_{x \in m\beta p} \{\max\{(\lambda_{\Xi_3} *_\Gamma \lambda_{\Xi_2})(m), \lambda_{\Xi_1}(p)\}\} \\
= & ((\lambda_{\Xi_3} *_\Gamma \lambda_{\Xi_2}) *_\Gamma \lambda_{\Xi_1})(x).
\end{aligned}$$

Hence $(\mathcal{C}(H), *_\Gamma)$ is an LA-Γ-semihypergroup. □ □

Corollary 3.3. *If H is an LA-Γ-semihypergroup, then the Γ-hypermedial law holds in $\mathcal{C}(H)$.*

Proof. Let $\Xi_1 = \langle \widetilde{\mu}_{\Xi_1}, \lambda_{\Xi_1} \rangle$, $\Xi_2 = \langle \widetilde{\mu}_{\Xi_2}, \lambda_{\Xi_2} \rangle$, $\Xi_3 = \langle \widetilde{\mu}_{\Xi_3}, \lambda_{\Xi_3} \rangle$ and $\Xi_4 = \langle \widetilde{\mu}_{\Xi_4}, \lambda_{\Xi_4} \rangle$ be in $\mathcal{C}(H)$. By successive use of left invertive law,

$$\begin{aligned}(\widetilde{\mu}_{\Xi_1} *_\Gamma \widetilde{\mu}_{\Xi_2}) *_\Gamma (\widetilde{\mu}_{\Xi_3} *_\Gamma \widetilde{\mu}_{\Xi_4}) &= ((\widetilde{\mu}_{\Xi_3} *_\Gamma \widetilde{\mu}_{\Xi_4}) *_\Gamma \widetilde{\mu}_{\Xi_2}) *_\Gamma \widetilde{\mu}_{\Xi_1} \\ &= ((\widetilde{\mu}_{\Xi_2} *_\Gamma \widetilde{\mu}_{\Xi_4}) *_\Gamma \widetilde{\mu}_{\Xi_3}) *_\Gamma \widetilde{\mu}_{\Xi_1} \\ &= (\widetilde{\mu}_{\Xi_1} *_\Gamma \widetilde{\mu}_{\Xi_3}) *_\Gamma (\widetilde{\mu}_{\Xi_2} *_\Gamma \widetilde{\mu}_{\Xi_4}).\end{aligned}$$

And

$$\begin{aligned}(\lambda_{\Xi_1} *_\Gamma \lambda_{\Xi_2}) *_\Gamma (\lambda_{\Xi_3} *_\Gamma \lambda_{\Xi_4}) &= ((\lambda_{\Xi_3} *_\Gamma \lambda_{\Xi_4}) *_\Gamma \lambda_{\Xi_2}) *_\Gamma \lambda_{\Xi_1} \\ &= ((\lambda_{\Xi_2} *_\Gamma \lambda_{\Xi_4}) *_\Gamma \lambda_{\Xi_3}) *_\Gamma \lambda_{\Xi_1} \\ &= (\lambda_{\Xi_1} *_\Gamma \lambda_{\Xi_3}) *_\Gamma (\lambda_{\Xi_2} *_\Gamma \lambda_{\Xi_4}).\end{aligned}$$

Hence this shows that the Γ-hypermedial law holds in $\mathcal{C}(H)$. □ □

Definition 3.4. Let H be an LA-Γ-semihypergroup. A cubic set $\Xi = \langle \widetilde{\mu}_\Xi, \lambda_\Xi \rangle$ in H is called a cubic sub LA-Γ-semihypergroup of H if for all $x, y \in H$ and $\gamma \in \Gamma$,

$$\operatorname*{rinf}_{z \in x\gamma y} \{\widetilde{\mu}_\Xi(z)\} \succeq \operatorname{rmin}\{\widetilde{\mu}_\Xi(x), \widetilde{\mu}_\Xi(y)\}$$

and

$$\sup_{z \in x\gamma y} \{\lambda_\Xi(z)\} \leq \max\{\lambda_\Xi(x), \lambda_\Xi(y)\}.$$

Definition 3.5. Let H be an LA-Γ-semihypergroup. A cubic set $\Xi = \langle \widetilde{\mu}_\Xi, \lambda_\Xi \rangle$ in H is called a left (resp. right) cubic Γ-hyperideal of H if for all $x, y \in H$ and $\gamma \in \Gamma$,

(1) $\operatorname*{rinf}_{z \in x\gamma y} \{\widetilde{\mu}_\Xi(z)\} \succeq \widetilde{\mu}_\Xi(y)$ (resp. $\operatorname*{rinf}_{z \in x\gamma y} \{\widetilde{\mu}_\Xi(z)\} \succeq \widetilde{\mu}_\Xi(x)$).

(2) $\sup_{z \in x\gamma y} \{\lambda_\Xi(y)\} \leq \lambda_\Xi(y)$ (resp. $\sup_{z \in x\gamma y} \{\lambda_\Xi(z)\} \leq \lambda_\Xi(x)$).

A cubic set Ξ in H is called a two-sided cubic Γ-hyperideal of H if it is both a left cubic and a right cubic Γ-hyperideal of H.

Example 3.6. Let $H = \{a, b, c, d\}$ and $\Gamma = \{\beta, \gamma\}$ be the sets of binary hyperoperations defined below:

β	a	b	c	d
a	b	b	$\{c,d\}$	d
b	b	b	$\{c,d\}$	d
c	$\{c,d\}$	$\{c,d\}$	c	d
d	d	d	d	d

γ	a	b	c	d
a	a	a	$\{c,d\}$	d
b	$\{a,b\}$	$\{a,b\}$	$\{c,d\}$	d
c	$\{c,d\}$	$\{c,d\}$	c	d
d	d	d	d	d

Clearly H is not a Γ-semihypergroup because $\{a, b\} = (a\beta a)\gamma b \neq a\beta(a\gamma b) = \{b\}$. Thus H is an LA-Γ-semihypergroup because it satisfies the left invertive law. Let $\Xi = \langle \widetilde{\mu}_\Xi, \lambda_\Xi \rangle$ be a cubic subset of H, which is defined as

H	$\widetilde{\mu}_\Xi$	λ_Ξ
a	$[0.15, 0.3]$	0.8
b	$[0.15, 0.3]$	0.8
c	$[0.45, 0.57]$	0.4
d	$[0.6, 0.67]$	0.1

By routine calculations, it can be seen that $\Xi = \langle \widetilde{\mu}_\Xi, \lambda_\Xi \rangle$ is a cubic Γ-hyperideal of H.

Proposition 3.7. *If $\{\Xi_i\}_{i\in I}$ is a family of left (resp. right) cubic Γ-hyperideals of an LA-Γ-semihypergroup H, then $\bigcap_{i\in I} \Xi_i$ is a left (resp. right) cubic Γ-hyperideal of H.*

Proof. Consider $\{\Xi_i\}_{i\in I}$ is a family of left cubic Γ-hyperideals of an LA-Γ-semihypergroup H. Let us suppose that $\Upsilon = \bigcap_{i\in I} \Xi_i$ and $x, y \in H$. Then for all $\gamma \in \Gamma$, we have:

$$\begin{aligned}
\widetilde{\mu}_{\Upsilon}(y) &= \operatorname{rinf}_{i\in I}\{\widetilde{\mu}_{\Xi_i}(y)\} \preceq \operatorname{rinf}_{i\in I}\left\{\operatorname{rinf}_{z\in x\gamma y}\{\widetilde{\mu}_{\Xi_i}(z)\}\right\} \\
&= \operatorname{rinf}_{z\in x\gamma y}\left\{\operatorname{rinf}_{i\in I}\{\widetilde{\mu}_{\Xi_i}(z)\}\right\} = \operatorname{rinf}_{z\in x\gamma y}\{\widetilde{\mu}_{\Upsilon}(z)\},
\end{aligned}$$

and

$$\begin{aligned}
\sup_{z\in x\gamma y}\{\lambda_{\Upsilon}(z)\} &= \sup_{z\in x\gamma y}\left\{\sup_{i\in I}\{\lambda_{\Xi_i}(z)\}\right\} = \sup_{i\in I}\left\{\sup_{z\in x\gamma y}\{\lambda_{\Xi_i}(z)\}\right\} \\
&\leq \sup_{i\in I}\{\lambda_{\Xi_i}(y)\} = \lambda_{\Upsilon}(y).
\end{aligned}$$

This completes the proof. □

For any $\widetilde{t} \in D[0,1]$ and $s \in [0,1]$. Let $\Xi = \langle \widetilde{\mu}_{\Xi}, \lambda_{\Xi}\rangle$ be a cubic set in H, the set

$$U(\Xi; \widetilde{t}, s) = \{x \in H : \widetilde{\mu}_{\Xi}(x) \succeq \widetilde{t},\ \ \lambda_{\Xi}(x) \leq s\}$$

is called the cubic level set of $\Xi = \langle \widetilde{\mu}_{\Xi}, \lambda_{\Xi}\rangle$.

Theorem 3.8. *Let H be an LA-Γ-semihypergroup and $\Xi = \langle \widetilde{\mu}_{\Xi}, \lambda_{\Xi}\rangle$ be a cubic set in H. Then, $\Xi = \langle \widetilde{\mu}_{\Xi}, \lambda_{\Xi}\rangle$ is a left (resp. right) cubic Γ-hyperideal of H if and only if for all $\widetilde{t} \in D[0,1]$ and $s \in [0,1]$, the set $U(\Xi; \widetilde{t}, s)$ is either empty or a left (resp. right) cubic Γ-hyperideal of H.*

Proof. Let us assume that the non-empty level set $U(\Xi; \widetilde{t}, s)$ is a left (resp. right) Γ-hyperideal of H. Let $x, y \in H$ and $\gamma \in \Gamma$. If $\widetilde{t}_1 = \widetilde{\mu}_{\Xi}(y)$ and $s_1 = \lambda_{\Xi}(y)$, then $y \in U(\Xi; \widetilde{t}_1, s_1)$. So $x\gamma y \subseteq U(\Xi; \widetilde{t}_1, s_1)$. Therefore, for all $z \in x\gamma y$, we have $\widetilde{\mu}_{\Xi}(z) \succeq \widetilde{t}_1$ and $\lambda_{\Xi}(z) \leq s_1$, and so

$$\operatorname{rinf}_{z\in x\gamma y}\{\widetilde{\mu}_{\Xi}(z)\} \succeq \widetilde{\mu}_{\Xi}(y) \quad \text{and} \quad \sup_{z\in x\gamma y}\{\lambda_{\Xi}(z)\} \leq \lambda_{\Xi}(y).$$

Hence, $\Xi = \langle \widetilde{\mu}_{\Xi}, \lambda_{\Xi}\rangle$ is a left cubic Γ-hyperideal of H.

Conversely, let $\Xi = \langle \widetilde{\mu}_{\Xi}, \lambda_{\Xi}\rangle$ be a left cubic Γ-hyperideal of H. Let $x \in H$, $\gamma \in \Gamma$ and $y \in U(\Xi; \widetilde{t}, s)$. We have $\operatorname{rinf}_{z\in x\gamma y}\{\widetilde{\mu}_{\Xi}(z)\} \succeq \widetilde{\mu}_{\Xi}(y) \succeq \widetilde{t}$ and $\sup_{z\in x\gamma y}\{\lambda_{\Xi}(z)\} \leq \lambda_{\Xi}(y) \leq s$. Therefore, for all $z \in x\gamma y$, we have $z \in U(\Xi; \widetilde{t}, s)$, and so $x\gamma y \subseteq U(\Xi; \widetilde{t}, s)$. This completes the proof. □

Definition 3.9. Let H be an LA-Γ-semihypergroup. Then the cubic characteristic function $\chi_{\Xi} = \langle \widetilde{\mu}_{\chi_{\Xi}}, \lambda_{\chi_{\Xi}}\rangle$ of $\Xi = \langle \widetilde{\mu}_{\Xi}, \lambda_{\Xi}\rangle$ is defined as

$$\widetilde{\mu}_{\chi_{\Xi}} = \begin{cases} [1,1] & \text{if } x \in \Xi \\ [0,0] & \text{if } x \notin \Xi \end{cases} \quad \text{and} \quad \lambda_{\chi_{\Xi}} = \begin{cases} 0 & \text{if } x \in \Xi \\ 1 & \text{if } x \notin \Xi \end{cases}$$

Theorem 3.10. *Let H be an LA-Γ-semihypergroup. The following statements are equivalent:*

(1) $\Xi = \langle \widetilde{\mu}_\Xi, \lambda_\Xi \rangle$ is a left (resp. right) cubic Γ-hyperideal of H.

(2) $\mathcal{H} *_\Gamma \Xi \subseteq \Xi$ (resp. $\Xi *_\Gamma \mathcal{H} \subseteq \Xi$), where $\mathcal{H} = \langle \widetilde{1}, 0 \rangle$, $\widetilde{1}(x) = \widetilde{1}$ and $0(x) = 0$ for all $x \in H$.

Proof. Let $\Xi = \langle \widetilde{\mu}_\Xi, \lambda_\Xi \rangle$ be a left cubic Γ-hyperideal of H and $a \in H$. Let us suppose that there exist $x, y \in H$ and $\gamma \in \Gamma$ such that $a \in x\gamma y$. Then, since Ξ is a left cubic Γ-hyperideal of H, we have

$$\begin{aligned}(\widetilde{1} *_\Gamma \widetilde{\mu}_\Xi)(a) &= \operatorname*{rsup}_{a \in x\gamma y}[\operatorname{rmin}\{\widetilde{1}(x), \widetilde{\mu}_\Xi(y)\}] \\ &= \operatorname*{rsup}_{a \in x\gamma y}[\operatorname{rmin}\{\widetilde{1}, \widetilde{\mu}_\Xi(y)\}] = \operatorname*{rsup}_{a \in x\gamma y} \widetilde{\mu}_\Xi(y)\end{aligned}$$

and

$$\begin{aligned}(0 *_\Gamma \lambda_\Xi)(a) &= \inf_{a \in x\gamma y}[\max\{0(x), \lambda_\Xi(y)\}] \\ &= \inf_{a \in x\gamma y}[\max\{0, \lambda_\Xi(y)\}] = \inf_{a \in x\gamma y} \lambda_\Xi(y).\end{aligned}$$

In case of $\Xi = \langle \widetilde{\mu}_\Xi, \lambda_\Xi \rangle$ is a left cubic Γ-hyperideal of H,

$$\operatorname*{rinf}_{z \in x\gamma y} \widetilde{\mu}_\Xi(z) \succeq \widetilde{\mu}_\Xi(y) \quad \text{and} \quad \sup_{z \in x\gamma y} \lambda_\Xi(z) \leq \lambda_\Xi(y).$$

So, in particular, $\widetilde{\mu}_\Xi(y) \preceq \widetilde{\mu}_\Xi(a)$ and $\lambda_\Xi(y) \geq \lambda_\Xi(a)$ for all $a \in x\gamma y$. Hence $\operatorname*{rsup}_{a \in x\gamma y} \widetilde{\mu}_\Xi(y) \preceq \widetilde{\mu}_\Xi(a)$ and $\inf_{a \in x\gamma y} \lambda_\Xi(y) \geq \lambda_\Xi(a)$. Thus, $\widetilde{\mu}_\Xi(a) \succeq (\widetilde{1} *_\Gamma \widetilde{\mu}_\Xi)(a)$ and $\lambda_\Xi(a) \leq (0 *_\Gamma \lambda_\Xi)(a)$. If there do not exist $x, y \in H$ and $\gamma \in \Gamma$ such that $a \in x\gamma y$, then $(\widetilde{1} *_\Gamma \widetilde{\mu}_\Xi)(a) = \widetilde{0} \preceq \widetilde{\mu}_\Xi(a)$ and $(0 *_\Gamma \lambda_\Xi)(a) = 1 \geq \lambda_\Xi(a)$. Hence we get $\mathcal{H} *_\Gamma \Xi \subseteq \Xi$.

Conversely, let $x, y \in H$, $\gamma \in \Gamma$ and $a \in x\gamma y$. Then, $\operatorname*{rinf}_{a \in x\gamma y} \widetilde{\mu}_\Xi(a) \succeq (\widetilde{1} *_\Gamma \widetilde{\mu}_\Xi)(a)$ and $\sup_{a \in x\gamma y} \lambda_\Xi(a) \leq (0 *_\Gamma \lambda_\Xi)(a)$. We have for all $\gamma \in \Gamma$,

$$\begin{aligned}(\widetilde{1} *_\Gamma \widetilde{\mu}_\Xi)(a) &= \operatorname*{rsup}_{a \in x\gamma y}[\operatorname{rmin}\{\widetilde{1}(x), \widetilde{\mu}_\Xi(y)\}] \succeq \operatorname{rmin}\{\widetilde{1}(x), \widetilde{\mu}_\Xi(y)\} \\ &= \operatorname{rmin}\{\widetilde{1}, \widetilde{\mu}_\Xi(y)\} = \widetilde{\mu}_\Xi(y)\end{aligned}$$

and

$$\begin{aligned}(0 *_\Gamma \lambda_\Xi)(a) &= \inf_{a \in x\gamma y}[\max\{0(x), \lambda_\Xi(y)\}] \leq \max\{0(x), \lambda_\Xi(y)\} \\ &= \max\{0, \lambda_\Xi(y)\} = \lambda_\Xi(y).\end{aligned}$$

Consequently, $\operatorname*{rinf}_{a \in x\gamma y} \widetilde{\mu}_\Xi(a) \succeq \widetilde{\mu}_\Xi(y)$ and $\sup_{a \in x\gamma y} \lambda_\Xi(a) \leq \lambda_\Xi(y)$. Hence, $\Xi = \langle \widetilde{\mu}_\Xi, \lambda_\Xi \rangle$ is a left cubic Γ-hyperideal of H. The other case can be seen in a similar way. □

Theorem 3.11. *Let H be an LA-Γ-semihypergroup. The following statements are equivalent:*

(1) $\Xi = \langle \widetilde{\mu}_\Xi, \lambda_\Xi \rangle$ is a cubic Γ-hyperideal of H.

(2) $\mathcal{H} *_\Gamma \Xi \subseteq \Xi$ and $\Xi *_\Gamma \mathcal{H} \subseteq \Xi$, $\mathcal{H} = \langle \widetilde{1}, 0 \rangle$ and $\widetilde{1}(x) = \widetilde{1}$ and $0(x) = 0$ for all $x \in H$.

Proof. The proof is similar to the proof of Theorem 3.10. □

Proposition 3.12. *Let $\Xi_1 = \langle \widetilde{\mu}_{\Xi_1}, \lambda_{\Xi_1} \rangle$ be a right cubic Γ-hyperideal of H and $\Xi_2 = \langle \widetilde{\mu}_{\Xi_2}, \lambda_{\Xi_2} \rangle$ be a left cubic Γ-hyperideal of H. Then $\Xi_1 *_\Gamma \Xi_2 \subseteq \Xi_1 \cap \Xi_2$.*

Proof. Let $\Xi_1 = \langle \widetilde{\mu}_{\Xi_1}, \lambda_{\Xi_1} \rangle$ be a right cubic Γ-hyperideal of H and $\Xi_2 = \langle \widetilde{\mu}_{\Xi_2}, \lambda_{\Xi_2} \rangle$ be a left cubic Γ-hyperideal of H. Let $x \in H$ and suppose that there exist $u, v \in H$ and $\gamma \in \Gamma$ such that $x \in u\gamma v$. Then

$$\begin{aligned}(\widetilde{\mu}_{\Xi_1} *_\Gamma \widetilde{\mu}_{\Xi_2})(x) &= \underset{x \in u\gamma v}{\text{rsup}} \{\text{rmin}\{\widetilde{\mu}_{\Xi_1}(u), \widetilde{\mu}_{\Xi_2}(v)\}\} \\ &\preceq \underset{x \in u\gamma v}{\text{rsup}} \left\{ \text{rmin}\{ \underset{x \in u\gamma v}{\text{rinf}} \{\widetilde{\mu}_{\Xi_1}(x)\}, \underset{x \in u\gamma v}{\text{rinf}} \{\widetilde{\mu}_{\Xi_2}(x)\}\} \right\} \\ &= \text{rmin}\{\widetilde{\mu}_{\Xi_1}(x), \widetilde{\mu}_{\Xi_2}(x)\} = (\widetilde{\mu}_{\Xi_1} \wedge \widetilde{\mu}_{\Xi_2})(x)\end{aligned}$$

and

$$\begin{aligned}(\lambda_{\Xi_1} *_\Gamma \lambda_{\Xi_2})(x) &= \inf_{x \in u\gamma v} \{\max\{\lambda_{\Xi_1}(u), \lambda_{\Xi_2}(v)\}\} \\ &\geq \inf_{x \in u\gamma v} \left\{ \max\{ \sup_{x \in u\gamma v} \{\lambda_{\Xi_1}(x)\}, \sup_{x \in u\gamma v} \{\lambda_{\Xi_2}(x)\}\} \right\} \\ &= \max\{\lambda_{\Xi_1}(x), \lambda_{\Xi_2}(x)\} = (\lambda_{\Xi_1} \vee \lambda_{\Xi_2})(x).\end{aligned}$$

Let us suppose there do not exist $u, v \in H$ such that $x \in u\gamma v$. Then, $(\widetilde{\mu}_{\Xi_1} *_\Gamma \widetilde{\mu}_{\Xi_2})(x) = [0, 0] \preceq (\widetilde{\mu}_{\Xi_1} \wedge \widetilde{\mu}_{\Xi_2})(x)$ and $(\lambda_{\Xi_1} *_\Gamma \lambda_{\Xi_2})(x) = 1 \geq (\lambda_{\Xi_1} \vee \lambda_{\Xi_2})(x)$. Hence the proof is completed. □

Definition 3.13. Let H be an LA-Γ-semihypergroup. A cubic sub LA-Γ-semihypergroup $\Xi = \langle \widetilde{\mu}_\Xi, \lambda_\Xi \rangle$ of H is called a cubic bi-Γ-hyperideal of H if

(1) $\underset{a \in (x\alpha y)\beta z}{\text{rinf}} \{\widetilde{\mu}_\Xi(a)\} \succeq \text{rmin}\{\widetilde{\mu}_\Xi(x), \widetilde{\mu}_\Xi(z)\}$,

(2) $\sup_{a \in (x\alpha y)\beta z} \{\lambda_\Xi(a)\} \leq \max\{\lambda_\Xi(x), \lambda_\Xi(z)\}$,

for all $x, y, z \in H$ and $\alpha, \beta \in \Gamma$.

Example 3.14. Let $H = \{a, b, c, d\}$ and $\Gamma = \{\alpha, \beta\}$ be the sets of binary hyperoperations defined below:

α	a	b	c	d
a	a	$\{a, d\}$	$\{a, d\}$	d
b	$\{a, d\}$	$\{b, c\}$	$\{b, c\}$	d
c	$\{a, d\}$	b	b	d
d	d	d	d	d

β	a	b	c	d
a	a	$\{a, d\}$	$\{a, d\}$	d
b	$\{a, d\}$	$\{b, c\}$	$\{b, c\}$	d
c	$\{a, d\}$	b	$\{b, c\}$	d
d	d	d	d	d

Clearly H is not a Γ-semihypergroup because $\{b, c\} = (c\alpha c)\beta b \neq c\alpha(c\beta b) = \{b\}$. Thus H is an LA-Γ-semihypergroup because it satisfies the left invertive law. Let $\Xi = \langle \widetilde{\mu}_\Xi, \lambda_\Xi \rangle$ be a cubic subset of H, which is defined as

H	$\widetilde{\mu}_\Xi$	λ_Ξ
a	$[0.4, 0.57]$	0.3
b	$[0.25, 0.29]$	0.6
c	$[0.25, 0.29]$	0.6
d	$[0.71, 0.77]$	0.2

By routine calculations, it can be seen that $\Xi = \langle \widetilde{\mu}_\Xi, \lambda_\Xi \rangle$ is a cubic bi-Γ-hyperideal of H.

Definition 3.15. Let H be an LA-Γ-semihypergroup. A cubic sub LA-Γ-semihypergroup $\Xi = \langle \widetilde{\mu}_\Xi, \lambda_\Xi \rangle$ of H is called a cubic $(1,2)$-Γ-hyperideal of H if

(1) $\underset{a\in(x\alpha w)\beta(y\gamma z)}{\text{rinf}} \{\widetilde{\mu}_\Xi(a)\} \succeq \text{rmin}\{\widetilde{\mu}_\Xi(x), \widetilde{\mu}_\Xi(y), \widetilde{\mu}_\Xi(z)\}$,

(2) $\sup\limits_{a\in(x\alpha w)\beta(y\gamma z)} \{\lambda_\Xi(a)\} \leq \max\{\lambda_\Xi(x), \lambda_\Xi(y), \lambda_\Xi(z)\}$,

for all $w, x, y, z \in H$ and $\alpha, \beta, \gamma \in \Gamma$.

Theorem 3.16. *Let H be an LA-Γ-semihypergroup. The following statements are equivalent:*

(1) $\Xi = \langle \widetilde{\mu}_\Xi, \lambda_\Xi \rangle$ is a cubic bi-Γ-hyperideal of H.

(2) $\Xi *_\Gamma \Xi \subseteq \Xi$ and $(\Xi *_\Gamma \mathcal{H}) *_\Gamma \Xi \subseteq \Xi$, where $\mathcal{H} = \langle \widetilde{1}, 0 \rangle$ and $\widetilde{1}(x) = \widetilde{1}$, and $0(x) = 0$ for all $x \in H$.

Proof. The proof is similar to the proof of Theorem 3.10. □

Theorem 3.17. *If $\{\Xi_i\}_{i\in\Lambda}$ is a family of cubic $(1,2)$-Γ-hyperideals of an LA-Γ-semihypergroup H, then $\bigcap\limits_{i\in\Lambda} \Xi_i$ is a cubic $(1,2)$-Γ-hyperideal of H, where $\bigcap\limits_{i\in\Lambda} \Xi_i = (\bigwedge\limits_{i\in\Lambda} \widetilde{\mu}_{\Xi_i}, \bigvee\limits_{i\in\Lambda} \lambda_\Xi)$ and*

$$\begin{aligned} \bigwedge_{i\in\Lambda} \widetilde{\mu}_{\Xi_i}(x) &= rinf\{\widetilde{\mu}_{\Xi_i}(x) : i \in \Lambda,\ x \in H\} \\ \bigvee_{i\in\Lambda} \lambda_{\Xi_i}(x) &= \sup\{\lambda_{\Xi_i}(x) : i \in \Lambda,\ x \in H\}. \end{aligned}$$

Proof. Let $x, y \in H$ and $\gamma \in \Gamma$. Then for every $z \in x\gamma y$, we have

$$\begin{aligned} \underset{z\in x\gamma y}{\text{rinf}} \left\{ \bigwedge_{i\in\Lambda} \widetilde{\mu}_{\Xi_i}(z) \right\} &= \bigwedge_{i\in\Lambda} \left\{ \underset{z\in x\gamma y}{\text{rinf}} \{\widetilde{\mu}_{\Xi_i}(z)\} \right\} \\ &\succeq \bigwedge_{i\in\Lambda} \{\text{rmin}\{\widetilde{\mu}_{\Xi_i}(x), \widetilde{\mu}_{\Xi_i}(y)\}\} \\ &= \text{rmin}\{\text{rmin}\{\widetilde{\mu}_{\Xi_i}(x)\}, \text{rmin}\{\widetilde{\mu}_{\Xi_i}(y)\}\} \\ &= \text{rmin}\left\{ \bigwedge_{i\in\Lambda} \widetilde{\mu}_{\Xi_i}(x), \bigwedge_{i\in\Lambda} \widetilde{\mu}_{\Xi_i}(y) \right\} \end{aligned}$$

and

$$\begin{aligned} \sup_{z\in x\gamma y} \left\{ \bigvee_{i\in\Lambda} \lambda_{\Xi_i}(z) \right\} &= \bigvee_{i\in\Lambda} \left\{ \sup_{z\in x\gamma y} \{\lambda_{\Xi_i}(z)\} \right\} \\ &\leq \bigvee_{i\in\Lambda} \{\max\{\lambda_{\Xi_i}(x), \lambda_{\Xi_i}(y)\}\} \\ &= \max\{\max\{\lambda_{\Xi_i}(x)\}, \max\{\lambda_{\Xi_i}(y)\}\} \\ &= \max\left\{ \bigvee_{i\in\Lambda} \lambda_{\Xi_i}(x), \bigvee_{i\in\Lambda} \lambda_{\Xi_i}(y) \right\}. \end{aligned}$$

Hence $\bigcap_{i\in\Lambda} \Xi_i$ is a cubic sub LA-Γ-semihypergroup. Now for $x, y, z, t \in H$ and $\alpha, \beta, \gamma \in \Gamma$, we have

$$\begin{aligned}
\operatorname*{rinf}_{m\in(x\alpha t)\beta(y\gamma z)} \left\{ \bigwedge_{i\in\Lambda} \widetilde{\mu}_{\Xi_i}(m) \right\} &= \bigwedge_{i\in\Lambda} \left\{ \operatorname*{rinf}_{m\in(x\alpha t)\beta(y\gamma z)} \{\widetilde{\mu}_{\Xi_i}(m)\} \right\} \\
&\succeq \bigwedge_{i\in\Lambda} \{\mathrm{rmin}\{\widetilde{\mu}_{\Xi_i}(x), \widetilde{\mu}_{\Xi_i}(y), \widetilde{\mu}_{\Xi_i}(z)\}\} \\
&= \mathrm{rmin}\{\mathrm{rmin}\{\widetilde{\mu}_{\Xi_i}(x)\}, \mathrm{rmin}\{\widetilde{\mu}_{\Xi_i}(y)\}, \mathrm{rmin}\{\widetilde{\mu}_{\Xi_i}(z)\}\} \\
&= \mathrm{rmin}\left\{ \bigwedge_{i\in\Lambda} \widetilde{\mu}_{\Xi_i}(x), \bigwedge_{i\in\Lambda} \widetilde{\mu}_{\Xi_i}(y), \bigwedge_{i\in\Lambda} \widetilde{\mu}_{\Xi_i}(z) \right\}
\end{aligned}$$

and

$$\begin{aligned}
\sup_{m\in(x\alpha t)\beta(y\gamma z)} \left\{ \bigvee_{i\in\Lambda} \lambda_{\Xi_i}(m) \right\} &= \bigvee_{i\in\Lambda} \left\{ \sup_{m\in(x\alpha t)\beta(y\gamma z)} \{\lambda_{\Xi_i}(m)\} \right\} \\
&\leq \bigvee_{i\in\Lambda} \{\max\{\lambda_{\Xi_i}(x), \lambda_{\Xi_i}(y), \lambda_{\Xi_i}(z)\}\} \\
&= \max\{\max\{\lambda_{\Xi_i}(x)\}, \max\{\lambda_{\Xi_i}(y)\}, \max\{\lambda_{\Xi_i}(z)\}\} \\
&= \max\left\{ \bigvee_{i\in\Lambda} \lambda_{\Xi_i}(x), \bigvee_{i\in\Lambda} \lambda_{\Xi_i}(y), \bigvee_{i\in\Lambda} \lambda_{\Xi_i}(z) \right\}.
\end{aligned}$$

Hence this shows that $\bigcap_{i\in\Lambda} \Xi_i$ is a cubic $(1, 2)$-Γ-hyperideal of H. □

Theorem 3.18. *Let H be an LA-Γ-semihypergroup. Then, every right cubic Γ-hyperideal of H is a cubic $(1, 2)$-Γ-hyperideal of H.*

Proof. Let $\Xi = \langle \widetilde{\mu}_\Xi, \lambda_\Xi \rangle$ be a right cubic Γ-hyperideal of H. It is easy to prove that every right cubic Γ-hyperideal of H is a cubic sub LA-Γ-semihypergroup of H. Now let $t, x, y, z \in H$ and $\alpha, \beta, \gamma \in \Gamma$. Then, for every $m \in (x\alpha t)\beta(y\gamma z)$, we have

$$\begin{aligned}
\operatorname*{rinf}_{m\in(x\alpha t)\beta(y\gamma z)} \{\widetilde{\mu}_\Xi(a)\} &\succeq \operatorname*{rinf}_{m\in c\beta(y\gamma z)} \{\widetilde{\mu}_\Xi(a)\} \text{ for each } c \in x\alpha t \\
&\succeq \mathrm{rmin}\{\widetilde{\mu}_\Xi(c), \widetilde{\mu}_\Xi(y), \widetilde{\mu}_\Xi(z)\} \\
&\succeq \mathrm{rmin}\{\mathrm{rmin}\{\widetilde{\mu}_\Xi(x)\}, \widetilde{\mu}_\Xi(y), \widetilde{\mu}_\Xi(z)\} \\
&\quad (\text{because } \operatorname*{rinf}_{c\in x\alpha t}\{\widetilde{\mu}_\Xi(c)\} \succeq \mathrm{rmin}\{\widetilde{\mu}_\Xi(x)\}) \\
&= \mathrm{rmin}\{\widetilde{\mu}_\Xi(x), \widetilde{\mu}_\Xi(y), \widetilde{\mu}_\Xi(z)\}
\end{aligned}$$

and

$$\begin{aligned}
\sup_{m\in(x\alpha t)\beta(y\gamma z)} \{\lambda_\Xi(a)\} &\leq \sup_{m\in c\beta(y\gamma z)} \{\lambda_\Xi(a)\} \text{ for each } c \in x\alpha t \\
&\leq \max\{\lambda_\Xi(c), \lambda_\Xi(y), \lambda_\Xi(z)\} \\
&\leq \max\{\max\{\lambda_\Xi(x)\}, \lambda_\Xi(y), \lambda_\Xi(z)\} \\
&\quad (\text{because } \sup_{c\in x\alpha t}\{\lambda_\Xi(c)\} \leq \max\{\lambda_\Xi(x)\}) \\
&= \max\{\lambda_\Xi(x), \lambda_\Xi(y), \lambda_\Xi(z)\}.
\end{aligned}$$

Hence $\Xi = \langle \widetilde{\mu}_\Xi, \lambda_\Xi \rangle$ is a cubic $(1,2)$-Γ-hyperideal of H. □

Definition 3.19. Let H be an LA-Γ-semihypergroup and $\Xi = \langle \widetilde{\mu}_\Xi, \lambda_\Xi \rangle$ be a cubic subset of H. Then, $\Xi = \langle \widetilde{\mu}_\Xi, \lambda_\Xi \rangle$ is called a cubic M-hypersystem of H if for all $x, y, z \in H$ and $\alpha, \beta \in \Gamma$, we have

$$\begin{aligned} \operatorname*{rinf}_{a \in x\alpha(y\beta z)} \widetilde{\mu}_\Xi(a) &\succeq \operatorname{rmin}\{\widetilde{\mu}_\Xi(x), \widetilde{\mu}_\Xi(z)\} \\ \sup_{a \in x\alpha(y\beta z)} \lambda_\Xi(a) &\leq \max\{\lambda_\Xi(x), \lambda_\Xi(z)\}. \end{aligned}$$

Definition 3.20. Let H be an LA-Γ-semihypergroup and $\Xi = \langle \widetilde{\mu}_\Xi, \lambda_\Xi \rangle$ be a cubic subset of H. Then, $\Xi = \langle \widetilde{\mu}_\Xi, \lambda_\Xi \rangle$ is called a cubic N-hypersystem of H if for all $x, y \in H$ and $\alpha, \beta \in \Gamma$, we have

$$\begin{aligned} \operatorname*{rinf}_{a \in x\alpha(y\beta x)} \widetilde{\mu}_\Xi(a) &\succeq \widetilde{\mu}_\Xi(x) \\ \sup_{a \in x\alpha(y\beta x)} \widetilde{\mu}_\Xi(a) &\leq \widetilde{\mu}_\Xi(x). \end{aligned}$$

Example 3.21. Let $H = \{a, b, c\}$ and $\Gamma = \{\alpha, \beta\}$ be the sets of binary hyperoperations defined below:

α	a	b	c
a	$\{a,b\}$	$\{a,b\}$	c
b	$\{a,c\}$	$\{a,c\}$	c
c	c	c	c

β	a	b	c
a	$\{a,b\}$	$\{a,b\}$	c
b	$\{a,b,c\}$	$\{a,b,c\}$	c
c	c	c	c

Clearly H is not a Γ-semihypergroup because $\{a,b,c\} = (a\alpha a)\beta b \neq a\alpha(a\beta b) = \{a,b\}$. Thus H is an LA-Γ-semihypergroup because it satisfies the left invertive law. Let $\Xi = \langle \widetilde{\mu}_\Xi, \lambda_\Xi \rangle$ be a cubic subset of H, which is defined as

H	$\widetilde{\mu}_\Xi$	λ_Ξ
a	$[0.1, 0.2]$	0.5
b	$[0.1, 0.2]$	0.5
c	$[0.3, 0.5]$	0.2

By routine calculations, it can be seen that $\Xi = \langle \widetilde{\mu}_\Xi, \lambda_\Xi \rangle$ is a cubic M-hypersystem and also cubic N-hypersystem of H.

Remark. Every cubic M-hypersystem of an LA-Γ-semihypergroup H is a cubic N-hypersystem.

Theorem 3.22. *If $\{\Xi_i\}_{i\in\Lambda}$ is a family of cubic M-hypersystems of an LA-Γ-semihypergroup H, then $\bigcap_{i\in\Lambda} \Xi_i = \left(\bigwedge_{i\in\Lambda} \widetilde{\mu}_{\Xi_i}, \bigvee_{i\in\Lambda} \lambda_\Xi \right)$ is also a cubic M-hypersystem of H, where*

$$\begin{aligned} \bigwedge_{i\in\Lambda} \widetilde{\mu}_{\Xi_i}(x) &= r\inf\{\widetilde{\mu}_{\Xi_i}(x) : i \in \Lambda,\ x \in H\} \\ \bigvee_{i\in\Lambda} \lambda_{\Xi_i}(x) &= \sup\{\lambda_{\Xi_i}(x) : i \in \Lambda,\ x \in H\}. \end{aligned}$$

Proof. Let $x, y, z \in H$ and $\alpha, \beta \in \Gamma$. Then, for every $a \in x\alpha(y\beta z)$, we have

$$\begin{aligned}
\underset{a\in x\alpha(y\beta z)}{\text{rinf}}\left\{\bigwedge_{i\in\Lambda}\widetilde{\mu}_{\Xi_i}(a)\right\} &= \underset{a\in x\alpha(y\beta z)}{\text{rinf}}\left\{\bigwedge_{i\in\Lambda}\{\widetilde{\mu}_{\Xi_i}(a)\}\right\} \\
&\succeq \left\{\bigwedge_{i\in\Lambda}(\text{rmin}\{\widetilde{\mu}_{\Xi_i}(x), \widetilde{\mu}_{\Xi_i}(z)\})\right\} \\
&= \text{rmin}\left\{\bigwedge_{i\in\Lambda}\widetilde{\mu}_{\Xi_i}(x), \bigwedge_{i\in\Lambda}\widetilde{\mu}_{\Xi_i}(z)\right\},
\end{aligned}$$

and

$$\begin{aligned}
\sup_{a\in x\alpha(y\beta z)}\left\{\bigvee_{i\in\Lambda}\lambda_{\Xi_i}(a)\right\} &= \sup_{a\in x\alpha(y\beta z)}\left\{\bigvee_{i\in\Lambda}\{\lambda_{\Xi_i}(a)\}\right\} \\
&\leq \left\{\bigvee_{i\in\Lambda}(\max\{\lambda_{\Xi_i}(x), \lambda_{\Xi_i}(z)\})\right\} \\
&= \max\left\{\bigvee_{i\in\Lambda}\lambda_{\Xi_i}(x), \bigvee_{i\in\Lambda}\lambda_{\Xi_i}(z)\right\}.
\end{aligned}$$

Hence, $\bigcap_{i\in\Lambda}\Xi_i$ is a cubic M-hypersystem of LA-Γ-semihypergroup H. □

Corollary 3.23. *If $\{\Xi_i\}_{i\in\Lambda}$ is a family of cubic N-hypersystems of an LA-Γ-semihypergroup H, then $\bigcap_{i\in\Lambda}\Xi_i$ is also a cubic N-hypersystem of H. Where $\bigcap_{i\in\Lambda}\Xi_i = (\bigwedge_{i\in\Lambda}\widetilde{\mu}_{\Xi_i}, \bigvee_{i\in\Lambda}\lambda_{\Xi})$ and*

$$\begin{aligned}
\bigwedge_{i\in\Lambda}\widetilde{\mu}_{\Xi_i}(x) &= r\inf\{\widetilde{\mu}_{\Xi_i}(x) : i\in\Lambda,\ x\in H\} \\
\bigvee_{i\in\Lambda}\lambda_{\Xi_i}(x) &= \sup\{\lambda_{\Xi_i}(x) : i\in\Lambda,\ x\in H\}.
\end{aligned}$$

Proof. The proof is similar to the proof of Theorem 3.22. □

Proposition 3.24. *Every two sided cubic Γ-hyperideal of an LA-Γ-semihypergroup H is a cubic M-hypersystem of H.*

Proof. The proof is straightforward. □

Corollary 3.25. *Every one sided cubic Γ-hyperideal of an LA-Γ- semihypergroup H is a cubic N-hypersystem of H.*

Proof. The proof is straightforward. □

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