Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 5, No. 1, (January 2013), pp. 147–155
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr

@FMI

Fixed point theorems in M-fuzzy metric spaces

Deepak Singh, Mayank Sharma, Ramakant Sharma

Received 5 February 2012; Accepted 9 April 2012

Abstract. The purpose of this paper is to prove common fixed point theorems containing rational term in M-fuzzy metric spaces, while proving our results, we utilize the idea of compatible mappings of type (*) due to J.H.Park et.al [12]. Our results suggest a path to rational version of various fixed point theorems in existing literature of M-fuzzy metric spaces.

2010 AMS Classification: 54H25, 47H10.

Keywords: M-fuzzy metric spaces, Fixed point, Compatible of type (*).

Corresponding Author: Deepak Singh (deepak.singh.2006@indiatimes.com)

1. Introduction

The theory of fuzzy sets is proposed by Zadeh [14] . Deng [2],Erceg [3], Kaleva and Seikkala [7] and Kramosil and Michalek [8] introduced the concepts of fuzzy metric spaces in different ways. George and Veeramani [4] modified the concept of fuzzy metric spaces due to Kramosil and Michalek [8] and defined the Hausdorff topology of fuzzy metric spaces. Recently, many authors (Chang et al. [1]; Jung et al. [6]; Mishra et al. [9]; Park et al. [10] have also studied the fixed point theory in these fuzzy metric spaces. Sedghi et al. [13] introduced the concept of M-fuzzy metric spaces which is a generalization of fuzzy metric spaces due to George and Veeramani [4] and proved common fixed point theorems for two mappings under the conditions of weak compatible and R-weakly commuting mappings in complete M-fuzzy metric spaces. In a paper, J. H.Park et al. [11] introduced the concept of compatible mapping of type (*) in M-fuzzy metric spaces and established common fixed point theorems for five mappings satisfying some conditions. In this paper we show that every D^*-metric and fuzzy metric induces a M-fuzzy metric respectively, further we proved the common fixed point theorems using the idea of compatible mappings of type (*) and rational inequality satisfying some conditions.

2. Preliminaries

Definition 2.1 ([13])**.** Let X be a nonempty set. A generalized metric (or D^*-metric) on X is a function $D : X^3 \to R^+$ satisfying the following conditions for all $x, y, z, a \in X$, satisfying the following conditions:

(D-1) $D^*(x, y, z) \geq 0$, for all $a \in [0, 1]$,
(D-2) $D^*(x, y, z) = 0$ if and only if $x = y = z$,
(D-3) $D^*(x, y, z) = D^*(px, y, z)$ (symmetry), where p is a permutation function
(D-4) $D^*(x, y, z) \leq D^*(x, y, a) + D^*(a, z, z)$ for all $a, b, c \in [0, 1]$.

The pair (X, D^*) is called a generalized metric (or D^*-metric) space. Immediate examples of D^*-metric are

(a) $D^*(x, y, z) = \max d(x, y), d(y, z), d(z, x)$,
(b) $D^*(x, y, z) = d(x, y) + d(y, z) + d(z, x)$, where d is the ordinary metric on X.

Definition 2.2 ([12])**.** A binary operation $* : [0, 1] \times [0, 1] \to [0, 1]$ is called a continuous t-norm if $([0, 1], *)$ is an abelian topological monoid with unit 1 such that $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0, 1]$.

Typical examples of continuous t-norm are $a * b = ab$ and $a * b = \min (a, b)$.

Definition 2.3 ([13])**.** The 3-tuple $(X, M, *)$ is called a M-fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm, and $M$is a fuzzy set on $X^3 \times (0, \infty)$ satisfying the following conditions for all $x, y, z, a \in X$ and $t, s > 0$,

(FM-1) $M(x, y, z, t) > 0$,
(FM-2) $M(x, y, z, t) = 1$ if and only if $x = y = z$,
(FM-3) $M(x, y, z, t) = M(px, y, z, t)$ (symmetry), where p is a permutation function,
(FM-4) $M(x, y, a, t) * M(a, z, z, s) \leq M(x, y, z, t + s)$,
(FM-5) $M(x, y, z, \grave{A}) : (0, \infty) \times [0, 1]$ is continuous.

In the following examples, we know that both D^*-metric and fuzzy metric induce a M-fuzzy metric.

Example 2.4. Let $(X, D, *)$ be a D^*-metric space, where $a * b = a \times b$ for all $a, b \in [0, 1]$ and for all $x, y, z \in X$ and $t > 0$, and $M(x, y, z, t) = \frac{t}{t+D^*(x,y,z)}$. Then $(X, M, *)$ is a M-fuzzy metric space.

Example 2.5. Let $(X, M, *)$ be a fuzzy metric space. If we define

$$M : X^3 \times (0, \infty) \to [0, 1]$$

by $M(x, y, z, t) = M(x, y, t) * M(y, z, t * M(z, x, t)$, then $(X, M, *)$ is a M-fuzzy metric space.

Lemma 2.6 ([13])**.** *Let $(X, M, *)$ be a M-fuzzy metric space. For any $x, y, z \in X$ and $t > 0$, we have*

(a) $M(x, x, y, t) = M(x, y, y, t)$.
(b) $M(x, y, z, \grave{A})$ *is nondecreasing.*

Definition 2.7 ([13])**.** Let $(X, M, *)$ be a M-fuzzy metric space and x_n be a sequence in X.

(a) x_n is said to be convergent to a point $x \in X$ (denoted by $\lim_{n\to\infty} x_n = x$) if

$\lim_{n\to\infty} M(x, x, x_n, t) = 1$ for all $t > 0$.

(b) x_n is called a Cauchy sequence if $\lim_{n\to\infty} M(x_(n+p), x_(n+p), x_n, t) = 1$ for all $t > 0$ and $p > 0$.

(c) A M-fuzzy metric in which every Cauchy sequence is convergent is said to be complete.

Remark 2.8. Since $*$ is continuous, it follows from (FM-4) that the limit of sequence is uniquely determined.

Let $(X, M, *)$ be a M-fuzzy metric space with the following condition:
(FM-6) $\lim_{n\to\infty} M(x, y, z, t) = 1$ for all $x, y, z \in X$ and $t > 0$.

Lemma 2.9 ([11]). *Let x_n be a sequence in a M-fuzzy metric space $(X, M, *)$ with the condition (FM-6). If there exists a number $k \in (0, 1)$ such that*
$M(x_{n+p}, x_{n+p}, x_n, kt) \geq M(x_{n+1}, x_n, x_n, t)$
for all $t > 0$ and $n = 1, 2, ...,$ then x_n is a Cauchy sequence in X.

Lemma 2.10 ([11]). *Let $(X, M, *)$ be a M-fuzzy metric space with the condition (FM-6). If, for all $x, y \in X$ and for a number $k \in (0, 1)$,*
$M(x, y, z, kt) \geq M(x, y, z, t)$,
then $x = y = z$.

3. Compatible mappings of type (*)

Definition 3.1 ([13]). Let A and$B$be mappings from a M-fuzzy metric space $(X, M, *)$ into itself. The mappings are said to be compatible if
$\lim_{n\to\infty} M(ABx_n, BAx_n, BAx_n, t) = 1$ for all $t > 0$,
whenever x_n is a sequence in X such that
$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = z$ for some$z \in X$.

J. H. Park et al. [11] introduced the concept of compatible mappings of type $(*)$ as follows.

Definition 3.2 ([11]). Let$A$ and B be mappings from a M-fuzzy metric space$(X, M, *)$ into itself. The mappings are said to be compatible of type $(*)$ if
$\lim_{n\to\infty} M(ABx_n, BAx_n, BAx_n, t) = 1$ and $\lim_{n\to\infty} M(BAx_n, AAx_n, AAx_n, t) = 1$ for all $t > 0$, whenever x_n is a sequence in Xsuch that
$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = z$ for some $z \in X$.

Proposition 3.3. *Let $(X, M, *)$ be a M-fuzzy metric space and A and B be continuous mappings from Xinto itself. Then A and B are compatible if and only if they are compatible of type $(*)$.*

Proof. Let x_n be a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = z$ for some $z \in X$. Since A is continuous, we have

$$\lim_{n\to\infty} AAx_n = \lim_{n\to\infty} ABx_n = Az.$$

Further, since A and B are compatible, we get

$$\lim_{n\to\infty} M(ABx_n, BAx_n, BAx_n, t) = 1,$$

for all $t > 0$. Thus, from the inequality

$M(AAx_n, BAx_n, BAx_n, t) \geq M(AAx_n, AAx_n, ABx_n, t)*M(ABx_n, BAx_n, BAx_n, t)$,

it follows that $M(AAx_n, BAx_n, BAx_n, t) = 1$. Similarly, we also obtain

$$M(BBx_n, BBx_n, ABx_n, t) = 1.$$

Hence, A and$B$ are compatible of type $(*)$.

Conversely, suppose that x_n is a sequence in X such that $\lim_{n\to\infty} Ax_n = z$and $\lim_{n\to\infty} Bx_n = z$ for some $z \in X$. Then, since B is continuous, we have

$$\lim_{n\to\infty} BAx_n = \lim_{n\to\infty} BBx_n = Bz.$$

Since A and B are compatible of type $(*)$, we get

$$\lim_{n\to\infty} M(ABx_n, BBx_n, BBx_n, t/2) = \lim_{n\to\infty} M(BAx_n, AAx_n, AAx_n, t/2) = 1$$

for all $t > 0$. Hence, from the inequality

$M(ABx_n, BAx_n, BAx_n, t) \geq M(ABx_n, BBx_n, BBx_n, t)*M(BBx_n, BBx_n, BAx_n, t)$,

it follows that $\lim_{n\to\infty} M(ABx_n, BAx_n, BAx_n, t) \geq 1 * 1 \geq 1$ and so

$$\lim_{n\to\infty} M(ABx_n, BAx_n, BAx_n, t) = 1.$$

Hence, A and B are compatible. □

Proposition 3.4. *Let $(X, M, *)$ be a M-fuzzy metric space and A and B be mappings from X into itself. If A and B are compatible of type $(*)$ and $Az = Bz$ for some $z \in X$, then $ABz = BBz = BAz = AAz$.*

Proof. Let x_n be a sequence in X defined by $x_n = z$ for some $z \in X$ and $n = 1, 2...$ and $Az = Bz$. Then we have $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = Az$. Since A and B are compatible of type (*), we get

$$M(ABz, BBz, BBz, t) = \lim_{n\to\infty} M(ABx_n, BBx_n, BBx_n, t) = 1$$

and hence $ABz = BBz$. Similarly, we have $BAz = AAz$. But, $Az = Bz$ implies $BBz = BAz$. Therefore, we obtain $ABz = BBz = BAz = AAz$. □

Proposition 3.5. *Let $(X, M, *)$ be a M-fuzzy metric space and A and B be mappings from X into itself. If A and B are compatible of type $(*)$ and x_n is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = z$ for some $z \in X$, then*

(a) $\lim_{n\to\infty} BAx_n = Az$ *if* A*is continuous at* z.

(b) $ABz = BAz and Az = Bz$ *if A and B are continuous at z.*

Proof. (a) Since A is continuous at z and $\lim_{n\to\infty} Ax_n = z, \lim_{n\to\infty} AAx_n = Az$. Since$A$ and B is compatible of type $(*)$, for all $t > 0$, we have

$$\lim_{n\to\infty} M(BAx_n, AAx_n, AAx_n, t) = 1$$

and thus from (FM-4) we get

$\lim_{n\to\infty} M(BAx_n, Az, Az, t)$
$\geq \lim_{n\to\infty} M(BAx_n, AAx_n, AAx_(n,)t/2) * \lim_{n\to\infty} M(AAx_n, Az, Az, t/2)$
≥ 1,

i.e., $\lim_{n\to\infty} M(BAx_n, Az, Az, t) = 1$. Hence we have$\lim_{n\to\infty} BAx_n = Az$.

(b) $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = z$ and A and B are continuous at z, by (a), we have

$$\lim_{n\to\infty} ABx_n = Az and \lim_{n\to\infty} BAx_n = Bz.$$

Thus we obtain $Az = Bz$ by the uniqueness of the limit and so by Proposition 2.2, we have $BAz = ABz$. □

4. Common fixed point theorems via rational terms

In this section, we prove some common fixed point theorems for mappings satisfying some rational conditions, leading to a path to obtain rational versions of fixed point theorems in M-fuzzy metric spaces.

Theorem 4.1. *Let $(X, M, *)$ be a complete M-fuzzy metric space with $t * t \geq t$ for all $t \in [0, 1]$ and the condition (FM-6). Let A, B, S, T and P be mappings from X into itself such that*
(i) *$P(X) \subset AB(X)$ and $P(X) \subset ST(X)$,*
(ii) *there exists a number $k \in (0, 1)$ such that*

$$M(Px, Py, Py, kt) \geq$$
$$M(ABx, Px, Px, t) * M(Px, STy, STy, t) * M(ABx, STy, STy, t) *$$
$$\frac{M(Px, ABx,, ABx, t) * M(Px, STy, STy, t)}{M(STy, ABx, ABx, t)} * M(ABx, Py, Py, (3-\alpha)t)$$

for all $x, y \in X$,$\alpha \in (0, 3)$ and $t > 0$,
(iii) *$PB = BP$, $PT = TP$, $AB = BA$ and $ST = TS$,*
(iv) *A and B are continuous,*
(v) *the pair P, AB are compatible of type $(*)$,*
(vi) *$M(x, STx, STx, t) \geq M(x, ABxABx, t)$ for all $x \in X$ and $t > 0$.*
Then A,B,S,T and P have a common fixed point in X.

Proof. Since $P(X) \subset AB(X)$, for ant $x_0 \in X$, we can choose a point $x_0 \in X$ such that $Px_0 = ABx_1$. Since $P(X) \subset ST(X)$, for this point x_1, we can choose a point $x_2 \in X$ such that $Px_1 = STx_2$. Thus by induction, we can define a sequence $y_n \in X$ as follows: $y_{2n} = Px_{2n} = ABx_{2n+1}$ and $y_{2n+1} = Px_{2n+1} = STx_{2n+1}$ for $n = 1, 2, ...$ By (ii), for all $t > 0$ and $\alpha = 2 - q$ with $q \in (0, 2)$, we have

$$\begin{aligned} M(y_{2n+1}, y_{2n+2}, y_{2n+2}, kt) &= M(Px_{2n+1}, Px_{2n+2}, Px_{2n+2}, kt) \\ &\geq M(y_{2n+1}, y_{2n+1}, y_{2n+1}, t) * M(y_{2n}, y_{2n+1}, y_{2n+1}, t) * M(y_{2n}, y_{2n+1}, y_{2n+1}, t) * \\ &\quad \frac{M(y_{2n+1}), y_{2n}, y_{2n}, t) * M(y_{2n+1}, y_{2n+1}, y_{2n+1}, t)}{M(y_{2n+1}, y_{2n}, y_{2n}, t)} * M(y_{2n}, y_{2n+2}, y_{2n+2}, (1+q)t), \end{aligned}$$

$$\begin{aligned} M(y_{2n+1}, y_{2n+2}, y_{2n+2}, kt) &\geq M(y_{2n}, y_{2n+1}, y_{2n+1}, t) * M(y_{2n}, y_{2n+2}, y_{2n+2}, (1+q)t) \\ &\geq M(y_{2n}, y_{2n+1}, y_{2n+1}, t) * M(y_{2n}, y_{2n+1}, y_{2n+1}, t) * M(y_{2n+1}, y_{2n+2}, y_{2n+2}, qt) \\ &\geq M(y_{2n}, y_{2n+1}, y_{2n+1}, t) * M(y_{2n+1}, y_{2n+2}, y_{2n+2}, t) \end{aligned}$$

as $q \to 1$. Since $*$ is continuous and $M(x, y, z, *)$ is continuous, letting $q \to 1$ in above equation,we get

$$\begin{aligned} &M(y_{2n+1}, y_{2n+2}, y_{2n+2}, kt) \\ &\quad \geq M(y_{2n}, y_{2n+1}, y_{2n+1}, t) * M(y_{2n+1}, y_{2n+2}, y_{2n+2}, t)...... \end{aligned} \tag{1}$$

Similarly, we have

$$\begin{aligned} &M(y_{2n+2}, y_{2n+3}, y_{2n+3}, kt) \\ &\quad \geq M(y_{2n+1}, y_{2n+2}, y_{2n+2}, t) * M(y_{2n+2}, y_{2n+2}, y_{2n+2}, t) \;...... \end{aligned} \tag{2}$$

Thus from (1) and (2), it follows that

$$M(y_{n+1}, y_{n+2}, y_{n+2}, kt) \geq M(y_n, y_{n+1}, y_{n+1}, t) * M(y_{n+1}, y_{n+2}, y_{n+2}, t)$$

for $n = 1, 2, ...$ and then for positive integers n and p,

$$M(y_{n+1}, y_{n+2}, y_{n+2}, kt) \geq M(y_n, y_{n+1}, y_{n+1}, t) * M(y_{n+1}, y_{n+2}, y_{n+2}, t/k^p).$$

Thus, since $M(y_{n+1}, y_{n+1}, y_{n+1}, t/k^p) \to 1$ as $p \to \infty$ we have

$$M(y_{n+1}, y_{n+2}, y_{n+2}, kt) \geq M(y_n, y_{n+1}, y_{n+1}, t).$$

By Lemma 2.9, y_n is a Cauchy sequence in X and since X is complete, y_n converges to a point $z \in X$. Since Px_n, ABx_{2n+1} and STx_{2n+2} are subsequences of y_n, they also converge to the point$z$. Since A, B are continuous and the pair P, AB is compatible of type $(*)$, by Proposition 3.5 (a), we have

$$\lim_{n\to\infty} PABx_{2n+1} = ABz \;\text{ and }\; \lim_{n\to\infty} (AB)^2 x_{2n+1} = ABz.$$

By (ii) with $\alpha = 2$, we get

$$\begin{aligned}&M(PABx_{2n+1}, Px_{2n+2}, Px_{2n+2}, kt) \geq M((AB)^2x_{2n+1}, PABx_{2n+1}, PABx_{2n+1}, t)*\\ &M(PABx_{2n+1}, STx_{2n+2}, STx_{2n+2}, t) * M((AB)^2x_{2n+1}, STx_{2n+2}, STx_{2n+2}, t)*\\ &\frac{M(PABx_{2n+1},(AB)^2x_{2n+1},(AB)^2x_{2n+1},t)*M(PABx_(2n+1),STx_{2n+2},STx_{2n+2},t)}{M(STx_(2n+2,(AB)^2x_(2n+1,(AB)^2x_{2n+1},t)}*\\ &M((AB)^2x_{2n+1}, Px_{2n+2}, Px_{2n+2}, t)\end{aligned}$$

which implies that
$M(ABz, z, z, kt) = \lim_{n\to\infty} M(PABx_{2n+2}, Px_{2n+2}, Px_{2n+2}, kt)$
$\geq 1 * M(ABz, z, z, t) * M(ABz, z, z, t) * (\frac{1*M(ABz,z,z,t)}{M(z,ABz,ABz,t)}) * M(ABz, z, z, t).$
By Lemma 2.10, we have $ABz = z$ By (vi), since $M(z, z, STz, t) \geq M(z, z, ABz, t) = 1$ for all $t > 0$, we get $STz = z$. Again, by (ii) with $\alpha = 2$, we have

$$\begin{aligned}M(PABx_{2n+1}, Pz, Pz, kt) \geq\; &M((AB)^2x_{2n+1}, PABx_{2n+1}, PABx_{2n+1}, t)*\\ &M(PABx_{2n+1}, STz, STz, t) * M((AB)^2x_{2n+1}, STz, STz, t)*\\ &\frac{M(PABx_{2n+1},(AB)^2x_{2n+1},(AB)^2x_{2n+1},t)*M(PABx_(2n+1),STz,STz,t)}{M(STz,(AB)^2x_{2n+1},(AB)^2x_{2n+1},t)}*\\ &M((AB)^2x_{2n+1}, Pz, Pz, t)\end{aligned}$$

which implies that
$M(ABz, Pz, Pz, kt) = \lim_{n\to\infty} M(PABx_{2n+1}, Pz, Pz, kt)$
$\geq 1 * 1 * 1 * 1 * M(ABz, Pz, Pz, t) \geq M(ABz, Pz, Pz, t).$
By Lemma 2.10, we have $ABz = Pz$. Now, we show that $Bz = z$. In fact, by (ii) with $\alpha = 2$ and (iii), we get

$$\begin{aligned}&M(Bz, z, z, kt) = M(BPz, Pz, Pz, kt) = M(PBz, Pz, Pz, kt)\\ &M(PBz, Pz, Pz, kt) \geq M(PBz, STz, STz, t) * M(ABBz, STz, STz, t)*\\ &\frac{M(PBz,ABBz,,ABBz,t)*M(PBz,z,z,t)}{M(z,PBz,PBz,t)} * M(PBz, z, z, t)\\ &= 1 * M(Bz, z, z, t) * M(Bz, z, z, t) * 1 * M(Bz, z, z, t)\\ &= M(Bz, z, z, t)\end{aligned}$$

which implies that $Bz = z$. Since $ABz = z$, we have $Az = z$. Next, we show that $Tz = z$. Indeed, by (ii) with $\alpha = 2$ and (iii), we get

$$\begin{aligned} &M(Tz, z, z, kt) = M(TPz, Pz, Pz, kt) = M(Pz, Pz, TPz, kt) \\ &\geq 1 * M(z, Tz, Tz, t) * M(z, Tz, Tz, t) * 1 * M(z, Tz, Tz, t) \\ &\geq M(Tz, z, z, t) \end{aligned}$$

which implies that $Tz = z$. Since $STz = z$, we have $Sz = STz = z$. Therefore, by combining the above results, we obtain $Az = Bz = Sz = Tz = Pz = z$, that is, z is the common fixed point of A, B, S, T and P. Finally, the uniqueness of the fixed point of A, B, S, T and P follows easily from (ii). □

From Theorem 4.1 with $B = T = IX$ (the identity mapping on X), we have the following.

Corollary 4.2. *Let $(X, M, *)$ be a complete M-fuzzy metric space with $t * t \geq t$ for all $t \in [0, 1]$ and the condition (FM-6). Let A, S, and P be mappings from X into itself such that*
(i) *$P(X) \subset A(X)$ and $P(X) \subset S(X)$,*
(ii) *there exists a number $k \in (0, 1)$ such that*

$$\begin{gathered} M(Px, Py, Py, kt) \geq \\ M(Ax, Px, Px, t) * M(Px, Sy, Sy, t) * M(Ax, Sy, Sy, t) * \\ \tfrac{M(Px, Ax,, Ax, t) * M(Px, Sy, STy, t)}{M(Sy, Ax, Ax, t)} * M(Ax, Py, Py, (3-\alpha)t) \end{gathered}$$

for all $x, y \in X$, $\alpha \in (0, 3)$ and $t > 0$,
(iii) *A is continuous,*
(iv) *the pair P, A are compatible of type $(*)$,*
(v) *$M(x, Sx, Sx, t) \geq M(x, AxAx, t)$ for all $x \in X$ and $t > 0$.*
Then A,S,and P have a common fixed point in X.

From Theorem 4.1 with $A = B = S = T = IX$ (the identity mapping on X), we have the following.

Corollary 4.3. *Let $(X, M, *)$ be a complete M-fuzzy metric space with $t * t \geq t$ for all $t \in [0, 1]$ and the condition $(FM-6)$. Let P be mappings from X into itself such that*

$$\begin{gathered} M(Px, Py, Py, kt) \geq \\ M(x, Px, Px, t) * M(Px, y, y, t) * M(x, Sy, Sy, t) * \\ \tfrac{M(Px, x,, x, t) * M(Px, y, y, t)}{M(y, x, x, t)} * M(x, Py, Py, (3-\alpha)t) \end{gathered}$$

for all $x, y \in X$, $\alpha \in (0, 3)$ and $t > 0$, Then P have a common fixed point in X.

Corollary 4.4. *Let $(X, M, *)$ be a complete M-fuzzy metric space with $t * t \geq t$ for all $t \in [0, 1]$ and the condition $(FM-6)$. Let P be mappings from X into itself such that there exists a number $k \in (0, 1)$ such that $M(Px, Py, Py, kt) \geq M(x, y, y, t)$ for all $x, y \in X$,$\alpha \in (0, 3)$ and $t > 0$, Then P has a common fixed point in X*

Remark 4.5. Corollary 4.4 is an extension of Banach contraction theorem (Grabiec [5]) in fuzzy metric spaces to a contractive mapping on complete M-fuzzy metric spaces.

By using Theorem 4.1, we have the following:

Theorem 4.6. *Let $(X, M, *)$ be a complete M-fuzzy metric space with $t * t \geq t$ for all $t \in [0, 1]$ and the condition (FM-6). Let A, B, S, T and $(P_i)_{i\in\wedge}$ be mappings from X into itself such that the conditions (iv) and (vi) of Theorem 3.1 holds and*
(i) $\cup_{i\in\wedge}(P_i)_{i\in\wedge} \subset AB(X)$ *and* $\cup_{i\in\wedge}(P_i)_{i\in\wedge} \subset ST(X)$,
(ii) *there exists a number $k \in (0, 1)$ such that*

$$M(P_ix, P_iy, P_iy, kt) \geq$$
$$M(ABx, P_ix, P_ix, t) * M(P_ix, STy, STy, t) * M(ABx, STy, STy, t)*$$
$$\frac{M(P_ix, ABx,, ABx, t) * M(P_ix, STy, STy, t)}{M(STy, ABx, ABx, t)} * M(ABx, P_iy, P_iy, (3-\alpha)t)$$

for all $x, y \in X$,$\alpha \in (0, 3)$ and $t > 0$,
(iii) $P_iB = BP_i$, $P_iT = TP_i$, $AB = BA$ *and* $ST = TS$,*for all* i
(iv) *A and B are continuous,*
(v) *the pair P, AB are compatible of type $(*)$,*
(vi) $M(x, STx, STx, t) \geq M(x, ABxABx, t)$ *for all $x \in X$ and $t > 0$.*
Then A,B,S,T and $(P_i)_{i\in\wedge}$ have a common fixed point in X.

Acknowledgements. We are thankful to the Prof. Young Bae Jun and the reviewers for their valuable suggestions to improve the presentation of paper.

References

[1] S. S. Chang, Y. J. Cho, B. S. Lee, J. S. Jung and S. M. Kang, Coincidence point and minimization theorems in fuzzy metric spaces , Fuzzy Sets and Systems 88 (1997) 119–128.
[2] Z. K. Deng, Fuzzy pseudo-metric spaces, J. Math. Anal. Appl. 86 (1982) 74–95.
[3] M. A. Erceg, Metric spaces in fuzzy set theory, J. Math. Anal. Appl. 69 (1979) 205–230.
[4] A. George and P. Veeramani, On some results in fuzzy metric spaces, Fuzzy Sets and Systems 64 (???) 395–399.
[5] M. Grabiec, Fixed point in fuzzy metric spaces, Fuzzy Sets and Systems 27 (1988) 385–389.
[6] J. S. Jung, Y. J. Cho and J. K. Kim, Minimization theorems for fixed point theorems in fuzzy metric spaces and applications, Fuzzy Sets and Systems 61 (1994) 199–207.
[7] O. Kaleva and S. Seikkala, On fuzzy metric spaces, Fuzzy Sets and Systems 12 (1984) 215–229.
[8] O. Kramosil and J. Michalek, Fuzzy metric and statistical metric spaces, Kybernetica 11 (1975) 336–344.
[9] S. N. Mishra, N. Sharma and S. L. Singh, Common fixed points of maps on fuzzy metric spaces., Int. J. Math. Math. Sci. 17 (1994) 253–258.
[10] J. H. Park, J. S. Park and Y. C. Kwun, A common fixed point theorem in the intuitionistic fuzzy metric spacesIn L. Jiao et al. (Eds.), Advances in natural computation data mining Xian: Xidian University (2006) 293–300.
[11] J. H. Park, J. S. Park, and Y. C. Kwun, Fixed points in M-fuzzy metric spaces, Fuzzy Optim. Decis. Mak. 7 (2008) 305–315.
[12] B. Schweizer and A. Sklar, Statistical metric spaces, Pacific J. Math. 10 (1960) 313–334.
[13] S. Sedghi and N. Shobe, Fixed Point Theorem inM-Fuzzy Metric Spaces with property (E), Advances in Fuzzy Mathematics. 1 (2006) 55–65.
[14] L. A. Zadeh, Fuzzy sets, Information and Control 8 (1965) 338–353.

Deepak Singh (deepak.singh.2006@indiatimes.com)
Department of Mathematics, Technocrats Institute of Technology, Bhopal-462010, M.P, INDIA

Mayank Sharma (mayank.math@rediffmail.com)
Department of Mathematics, Scope College of Engineering, Bhopal-462026, M.P, INDIA

Ramakant Sharma (ramakantsharma.ask@gmail.com)
Department of Mathematics, Bansal College of Engineering, Mandideep, Bhopal-462046,M.P, INDIA