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Saturated T-syntopogenous spaces

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Abstract. In this paper, we introduce the concepts of saturated T-syntopogenous spaces and we prove basic properties of this spaces. We show that the category of perfect saturated T-syntopogenous spaces is isomorphic to category of I-topological spaces

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1. Introduction

In [1], Császár first gave his theory of syntopogenous structures. It is well known that the category of syntopogenous spaces and continuous maps is isomorphic to category of topological spaces and continuous maps. In [3], Hashem deduced the T-syntopogenous spaces, for each continuous triangular norm T. In this manuscript, we introduce the notions of saturated T-syntopogenous spaces.

We proceed as follows :

In Section 2, we present some basic definitions and ideas of fuzzy sets, I-topological spaces and T-syntopogenous spaces.

In Section 3, we supply three propositions of T-topogenous orders.

In Section 4, we introduce our definition of saturated T-syntopogenous spaces and we prove basic properties of this spaces. Also, we show that the category of perfect saturated T-syntopogenous spaces is isomorphic to category of I-topological spaces.

2. Preliminaries

A triangular norm (cf. [8]) is a binary operation on the unit interval I= [0, 1] that is associative, symmetric, monotone in each argument and has the neutral element 1. The triangular conorm of a triangular norm T is the binary operation T^* on the unite interval I given by : $\alpha\ T^*\ \beta = 1 - [(1-\alpha)\ T\ (1-\beta)], \quad \alpha, \beta \in I.$

For a continuous triangular norm T, the following binary operation on I,

$\hat{\mathrm{J}}(\alpha,\gamma) = \sup\{\theta \in I \quad : \quad \alpha\ T\ \theta \leq \gamma\}, \quad \alpha,\gamma \in I,$
is called the residual implication of T [6]. For this implication,we shall use the following property :
(2.1) $\qquad \alpha\ T\ \beta > \gamma \quad$ if and only if $\quad \alpha > \hat{\mathrm{J}}(\beta,\gamma).$

A fuzzy set λ in a universe set X, introduced by Zadeh in [9], is a function λ :X $\longrightarrow I$. The collection of all fuzzy sets of X is denoted by I^X. The height of a fuzzy set λ is the following real number : hgt $\lambda = \sup\{\lambda(\mathrm{x}) \quad : \quad \mathrm{x} \in \mathrm{X}\}$.
If H is a subset of X, then we shall denote to its characteristic function by the symbol $\mathbf{1}_H$, said to be a crisp fuzzy subset of X. We also denote the constant fuzzy set of X with value $\alpha \in I$ by $\underline{\alpha}$.

Given a fuzzy set $\lambda \in I^X$ and a real number $\alpha \in I_1 = [0, 1[$, the strong α-cut of λ is the following subset of X : $\lambda^\alpha = \{\mathrm{x} \in \mathrm{X} \quad : \quad \lambda(\mathrm{x}) > \alpha\}$;
and the weak α-cut of λ is the subset of X : $\lambda_{\alpha *} = \{\mathrm{x} \in \mathrm{X} \quad : \quad \lambda(\mathrm{x}) \geq \alpha\}$.
It is direct to verify that every $\lambda \in I^X$ has the following resolutions
(2.2) $\qquad \lambda = \vee_{\alpha \in I}\ [\underline{\alpha}\ \wedge \mathbf{1}_{\lambda_{\alpha *}}] = \wedge_{\alpha \in I}\ [\underline{\alpha}\ \vee \mathbf{1}_{\lambda_{\alpha *}}]$.

Given two fuzzy sets $\mu, \lambda \in I^X$ we denote by $\mu T \lambda$ the following fuzzy set of X : $(\mu\ T\ \lambda)(\mathrm{x}) = \mu(\mathrm{x})\ T\ \lambda(\mathrm{x}),\ \mathrm{x} \in \mathrm{X}$.

Lemma 2. 1 [2]. For every $\mu, \lambda \in I^X$ and $\alpha \in I_1$,
$(\mu\ T\ \lambda)_{\alpha *} = \cup_{\theta T \beta \geq \alpha}\ (\mu_{\theta *}\ \cap\ \lambda_{\beta *}).$

Definition 2. 1 [3]. A T-topogenous order on a set X is a function $\zeta : I^X \times I^X \longrightarrow I$, which satisfies, for any $\mu,\ \lambda,\ \nu \in I^X$ and $\alpha \in I$, the following :
(TT1) $\zeta(\underline{1}, \underline{\alpha}) = \alpha$ and $\zeta(\underline{\alpha}, \underline{0}) = 1 - \alpha$;
(TT2) $\zeta(\mu \vee \lambda,\ \nu) = \zeta(\mu,\ \nu)\ \wedge\ \zeta(\lambda,\ \nu)$ and $\zeta(\mu,\ \lambda\ \wedge\ \nu) = \zeta(\mu,\ \lambda)\ \wedge\ \zeta(\mu,\ \nu)$;
(TT3) If $\zeta(\mu,\ \lambda) > 1 - (\theta T \beta)$ for some $\theta, \beta \in I_0$, there is $\mathrm{C} \subseteq \mathrm{X}$ such that $\zeta(\mu,\ \mathbf{1}_C) \geq 1 - \theta$ and $\zeta(\mathbf{1}_C,\ \lambda) \geq 1 - \beta$;
(TT4) $\zeta(\mu,\ \lambda) \leq 1 -$ hgt $[\mu\ T\ (\underline{1}\ - \lambda)]$;
(TT5) $\zeta(\underline{\alpha}\ T\ \mu,\ \lambda) = (1-\alpha)\ T^*\ \zeta(\mu,\ \lambda) = \zeta(\mu,\ (\underline{1-\alpha})\ T^*\ \lambda)$.
The I-topology [5], determined by a T-topogenous order ζ, denoted by $\tau(\zeta)$ [6], is defined through its fuzzy closure operator $^-$ given by :
(2.3) $\qquad \mu^-(\mathrm{x}) = 1 - \zeta(\mathbf{1}_x,\ \underline{1} - \mu), \qquad \mu \in I^X,\ \mathrm{x} \in \mathrm{X}.$

Definition 2.2 [3]. A T-topogenous order ζ on a set X is said to be :
(i) perfect if $\zeta(\vee_{j \in J}\ \mu_j,\ \lambda) = \wedge_{j \in J}\ \zeta(\mu_j,\ \lambda), \qquad \mu_j, \lambda \in I^X$;
(ii) biperfect if it is perfect and $\zeta(\mu,\ \wedge_{j \in J}\ \lambda_j) = \wedge_{j \in J}\ \zeta(\mu,\ \lambda_j), \qquad \mu, \lambda_j \in I^X$;
(iii) symmetrical if $\zeta(\mu,\ \lambda) = \zeta(\underline{1} - \lambda,\ \underline{1} - \mu), \qquad \mu, \lambda \in I^X$.

Definition 2.3 [3]. For T-topogenous orders ζ and η on X, we define the T-composition of ζ and η by :
$$(\zeta\ \mathrm{o}_T\ \eta)(\mu,\ \lambda) = \sup_{C \subseteq X}[\eta(\mu,\ \mathbf{1}_C)\ T\ \zeta(\mathbf{1}_C,\ \lambda)], \qquad \mu, \lambda \in I^X.$$

Definition 2.4 [3]. A T-syntopogenous structure on a set X is a family $\wp$ of

T-topogenous orders on X satisfying the following conditions
(TS1) $\wp$ is directed in the sence that, given $\zeta, \eta \in \wp$ there is $\xi \in \wp$ such that $\xi \geq \zeta \vee \eta$;
(TS2) for every $\zeta \in \wp$ and $\varepsilon \in I_0$, there is $\zeta_\varepsilon \in \wp$ such that $(\zeta_\varepsilon \ \mathrm{o}_T \ \zeta_\varepsilon) + \underline{\varepsilon} \geq \zeta$.
The pair (X, $\wp$) is said to be a T-syntopogenous space. If $\wp$ consisting of a single T-topogenous order, then (X, $\wp$) is called a T-topogenous space.

Definition 2.5 [3]. Let (X, $\wp$) and (Y, $\Im$) be T-syntopogenous spaces. A function f : X $\longrightarrow$ Y is called syntopogenously continuous, if for every $\eta \in \Im$ there is $\zeta \in \wp$ such that

(2.4) $$\eta(f(\mu), \underline{1}-f(\lambda)) \leq \zeta(\mu, \underline{1}-\lambda), \qquad \mu, \lambda \in I^X.$$

3. Results on T-topogenous orders

In this section, we deduce some properties of the T-topogenous orders.

Proposition 3.1. Let ζ be a T-topogenous order on a set X, $\mu \in I^X$ and $\alpha \in I$. Then
(i) $\alpha \leq \zeta(\mu, \underline{\alpha}) = \alpha\ T^* \ [1-\text{ hgt } \mu]$;
(ii) $1-\alpha \leq \zeta(\underline{\alpha}, \mu) = (1-\alpha)\ T^* \inf_{x\in X} \mu(\text{x})$;
(iii) $\zeta(\underline{1}, \mu) = \inf_{x\in X} \mu(\text{x})$ and $\zeta(\mu, \underline{1}) = 1$;
(iv) $\zeta(\mathbf{1}_x, \mu) \leq \mu(\text{x})$, $\quad$ x$\in$X.
Proof. Let $\mu \in I^X$ and $\alpha \in I$. Then
(i) By (TT1) and (TT2), we have $\alpha = \zeta(\underline{1}, \underline{\alpha}) \leq \zeta(\mu, \underline{\alpha})$, since $\mu \leq \underline{1}$.
On the other hand,

$$\begin{aligned}
\zeta(\mu, \underline{\alpha}) &\leq 1 - \text{hgt } [\mu\ T\ \underline{1-\alpha}], && \text{by (TT4)}\\
&= 1 - [(1-\alpha)\ T \text{ hgt } \mu], && \text{by continuity of } T\\
&= \alpha\ T^* \ [1-\text{ hgt } \mu], &&\\
&= \alpha\ T^* \ \zeta(\underline{\text{hgt } \mu}, \underline{0}), && \text{by (TT1)}\\
&= \zeta(\underline{\text{hgt } \mu}, \underline{\alpha\ T^*\ 0}), && \text{by (TT5)}\\
&\leq \zeta(\mu, \underline{\alpha}), && \text{by (TT2)}
\end{aligned}$$

Which proves the required equality.
(ii) As the same manner of (i).
(iii) Put hgt $(\underline{1}-\mu) = \gamma$, so $\underline{1}-\mu \leq \gamma$. Consequently,

$$\begin{aligned}
\inf_{x\in X} \mu(\text{x}) &= 1-\text{ hgt } (\underline{1}-\mu)\\
&= 1-\text{ hgt } [\underline{1}\ T\ (\underline{1}-\mu)]\\
&\geq \zeta(\underline{1}, \mu), && \text{by (TT4)}\\
&\geq \zeta(\underline{1}, \underline{1-\gamma}), && \text{by hypothesis and (TT2)}\\
&= 1-\gamma, && \text{by (TT1)}\\
&= 1-\text{ hgt } (\underline{1}-\mu)\\
&= \inf_{x\in X} \mu(\text{x}).
\end{aligned}$$

Hence, equality holds.
Also, since $\mu \leq 1$, then by (TT1) and (TT2), we have
$\zeta(\mu, \underline{1}) \leq 1 = \zeta(\underline{1}, \underline{1}) \leq \zeta(\mu, \underline{1})$.
(iv) Follows immediately from (TT4).
Which completes the proof.

Proposition 3.2. Let ζ be a T-topogenous order on a set X, $\mu, \lambda \in I^X$ and H$\subseteq$X. Then
(i) For every $\alpha \in I_0$ with $\alpha > \zeta(\mathbf{1}_H, \mu)$, we have $\zeta(\mathbf{1}_H, \mu) \geq \zeta(\mathbf{1}_H, \mathbf{1}_{\mu_{\alpha*}})$;
(ii) For every $\alpha \in I_0$ with $\alpha > \zeta(\mu, \mathbf{1}_H)$, we have $\zeta(\mu, \mathbf{1}_H) \geq \zeta(\mathbf{1}_{\mu_{(1-\alpha)*}}, \mathbf{1}_H)$;
(iii) For every $\alpha \in I_1$ with $\alpha < \zeta(\mu, \lambda)$, we have $\mu_{\theta*} \subseteq \lambda_{(1-\beta)*}$, $\forall \ \theta, \beta \in I$ such that $\theta T \beta \geq 1 - \alpha$.
Proof. Let $\mu, \lambda \in I^X$ and H $\subseteq$ X.
(i) If $\alpha \in I_0$ with $\alpha > \zeta(\mathbf{1}_H, \mu)$, then we get

$$\begin{aligned}
\alpha \wedge \zeta(\mathbf{1}_H, \mathbf{1}_{\mu_{\alpha*}}) &\leq \zeta(\mathbf{1}_H, \underline{\alpha}) \wedge \zeta(\mathbf{1}_H, \mathbf{1}_{\mu_{\alpha*}}), && \text{by Proposition 3.1. (i)}\\
&= \zeta(\mathbf{1}_H, \underline{\alpha} \wedge \mathbf{1}_{\mu_{\alpha*}}), && \text{by (TT2)}\\
&\leq \zeta(\mathbf{1}_H, \mu), && \text{by (2.2), (TT2)}\\
&< \alpha, && \text{by hypothesis}
\end{aligned}$$

Which implies $\zeta(\mathbf{1}_H, \mathbf{1}_{\mu_{\alpha*}}) < \alpha$ and hence $\zeta(\mathbf{1}_H, \mu) \geq \zeta(\mathbf{1}_H, \mathbf{1}_{\mu_{\alpha*}})$.
(ii) It can be proved along similar lines of (i).
(iii) Let $\alpha \in I_1$ be such that $\alpha < \zeta(\mu, \lambda)$. Then by (TT4), $\alpha < 1-$ hgt $[\mu \ T \ (\underline{1} - \lambda)]$ thus $\varnothing = [\mu \ T \ (\underline{1} - \lambda)]_{(1-\alpha)*} = \cup_{\theta T \beta \geq 1-\alpha} [\mu_{\theta*} \cap (\underline{1} - \lambda)_{\beta*}]$, by Lemma 2.1
Hence, for every $\theta, \beta \in I$ such that $\theta T \beta \geq 1 - \alpha$, we have
$\mu_{\theta*} \subseteq \text{X} - (\underline{1} - \lambda)_{\beta*} = \lambda^{(1-\beta)} \subseteq \lambda_{(1-\beta)*}$.

Proposition 3.3. Let ζ be a T-topogenous order on a set X and $\mu, \lambda \in I^X$. Then
(i) $\zeta(\mu, \lambda) \geq \vee_{\alpha \in I} [\alpha \wedge \zeta(\mathbf{1}_{\mu_{(1-\alpha)*}}, \mathbf{1}_{\lambda_{\alpha*}})]$;
(ii) $\zeta(\mu, \lambda) \geq [1 - \text{hgt } \mu] \vee [\inf_{x \in X} \lambda(\text{x})]$.
Proof. Let $\mu, \lambda \in I^X$. Then
(i) By (TT2) and (2.2), we have for every $\alpha \in I$,

$$\begin{aligned}
\zeta(\mu, \lambda) &\geq \zeta(\underline{1-\alpha} \vee \mathbf{1}_{\mu_{(1-\alpha)*}}, \underline{\alpha} \wedge \mathbf{1}_{\lambda_{\alpha*}})\\
&= \zeta(\underline{1-\alpha}, \underline{\alpha} \wedge \mathbf{1}_{\lambda_{\alpha*}}) \wedge \zeta(\mathbf{1}_{\mu_{(1-\alpha)*}}, \underline{\alpha} \wedge \mathbf{1}_{\lambda_{\alpha*}})\\
&= \zeta(\underline{1-\alpha}, \underline{\alpha}) \wedge \zeta(\underline{1-\alpha}, \mathbf{1}_{\lambda_{\alpha*}}) \wedge \zeta(\mathbf{1}_{\mu_{(1-\alpha)*}}, \underline{\alpha}) \wedge \zeta(\mathbf{1}_{\mu_{(1-\alpha)*}}, \mathbf{1}_{\lambda_{\alpha*}})\\
&\geq \alpha \wedge \alpha \wedge \alpha \wedge \zeta(\mathbf{1}_{\mu_{(1-\alpha)*}}, \mathbf{1}_{\lambda_{\alpha*}}), \quad \text{by Proposition 3.1}\\
&= \alpha \wedge \zeta(\mathbf{1}_{\mu_{(1-\alpha)*}}, \mathbf{1}_{\lambda_{\alpha*}}).
\end{aligned}$$

Hence, $\zeta(\mu, \lambda) \geq \vee_{\alpha \in I} [\alpha \wedge \zeta(\mathbf{1}_{\mu_{(1-\alpha)*}}, \mathbf{1}_{\lambda_{\alpha*}})]$.
Rendering (i).
(ii) By (TT2) and Proposition 3.1 (i),(ii) we have
$\zeta(\mu, \lambda) \geq \zeta(\mu, \underline{0}) = 1-$ hgt μ and $\zeta(\mu, \lambda) \geq \zeta(\underline{1}, \lambda) = \inf_{x \in X} \lambda(\text{x})$. Hence,
$\zeta(\mu, \lambda) \geq [1-$ hgt $\mu] \vee [\inf_{x \in X} \lambda(\text{x})]$ and winds up the proof.

4. Saturated T-syntopogenous spaces

In this section, we introduce the concepts of saturated T-topogenous orders on a set. Also, we show that the category of perfect saturated T-syntopogenous spaces is isomorphic to category of I-topological spaces.

Let X be a set and ζ be a T-topogenous order on X. We define a mapping
$\zeta^{\sim} : I^X \times I^X \longrightarrow I$, as follows :

$$\zeta^{\sim}(\mu,\ \lambda) = \inf\{\zeta(\underline{\mu(x)} \wedge \mathbf{1}_x,\ \underline{\lambda(y)} \vee \mathbf{1}_{(X-y)}) \quad : \quad x,y\in X\},\quad \mu,\lambda\in I^X.$$

Theorem 4.1. If ζ is a T-topogenous order on a set X, then $\zeta^{\sim}$ is a T-topogenous order on X. Moreover, if ζ is perfect (resp. biperfect, resp. symmetrical), then so is $\zeta^{\sim}$. Proof. Let $\mu,\lambda,\nu\in I^X$ and $\alpha\in I$. Then

(TT1) $\zeta^{\sim}(\underline{1},\ \underline{\alpha}) = \inf\{\zeta(\underline{1(x)} \wedge \mathbf{1}_x,\ \underline{\alpha(y)} \vee \mathbf{1}_{(X-y)}) \quad : \quad x,y\in X\}$
$= \inf\{\zeta(\mathbf{1}_x,\ \underline{\alpha} \vee \mathbf{1}_{(X-y)}) \quad : \quad x,y\in X\}$
$\leq \inf\{1-\text{hgt}\ [\mathbf{1}_x\ T(\underline{1} - (\underline{\alpha} \vee \mathbf{1}_{(X-y)}))] \quad : \quad x,y\in X\}$, by (TT4)
$= 1-\sup\{\text{hgt}\ [\mathbf{1}_x\ T(\underline{1-\alpha} \wedge \mathbf{1}_y)] \quad : \quad x,y\in X\}$
$= \alpha.$

For the opposite inequality, we have
$\zeta^{\sim}(\underline{1},\ \underline{\alpha}) = \inf\{\zeta(\underline{1(x)} \wedge \mathbf{1}_x,\ \underline{\alpha(y)} \vee \mathbf{1}_{(X-y)}) \quad : \quad x,y\in X\}$
$= \inf\{\zeta(\mathbf{1}_x,\ \underline{\alpha} \vee \mathbf{1}_{(X-y)}) \quad : \quad x,y\in X\}$
$\geq \inf\{\zeta(\mathbf{1}_x,\ \underline{\alpha}) \quad : \quad x\in X\}$, by (TT2)
$\geq \alpha$, by Proposition 3.1 (i)

Hence, $\zeta^{\sim}(\underline{1},\ \underline{\alpha}) = \alpha$.
Similarly, we can show $\zeta^{\sim}(\underline{\alpha},\ \underline{0}) = 1-\alpha$.

(TT2) $\zeta^{\sim}(\mu\ \vee\ \lambda,\ \nu) = \inf\{\zeta(\underline{(\mu\vee\lambda)(x)} \wedge \mathbf{1}_x,\ \underline{\nu(y)} \vee \mathbf{1}_{(X-y)}) \quad : \quad x,y\in X\}$
$= \inf\{\zeta((\underline{\mu(x)} \wedge \mathbf{1}_x) \vee (\underline{\lambda(x)} \wedge \mathbf{1}_x),\ \underline{\nu(y)} \vee \mathbf{1}_{(X-y)}) \quad : \quad x,y\in X\}$
$= \inf\{\zeta(\underline{\mu(x)} \wedge \mathbf{1}_x,\ \underline{\nu(y)} \vee \mathbf{1}_{(X-y)}) \wedge \zeta(\underline{\lambda(x)} \wedge \mathbf{1}_x,\ \underline{\nu(y)} \vee \mathbf{1}_{(X-y)}) \quad : \quad x,y\in X\}$, by (TT2)
$= \inf\{\zeta(\underline{\mu(x)} \wedge \mathbf{1}_x,\ \underline{\nu(y)} \vee \mathbf{1}_{(X-y)}) \quad : \quad x,y\in X\} \wedge$
$\inf\{\zeta(\underline{\lambda(x)} \wedge \mathbf{1}_x,\ \underline{\nu(y)} \vee \mathbf{1}_{(X-y)}) \quad : \quad x,y\in X\}$
$= \zeta^{\sim}(\mu,\ \nu)\ \wedge\ \zeta^{\sim}(\lambda,\ \nu).$

Analogously, $\zeta^{\sim}(\mu,\ \lambda\ \wedge\ \nu) = \zeta^{\sim}(\mu,\ \lambda)\ \wedge\ \zeta^{\sim}(\mu,\ \nu)$.

(TT3) Let $\zeta^{\sim}(\mu,\ \lambda) > 1-(\theta T\beta)$ for some $\theta,\beta\in I_0$. Then for every x,y$\in$ X, we have

$$\zeta(\underline{\mu(x)} \wedge \mathbf{1}_x,\ \underline{\lambda(y)} \vee \mathbf{1}_{(X-y)}) > 1-(\theta T\beta),$$

thus there is $C_{xy}\subseteq X$ such that
$\zeta(\underline{\mu(x)} \wedge \mathbf{1}_x,\ \mathbf{1}_{C_{xy}}) \geq 1-\theta$ and $\zeta(\mathbf{1}_{C_{xy}},\ \underline{\lambda(y)} \vee \mathbf{1}_{(X-y)}) \geq 1-\beta$.
Hence, by taking $C = \cup_x \cap_y C_{xy} \subseteq X$, we have
$\zeta^{\sim}(\mu,\ \mathbf{1}_C) = \inf\{\zeta(\underline{\mu(x)} \wedge \mathbf{1}_x,\ \underline{\mathbf{1}_C(y)} \vee \mathbf{1}_{(X-y)}) \quad : \quad x,y\in X\}$
$\geq \inf\{\zeta(\underline{\mu(x)} \wedge \mathbf{1}_x,\ \wedge_y\ (\underline{\mathbf{1}_{Cxy}(y)} \vee \mathbf{1}_{(X-y)})) \quad : \quad x,y\in X\}$, by (TT2)
$= \inf\{\zeta(\underline{\mu(x)} \wedge \mathbf{1}_x,\ \mathbf{1}_{Cxy}) \quad : \quad x,y\in X\}$, clear
$\geq 1-\theta$, and
$\zeta^{\sim}(\mathbf{1}_C,\ \lambda) = \inf\{\zeta(\underline{\mathbf{1}_C(x)} \wedge \mathbf{1}_x,\ \underline{\lambda(y)} \vee \mathbf{1}_{(X-y)}) \quad : \quad x,y\in X\}$
$\geq \inf\{\zeta(\vee_x\ (\underline{\mathbf{1}_{Cxy}(x)} \wedge \mathbf{1}_x),\ \underline{\lambda(y)} \vee \mathbf{1}_{(X-y)})) \quad : \quad x,y\in X\}$, by (TT2)
$= \inf\{\zeta(\mathbf{1}_{Cxy},\ \underline{\lambda(y)} \vee \mathbf{1}_{(X-y)}) \quad : \quad x,y\in X\}$
$\geq 1-\beta$.

(TT4) $\zeta^{\sim}(\mu,\ \lambda) = \inf\{\zeta(\underline{\mu(x)} \wedge \mathbf{1}_x,\ \underline{\lambda(y)} \vee \mathbf{1}_{(X-y)}) \quad : \quad x,y\in X\}$
$\leq \inf\{1-\text{hgt}\ [(\underline{\mu(x)} \wedge \mathbf{1}_x)\ T\ (1-(\underline{\lambda(y)} \vee \mathbf{1}_{(X-y)}))] \quad : \quad x,y\in X\}$
$= 1-\sup\{\text{hgt}\ [(\underline{\mu(x)} \wedge \mathbf{1}_x)\ T\ (1-(\underline{\lambda(y)} \vee \mathbf{1}_{(X-y)}))] \quad : \quad x,y\in X\}$
$= 1-\text{hgt}\ \{[\ \vee_{x\in X}\ (\underline{\mu(x)} \wedge \mathbf{1}_x)]\ T\ [\vee_{y\in X}(1-(\underline{\lambda(y)} \vee \mathbf{1}_{(X-y)}))]\}$
$= 1-\text{hgt}\ \{[\ \vee_{x\in X}\ (\underline{\mu(x)} \wedge \mathbf{1}_x)]\ T\ [\vee_{y\in X}(\underline{(1-\lambda)(y)} \wedge \mathbf{1}_y)]\}$

$= 1-$ hgt $[\mu\ T\ (1-\lambda)]$, clear

(TT5) $\zeta^{\sim}(\underline{\alpha}\ T\mu,\ \lambda) = \inf\{\zeta(\underline{(\alpha T\mu)(x)} \wedge \mathbf{1}_x,\ \underline{\lambda(y)} \vee \mathbf{1}_{(X-y)}) \quad : \quad x,y\in X\}$

$= \inf\{\zeta(\underline{\alpha}\ T\ (\underline{\mu(x)} \wedge \mathbf{1}_x),\ \underline{\lambda(y)} \vee \mathbf{1}_{(X-y)}) \quad : \quad x,y\in X\}$

$= \inf\{(1-\alpha)\ T^*\ \zeta(\underline{\mu(x)} \wedge \mathbf{1}_x,\ \underline{\lambda(y)} \vee \mathbf{1}_{(X-y)}) \ : \ x,y\in X\}$, by (TT5)

$= (1-\alpha)\ T^*\ \inf\{\zeta(\underline{\mu(x)} \wedge \mathbf{1}_x,\ \underline{\lambda(y)} \vee \mathbf{1}_{(X-y)}) \ : \ x,y\in X\}$

$= (1-\alpha)\ T^*\ \zeta^{\sim}(\mu,\ \lambda)$.

Similarly, $\zeta^{\sim}(\mu,\ \underline{(1-\alpha)}\ T^*\lambda) = (1-\alpha)\ T^*\ \zeta^{\sim}(\mu,\ \lambda)$.

Now, we show that $\zeta^{\sim}$ is a perfect as follows :

$\zeta^{\sim}(\vee_{j\in J}\ \mu_j,\ \lambda) = \inf\{\zeta(\underline{(\vee_{j\in J}\ \mu_j)(x)} \wedge \mathbf{1}_x,\ \underline{\lambda(y)} \vee \mathbf{1}_{(X-y)}) \quad : \quad x,y\in X\}$

$= \inf\{\zeta(\vee_{j\in J}\ (\underline{\mu_j(x)} \wedge \mathbf{1}_x),\ \underline{\lambda(y)} \vee \mathbf{1}_{(X-y)}) \quad : \quad x,y\in X\}$

$= \inf\{\wedge_{j\in J}\ \zeta(\underline{\mu_j(x)} \wedge \mathbf{1}_x,\ \underline{\lambda(y)} \vee \mathbf{1}_{(X-y)}) \quad : \quad x,y\in X\}$, since ζ perfect

$= \wedge_{j\in J}\ \inf\{\zeta(\underline{\mu_j(x)} \wedge \mathbf{1}_x,\ \underline{\lambda(y)} \vee \mathbf{1}_{(X-y)}) \quad : \quad x,y\in X\}$

$= \wedge_{j\in J}\ \zeta^{\sim}(\mu_j, \lambda)$.

As the same manner, we can show that $\zeta^{\sim}$ is a biperfect.

Finally, we show that $\zeta^{\sim}$ is a symmetrical as :

$\zeta^{\sim}(\mu,\ \lambda) = \inf\{\zeta(\underline{\mu(x)} \wedge \mathbf{1}_x,\ \underline{\lambda(y)} \vee \mathbf{1}_{(X-y)}) \quad : \quad x,y\in X\}$

$= \inf\{\zeta(\underline{1}-\ (\underline{\lambda(y)} \vee \mathbf{1}_{(X-y)}),\ \underline{1}-\ (\underline{\mu(x)} \wedge \mathbf{1}_x)) \ : \ x,y\in X\}$, by symmetrical of ζ

$= \inf\{\zeta(\underline{(1-\lambda)(y)} \wedge \mathbf{1}_y,\ \underline{(1-\ \mu)(x)} \vee \mathbf{1}_{(X-x)}) \ : \ x,y\in X\}$

$= \zeta^{\sim}(\underline{1}-\lambda,\ \underline{1}-\mu)$.

Which completes the proof.

Definition 4.1. (i) Let ζ be a T-topogenous order on a set X. Then $\zeta^{\sim}$ will be called saturation of ζ. If $\zeta = \zeta^{\sim}$, then we say that ζ is a saturated T-topogenous order.

(ii) A T-syntopogenous structure $\wp$ on a set X is said to be saturated T-syntopogenous structure on X, if each member of $\wp$ is saturated. In this case, (X, $\wp$) is called a saturated T-syntopogenous space.

The following proposition leads to the proof of Theorem 4.2, below, which states that the saturation of T-syntopogenous structure is again T-syntopogenous structure.

Proposition 4.1. Let $\zeta,\ \eta$ be T-topogenous orders on a set X. Then one has the following :

(i) For every x,y $\in$ X and $\mu, \lambda \in I^X$, we have

$$\zeta^{\sim}(\underline{\mu(x)} \wedge \mathbf{1}_x,\ \underline{\lambda(y)} \vee \mathbf{1}_{(X-y)}) = \zeta(\underline{\mu(x)} \wedge \mathbf{1}_x,\ \underline{\lambda(y)} \vee \mathbf{1}_{(X-y)});$$

(ii) If $(C_i)_{i\in\Lambda}$ is a finite family of subsets of X and $\lambda \in I^X$, then

$$\zeta^{\sim}(\cup_{i\in\Lambda}\ \mathbf{1}_{C_i},\ \lambda) = \wedge_{i\in\Lambda}\ \zeta^{\sim}(\mathbf{1}_{C_i},\ \lambda);$$

(iii) $\zeta\ \leq\ \zeta^{\sim}$;

(iv) If $\zeta\ \leq\ \eta$, then $\zeta^{\sim}\ \leq\ \eta^{\sim}$;

(v) $(\zeta^{\sim})^{\sim} = \zeta^{\sim}$;

(vi) $(\zeta\ \wedge\ \eta)^{\sim} = \zeta^{\sim}\ \wedge\ \eta^{\sim}$ and $(\zeta + \underline{\varepsilon})^{\sim} = \zeta^{\sim} + \underline{\varepsilon}$;

(vii) $(\zeta\ o_T\ \eta)^{\sim} \leq\ (\zeta^{\sim}\ o_T\ \eta^{\sim})^{\sim} = \zeta^{\sim}\ o_T\ \eta^{\sim}$.
Proof. Let $\mu, \lambda \in I^X$. Then
(i) For every x,y $\in$X, we have
$\zeta^{\sim}(\underline{\mu(x)} \wedge \mathbf{1}_x,\ \underline{\lambda(y)} \vee \mathbf{1}_{(X-y)})$
$= \inf\ \{\zeta((\underline{\mu(x) \wedge \mathbf{1}_x)(s)} \wedge \mathbf{1}_s,\ (\underline{\lambda(y) \vee \mathbf{1}_{(X-y)})(t)} \vee \mathbf{1}_{(X-t)})\quad :$
s,t$\in$X$\}$
$= \zeta(\underline{\mu(x)} \wedge \mathbf{1}_x,\ \underline{\lambda(y)} \vee \mathbf{1}_{(X-y)})$.
(ii) $\zeta^{\sim}(\cup_{i\in\Lambda}\ \mathbf{1}_{C_i},\ \lambda) = \inf\ \{\zeta(\underline{\cup_{i\in\Lambda}\ \mathbf{1}_{C_i}(x)} \wedge \mathbf{1}_x,\ \underline{\lambda(y)} \vee \mathbf{1}_{(X-y)})\quad :\quad$ x,y$\in$X$\}$
$= \wedge_{i\in\Lambda}\ \inf\ \{\zeta(\underline{\mathbf{1}_{C_i}(x)} \wedge \mathbf{1}_x,\ \underline{\lambda(y)} \vee \mathbf{1}_{(X-y)})\quad :\ $ x,y$\in$X$\}$, by (TT2)
$= \wedge_{i\in\Lambda}\ \zeta^{\sim}(\mathbf{1}_{C_i},\ \lambda)$.
(iii) It is immediately follows by (TT2), from the fact that for every x,y$\in$ X,
$\underline{\mu(x)} \wedge \mathbf{1}_x\ \leq\ \mu$ and $\underline{\lambda(y)} \vee \mathbf{1}_{(X-y)}\ \geq\ \lambda$.
(iv) Trivially holds.
(v) It is immediately follows from (i).
(vi)$(\zeta\ \wedge\ \eta)^{\sim}(\mu,\ \lambda) = \inf\ \{(\zeta\ \wedge\ \eta)(\underline{\mu(x)} \wedge \mathbf{1}_x,\ \underline{\lambda(y)} \vee \mathbf{1}_{(X-y)})\quad :\ $ x,y$\in$X$\}$
$= \inf\ \{\zeta(\underline{\mu(x)} \wedge \mathbf{1}_x,\ \underline{\lambda(y)} \vee \mathbf{1}_{(X-y)})\ \wedge\ \eta(\underline{\mu(x)} \wedge \mathbf{1}_x,\ \underline{\lambda(y)} \vee \mathbf{1}_{(X-y)})\quad :$
x,y$\in$X$\}$
$= \inf\ \{\zeta(\underline{\mu(x)} \wedge \mathbf{1}_x,\ \underline{\lambda(y)} \vee \mathbf{1}_{(X-y)})\quad :\quad$ x,y$\in$X$\}$ $\wedge$
$\inf\ \{\eta(\underline{\mu(x)} \wedge \mathbf{1}_x,\ \underline{\lambda(y)} \vee \mathbf{1}_{(X-y)})\quad :\quad$ x,y$\in$X$\}$
$= \zeta^{\sim}(\mu,\ \lambda)\ \wedge\ \eta^{\sim}(\mu,\ \lambda)$.
(vii) $(\zeta\ o_T\ \eta)^{\sim} \leq\ (\zeta^{\sim}\ o_T\ \eta^{\sim})^{\sim}$ follows from (iii).
Now, let $\varepsilon \in I_1$ be such that $\varepsilon\ T\ \varepsilon < (\zeta^{\sim}\ o_T\ \eta^{\sim})^{\sim}$.
Then for any x,y$\in$X, there is $C_{xy} \subseteq$ X such that
$\varepsilon\ T\ \varepsilon < \eta^{\sim}(\underline{\mu(x)} \wedge \mathbf{1}_x,\ \mathbf{1}_{C_{xy}})\ T\ \zeta^{\sim}(\mathbf{1}_{C_{xy}},\ \underline{\lambda(y)} \vee \mathbf{1}_{(X-y)})$.
By taking C $= \cup_{x,y\in X}\ C_{xy} \subseteq$ X, we have for any x$\in$ X,
$\eta^{\sim}(\underline{\mu(x)} \wedge \mathbf{1}_x,\ \mathbf{1}_C) \geq \eta^{\sim}(\underline{\mu(x)} \wedge \mathbf{1}_x,\ \mathbf{1}_{C_{xy}}) > \varepsilon$, (y$\in X$).
$\eta^{\sim}(\mu,\ \mathbf{1}_C) = (\eta^{\sim})^{\sim}(\mu,\ \mathbf{1}_C)$, by (v)
$= \inf\ \{\eta^{\sim}(\underline{\mu(x)} \wedge \mathbf{1}_x,\ \mathbf{1}_C)\quad :\quad$ x$\in X)\} \geq \varepsilon$,
$\zeta^{\sim}(\mathbf{1}_C,\ \lambda) = (\zeta^{\sim})^{\sim}(\mathbf{1}_C,\ \lambda)$, by (v)
$= \inf\ \{\zeta^{\sim}(\mathbf{1}_{C_{xy}},\ \lambda)\quad :\quad$ x,y$\in X)\}$
$= \inf\ \{\inf\ \{\zeta^{\sim}(\mathbf{1}_{C_{xy}},\ \underline{\lambda(y)} \vee \mathbf{1}_{(X-y)})\quad :\quad$ x$\in X)\}\quad :\quad$ y$\in X)\} \geq \varepsilon$.
Thus we have
$(\zeta^{\sim}\ o_T\ \eta^{\sim})(\mu,\ \lambda) = \sup_{C\subseteq X}\ [\eta^{\sim}(\mu,\ \mathbf{1}_C)\ T\ \zeta^{\sim}(\mathbf{1}_C,\ \lambda)] \geq \varepsilon\ T\ \varepsilon$.
Which shows that, $(\zeta^{\sim}\ o_T\ \eta^{\sim})^{\sim} \leq\ (\zeta^{\sim}\ o_T\ \eta^{\sim})$.
The opposite inequality, follows from (iii), which rendering (vii) and winds up the proof.

Theorem 4.2. If $\wp$ is a T-syntopogenous structure on a set X, then so $\wp^{\sim} = \{\zeta^{\sim}\quad :\quad \zeta \in \wp\}$ is a T-syntopogenous structure on X.
Proof. Let $\wp$ be a T-syntopogenous structure on X. Then :
(TS1) We show that $\wp^{\sim}$ is directed, given $\zeta^{\sim}, \eta^{\sim} \in \wp^{\sim}$ this means that $\zeta, \eta \in \wp$. Since
$\wp$ is directed, then there is $\xi \in \wp$ such that $\xi \geq \zeta \vee \eta$. This implies that, $\xi^{\sim} \in \wp^{\sim}$

which satisfies by Proposition 4.1 (iv), that $\xi^{\sim} \geq \zeta^{\sim} \vee \eta^{\sim}$.

(TS2) Let $\zeta^{\sim} \in \wp^{\sim}$ and $\varepsilon \in I_0$, that is $\zeta \in \wp$. Then there is $\zeta_\varepsilon \in \wp$ such that $\zeta \leq (\zeta_\varepsilon \ o_T\ \zeta_\varepsilon) + \underline{\varepsilon}$. Hence, $\zeta_\varepsilon^{\sim} \in \wp^{\sim}$ which satisfies by Proposition 4.1, that $\zeta^{\sim} \leq [(\zeta_\varepsilon \ o_T\ \zeta_\varepsilon) + \underline{\varepsilon}\,]^{\sim} = (\zeta_\varepsilon \ o_T\ \zeta_\varepsilon)^{\sim} + \underline{\varepsilon} \leq (\zeta_\varepsilon^{\sim} \ o_T\ \zeta_\varepsilon^{\sim}) + \underline{\varepsilon}$.

Which completes the proof.

Let $f : X \longrightarrow Y$ be a function and ζ be a T-topogenous order on X. Then the mapping $f(\zeta) : I^Y \times I^Y \longrightarrow I$, defined by :

$$(f(\zeta))(\nu,\ \rho) = \zeta(f^{\leftarrow}(\nu),\ f^{\leftarrow}(\rho)), \qquad \nu, \rho \in I^Y.$$

is a T-topogenous order on Y (cf. [3]). It is called the image of ζ by the function f. Also, if η is a T-topogenous order on Y, then the mapping $f^{\leftarrow}(\eta) : I^X \times I^X \longrightarrow I$, defined by :

$$(f^{\leftarrow}(\eta))(\mu,\ \lambda) = \eta(f(\mu), \underline{1} - f(\underline{1} - \lambda)), \qquad \mu, \lambda \in I^X.$$

is a T-topogenous order on X [3]. It is called the inverse image of η under the function f.

Proposition 4.2. Let $f : X \longrightarrow Y$ be a function and ζ be a T-topogenous order on X. Then one has the following :

(i) $f(\zeta^{\sim}) \leq [f(\zeta)]^{\sim}$ and equality holds when f is injective;

(ii) If f is an injective and ζ is saturated, then so is $f(\zeta)$.

Proof. (i) Let $\nu, \rho \in I^Y$ and for all x,s$\in$ X we put f(x) = y, f(s) = t $\in$ Y, so $\underline{(f^{\leftarrow}(\nu))(x)} = \underline{\nu(f(x))} = \underline{\nu(y)} = \underline{f^{\leftarrow}(\nu(y))}$, similarly $\underline{(f^{\leftarrow}(\rho))(s)} = \underline{f^{\leftarrow}(\rho(t))}$. Then

$$\begin{aligned}
[f(\zeta)]^{\sim}(\nu,\ \rho) &= \inf\{f(\zeta)(\underline{\nu(y)} \wedge \mathbf{1}_y, \underline{\rho(t)} \vee \mathbf{1}_{(Y-t)}) &&: \text{y,t}\in\text{Y}\}\\
&= \inf\{\zeta(f^{\leftarrow}(\underline{\nu(y)} \wedge \mathbf{1}_y), f^{\leftarrow}(\underline{\rho(t)} \vee \mathbf{1}_{(Y-t)})) &&: \text{y,t}\in\text{Y}\}\\
&= \inf\{\zeta(\underline{f^{\leftarrow}(\nu(y))} \wedge f^{\leftarrow}(\mathbf{1}_y), \underline{f^{\leftarrow}(\rho(t))} \vee f^{\leftarrow}(\mathbf{1}_{(Y-t)})) &&: \text{y,t}\in\text{Y}\}\\
&= \inf\{\zeta(\underline{f^{\leftarrow}(\nu(y))} \wedge \mathbf{1}_{f^{\leftarrow}(y)}, \underline{f^{\leftarrow}(\rho(t))} \vee \mathbf{1}_{f^{\leftarrow}(Y-t)}) &&: \text{y,t}\in\text{Y}\}\\
&\geq \inf\{\zeta(\underline{(f^{\leftarrow}(\nu))(x)} \wedge \mathbf{1}_x, \underline{(f^{\leftarrow}(\rho))(s)} \vee \mathbf{1}_{(X-s)}) &&: \text{x,s}\in\text{X}\}\\
&= \zeta^{\sim}(f^{\leftarrow}(\nu),\ f^{\leftarrow}(\rho))\\
&= [f(\zeta)]^{\sim}(\nu,\ \rho).
\end{aligned}$$

That is, $f(\zeta^{\sim}) \leq [f(\zeta)]^{\sim}$.

(ii) It is immediate from (i).

Proposition 4.3. If $f : X \longrightarrow Y$ is a function and η is a T-topogenous order on Y, then one has the following:

(i) $[f^{\leftarrow}(\eta)]^{\sim} \leq f^{\leftarrow}(\eta^{\sim})$ and equality holds when f is injective;

(ii) If f is an injective and η is saturated, then so is $f^{\leftarrow}(\eta)$.

Proof. (i) Let $\mu, \lambda \in I^X$ and notice for all x,s$\in$ X with f(x) = y, f(s) = t $\in$ Y, that $\underline{(f(\mu))(y)} = \underline{\sup\{\mu(x) : x \in f^{\leftarrow}(y)\}} = \underline{\mu(x)}$ (for f injective) $= \underline{f(\mu(x))}$, similarly $\underline{[\underline{1} - f(\underline{1} - \lambda)](t)} = \underline{\underline{1} - f(\underline{1} - \lambda(s))}$. Then

$$\begin{aligned}
[f^{\leftarrow}(\eta)]^{\sim}(\mu,\ \lambda) &= \inf\{f^{\leftarrow}(\eta)(\underline{\mu(x)} \wedge \mathbf{1}_x, \underline{\lambda(s)} \vee \mathbf{1}_{(X-s)}) &&: \text{x,s}\in\text{X}\}\\
&= \inf\{\eta(f(\underline{\mu(x)} \wedge \mathbf{1}_x), \underline{1} - f[\underline{1} - (\underline{\lambda(s)} \vee \mathbf{1}_{(X-s)})]) &&: \text{x,s}\in\text{X}\}\\
&= \inf\{\eta(f(\underline{\mu(x)} \wedge \mathbf{1}_x), \underline{1} - f[(\underline{1} - \underline{\lambda(s)}) \wedge \mathbf{1}_s]) &&: \text{x,s}\in\text{X}\}\\
&\leq \inf\{\eta(f(\underline{\mu(x)}) \wedge f(\mathbf{1}_x), [\underline{1} - f(\underline{1} - \underline{\lambda(s)})] \vee [\underline{1} - f(\mathbf{1}_s)]) &&: \text{x,s}\in\text{X}\}\\
&= \inf\{\eta(\underline{(f(\mu)(y)} \wedge \mathbf{1}_y, \underline{[1 - f(\underline{1} - \lambda)](t)} \vee \mathbf{1}_{(Y-t)})) &&: \text{y,t}\in\text{Y}\}\\
&= \eta^{\sim}(f(\mu), \underline{1} - f(\underline{1} - \lambda))
\end{aligned}$$

$= [f^{\leftarrow}(\eta^{\sim})](\mu,\ \lambda)$.
That is, $[f^{\leftarrow}(\eta)]^{\sim} \leq f^{\leftarrow}(\eta^{\sim})$.
(ii) It is immediately from (i).

We denote by [I-Tops] the category of I-topological spaces and by [Pers T-TS] the category of perfect saturated T-topogenous spaces.

Theorem 4.3 [7]. The map $\tau \vdash\!\longrightarrow \zeta_\tau$ from the set of all I-topologies on a set X which are given by T-neighbourhood structures to the set of all perfect saturated T-topogenous structures on X, is a one-to-one correspondence.

Theorem 4.4. Let $f :$ X $\longrightarrow$ Y be a function. Then
(i) If $f : (\mathrm{X},\ \zeta) \longrightarrow (\mathrm{Y},\ \eta) \in$ [Pers T-TS], then $f : (\mathrm{X},\ \tau_\zeta) \longrightarrow (\mathrm{Y},\ \tau_\eta) \in$ [I-Tops].
(ii) If $f : (\mathrm{X},\ \tau) \longrightarrow (\mathrm{Y},\ \sigma) \in$ [I-Tops], then $f : (\mathrm{X},\ \zeta_\tau) \longrightarrow (\mathrm{Y},\ \zeta_\sigma) \in$ [Pers T-TS].
Proof. Let $\mu, \lambda \in I^X$ and y$\in$Y. Then
(i) $[\mathrm{cl}_\eta\ (f(\mu))](\mathrm{y}) = 1 - \eta(\mathbf{1}_y,\ \underline{1}{-}f(\mu))$, by (2.3)
$\geq 1 - \eta(f(f^{\leftarrow}(\mathbf{1}_y)),\ \underline{1}{-}f(\mu))$, by (TT2)
$\geq 1 - \zeta(f^{\leftarrow}(\mathbf{1}_y),\ \underline{1}{-}\mu)$, by (2.4)
$\geq 1 - \wedge\ \{\zeta(\mathbf{1}_x,\ \underline{1}{-}\mu)\ :\ \mathrm{x}\in f^{\leftarrow}(\mathrm{y})\}$
$= \vee\ \{1 -\ \zeta(\mathbf{1}_x,\ \underline{1}{-}\mu)\ :\ \mathrm{x}\in f^{\leftarrow}(\mathrm{y})\}$
$= \vee\ \{(\mathrm{cl}_\zeta\ (\mu))(\mathrm{x})\ :\ \mathrm{x}\in f^{\leftarrow}(\mathrm{y})\}$
$= [f(\mathrm{cl}_\zeta\ (\mu))](\mathrm{y})$.
Thus, $f \in$ [I-Tops].
(ii) $\zeta_\sigma(f(\mu),\ \underline{1}{-}f(\lambda)) = 1 -$ hgt $[f(\mu)\ T\ \mathrm{cl}_\sigma(f(\lambda))]$, by [7. Theorem 2.3]
$= 1 - \sup\ \{f(\mu)(\mathrm{y})\ T\ \mathrm{cl}_\sigma(f(\lambda))(\mathrm{y})\ :\ \mathrm{y}\in \mathrm{Y}\}$
$\leq 1 - \sup\ \{f(\mu)(\mathrm{y})\ T\ f(\mathrm{cl}_\tau(\lambda))(\mathrm{y})\ :\ \mathrm{y}\in \mathrm{Y}\}$, by continuity of f
$= 1 - \sup\ \{\sup_{x\in\ f^{\leftarrow}(y)}\ \mu(\mathrm{x})\ \ T\ \sup_{x\in\ f^{\leftarrow}(y)}\ (\mathrm{cl}_\tau(\lambda))(\mathrm{x})\ :\ \mathrm{y}\in \mathrm{Y}\}$
$\leq 1 - \sup\ \{\mu(\mathrm{x})\ \ T\ (\mathrm{cl}_\tau(\lambda))(\mathrm{x})\ :\ \mathrm{x}\in \mathrm{X}\}$, clear
$= 1 -$ hgt $[\mu\ T\ \mathrm{cl}_\tau(\lambda)]$
$= \zeta_\tau(\mu,\ \underline{1}{-}\lambda)$, by [7. Theorem 2.3] again
Thus, by (2.4), $f \in$ [Pers T-TS].
Which completes the proof.

We get from Theorems 4.3 and 4.4 the following
Theorem 4.5. [Pers T-TS] and [I-Tops] are isomorphic categories.

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