Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 5, No. 1, (January 2013), pp. 107–114
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr

Common fixed point theorems using implicit relation and property (E.A) in fuzzy metric spaces

Suneel Kumar, Sunny Chauhan

Received 15 January 2012; Accepted 9 April 2012

Abstract. The aim of this paper is to prove a common fixed point theorem for six maps via notion of pairwise commuting maps in fuzzy metric space satisfying contractive type implicit relation. Our main result extends the result of Aalam, Kumar and Pant [A common fixed point theorem in fuzzy metric space, Bull. Math. Anal. Appl. 2(4) (2010) 76–82].

2010 AMS Classification: 47H10, 54H25.

Keywords: Fuzzy metric space, Weakly compatible maps, Implicit relation, Property (E.A), Fixed point.

Corresponding Author: Sunny Chauhan (sun.gkv@gmail.com)

1. Introduction

Zadeh [21] introduced the concept of fuzzy sets in 1965 and in the next decade Kramosil and Michalek [10] introduced the concept of fuzzy metric spaces (briefly, FM-spaces) in 1975, which opened an avenue for further development of analysis in such spaces. Consequently in due course of time some metric fixed point results were generalized to FM-spaces by various authors viz George and Veeramani [5], Grabiec [6] and others.

For the last quarter of the twentieth century, there has been considerable interest to study the common fixed points of commuting maps and its weaker forms. In 1994, Mishra et al. [14] extended the notion of compatible maps (introduced by Jungck [8] in metric space) under the name of asymptotically commuting maps and Singh and Jain [19] extended the notion of weakly compatible maps (introduced by Jungck [9] in metric space) to FM-spaces. In 2007, Pant and Pant [16] extended the study of common fixed points of a pair of non-compatible maps (studied by Pant [15] in metric space) and the property (E.A) to FM-spaces. Note that the study of property (E.A) has been initiated by Aamri and Moutawakil [2] as a generalization of the concept of non-compatible maps in metric spaces. Employing property (E.A), several results have been obtained in fuzzy metric space (see [1], [3], [11], [13]). In 2009, Imdad et al. [7] introduced the notion of pairwise commuting maps.

Implicit relations are used as a tool for finding common fixed point of contraction maps. Recently, Aalam, Kumar and Pant [1] proved a common fixed point theorem without completeness of space and continuity of involved mappings in FM-space, which generalizes the result of Singh and Jain [19].

In the present paper, we prove a common fixed point theorem for six self-maps in FM-space satisfying contractive type implicit relations. As an application, we extend our main result to four finite families of self-maps in FM-space.

2. Preliminaries

Definition 2.1 ([21])**.** Let X be any set. A fuzzy set A in X is a function with domain X and values in $[0, 1]$.

Definition 2.2 ([18])**.** A binary operation $* : [0, 1] \times [0, 1] \to [0, 1]$ is called a continuous t-norm if $([0, 1], *)$ is an abelian topological monoid with the unit 1 such that $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0, 1]$.

Definition 2.3 ([10])**.** The triplet $(X, M, *)$ is an FM-space if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set in $X^2 \times [0, \infty)$ satisfying the following conditions for all $x, y, z \in X$ and $t, s > 0$,

(1) $M(x, y, t) = 1$ for all $t > 0$ if and only if $x = y$;
(2) $M(x, y, 0) = 0$;
(3) $M(x, y, t) = M(y, x, t)$;
(4) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$;
(5) $M(x, y, \cdot) : [0, \infty) \to [0, 1]$ is left continuous.

In the following example (see [5]), we know that every metric induces a fuzzy metric:

Example 2.4. Let (X, d) be a metric space. Define $a * b = ab$ (or $a * b = \min\{a, b\}$) for all $x, y \in X$ and $t > 0$,

$$M(x, y, t) = \frac{t}{t + d(x, y)}.$$

Then $(X, M, *)$ is an FM-space and the fuzzy metric M induced by the metric d is often referred to as the standard fuzzy metric.

Definition 2.5 ([6])**.** Let $(X, M, *)$ be an FM-space. Then

(1) a sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ (denoted by $\lim_{n\to\infty} x_n = x$) if $\lim_{n\to\infty} M(x_n, x, t) = 1$ for all $t > 0$.
(2) a sequence $\{x_n\}$ in X is called a Cauchy sequence if $\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1$ for all $t > 0$ and $p > 0$.
(3) an FM-space in which every Cauchy sequence is convergent is called complete.

Lemma 2.6 ([6])**.** *For all, $x, y \in X$, $M(x, y, \cdot)$ is non-decreasing.*

Lemma 2.7 ([12]). *Let $M(x, y, *)$ be an FM-space. Then M is a continuous function on $X^2 \times (0, \infty)$.*

Definition 2.8 ([14]). Let A and S maps from an FM-space $(X, M, *)$ into itself. The maps A and S are said to be compatible (or asymptotically commuting), if for all t

$$\lim_{n\to\infty} M(ASx_n, SAx_n, t) = 1,$$

whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = z = \lim_{n\to\infty} Sx_n$ for some $z \in X$.

Definition 2.9 ([20]). Let A and S be maps from an FM-space $(X, M, *)$ into itself. The maps are said to be weakly compatible if they commute at their coincidence points, that is, $Az = Sz$ implies that $ASz = SAz$.

Remark 2.10. Every pair of compatible maps is weakly compatible but reverse is not always true.

Definition 2.11 ([16]). Let A and S be two self-maps of an FM-space $(X, M, *)$. We say that A and S satisfy the property (E.A) if there exists a sequence $\{x_n\}$ such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$ for some $z \in X$.

Notice that weakly compatible and property (E.A) are independent to each other (see [17], Example 2.2).

Remark 2.12. From Definition 2.11, it is inferred that two self maps A and S on an FM-space $(X, M, *)$ are non-compatible if and only if there exists at least one sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = z = \lim_{n\to\infty} Sx_n$ for some $z \in X$, but for some $t > 0$, either $\lim_{n\to\infty} M(ASx_n,\ SAx_n, t) \neq 1$ or the limit does not exist. Therefore, it is easy to see that any two non-compatible self-maps of $(X, M, *)$ satisfy the property (E.A) from Definition 2.11. But, two maps satisfying the property (E.A) need not be noncompatible (see [4], Example 1).

Definition 2.13 ([7]). Two families of self-maps $\{A_i\}$ and $\{B_j\}$ are said to be pairwise commuting if:

(1) $A_iA_j = A_jA_i$, $i, j \in \{1, 2, \ldots, m\}$,
(2) $B_iB_j = B_jB_i$, $i, j \in \{1, 2, \ldots, n\}$,
(3) $A_iB_j = B_jA_i$, $i \in \{1, 2, \ldots, m\}$, $j \in \{1, 2, \ldots, n\}$.

3. Implicit relation

In our results, we deal with implicit relation used in [19]. Let Φ be the set of all real continuous functions $\phi : (\mathbb{R}^+)^4 \to \mathbb{R}$, non-decreasing in the first argument and satisfying the following conditions:

(ϕ_1) For $u, v \geq 0$, $\phi(u, v, u, v) \geq 0$ or $\phi(u, v, v, u) \geq 0$ implies that $u \geq v$.
(ϕ_2) $\phi(u, u, 1, 1) \geq 0$ implies that $u \geq 1$.

Example 3.1. Define $\phi(t_1, t_2, t_3, t_4) = at_1 + bt_2 + ct_3 + dt_4$, where a, b, c, d are real constants. If $a > \max\{b, d\}$ and $a + c = b + d > 0$, then $\phi \in \Phi$.

4. Results

In our main result, we utilize the notion of commuting pairwise due to Imdad et al. [7].

Theorem 4.1. *Let A, B, R, S, H and T be self-maps of a FM-space $(X, M, *)$ satisfying*

(1) *(A, SR) or (B, TH) satisfies the property (E.A);*
(2) $\phi\begin{pmatrix} M(Ax, By, t), M(SRx, THy, t), \\ M(Ax, SRx, t), M(By, THy, t) \end{pmatrix} \geq 0$, *for all $t > 0$, $x, y \in X$ and for some $\phi \in \Phi$;*
(3) *$A(X) \subseteq TH(X)$, $B(X) \subseteq SR(X)$;*
(4) *One of $A(X)$, $B(X)$, $SR(X)$ and $TH(X)$ is a complete subspace of X.*

Then the pairs (A, SR) and (B, TH) have a point of coincidence each. Moreover, A, B, R, S, H and T have a unique common fixed point provided the pairs (A, SR) and (B, TH) commute pairwise (i.e. $AS = SA$, $AR = RA$, $SR = RS$, $BT = TB$, $BH = HB$ and $TH = HT$).

Proof. If the pair (B, TH) satisfies the property (E.A), then there exists a sequence $\{y_n\}$ in X such that $By_n \to z$ and $THy_n \to z$, for some $z \in X$ as $n \to \infty$. Since $B(X) \subseteq SR(X))$, there exists a sequence $\{x_n\}$ in X such that $By_n = SRx_n$. Hence, $SRx_n \to z$ as $n \to \infty$.

Now we show that $Ax_n \to z$ as $n \to \infty$. By putting $x = x_n$ and $y = y_n$ in (2), we have

$$\phi\begin{pmatrix} M(Ax_n, By_n, t), M(SRx_n, THy_n, t), \\ M(Ax_n, SRx_n, t), M(By_n, THy_n, t) \end{pmatrix} \geq 0.$$

Let $Ax_n \to l(\neq z)$ for $t > 0$ as $n \to \infty$. Then, passing to limit as $n \to \infty$, we get

$$\phi\left(M(l, z, t), M(z, z, t), M(l, z, t), M(z, z, t)\right) \geq 0,$$

or

$$\phi\left(M(l, z, t), 1, M(l, z, t), 1\right) \geq 0,$$

Using (ϕ_1), we get $M(l, z, t) \geq 1$ for all $t > 0$. Hence, $M(l, z, t) = 1$ i.e. $l = z$. It follows that $Ay_n \to z$ as $n \to \infty$. Suppose that $SR(X)$ is a complete subspace of X. Then, $z = SRu$ for some $u \in X$. Putting $x = u$ and $y = y_n$ in (2), we have

$$\phi\begin{pmatrix} M(Au, By_n, t), M(SRu, THy_n, t), \\ M(Au, SRu, t), M(By_n, THy_n, t) \end{pmatrix} \geq 0.$$

Letting $n \to \infty$,

$$\phi(M(Au, z, t), M(z, z, t), M(Au, z, t), M(z, z, t)) \geq 0,$$

or

$$\phi(M(Au, z, t), 1, M(Au, z, t), 1) \geq 0.$$

Using (ϕ_1), we get $M(Au, z, t) \geq 1$ for all $t > 0$. Hence, $M(Au, z, t) = 1$ i.e. $Au = z$. Thus $Au = SRu = z$ which shows that the pair (A, S) has a point of coincidence. On the other hand, since $A(X) \subseteq TH(X)$ and $Au = z$, there exists a point $v \in X$

such that $THv = z$. Now we show that $THv = Bv$. By putting $x = u$ and $y = v$ in (2), we have

$$\phi\left(\begin{array}{c} M(Au, Bv, t), M(SRu, THv, t), \\ M(Au, SRu, t), M(Bv, THv, t) \end{array}\right) \geq 0,$$

or

$$\phi(M(z, Bv, t), 1, 1, M(Bv, z, t)) \geq 0.$$

Using (ϕ_1), we get $M(z, Bv, t) \geq 1$ for all $t > 0$. Hence, $M(z, Bv, t) = 1$ i.e. $Bv = z$. Thus $Bv = THv = z$ which shows that the pair (B, T) has a point of coincidence. Since the pairs (A, SR) and (B, TH) are commuting pairwise i.e. $AS = SA$, $AR = RA$, $SR = RS$, $BT = TB$, $BH = HB$ and $TH = HT$. It implies that both the pairs (A, SR) and (B, TH) are weakly compatible at u and v respectively, i.e. $z = Au = SRu = Bv = THv$, therefore $Az = A(SR)u = (SR)Au = (SR)z$ and $Bz = B(TH)v = (TH)Bv = THz$. Now we assert that z is a fixed point of the self-maps A, S and R. Putting $x = Rz$ and $y = z$ in (2), we have

$$\phi\left(\begin{array}{c} M(A(Rz), Bz, t), M(SR(Rz), THz, t), \\ M(A(Rz), SR(Rz), t), M(Bz, THz, t) \end{array}\right) \geq 0,$$

and so

$$\phi(M(Rz, z, t), M(Rz, z, t), M(Rz, Rz, t), M(z, z, t)) \geq 0,$$

or

$$\phi(M(Rz, z, t), M(Rz, z, t), 1, 1) \geq 0.$$

Using (ϕ_2), we get $M(Rz, z, t) \geq 1$ for all $t > 0$. Hence, $M(Rz, z, t) = 1$. Thus, $Rz = z$. Hence $S(z) = S(Rz) = z$. Therefore, $z = Az = Sz = Rz$. On using (2) with $x = z$, $y = Hz$, we have

$$\phi\left(\begin{array}{c} M(Az, B(Hz), t), M(SRz, TH(Hz), t), \\ M(Az, SRz, t), M(B(Hz), TH(Hz), t) \end{array}\right) \geq 0,$$

and so

$$\phi(M(z, Hz, t), M(z, Hz, t), M(z, z, t), M(Hz, Hz, t)) \geq 0,$$

or

$$\phi(M(z, Hz, t), M(z, Hz, t), 1, 1) \geq 0,$$

Using (ϕ_2), we get $M(z, Hz, t) \geq 1$ for all $t > 0$. Hence, $M(z, Hz, t) = 1$. Thus, $Hz = z$. Hence $T(z) = T(Hz) = z$. Therefore, $z = Bz = Tz = Hz$. Thus we conclude that z is a common fixed point of self-maps A, B, R, S, H and T. Let w be another common fixed point of self-maps A, B, R, S, H and T then on using (2) with $x = z$, $y = w$, we have

$$\phi(M(Az, Bw, t), M(Sz, Tw, t), M(Az, Sz, t), M(Bw, Tw, t)) \geq 0,$$

and so

$$\phi(M(z, w, t), M(z, w, t), M(z, z, t), M(w, w, t)) \geq 0,$$

or

$$\phi(M(z, w, t), M(z, w, t), 1, 1)) \geq 0.$$

Using (ϕ_2), we get $M(z, w, t) \geq 1$ for all $t > 0$. Hence, $M(z, w, t) = 1$. Therefore, $z = w$ and the common fixed point is unique. We can also prove the same result if the pair (A, S) satisfies the property (E.A). The proof is similar when $TH(X)$ is assumed to be a complete subspace of X. The remaining two cases pertain essentially to the previous cases. If we assume that $A(X)$ is a complete subspace of X, then $z \in A(X) \subseteq TH(X)$ or $B(X)$ is a complete subspace of X, then $z \in B(X) \subseteq SR(X)$. Thus we can establish that both the pairs (A, SR) and (B, TH) have a point of coincidence each. This completes our proof. □

On taking $R = H = I_X$ (the identity maps on X) in Theorem 4.1, we get the result of Aalam et al. [1]:

Corollary 4.2. *([1], Theorem 3.1) Let A, B, S and T be self-maps of an FM-space $(X, M, *)$ satisfying*

(1) *(A, S) or (B, T) satisfies the property (E.A);*
(2) *$\phi(M(Ax, By, t), M(Sx, Ty, t), M(Ax, Sx, t), M(By, Ty, t)) \geq 0$, for all $t > 0$, $x, y \in X$ and for some $\phi \in \Phi$;*
(3) *$A(X) \subseteq T(X)$, $B(X) \subseteq S(X)$;*
(4) *One of $A(X)$, $B(X)$, $S(X)$ and $T(X)$ is a complete subspace of X.*

Then the pairs (A, S) and (B, T) have a point of coincidence each. Moreover, A, B, S and T have a unique common fixed point provided both the pairs (A, S) and (B, T) are weakly compatible.

Now we give an example which illustrates Corollary 4.2.

Example 4.3. Let $X = [2, 20)$ and d be the usual metric on X. For each $t \in [0, \infty)$, define

$$M(x, y, t) = \begin{cases} \frac{t}{t+|x-y|}, & \text{if } t > 0; \\ 0, & \text{if } t = 0. \end{cases}$$

for all $x, y \in X$. Clearly $(X, M, *)$ is an FM-space, where $*$ is defined by $a * b = ab$. Let $\phi : (\mathbb{R}^+)^4 \to \mathbb{R}$ be defined as in Example 3.1 and define the self-maps A, B, S and T by

$$A(x) = \begin{cases} 2, & \text{if } x \geq 2. \end{cases} \quad S(x) = \begin{cases} 2, & \text{if } x = 2; \\ 6, & \text{if } x > 2. \end{cases}$$

$$B(x) = \begin{cases} 2, & \text{if } x = 2 \text{ or } x > 5; \\ 6, & \text{if } 2 < x \leq 5. \end{cases} \quad T(x) = \begin{cases} 2, & \text{if } x = 2; \\ 12, & \text{if } 2 < x \leq 5; \\ \frac{(x+1)}{3}, & \text{if } x > 5. \end{cases}$$

Then A, B, S and T satisfy all the conditions of Corollary 4.2 and have a unique common fixed point $x = 2$. Clearly, the pairs (A, S) and (B, T) are noncompatible if we assume that $\{x_n\}$ is a sequence defined as $x_n = 5 + \frac{1}{n}$, $n \geq 1$. Also, the pairs (A, S) and (B, T) are weakly compatible since they commute at their coincidence points. It can also be seen that B and T satisfy the property (E.A) and all the maps A, B, S and T are discontinuous at the common fixed point.

On taking $A = B$ and $S = T$ in Corollary 4.2, we get the following result:

Corollary 4.4. *Let A and S be self-maps of an FM-space $(X, M, *)$ satisfying*

(1) *(A, S) satisfies the property (E.A);*
(2) *$\phi(M(Ax, Ay, t), M(Sx, Sy, t), M(Ax, Sx, t), M(Ay, Sy, t)) \geq 0$, for all $t > 0$, $x, y \in X$ and for some $\phi \in \Phi$;*
(3) *$A(X) \subseteq S(X)$;*
(4) *One of $A(X)$ and $S(X)$ is a complete subspace of X.*

Then the pair (A, S) has a point of coincidence. Moreover, A and S have a unique common fixed point provided the pair (A, S) is weakly compatible.

As an application of Corollary 4.2, we extend the related result to four finite families of self-maps on FM-spaces.

Theorem 4.5. *Let $\{A_1, A_2, \ldots A_m\}$, $\{B_1, B_2, \ldots B_p\}$, $\{S_1, S_2, \ldots S_n\}$ and $\{T_1, T_2, \ldots T_q\}$ be four finite families of self-maps of an FM-space $(X, M, *)$ such that $A = A_1A_2 \ldots A_m$, $B = B_1B_2 \ldots B_p$, $S = S_1S_2 \ldots S_n$ and $T = T_1T_2 \ldots T_q$ which also satisfy conditions (1)-(4) of Corollary 4.2. Then the pairs (A, S) and (B, T) have a point of coincidence each. Moreover, if the family $\{A_i\}_{i=1}^m$ commute pairwise with the family $\{S_i\}_{j=1}^n$ whereas the family $\{B_r\}_{r=1}^p$ commute pairwise with the family $\{T_k\}_{k=1}^q$, then (for all $i \in \{1, 2, \ldots, m\}$, $j \in \{1, 2, \ldots, n\}$, $r \in \{1, 2, \ldots, p\}$ and $k \in \{1, 2, \ldots, q\}$) A_i, B_j, S_r and T_k have a common fixed point.*

Proof. Using the terminology of Theorem 4.1, the proof of this theorem is similar to that of Theorem 3.1 contained in [7], hence it is omitted. □

Remark 4.6. Theorem 4.5 improves and extends the results of Singh and Jain [19] and Aalam et al. [1] to four finite families of self-maps.

By setting $A_1 = A_2 = \ldots = A_m = A$, $B_1 = B_2 = \ldots = B_p = B$, $S_1 = S_2 = \ldots = S_n = S$ and $T_1 = T_2 = \ldots = T_q = T$ in Theorem 4.5, we deduce the following:

Corollary 4.7. *Let A, B, S and T be self-maps of an FM-space $(X, M, *)$ satisfying*

(1) *(A^m, S^n) or (B^p, T^q) satisfies the property (E.A);*

(2) $\phi\begin{pmatrix} M(A^m x, B^p y, t), M(S^n x, T^q y, t), \\ M(A^m x, S^n x, t), M(B^p y, T^q y, t) \end{pmatrix} \geq 0$, *for all $t > 0$, $x, y \in X$, for some $\phi \in \Phi$ and m, n, p and q are fixed positive integers;*

(3) *$A^m(X) \subseteq T^q(X)$, $B^p(X) \subseteq S^n(X)$;*

(4) *One of $A^m(X)$, $B^p(X)$, $S^n(X)$ and $T^q(X)$ is a complete subspace of X.*

Then the pairs (A^m, S^n) and (B^p, T^q) have a point of coincidence each. Moreover, A, B, S and T have a unique common fixed point provided both the pairs (A^m, S^n) and (B^p, T^q) commute.

Remark 4.8. From the results, it is asserted that property (E.A) buys containment of ranges without any continuity requirements, besides minimize the commutativity conditions of the maps to the commutativity at their points of coincidence. Moreover, property (E.A) allows replacing the completeness requirement of the whole space with a more natural condition of completeness of the range space.

REFERENCES

[1] I. Aalam, S. Kumar and B. D. Pant, A common fixed point theorem in fuzzy metric space, Bull. Math. Anal. Appl. 2(4) (2010) 76–82. MR2747889

[2] M. Aamri and D. El Moutawakil, Some new common fixed point theorems under strict contractive conditions, J. Math. Anal. Appl. 270(1) (2002) 181–188. MR1911759 (2003d:54057)

[3] M. Abbas, I. Altun and D. Gopal, Common fixed point theorems for non compatible mappings in fuzzy metric spaces, Bull. Math. Anal. Appl. 1(2) (2009) 47–56. MR2578110

[4] J. X. Fang and Y. Gao, Common fixed point theorems under strict contractive conditions in Menger spaces, Nonlinear Anal. 70(1) (2009) 184–193. MR2468228 (2009k:47164)

[5] A. George and P. Veeramani, On some results in fuzzy metric spaces, Fuzzy Sets and Systems 64(3) (1994) 395–399. MR1289545 (95e:54010)
[6] M. Grabiec, Fixed points in fuzzy metric space, Fuzzy Sets and Systems 27(3) (1988) 385–389. MR0956385 (89j:54007)
[7] M. Imdad, J. Ali and M. Tanveer, Coincidence and common fixed point theorems for nonlinear contractions in Menger PM spaces, Chaos Solitons Fractals 42(5) (2009) 3121–3129. MR2562820 (2010j:54064)
[8] G. Jungck, Compatible mappings and common fixed points, Int. J. Math. Math. Sci. 9(4) (1986) 771–779. MR0870534 (87m:54122)
[9] G. Jungck, Common fixed points for noncontinuous nonself maps on nonmetric spaces, Far East J. Math. Sci. 4(2) (1996) 199–215. MR1426938
[10] I. Kramosil and J. Michálek, Fuzzy metrics and statistical metric spaces, Kybernetika (Prague) 11(5) (1975) 336–344. MR0410633 (53 #14381)
[11] S. Kumar and B. Fisher, A common fixed point theorem in fuzzy metric space using property (E.A.) and implicit relation, Thai J. Math. 8(3) (2010) 439–446. MR2763666 (2011m:54045)
[12] J. R. López and S. Romaguera, The Hausdorff fuzzy metric on compact sets, Fuzzy sets and Systems 147(2) (2004) 273–283. MR2089291
[13] D. Miheţ, Fixed point theorems in fuzzy metric spaces using property E.A., Nonlinear Anal. 73(7) (2010) 2184–2188. MR2674194
[14] S. N. Mishra, N. Sharma and S. L. Singh, Common fixed points of maps on fuzzy metric spaces, Int. J. Math. Math. Sci. 17(2) (1994) 253–258. MR1261071
[15] R. P. Pant, Common fixed points of contractive maps, J. Math. Anal. Appl. 226(1) (1998) 251–258. MR1646430 (99h:54063)
[16] V. Pant and R. P. Pant, Fixed points in fuzzy metric space for noncompatible maps, Soochow J. Math. 33(4) (2007) 647–655. MR2404591
[17] H. K. Pathak, R. R. López and R. K. Verma, A common fixed point theorem using implicit relation and property (E.A) in metric spaces, Filomat 21(2) (2007) 211–234. MR2360891
[18] B. Schweizer and A. Sklar, Probabilistic Metric Spaces, North-Holland Series in Probability and Applied Mathematics, North-Holland Publishing Co., New York, 1983. ISBN: 0-444-00666-4 MR0790314 (86g:54045)
[19] B. Singh and S. Jain, Semicompatibility and fixed point theorems in fuzzy metric space using implicit relation, Internat. J. Math. Math. Sci. 2005(16) (2005) 2617–2629. MR2184754
[20] B. Singh and S. Jain, Weak compatibility and fixed point theorems in fuzzy metric spaces, Ganita 56(2) (2005) 167–176. MR2251296
[21] L. A. Zadeh, Fuzzy sets, Information and Control 8 (1965) 338–353.

Suneel Kumar (ksuneel_math@rediffmail.com)
Department of Mathematics, R. H. Government Postgraduate College, Kashipur-244713, (U. S. Nagar), Uttarakhand, India.

Sunny Chauhan (sun.gkv@gmail.com)
Department of Mathematics, R. H. Government Postgraduate College, Kashipur-244713, (U. S. Nagar), Uttarakhand, India.

Sunny Chauhan (sun.gkv@gmail.com)
Near Nehru Training Centre, H. No. 274, Nai Basti B-14, Bijnor-246701, Uttar Pradesh, India.