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Reliability evaluation of condensate system using fuzzy Markov model

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Abstract. Traditional reliability studies assume that transition rates or probabilities in Markov models are accurate. However, in reality, reliability data is either insufficient or mixed with uncertainty. The purpose of this paper is to evaluate the fuzzy reliability of condensate system. In this paper, the fuzzy Kolmogorov's differential equations are developed by using fuzzy Markov model of condensate system and to evaluate the fuzzy reliability of condensate system, the fuzzy Kolmogorov's differential equations are solved by an existing analytical method for solving nth order fuzzy linear differential equations.

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Keywords: fuzzy reliability; condensate system; fuzzy Markov model; trapezoidal fuzzy number; fuzzy differential equations.

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1. Introduction

The conventional reliability of a system is defined as the probability that a system performs its function properly during a predefined period of time under the condition that the system behavior is fully characterized in the context of probability measures. In various engineering problems, the binary state assumption in conventional reliability theory is not extensively acceptable. Since 1965, a higher importance in scientific environment has been given to fuzzy theory due to L. A. Zadeh, when he presented the basic concepts of fuzzy set theory [13]. This theory can handle all the possible states between a fully working state and completely failed state. Thus binary state assumption in conventional reliability is replaced by fuzzy state assumption.

Kumar et al. [9] described a method of fuzzy Markov model for determination of fuzzy state probabilities of generating units including the effect of maintenance scheduling. Binh and Khoa [2] discussed the application of fuzzy markov in calculating reliability of power systems. Chongshan [4] calculated fuzzy availability of a repairable consecutive-2-out-of-3: F-system. Kumar et al. [10] calculated fuzzy reliability and fuzzy availability of the serial process in butter-oil processing plant. Uprety and Zaheeruddin [12] evaluated the fuzzy reliability of gracefully degradable computing systems. Kumar and Kumar [7] computed the fuzzy reliability of the stainless steel utensil manufacturing unit for the constant failure and repair rates. Liu and Huang [11] introduced a modified fuzzy multi-state system availability assessment approach to compute the system availability under the fuzzy user demand. Aminifar et al. [1] proposed reliability modeling of PMU and the Markov process is employed to analyze the proposed model. Kumar and Kumar [8] used the concept of fuzzy approach in the evaluation of the reliability of biscuit manufacturing plant.

To evaluate the fuzzy reliability using fuzzy Markov model there is need to solve fuzzy Kolmogorov's differential equations. To the best of our knowledge till now there are only two analytical methods for solving n^{th} order fuzzy linear differential equations, introduced by Buckley and Feuring [3]. Their first method of solution was to fuzzify the crisp solution and then checked to see if it satisfies the differential equation with fuzzy initial conditions and the second method was the reverse of the first method, in that they first solved the fuzzy initial value problem and then checked to see if it defines a fuzzy function.

Gupta and Tewari [5] used crisp Markov model to evaluate the crisp reliability of condensate system. In this paper, a fuzzy Markov model is constructed with the help of an existing crisp Markov model of condensate system and to evaluate the fuzzy reliability of condensate system, the fuzzy Kolmogorov's differential equations, obtained by using the constructed fuzzy Markov model, are solved with the help of existing method [3].

This paper is organized as follows. In Section 2, some basic definitions and arithmetic operations between α-cut of trapezoidal fuzzy numbers are presented. In Section 3, the existing method for solving n^{th} order fuzzy linear differential equations is presented. In Section 4, the fuzzy Kolmogorov's differential equations, developed by using fuzzy Markov model of condensate system, are solved with the help of existing method and the obtained solution is used to evaluate the fuzzy reliability of condensate system. The conclusion is discussed in Section 5.

2. Preliminaries

In this section, some basic definitions and arithmetic operations between α-cut of trapezoidal fuzzy numbers are presented.

2.1. Basic definitions.

In this section, some basic definitions are presented.

Definition 2.1 ([6])**.** A fuzzy number $\tilde{A} = (a, b, c, d)$ is said to be a trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x)=\begin{cases} \frac{(x-a)}{(b-a)}, & a \le x < b, \\ 1, & b \le x \le c \\ \frac{(x-d)}{(c-d)}, & c < x \le d, \\ 0, & \text{otherwise.} \end{cases}$$

Definition 2.2 ([6]). A trapezoidal fuzzy number $\tilde{A} = (a, b, c, d)$ is said to be zero trapezoidal fuzzy number if and only if $a = 0, b = 0, c = 0, d = 0$.

Definition 2.3 ([6]). An α-cut of a fuzzy number $\tilde{A}$ is defined as a crisp set $A_\alpha = \{x : \mu_{\tilde{A}}(x) \geq \alpha, x \in X\}$, where $\alpha \in [0, 1]$. For a trapezoidal fuzzy number $\tilde{A} = (a, b, c, d)$ the α-cut is $A_\alpha = [a + (b - a)\alpha, d - (d - c)\alpha]$.

Definition 2.4 ([6]). Two α-cuts $A_\alpha = [a_1, b_1]$ and $B_\alpha = [a_2, b_2]$ are said to be equal i.e., $A_\alpha = B_\alpha$ if and only if $a_1 = a_2$ and $b_1 = b_2$.

2.2. **Arithmetic operations between α-cut of trapezoidal fuzzy numbers.** In this section, some arithmetic operations between α-cut of trapezoidal fuzzy numbers are presented.

Let $A = [a_1, b_1]$ and $B = [a_2, b_2]$ be two α-cuts of trapezoidal fuzzy numbers $\tilde{A}$ and $\tilde{B}$ respectively. Then

(i) $A + B = [a_1 + a_2, b_1 + b_2]$

(ii) $A - B = [a_1 - b_2, b_1 - a_2]$

(iii) $\lambda A = \begin{cases} [\lambda a_1, \lambda b_1], & \lambda \geq 0 \\ [\lambda b_1, \lambda a_1], & \lambda \leq 0 \end{cases}$

(iv) $AB = [a, b]$, where

$$a = \text{minimum}(a_1a_2, a_1b_2, a_2b_1, b_1b_2) \text{ and } b = \text{maximum}(a_1a_2, a_1b_2, a_2b_1, b_1b_2).$$

3. Existing method

Buckley and Feuring [3] introduced two analytical methods for solving n^{th} order fuzzy linear differential equations. In this section, one of these existing methods, for solving n^{th} order fuzzy linear differential equations, is presented.

The solution of n^{th} order fuzzy linear differential equation

$$\sum_{j=0}^{n} \tilde{a}_j \tilde{y}^{(j)} = \tilde{g}(x), \tilde{y}^{(j)}(0) = \tilde{\gamma}_j,\ j = 0, 1, ..., n-1 \tag{1}$$

where, $\tilde{y}^{(j)} = \frac{d^j \tilde{y}}{dx^j}$ and $\tilde{a}_j$ are trapezoidal fuzzy numbers, can be obtained by using the following steps:

Step 1: Find the α-cut

$[a_{j(1)}(x, \alpha), a_{j(2)}(x, \alpha)]$, $[y_1^{(j)}(x, \alpha), y_2^{(j)}(x, \alpha)]$, $[\gamma_{j(1)}(0, \alpha), \gamma_{j(2)}(0, \alpha)]$

and $[g(x, \alpha), g(x, \alpha)]$, corresponding to fuzzy parameters $\tilde{a}_j, \tilde{y}^{(j)}, \tilde{\gamma}_j$ and $\tilde{g}(x)$ respectively.

Step 2: Convert the n^{th} order fuzzy linear differential equation (1), into the following n^{th} order linear differential equation:

$$\sum_{j=0}^{n} [a_{j(1)}(x, \alpha), a_{j(2)}(x, \alpha)][y_1^{(j)}(x, \alpha), y_2^{(j)}(x, \alpha)] = [g(x), g(x)],$$

$$[y_1^{(j)}(0, \alpha), y_2^{(j)}(0, \alpha)] = [\gamma_{j(1)}(0, \alpha), \gamma_{j(2)}(0, \alpha)], j = 0, 1, ..., n-1 \tag{2}$$

Step 3: Using Definition 2.4 and Section 2.2, the fuzzy differential equation, obtained in Step 2, can be split into following ordinary differential equations:

$$\sum_{j=0}^{n} \text{Minimum}\Big(a_{j(1)}(x,\alpha)y_1^{(j)}(x,\alpha), a_{j(1)}(x,\alpha)y_2^{(j)}(x,\alpha),$$
$$a_{j(2)}(x,\alpha)y_1^{(j)}(x,\alpha), a_{j(2)}(x,\alpha)y_2^{(j)}(x,\alpha)\Big)$$
$$\sum_{j=0}^{n} \text{Maximum}\Big(a_{j(1)}(x,\alpha)y_1^{(j)}(x,\alpha), a_{j(1)}(x,\alpha)y_2^{(j)}(x,\alpha),$$
$$a_{j(2)}(x,\alpha)y_1^{(j)}(x,\alpha), a_{j(2)}(x,\alpha)y_2^{(j)}(x,\alpha))$$

$$y_1^{(j)}(0,\alpha) = \gamma_{j(1)}(0,\alpha), y_2^{(j)}(0,\alpha) = \gamma_{j(2)}(0,\alpha)$$

Step 4: Solve the ordinary differential equations, obtained in Step 3, to find the values of $y_1(x_0,\alpha)$ and $y_2(x_0,\alpha)$ corresponding to $x = x_0$, where x_0 is any real number.

Step 5: Check that $[y_1(x_0,\alpha), y_2(x_0,\alpha)]$ defines the α-cut of a fuzzy number or not i.e., for the values of $y_1(x_0,\alpha)$ and $y_2(x_0,\alpha)$, the following conditions are satisfied or not.

(i) $y_1(x_0,\alpha)$ is monotonically increasing function for $\alpha \in [0,1]$
(ii) $y_2(x_0,\alpha)$ is monotonically decreasing function for $\alpha \in [0,1]$
(iii) $y_1(x_0,1) = y_2(x_0,1)$

Case 1: If $[y_1(x_0,\alpha), y_2(x_0,\alpha)]$ defines the α-cut of a fuzzy number then the fuzzy solution $\tilde{y}(x_0)$ of fuzzy differential equation (1) exist and $[y_1(x_0,\alpha), y_2(x_0,\alpha)]$ represents the α-cut corresponding to fuzzy solution $\tilde{y}(x_0)$.

Case 2: If $[y_1(x_0,\alpha), y_2(x_0,\alpha)$ does not define the α-cut of a fuzzy number then the fuzzy solution $\tilde{y}(x_0)$ of fuzzy differential equation (1) does not exist.

4. Case study

Gupta and Tewari [5] used Markov model with crisp parameters to evaluate the crisp reliability of condensate system. In this paper, the crisp parameters of the same Markov model are replaced by fuzzy parameters and then the fuzzy Kolmogorov's differential equations, obtained with the help of fuzzy Markov model, are used to evaluate the fuzzy reliability of condensate system.

4.1. **Fuzzy Markov modeling of condensate system.** Condensate system helps the power plants to function efficiently and keeps them in continuous operation for optimal performance. Condensate system consists of six sub-systems namely A, B, C, D, E and F in series. Fuzzy Markov model of condensate system is shown in Figure 1.

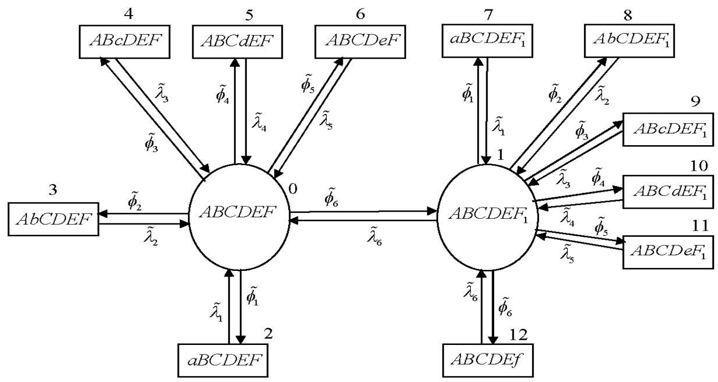

Figure 1: Fuzzy Markov model of condensate system

1. Sub-system A consists of condenser. It is single unit arranged in series. Failure of this unit causes the complete failure of the system.

2. Sub-system B consists of gland steam condenser arranged in series. Failure of this unit causes the complete failure of the system.

3. Sub-system C consists of one drain cooler arranged in series. Failure of this unit causes the complete failure of the system.

4. Sub-system D consists of three low pressure heaters arranged in series. Failure of any one unit causes the complete failure of the system.

5. Sub-system E consists of deaerator arranged in series. Failure of this unit causes the complete failure of the system.

6. Sub-system F consists of two condensate extraction pumps arranged in parallel; one operative and other in cold standby. Complete failure of the system will occur when both failed at a time.

4.2. **Assumptions.** In this section, the assumptions that are used for analyzing the fuzzy reliability of condensate system are presented.

1. The states of all components are mutually independent (statistically independent).
2. Components do not fail simultaneously and the probability that two or more failed components could be repaired and switched to operation at the same time is zero.
3. When one component fails, it is instantaneously replaced by one of the standby subsystems if there is one.
4. A repaired system is as good as new, performance wise, for a specified duration and standby sub-systems if any are of the same nature and capacity as that of active systems.
5. Failure rates and repair rates are represented by trapezoidal fuzzy numbers and are independent with each other.
6. At any given time, the system is either in operating state or in the failed state.

4.3. **Notation.** In this section, notation that is used for analyzing the fuzzy reliability of condensate system are presented.

◯	: Indicates the system is in full working state.
▭	: Indicates the system is in failed state.
A, B, C, D, E, F	: Represent full working states of sub-systems.
F_1	: Denote that the subsystem F is working on standby unit.
a, b, c, d, e, f	: Represent failed states of sub-systems.
$\tilde{P}_0(t)$	: Fuzzy probability of the system working with full capacity at time t.
$\tilde{P}_1(t)$	: Fuzzy probability of the system in cold standby state.
$\tilde{P}_2(t)$ to $\tilde{P}_{12}(t)$	: Fuzzy probability of the system in failed state.
$\tilde{\phi}_i, i = 1$ to 6	: Fuzzy failure rates of sub-systems A, B, C, D, E and F respectively.
$\tilde{\lambda}_i, i = 1$ to 6	: Fuzzy repair rates of sub-systems A, B, C, D, E and F respectively.
d/dt	: Represents the derivative with respect to time t.

4.4. **Data.** The fuzzy failure rates and fuzzy repair rates, represented by trapezoidal fuzzy numbers, that are assumed for evaluating the fuzzy reliability of condensate system are shown in Table 1.

Table 1: Fuzzy failure rate and fuzzy repair rate for the different sub-systems of condensate system

Fuzzy failure rate	Fuzzy repair rate
$\tilde{\phi}_1$ =(0.00615,0.00684,0.00836,0.00919)	$\tilde{\lambda}_1$ =(0.243,0.27,0.33,0.363)
$\tilde{\phi}_2$ =(0.00818,0.00909,0.01111,0.01222)	$\tilde{\lambda}_2$ =(0.122,0.135,0.165,0.182)
$\tilde{\phi}_3$ =(0.00332,0.00369,0.00451,0.00496)	$\tilde{\lambda}_3$ =(0.284,0.315,0.385,0.424)
$\tilde{\phi}_4$ =(0.00616,0.00684,0.00836,0.00919)	$\tilde{\lambda}_4$ =(0.203,0.225,0.275,0.303)
$\tilde{\phi}_5$ =(0.00267,0.00297,0.00363,0.00399)	$\tilde{\lambda}_5$ =(0.151,0.168,0.206,0.226)
$\tilde{\phi}_6$ =(0.0243,0.027,0.033,0.0363)	$\tilde{\lambda}_6$ =(0.223,0.248,0.303,0.333)

4.5. **Fuzzy Kolmogorov's differential equations for the condensate system.** In this section, fuzzy Kolmogorov's differential equations are developed by using fuzzy Markov model of the condensate system.

Fuzzy Kolmogorov's differential equations, developed by using fuzzy Markov model of condensate system, shown in Figure 1, are:

$$\frac{d\tilde{P}_0(t)}{dt} \oplus \tilde{\delta}_1 \tilde{P}_0(t) = \tilde{\lambda}_1 \tilde{P}_2(t) \oplus \tilde{\lambda}_2 \tilde{P}_3(t) \oplus \tilde{\lambda}_3 \tilde{P}_4(t) \oplus \tilde{\lambda}_4 \tilde{P}_5(t) \oplus \tilde{\lambda}_5(t) \tilde{P}_6(t) \oplus \tilde{\lambda}_6(t) \tilde{P}_1(t) \quad (3)$$

$$\frac{d\tilde{P}_1(t)}{dt} \oplus \tilde{\delta}_2 \tilde{P}_1(t) = \tilde{\lambda}_1 \tilde{P}_7(t) \oplus \tilde{\lambda}_2 \tilde{P}_8(t) \oplus \tilde{\lambda}_3 \tilde{P}_9(t) \oplus \tilde{\lambda}_4 \tilde{P}_{10}(t) \oplus \tilde{\lambda}_5 \tilde{P}_{11}(t) \oplus \tilde{\lambda}_6 \tilde{P}_{12}(t) \oplus \tilde{\phi}_6 \tilde{P}_0(t) \quad (4)$$

$$\frac{d\tilde{P}_{1+i}(t)}{dt} \oplus \tilde{\lambda}_i \tilde{P}_{1+i}(t) = \tilde{\phi}_i \tilde{P}_0(t), i = 1, 2, 3, 4, 5 \quad (5)$$

$$\frac{d\tilde{P}_{6+i}(t)}{dt} \oplus \tilde{\lambda}_i \tilde{P}_{6+i}(t) = \tilde{\phi}_i \tilde{P}_1(t), i = 1, 2, 3, 4, 5, 6 \quad (6)$$

where $\tilde{\delta}_1 = \tilde{\phi}_1 \oplus \tilde{\phi}_2 \oplus \tilde{\phi}_3 \oplus \tilde{\phi}_4 \oplus \tilde{\phi}_5 \oplus \tilde{\phi}_6$ and $\tilde{\delta}_2 = \tilde{\phi}_1 \oplus \tilde{\phi}_2 \oplus \tilde{\phi}_3 \oplus \tilde{\phi}_4 \oplus \tilde{\phi}_5 \oplus \tilde{\phi}_6 \oplus \tilde{\lambda}_6$ with fuzzy initial conditions

$\tilde{P}_0(0)$=(0.95, 0.955, 0.965, 0.97), $\tilde{P}_1(0)$=(0.004, 0.0045, 0.0055, 0.006) and $\tilde{P}_j(0)$=(0, 0, 0, 0), j=2 to 12.

4.6. **Solution of fuzzy Kolmogorov's differential equations of condensate system.** The solution of fuzzy Kolmogorov's differential equations of condensate system, developed in Section 4.5, is obtained by using the existing method [3], discussed in Section 3, for $\alpha = 0, 0.2, 0.4, 0.6, 0.8, 1$ at $t = 48$ and the solution is shown in Table 2.

Table 2: Solution of fuzzy Kolmogorov's differential equations for condensate system at $t = 48$

	$\tilde{p}_j(t)$ for $\alpha = 0$		$\tilde{p}_j(t)$ for $\alpha = 0.2$		$\tilde{p}_j(t)$ for $\alpha = 0.4$		$\tilde{p}_j(t)$ for $\alpha = 0.6$		$\tilde{p}_j(t)$ for $\alpha = 0.8$		$\tilde{p}_j(t)$ for $\alpha = 1$	
j	$\tilde{p}_{j1}(t,\alpha)$	$\tilde{p}_{j2}(t,\alpha)$	$\tilde{p}_{j1}(t,\alpha)$	$\tilde{p}_{j2}(t,\alpha)$	$\tilde{p}_{j1}(t,\alpha)$	$\tilde{p}_{j2}(t,\alpha)$	$\tilde{p}_{j1}(t,\alpha)$	$\tilde{p}_{j2}(t,\alpha)$	$\tilde{p}_{j1}(t,\alpha)$	$\tilde{p}_{j2}(t,\alpha)$	$\tilde{p}_{j1}(t,\alpha)$	$\tilde{p}_{j2}(t,\alpha)$
0	0.739958	0.756819	0.740764	0.755949	0.741571	0.755080	0.742377	0.754211	0.743184	0.753342	0.743991	0.752473
1	0.080578	0.082496	0.080656	0.082385	0.080734	0.082275	0.080813	0.082165	0.080891	0.082055	0.08097	0.081945
2	0.018731	0.01916	0.018754	0.019140	0.018778	0.019121	0.018801	0.019101	0.018825	0.019082	0.018849	0.019063
3	0.04955	0.050812	0.049652	0.050781	0.049755	0.050751	0.049858	0.05072	0.049961	0.05069	0.050064	0.05066
4	0.008651	0.008853	0.008663	0.008845	0.008676	0.008837	0.008689	0.008829	0.008702	0.008821	0.008715	0.008814
5	0.02246	0.022954	0.022492	0.022938	0.022524	0.022922	0.022556	0.022906	0.022588	0.02289	0.02262	0.022875
6	0.013085	0.013361	0.013098	0.01334	0.013112	0.01332	0.013126	0.013300	0.013140	0.013280	0.013154	0.01326
7	0.002038	0.002088	0.002040	0.002085	0.002042	0.002082	0.002045	0.002080	0.002047	0.002077	0.00205	0.002075
8	0.005365	0.005536	0.005351	0.005531	0.005338	0.005526	0.005325	0.005521	0.005312	0.005516	0.005299	0.005512
9	0.000941	0.000965	0.000942	0.000963	0.000943	0.000962	0.000945	0.000961	0.000946	0.000960	0.000948	0.000959
10	0.002442	0.0025	0.002445	0.002498	0.002449	0.002496	0.002452	0.002494	0.002456	0.002492	0.00246	0.00249
11	0.00142	0.001456	0.001421	0.001453	0.001423	0.001450	0.001425	0.001448	0.001427	0.001445	0.001429	0.001443
12	0.008774	0.008992	0.008781	0.008978	0.008789	0.008964	0.008796	0.008951	0.008804	0.008937	0.008812	0.008924

4.7. **Fuzzy reliability evaluation of condensate system.** In this section, the results of fuzzy Kolmogorov's differential equations, shown in Table 2, are used to evaluate the fuzzy reliability of condensate system.

Using the fuzzy probabilities for the condensate system, shown in Table 2, the α-cuts corresponding to fuzzy reliability $\tilde{R}(t) = \tilde{p}_1(t) \oplus \tilde{p}_2(t)$ of condensate system are computed for $\alpha = 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1$ at $t = 48$ and are shown in Table 3.

Table 3: Fuzzy reliability of condensate system at $t = 48$

Fuzzy Reliability $\longrightarrow$	$\tilde{R}(t)$	
$\alpha \downarrow$	$R_1(t,\alpha)$	$R_2(t,\alpha)$
0	0.820536	0.839315
0.1	0.820978	0.838825
0.2	0.821421	0.838335
0.3	0.821863	0.837845
0.4	0.822306	0.837356
0.5	0.822748	0.836866
0.6	0.823191	0.836376
0.7	0.823633	0.835887
0.8	0.824076	0.835397
0.9	0.824518	0.834907
1	0.824961	0.834418

The variation in reliability of condensate system at $t = 48$ corresponding to different presumption levels is shown in Figure 2.

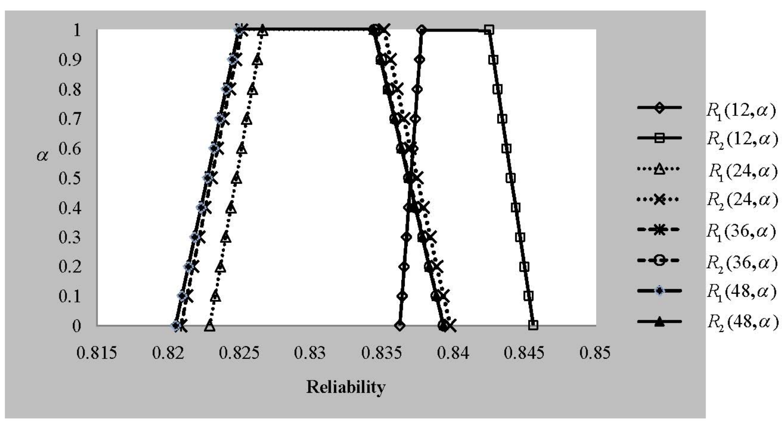

Figure 2: Trapezoidal fuzzy number representing fuzzy reliability of condensate system at $t = 48$ corresponding to different levels of presumptions

Also, the reliability curves, shown in Figure 3 to Figure 5, represents the variation in reliability of condensate system with time at presumption level 0, 0.6, 1 respectively.

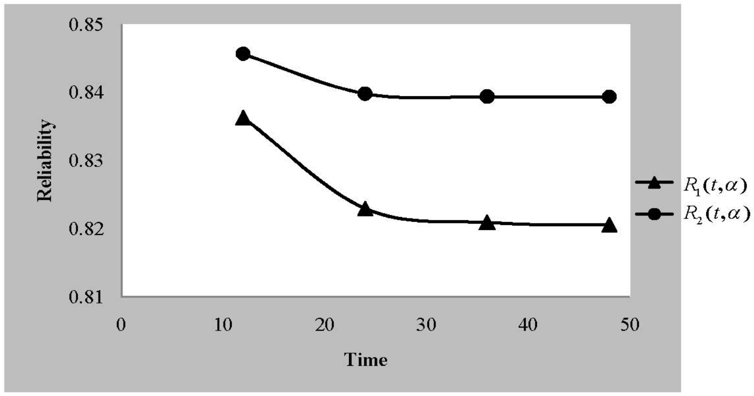

Figure 3: Variation in reliability with time at presumption level 0

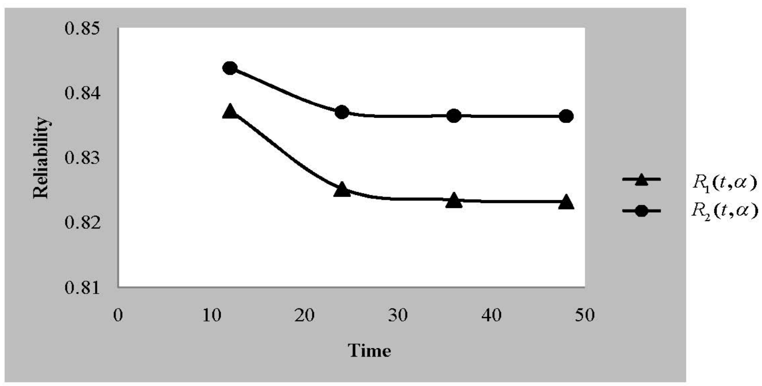

Figure 4: Variation in reliability with time at presumption level 0.6

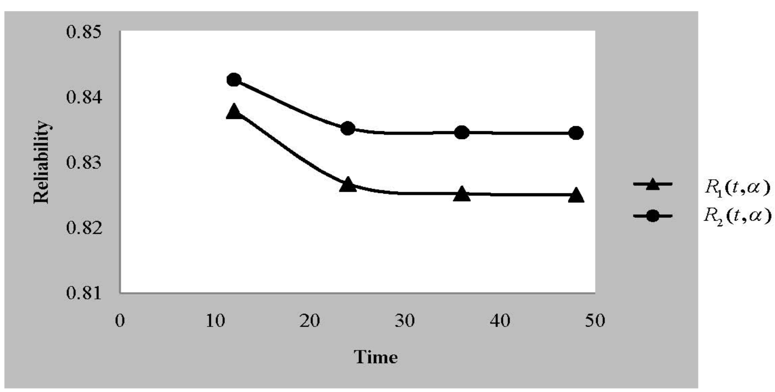

Figure 5: Variation in reliability with time at presumption level 1

5. Conclusions

The fuzzy reliability of condensate system is evaluated by solving the fuzzy Kolmogorov's differential equations, developed by fuzzy Markov model of the condensate system. The variation in reliability of condensate system corresponding to different presumption levels is shown with the help of table and graph. Also, the reliability curves are shown to represent the variation in reliability of condensate system with time at different presumption levels.

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References

[1] F. Aminifar, S. B. Shouraki, M. F. Firuzabad and M. Shahidehpour, Reliability modeling of PMUs using fuzzy sets, IEEE T. Power Deliver. 25 (2010) 2384–2391.
[2] P. T. T. Binh and T. Q. D. Khoa, Application of fuzzy markov in calculating reliability of power systems, Proceedings of IEEE PES Transmission and Distribution Conference and Exposition Latin America, Venezuela (2006) 1–4.
[3] J. J. Buckley and T. Feuring, Fuzzy initial value problem for N^{th}-order fuzzy linear differential equations, Fuzzy Sets and Systems 121 (2001) 247–255.
[4] G. Chongshan, Fuzzy availability analysis of a repairable consecutive-2-out-of-3:F System, Proceedings of IEEE International Conference on Grey Systems and Intelligent Services (2009) 434–437.
[5] S. Gupta and P. C. Tewari, Simulation model for stochastic analysis and performance evaluation of condensate system of a thermal power plant, Bangladesh J. Sci. Ind. Res. 44 (2009) 387–398.
[6] A. Kaufmann and M. M. Gupta, Introduction to Fuzzy Arithmetics: Theory and Applications, Van Nostrand Reinhold, New York (1985).
[7] K. Kumar and P. Kumar, Mathematical modeling and analysis of stainless steel utensil manufacturing unit using fuzzy reliability, International Journal of Engineering Science and Technology 2 (2010) 2370–2376.
[8] K. Kumar and P. Kumar, Fuzzy availability modeling and analysis of biscuit manufacturing plant: a case study, International Journal of Systems Assurance Engineering and Management 2 (2011) 193–204.
[9] D. Kumar, P. K. Sadhu and R. Chakrabarti, Fuzzy markov model for determination of fuzzy state probabilities of generating units including the effect of maintenance scheduling, IEEE Trans. Power Syst. 20 (2005) 2117–2124.
[10] K. Kumar, J. Singh and P. Kumar, Fuzzy reliability and fuzzy availability of the serial process in butter oil processesing plant, J. Math. Stat. 5 (2009) 65–71.
[11] Y. Liu and H. Z. Huang, Reliability assessment for fuzzy multi-state systems, Internat. J. Systems Sci. 41 (2010) 365–379.
[12] I. Uprety and Zaheeruddin, Fuzzy reliability of gracefully degradable computing systems, Proceedings of International Conference on Methods and Models on Computer Science (2009) 1–4.
[13] L. A. Zadeh, Fuzzy Sets, Information and Control 8 (1965) 338–353.

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