Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 4, No. 2, (October 2012), pp. 253–265
ISSN 2093–9310
http://www.afmi.or.kr

Bipolar fuzzy ideals with operators in semigroups

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Received 1 December 2011; Revised 19 January 2012; Accepted 21 January 2012

Abstract. Given a set Ω and the notion of bipolar valued fuzzy sets, the notions of a bipolar Ω-fuzzy left (right) ideal, a bipolar Ω-fuzzy bi-ideal and a bipolar Ω-fuzzy $(1, 2)$-ideal in semigroups are introduced, and related properties are investigated. Using bipolar fuzzy bi-ideals, a bipolar Ω-fuzzy bi-ideal is constructed. Conversely, a bipolar fuzzy bi-ideal is established by using bipolar Ω-fuzzy bi-ideals. Relations among a bipolar Ω-fuzzy bi-ideal, a bipolar Ω-fuzzy left (right) ideal and a bipolar Ω-fuzzy $(1, 2)$-ideal are discussed. Conditions for a bipolar Ω-fuzzy $(1, 2)$-ideal to be a bipolar Ω-fuzzy bi-ideal are provided.

2010 AMS Classification: 20M12, 08A72

Keywords: Bipolar fuzzy sub-semigroup, Bipolar Ω-fuzzy sub-semigroup, Bipolar Ω-fuzzy left (right) ideal, Bipolar Ω-fuzzy bi-ideal, Bipolar Ω-fuzzy $(1, 2)$-ideal, Negative s-cut, Positive t-cut.

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1. Introduction

In the traditional fuzzy sets, the membership degrees of elements range over the interval $[0, 1]$. The membership degree expresses the degree of belongingness of elements to a fuzzy set. The membership degree 1 indicates that an element completely belongs to its corresponding fuzzy set, and the membership degree 0 indicates that an element does not belong to the fuzzy set. The membership degrees on the interval $(0, 1)$ indicate the partial membership to the fuzzy set. Sometimes, the membership degree means the satisfaction degree of elements to some property or constraint corresponding to a fuzzy set (see [1, 7]) In the viewpoint of satisfaction degree, the membership degree 0 is assigned to elements which do not satisfy some property. The elements with membership degree 0 are usually regarded as having the same characteristics in the fuzzy set representation. By the way, among such elements, some have irrelevant characteristics to the property corresponding to a fuzzy set and the others have contrary characteristics to the property. The traditional fuzzy set

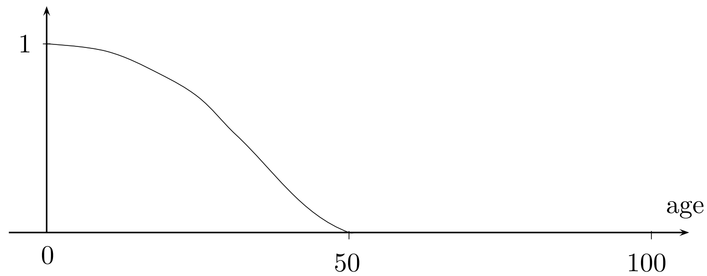

FIGURE 1. A fuzzy set ''*young*''

representation cannot tell apart contrary elements from irrelevant elements. Consider a fuzzy set "*young*" defined on the *age* domain $[0, 100]$ (see Figure 1) Now consider two ages 50 and 95 with membership degree 0. Although both of them do not satisfy the property "*young*", we may say that age 95 is more apart from the property rather than age 50 (see [5]). Only with the membership degrees ranged on the interval [0, 1], it is difficult to express the difference of the irrelevant elements from the contrary elements in fuzzy sets. If a set representation could express this kind of difference, it would be more informative than the traditional fuzzy set representation. Based on these observations, Lee [5] introduced an extension of fuzzy sets named bipolar-valued fuzzy sets. Kim et al. [4] studied ideal theory of semigroups based on the bipolar valued fuzzy set theory. Hur et al. [2] discussed fuzzy sub-semigroups and fuzzy ideals with operators in semigroups.

In this paper, we deal with bipolar valued fuzzy theory with operators applied to sub-semigroups in semigroups. Using a set Ω and the notion of bipolar valued fuzzy sets, we introduce the notions of a bipolar Ω-fuzzy left (right) ideal, a bipolar Ω-fuzzy bi-ideal and a bipolar Ω-fuzzy $(1, 2)$-ideal in semigroups, and investigate related properties. Using bipolar fuzzy bi-ideals, we construct a bipolar Ω-fuzzy bi-ideal, and conversely, we establish a bipolar fuzzy bi-ideal by using bipolar Ω-fuzzy bi-ideals. We discuss relations among a bipolar Ω-fuzzy bi-ideal, a bipolar Ω-fuzzy left (right) ideal and a bipolar Ω-fuzzy $(1, 2)$-ideal. We provide conditions for a bipolar Ω-fuzzy $(1, 2)$-ideal to be a bipolar Ω-fuzzy bi-ideal.

2. PRELIMINARIES

Fuzzy sets are a kind of useful mathematical structure to represent a collection of objects whose boundary is vague. There are several kinds of fuzzy set extensions in the fuzzy set theory, for example, intuitionistic fuzzy sets, interval-valued fuzzy sets, vague sets etc. Bipolar-valued fuzzy sets are an extension of fuzzy sets whose membership degree range is enlarged from the interval $[0, 1]$ to $[-1, 1]$. Bipolar-valued fuzzy sets have membership degrees that represent the degree of satisfaction to the property corresponding to a fuzzy set and its counter-property. In a bipolar-valued fuzzy set, the membership degree 0 means that elements are irrelevant to the corresponding property, the membership degrees on $(0, 1]$ indicate that elements somewhat satisfy the property, and the membership degrees on $[-1, 0)$ indicate that

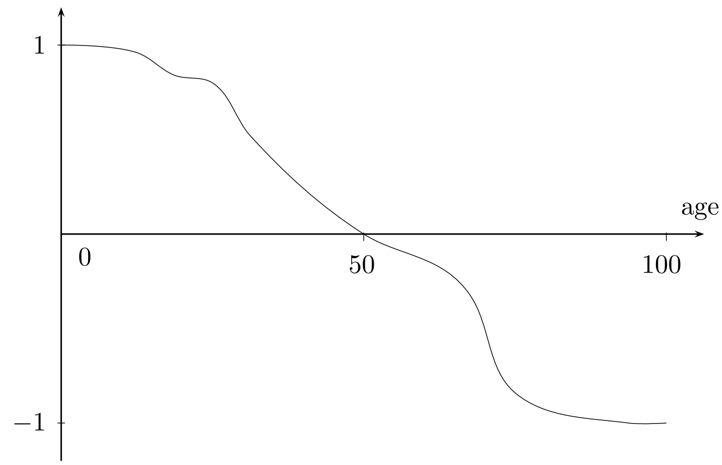

FIGURE 2. A bipolar fuzzy set ''*young*''

elements somewhat satisfy the implicit counter-property (see [5]). Figure 2 shows a bipolar-valued fuzzy set redefined for the fuzzy set "*young*" of Figure 1. The negative membership degrees indicate the satisfaction extent of elements to an implicit counter-property (e.g., old against the property young). This kind of bipolar-valued fuzzy set representation enables the elements with membership degree 0 in traditional fuzzy sets, to be expressed into the elements with membership degree 0 (irrelevant elements) and the elements with negative membership degrees (contrary elements). The age elements 50 and 95, with membership degree 0 in the fuzzy sets of Figure 1, have 0 and a negative membership degree in the bipolar-valued fuzzy set of Figure 2, respectively. Now it is manifested that 50 is an irrelevant age to the property young and 95 is more apart from the property young than 50, i.e., 95 is a contrary age to the property young (see [5]). Let S be the universe of discourse. A bipolar-valued fuzzy set f in S is an object having the form

$$f = \{(x, f_n(x), f_p(x)) \mid x \in S\}$$

where $f_n : S \to [-1, 0]$ and $f_p : S \to [0, 1]$ are mappings. The positive membership degree $f_p(x)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar-valued fuzzy set $f = \{(x, f_n(x), f_p(x)) \mid x \in S\}$, and the negative membership degree $f_n(x)$ denotes the satisfaction degree of x to some implicit counter-property of $f = \{(x, f_n(x), f_p(x)) \mid x \in S\}$. If $f_p(x) \neq 0$ and $f_n(x) = 0$, it is the situation that x is regarded as having only positive satisfaction for $f = \{(x, f_n(x), f_p(x)) \mid x \in S\}$. If $f_p(x) = 0$ and $f_n(x) \neq 0$, it is the situation that x does not satisfy the property of $f = \{(x, f_n(x), f_p(x)) \mid x \in S\}$ but somewhat satisfies the counter-property of $f = \{(x, f_n(x), f_p(x)) \mid x \in S\}$. It is possible for an element x to be $f_p(x) \neq 0$ and $f_n(x) \neq 0$ when the membership function of the property overlaps that of its counter-property over some portion of the domain (see [6]). For the sake of simplicity, we shall use the symbol $f = (S; f_n, f_p)$ for the bipolar-valued fuzzy set $f = \{(x, f_n(x), f_p(x)) \mid x \in S\}$, and use the notion of bipolar fuzzy sets instead of the notion of bipolar-valued fuzzy sets.

3. Bipolar Ω-fuzzy ideals

By a sub-semigroup of a semigroup S we mean a nonempty subset A of S such that $A^2 \subseteq A$. Let S be a semigroup. An element $x \in S$ is said to be regular if there exists an element $a \in S$ such that $x = xax$. A semigroup S is said to be regular if every element of S is regular. For any $x \in S$, we write

$$R_x := \{a \in S \mid x = xax\}.$$

A fuzzy set in S is a function μ from S into the unit interval $[0, 1]$. A fuzzy set μ in S is called a fuzzy sub-semigroup of S if it satisfies

$$(\forall x, y \in S)\,(\mu(xy) \geq \min\{\mu(x), \mu(y)\}).$$

For any family $\{a_i \mid i \in \Lambda\}$ of real numbers, we define

$$\bigvee\{a_i \mid i \in \Lambda\} := \begin{cases} \max\{a_i \mid i \in \Lambda\} & \text{if } \Lambda \text{ is finite,} \\ \sup\{a_i \mid i \in \Lambda\} & \text{otherwise,} \end{cases}$$

$$\bigwedge\{a_i \mid i \in \Lambda\} := \begin{cases} \min\{a_i \mid i \in \Lambda\} & \text{if } \Lambda \text{ is finite,} \\ \inf\{a_i \mid i \in \Lambda\} & \text{otherwise.} \end{cases}$$

A bipolar fuzzy set $f = (S; f_n, f_p)$ in S is called a bipolar fuzzy sub-semigroup of a semigroup S (see [4]) if it satisfies the following condition:

$$(\forall x, y \in S) \begin{pmatrix} f_n(xy) \leq \bigvee\{f_n(x), f_n(y)\} \\ f_p(xy) \geq \bigwedge\{f_p(x), f_p(y)\} \end{pmatrix}. \tag{3.1}$$

In what follows let S and Ω denote a semigroup and a nonempty set, respectively, unless otherwise specified.

A bipolar Ω-fuzzy set F_Ω in S is defined to be an object having the form

$$F_\Omega := \left\{\langle (x, \alpha); f_n^\Omega(x, \alpha), f_p^\Omega(x, \alpha)\rangle \mid (x, \alpha) \in S \times \Omega\right\}$$

where the function $f_n^\Omega : S \times \Omega \to [-1, 0]$ and $f_p^\Omega : S \times \Omega \to [0, 1]$.

We shall use the symbol $F_\Omega = \left\langle S \times \Omega; f_n^\Omega, f_p^\Omega \right\rangle$ for the bipolar Ω-fuzzy set

$$F_\Omega := \left\{\langle (x, \alpha); f_n^\Omega(x, \alpha), f_p^\Omega(x, \alpha)\rangle \mid (x, \alpha) \in S \times \Omega\right\}.$$

Definition 3.1. A bipolar Ω-fuzzy set $F_\Omega = \left\langle S \times \Omega; f_n^\Omega, f_p^\Omega \right\rangle$ in S is called a bipolar Ω-fuzzy left (resp. right) ideal of S if it satisfies

$$\begin{gathered} \left(f_n^\Omega(xy, \alpha) \leq f_n^\Omega(y, \alpha) \text{ and} f_p^\Omega(xy, \alpha) \geq f_p^\Omega(y, \alpha)\right), \\ \left(\text{resp. } f_n^\Omega(xy, \alpha) \leq f_n^\Omega(x, \alpha) \text{ and} f_p^\Omega(xy, \alpha) \geq f_p^\Omega(x, \alpha)\right) \end{gathered} \tag{3.2}$$

for all $x, y \in S$ and $\alpha \in \Omega$.

If $F_\Omega = \left\langle S \times \Omega; f_n^\Omega, f_p^\Omega \right\rangle$ is both a bipolar Ω-fuzzy left ideal and a bipolar Ω-fuzzy right ideal of S, we say that $F_\Omega = \left\langle S \times \Omega; f_n^\Omega, f_p^\Omega \right\rangle$ is a bipolar Ω-fuzzy ideal of S.

Let $S^\Omega := \{u \mid u : \Omega \to S\}$. For any $u, v \in S^\Omega$, we define $(uv)(\alpha) = u(\alpha)v(\alpha)$ for all $\alpha \in \Omega$. Then S^Ω is a semigroup.

Theorem 3.2. *Let $F_\Omega = \left\langle S \times \Omega; f_n^\Omega, f_p^\Omega \right\rangle$ be a bipolar Ω-fuzzy left* (resp. *right*) *ideal of S and let $\Phi_\Omega = \left\langle S^\Omega; \Phi_n^\Omega, \Phi_p^\Omega \right\rangle$ be a bipolar fuzzy set in S^Ω defined by*

$$\Phi_n^\Omega(u) = \bigvee \left\{ f_n^\Omega(u(\alpha), \alpha) \mid \alpha \in \Omega \right\}$$

and

$$\Phi_p^\Omega(u) = \bigwedge \left\{ f_p^\Omega(u(\alpha), \alpha) \mid \alpha \in \Omega \right\}$$

for all $u \in S^\Omega$. Then $\Phi_\Omega = \left\langle S^\Omega; \Phi_n^\Omega, \Phi_p^\Omega \right\rangle$ is a bipolar fuzzy left (resp. *right*) *ideal of S^Ω.*

Proof. Assume that $F_\Omega = \left\langle S \times \Omega; f_n^\Omega, f_p^\Omega \right\rangle$ is a bipolar Ω-fuzzy left ideal of S. For any $u, v \in S^\Omega$, we have

$$\begin{aligned} \Phi_n^\Omega(uv) &= \bigvee \left\{ f_n^\Omega((uv)(\alpha), \alpha) \mid \alpha \in \Omega \right\} \\ &= \bigvee \left\{ f_n^\Omega(u(\alpha)v(\alpha), \alpha) \mid \alpha \in \Omega \right\} \\ &\leq \bigvee \left\{ f_n^\Omega(u(\alpha), \alpha) \mid \alpha \in \Omega \right\} \\ &= \Phi_n^\Omega(v) \end{aligned}$$

and

$$\begin{aligned} \Phi_p^\Omega(uv) &= \bigwedge \left\{ f_p^\Omega((uv)(\alpha), \alpha) \mid \alpha \in \Omega \right\} \\ &= \bigwedge \left\{ f_p^\Omega(u(\alpha)v(\alpha), \alpha) \mid \alpha \in \Omega \right\} \\ &\geq \bigwedge \left\{ f_p^\Omega(u(\alpha), \alpha) \mid \alpha \in \Omega \right\} \\ &= \Phi_p^\Omega(v). \end{aligned}$$

Similarly, if $F_\Omega = \left\langle S \times \Omega; f_n^\Omega, f_p^\Omega \right\rangle$ is a bipolar Ω-fuzzy right ideal of S then

$$\Phi_n^\Omega(uv) \leq \Phi_n^\Omega(u) \text{ and } \Phi_p^\Omega(uv) \geq \Phi_p^\Omega(u)$$

for all $u, v \in S^\Omega$. This completes the proof. □

Definition 3.3. A bipolar Ω-fuzzy set $F_\Omega = \left\langle S \times \Omega; f_n^\Omega, f_p^\Omega \right\rangle$ in S is called a bipolar Ω-fuzzy bi-ideal of S if it is a bipolar Ω-fuzzy sub-semigroup of S that satisfies:

$$(\forall \alpha \in \Omega)\,(\forall x, a, y \in S) \begin{pmatrix} f_n^\Omega(xay, \alpha) \leq \bigvee \left\{ f_n^\Omega(x, \alpha), f_n^\Omega(y, \alpha) \right\} \\ f_p^\Omega(xay, \alpha) \geq \bigwedge \left\{ f_p^\Omega(x, \alpha), f_p^\Omega(y, \alpha) \right\} \end{pmatrix}. \tag{3.3}$$

If $F_\Omega = \left\langle S \times \Omega; f_n^\Omega, f_p^\Omega \right\rangle$ satisfies the condition (3.3) only, we say that $F_\Omega = \left\langle S \times \Omega; f_n^\Omega, f_p^\Omega \right\rangle$ is a generalized bipolar Ω-fuzzy bi-ideal of S. It is clear that every bipolar Ω-fuzzy bi-ideal of S is a generalized bipolar Ω-fuzzy bi-ideal of S, but not conversely as seen in the following example.

Example 3.4. Let S be a semigroup of four elements $\{a, b, c, d\}$ with the multiplication table which is given in Table 1. Let $\Omega = \{1, 2\}$ and let $F_\Omega = \left\langle S \times \Omega; f_n^\Omega, f_p^\Omega \right\rangle$ be a bipolar Ω-fuzzy set in S defined by

$$\begin{aligned} F_\Omega = \{ &\langle (a,1); -0.8, 0.9 \rangle, \langle (a,2); -0.5, 0.6 \rangle, \langle (b,1); -0.7, 0.4 \rangle, \langle (b,2); -0.2, 0.3 \rangle, \\ &\langle (c,1); -0.3, 0.8 \rangle, \langle (c,2); -0.5, 0.3 \rangle, \langle (d,1); -0.7, 0.5 \rangle, \langle (d,2); -0.3, 0.5 \rangle \}. \end{aligned}$$

Table 1. Multiplication table

	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	b	a
d	a	a	b	b

Table 2. Multiplication table

	a	b	c	d	e
a	a	a	a	a	a
b	a	a	a	a	a
c	a	a	c	c	e
d	a	a	c	d	e
e	a	a	c	c	e

Then, as is easily seen, $F_\Omega = \left\langle S \times \Omega; f_n^\Omega, f_p^\Omega \right\rangle$ is a generalized bipolar Ω-fuzzy bi-ideal of S. But it is not a bipolar Ω-fuzzy bi-ideal of S since

$$f_n^\Omega(cc, 2) = f_n^\Omega(b, 2) = -0.2 > -0.5 = \bigvee \left\{ f_n^\Omega(c, 2), f_n^\Omega(c, 2) \right\}$$

and/or

$$f_p^\Omega(dc, 1) = f_p^\Omega(b, 1) = 0.4 < 0.5 = \bigwedge \left\{ f_p^\Omega(d, 1), f_p^\Omega(c, 1) \right\}.$$

Example 3.5. Let $S^\Omega := \{u \mid u : \Omega \to S\}$. For any $u, v \in S^\Omega$, we define $(uv)(\alpha) = u(\alpha)v(\alpha)$ for all $\alpha \in \Omega$. Then S^Ω is a semigroup. Let $f = (S; f_n, f_p)$ be a bipolar fuzzy left (right) ideal of S and let $\Phi_\Omega = \left\langle S^\Omega \times \Omega; \Phi_n^\Omega, \Phi_p^\Omega \right\rangle$ where

$$\Phi_n^\Omega : S^\Omega \times \Omega \to [-1, 0], \ (u, \alpha) \mapsto f_n(u(\alpha))$$

and

$$\Phi_p^\Omega : S^\Omega \times \Omega \to [0, 1], \ (u, \alpha) \mapsto f_p(u(\alpha)).$$

Then $\Phi_\Omega = \left\langle S^\Omega \times \Omega; \Phi_n^\Omega, \Phi_p^\Omega \right\rangle$ is a bipolar Ω-fuzzy sub-semigroup of S^Ω (see [3]). It is easy to verify that $\Phi_\Omega = \left\langle S^\Omega \times \Omega; \Phi_n^\Omega, \Phi_p^\Omega \right\rangle$ is a bipolar Ω-fuzzy left (or, right) ideal of S^Ω. Also, if $f = (S; f_n, f_p)$ is a bipolar fuzzy bi-ideal of S then Φ_Ω is a bipolar Ω-fuzzy bi-ideal of S.

Example 3.6. Let $S = \{a, b, c, d, e\}$ be a semigroup with the multiplication table which is given in Table 2. Let $\Omega = \{1, 2\}$ and let $F_\Omega = \left\langle S \times \Omega; f_n^\Omega, f_p^\Omega \right\rangle$ be a bipolar Ω-fuzzy set in S defined by

$$\begin{aligned} F_\Omega =\{ &\langle (a,1); -0.7, 0.7\rangle, \langle (a,2); -0.9, 0.5\rangle, \langle (b,1); -0.5, 0.2\rangle, \langle (b,2); -0.8, 0.4\rangle, \\ &\langle (c,1); -0.3, 0.5\rangle, \langle (c,2); -0.6, 0.3\rangle, \langle (d,1); -0.4, 0.4\rangle, \langle (d,2); -0.4, 0.4\rangle, \\ &\langle (e,1); -0.2, 0.6\rangle, \langle (e,2); -0.3, 0.2\rangle\}. \end{aligned}$$

Then $F_\Omega = \left\langle S \times \Omega; f_n^\Omega, f_p^\Omega \right\rangle$ is a bipolar Ω-fuzzy bi-ideal of S.

A bipolar fuzzy set $f = (S; f_n, f_p)$ in S is called a bipolar fuzzy bi-ideal of S (see [4]) if it is a bipolar fuzzy sub-semigroup of S that satisfies the following condition:

$$(\forall a, x, y \in S) \begin{pmatrix} f_n(xay) \leq \bigvee \{f_n(x), f_n(y)\}, \\ f_p(xay) \geq \bigwedge \{f_p(x), f_p(y)\} \end{pmatrix}. \tag{3.4}$$

Theorem 3.7. *If $f = \left(S; f_n^\alpha, f_p^\alpha\right)$, $\alpha \in \Omega$, is a bipolar fuzzy bi-ideal of S, then a bipolar Ω-fuzzy set $F_\Omega = \left\langle S \times \Omega; f_n^\Omega, f_p^\Omega \right\rangle$ where*

$$f_n^\Omega : S \times \Omega \to [-1, 0], \; (x, \alpha) \mapsto f_n^\alpha(x)$$

and

$$f_p^\Omega : S \times \Omega \to [0, 1], \; (x, \alpha) \mapsto f_p^\alpha(x)$$

is a bipolar Ω-fuzzy bi-ideal of S.

Proof. If $f = \left(S; f_n^\alpha, f_p^\alpha\right)$, $\alpha \in \Omega$, is a bipolar fuzzy bi-ideal of S, then it is a bipolar fuzzy sub-semigroup of S. It follows from [3, Proposition 3.5] that $F_\Omega = \left\langle S \times \Omega; f_n^\Omega, f_p^\Omega \right\rangle$ is a bipolar Ω-fuzzy sub-semigroup of S. Let $x, a, y \in S$ and $\alpha \in \Omega$. Then

$$f_n^\Omega(xay, \alpha) = f_n^\alpha(xay) \leq \bigvee \{f_n^\alpha(x), f_n^\alpha(y)\} = \bigvee \left\{f_n^\Omega(x, \alpha), f_n^\Omega(y, \alpha)\right\}$$

and

$$f_p^\Omega(xay, \alpha) = f_p^\alpha(xay) \geq \bigwedge \{f_p^\alpha(x), f_p^\alpha(y)\} = \bigwedge \left\{f_p^\Omega(x, \alpha), f_p^\Omega(y, \alpha)\right\}.$$

Hence $F_\Omega = \left\langle S \times \Omega; f_n^\Omega, f_p^\Omega \right\rangle$ is a bipolar Ω-fuzzy bi-ideal of S. □

Lemma 3.8 ([3])**.** *Let Ω be the set of all bipolar fuzzy sub-semigroups of S and let $F_\Omega = \left\langle S \times \Omega; f_n^\Omega, f_p^\Omega \right\rangle$ be a bipolar Ω-fuzzy set in S where $f_n^\Omega(x, f) = f_n(x)$ and $f_p^\Omega(x, f) = f_p(x)$ for all $x \in S$ and $f = (S; f_n, f_p) \in \Omega$. Then $F_\Omega = \left\langle S \times \Omega; f_n^\Omega, f_p^\Omega \right\rangle$ is a bipolar Ω-fuzzy sub-semigroup of S.*

Theorem 3.9. *Let Ω be the set of all bipolar fuzzy bi-ideals of S and let $F_\Omega = \left\langle S \times \Omega; f_n^\Omega, f_p^\Omega \right\rangle$ be a bipolar Ω-fuzzy set in S where $f_n^\Omega(x, f) = f_n(x)$ and $f_p^\Omega(x, f) = f_p(x)$ for all $x \in S$ and $f = (S; f_n, f_p) \in \Omega$. Then $F_\Omega = \left\langle S \times \Omega; f_n^\Omega, f_p^\Omega \right\rangle$ is a bipolar Ω-fuzzy bi-ideal of S.*

Proof. Let $f = (S; f_n, f_p) \in \Omega$. Then $f = (S; f_n, f_p)$ is a bipolar fuzzy sub-semigroup of S. Hence $F_\Omega = \left\langle S \times \Omega; f_n^\Omega, f_p^\Omega \right\rangle$ is a bipolar Ω-fuzzy sub-semigroup of S by Lemma 3.8. Let $x, ay \in S$. Then

$$f_n^\Omega(xay, f) = f_n(xay) \leq \bigvee \{f_n(x), f_n(y)\} = \bigvee \left\{f_n^\Omega(x, f), f_n^\Omega(y, f)\right\}$$

and

$$f_p^\Omega(xay, f) = f_p(xay) \geq \bigwedge \{f_p(x), f_p(y)\} = \bigwedge \left\{f_p^\Omega(x, f), f_p^\Omega(y, f)\right\}.$$

Therefore $F_\Omega = \left\langle S \times \Omega; f_n^\Omega, f_p^\Omega \right\rangle$ is a bipolar Ω-fuzzy bi-ideal of S. □

Lemma 3.10 ([3])**.** *Let $F_\Omega = \left\langle S \times \Omega; f_n^\Omega, f_p^\Omega \right\rangle$ be a bipolar Ω-fuzzy sub-semigroup of S. For any $\alpha \in \Omega$, let $f = \left(S; f_n^\alpha, f_p^\alpha\right)$ be a bipolar fuzzy set in S where*

$$f_n^\alpha : S \to [-1, 0], \; x \mapsto f_n^\Omega(x, \alpha)$$

and

$$f_p^{\alpha} : S \to [0,1],\ x \mapsto f_p^{\Omega}(x,\alpha).$$

Then $f = \left(S; f_n^{\alpha}, f_p^{\alpha}\right)$ is a bipolar fuzzy sub-semigroup of S.

Theorem 3.11. *Let $F_{\Omega} = \left\langle S \times \Omega; f_n^{\Omega}, f_p^{\Omega}\right\rangle$ be a bipolar Ω-fuzzy bi-ideal of S. For any $\alpha \in \Omega$, let $f = \left(S; f_n^{\alpha}, f_p^{\alpha}\right)$ be a bipolar fuzzy set in S where*

$$f_n^{\alpha} : S \to [-1,0],\ x \mapsto f_n^{\Omega}(x,\alpha)$$

and

$$f_p^{\alpha} : S \to [0,1],\ x \mapsto f_p^{\Omega}(x,\alpha).$$

Then $f = \left(S; f_n^{\alpha}, f_p^{\alpha}\right)$ is a bipolar fuzzy bi-ideal of S.

Proof. Let $F_{\Omega} = \left\langle S \times \Omega; f_n^{\Omega}, f_p^{\Omega}\right\rangle$ be a bipolar Ω-fuzzy bi-ideal of S. Then $F_{\Omega} = \left\langle S \times \Omega; f_n^{\Omega}, f_p^{\Omega}\right\rangle$ be a bipolar Ω-fuzzy sub-semigroup of S, and so $f = \left(S; f_n^{\alpha}, f_p^{\alpha}\right)$ is a bipolar fuzzy sub-semigroup of S by Lemma 3.10. For any $x, a, y \in S$, we have

$$f_n^{\alpha}(xay) = f_n^{\Omega}(xay,\alpha) \leq \bigvee \left\{f_n^{\Omega}(x,\alpha), f_n^{\Omega}(y,\alpha)\right\} = \bigvee \left\{f_n^{\alpha}(x), f_n^{\alpha}(y)\right\}$$

and

$$f_p^{\alpha}(xay) = f_p^{\Omega}(xay,\alpha) \geq \bigwedge \left\{f_p^{\Omega}(x,\alpha), f_p^{\Omega}(y,\alpha)\right\} = \bigwedge \left\{f_p^{\alpha}(x), f_p^{\alpha}(y)\right\}.$$

Hence $f = \left(S; f_n^{\alpha}, f_p^{\alpha}\right)$ is a bipolar fuzzy sub-semigroup of S. □

Theorem 3.12. *If S is a group, then every bipolar Ω-fuzzy bi-ideal*

$$F_{\Omega} = \left\langle S \times \Omega; f_n^{\Omega}, f_p^{\Omega}\right\rangle$$

of S is constant, that is, f_n^{Ω} and f_p^{Ω} are constant functions on $S \times \Omega$.

Proof. Assume that S is a group and let $F_{\Omega} = \left\langle S \times \Omega; f_n^{\Omega}, f_p^{\Omega}\right\rangle$ be a bipolar Ω-fuzzy bi-ideal of S. For any $x \in S$ and $\alpha \in \Omega$, we have

$$\begin{aligned} f_n^{\Omega}(x,\alpha) &= f_n^{\Omega}(exe,\alpha) \leq \bigvee \left\{f_n^{\Omega}(e,\alpha), f_n^{\Omega}(e,\alpha)\right\} \\ &= f_n^{\Omega}(e,\alpha) = f_n^{\Omega}(ee,\alpha) = f_n^{\Omega}\left((xx^{-1}(x^{-1}x),\alpha\right) \\ &= f_n^{\Omega}\left(x(x^{-1}x^{-1})x,\alpha\right) \leq \bigvee \left\{f_n^{\Omega}(x,\alpha), f_n^{\Omega}(x,\alpha)\right\} = f_n^{\Omega}(x,\alpha) \end{aligned}$$

and

$$\begin{aligned} f_p^{\Omega}(x,\alpha) &= f_p^{\Omega}(exe,\alpha) \geq \bigwedge \left\{f_p^{\Omega}(e,\alpha), f_p^{\Omega}(e,\alpha)\right\} = f_p^{\Omega}(e,\alpha) = f_p^{\Omega}(ee,\alpha) \\ &= f_p^{\Omega}\left((xx^{-1}(x^{-1}x),\alpha\right) = f_p^{\Omega}\left(x(x^{-1}x^{-1})x,\alpha\right) \\ &\geq \bigwedge \left\{f_p^{\Omega}(x,\alpha), f_p^{\Omega}(x,\alpha)\right\} = f_p^{\Omega}(x,\alpha) \end{aligned}$$

where e is the identity element of S. Hence $f_n^{\Omega}(x,\alpha) = f_n^{\Omega}(e,\alpha)$ and $f_p^{\Omega}(x,\alpha) = f_p^{\Omega}(e,\alpha)$ for all $x \in S$ and $\alpha \in \Omega$. This completes the proof. □

Theorem 3.13. *If S is a regular semigroup, then every generalized bipolar Ω-fuzzy bi-ideal is a bipolar Ω-fuzzy bi-ideal.*

Proof. Let $F_\Omega = \left\langle S \times \Omega; f_n^\Omega, f_p^\Omega \right\rangle$ be a generalized bipolar Ω-fuzzy bi-ideal of a regular semigroup S. Let $x, y \in S$ and $\alpha \in \Omega$. Since S is regular, there exists $a \in S$ such that $y = yay$. It follows from (3.3) that

$$f_n^\Omega(xy, \alpha) = f_n^\Omega(x(yay), \alpha) = f_n^\Omega(x(ya)y, \alpha) \leq \bigvee \left\{ f_n^\Omega(x, \alpha), f_n^\Omega(y, \alpha) \right\}$$

and

$$f_p^\Omega(xy, \alpha) = f_p^\Omega(x(yay), \alpha) = f_p^\Omega(x(ya)y, \alpha) \geq \bigwedge \left\{ f_p^\Omega(x, \alpha), f_p^\Omega(y, \alpha) \right\}.$$

Therefore $F_\Omega = \left\langle S \times \Omega; f_n^\Omega, f_p^\Omega \right\rangle$ is a bipolar Ω-fuzzy bi-ideal of S. □

Theorem 3.14. *Every bipolar Ω-fuzzy left* (resp. *right*) *ideal is a bipolar Ω-fuzzy bi-ideal.*

Proof. Let $F_\Omega = \left\langle S \times \Omega; f_n^\Omega, f_p^\Omega \right\rangle$ be a bipolar Ω-fuzzy left ideal of S. Then $F_\Omega = \left\langle S \times \Omega; f_n^\Omega, f_p^\Omega \right\rangle$ is a bipolar Ω-fuzzy sub-semigroup of S. For any $x, a, y \in S$ and $\alpha \in \Omega$, we have

$$f_n^\Omega(xay, \alpha) = f_n^\Omega((xa)y, \alpha) \leq f_n^\Omega(y, \alpha) \leq \bigvee \left\{ f_n^\Omega(x, \alpha), f_n^\Omega(y, \alpha) \right\}$$

and

$$f_p^\Omega(xay, \alpha) = f_p^\Omega((xa)y, \alpha) \geq f_p^\Omega(y, \alpha) \geq \bigwedge \left\{ f_p^\Omega(x, \alpha), f_p^\Omega(y, \alpha) \right\}.$$

Hence $F_\Omega = \left\langle S \times \Omega; f_n^\Omega, f_p^\Omega \right\rangle$ is a bipolar Ω-fuzzy bi-ideal of S. The right case is proved in an analogous way. □

The following example shows that the converse of Theorem 3.14 is not true.

Example 3.15. The bipolar Ω-fuzzy bi-ideal $F_\Omega = \left\langle S \times \Omega; f_n^\Omega, f_p^\Omega \right\rangle$ of S in Example 3.6 is neither a bipolar Ω-fuzzy left ideal nor a bipolar Ω-fuzzy right ideal of S because

$$f_n^\Omega(cd, 1) = f_n^\Omega(c, 1) = -0.3 > -0.4 = f_n^\Omega(d, 1)$$

and

$$f_p^\Omega(dc, 2) = f_p^\Omega(c, 2) = 0.3 < 0.4 = f_p^\Omega(d, 2).$$

Now, we provide a condition for a bipolar Ω-fuzzy bi-ideal to be a bipolar Ω-fuzzy right ideal.

Theorem 3.16. *Let S be a regular semigroup in which every bi-ideal is a right ideal. Then every bipolar Ω-fuzzy bi-ideal of S is a bipolar Ω-fuzzy right ideal of S.*

Proof. Let $F_\Omega = \left\langle S \times \Omega; f_n^\Omega, f_p^\Omega \right\rangle$ be a bipolar fuzzy bi-ideal of S and let $x, y \in S$. Then xSx is a bi-ideal of S, and so xSx is a right ideal of S. Since S is regular, $xy \in (xSx)S \subseteq xSx$. Hence $xy = xax$ for some $a \in S$. Since $F_\Omega = \left\langle S \times \Omega; f_n^\Omega, f_p^\Omega \right\rangle$ is a bipolar fuzzy bi-ideal of S, it follows that

$$f_n^\Omega(xy, \alpha) = f_n^\Omega(xax, \alpha) \leq \bigvee \left\{ f_n^\Omega(x, \alpha), f_n^\Omega(x, \alpha) \right\} = f_n^\Omega(x, \alpha)$$

and

$$f_p^\Omega(xy, \alpha) = f_p^\Omega(xax, \alpha) \geq \bigwedge \left\{ f_p^\Omega(x, \alpha), f_p^\Omega(x, \alpha) \right\} = f_p^\Omega(x, \alpha).$$

Therefore $F_\Omega = \left\langle S \times \Omega; f_n^\Omega, f_p^\Omega \right\rangle$ is a bipolar fuzzy right ideal of S. □

For a bipolar Ω-fuzzy set $F_\Omega = \left\langle S \times \Omega; f_n^\Omega, f_p^\Omega \right\rangle$ in S and $(s,t) \in [-1,0] \times [0,1]$, we define

$$\begin{aligned} N(F_\Omega; s) &= \left\{ x \in S \mid f_n^\Omega(x,\alpha) \le s,\ \forall \alpha \in \Omega \right\}, \\ P(F_\Omega; t) &= \left\{ x \in S \mid f_p^\Omega(x,\alpha) \ge t,\ \forall \alpha \in \Omega \right\} \end{aligned} \tag{3.5}$$

which are called the *negative s-cut* of $F_\Omega = \left\langle S \times \Omega; f_n^\Omega, f_p^\Omega \right\rangle$ and the *positive t-cut* of $F_\Omega = \left\langle S \times \Omega; f_n^\Omega, f_p^\Omega \right\rangle$, respectively. The set

$$C(F_\Omega; (s,t)) := N(F_\Omega; s) \cap P(F_\Omega; t)$$

is called the *(s,t)-cut* of $F_\Omega = \left\langle S \times \Omega; f_n^\Omega, f_p^\Omega \right\rangle$. For every $k \in [0,1]$, if $(s,t) = (-k,k)$ then the set

$$C(F_\Omega; k) := N(F_\Omega; -k) \cap P(F_\Omega; k)$$

is called the *k-cut* of $F_\Omega = \left\langle S \times \Omega; f_n^\Omega, f_p^\Omega \right\rangle$.

Lemma 3.17 ([3])**.** *If a bipolar Ω-fuzzy set $F_\Omega = \left\langle S \times \Omega; f_n^\Omega, f_p^\Omega \right\rangle$ in S is a bipolar Ω-fuzzy sub-semigroup of S, then the following assertions are valid:*

(1) $(\forall s \in [-1,0])\,(N(F_\Omega; s) \neq \varnothing \ \Rightarrow\ N(F_\Omega; s)$ *is a sub-semigroup of* $S)$.
(2) $(\forall t \in [0,1])\,(P(F_\Omega; t) \neq \varnothing \ \Rightarrow\ P(F_\Omega; t)$ *is a sub-semigroup of* $S)$.

Theorem 3.18. *If a bipolar Ω-fuzzy set $F_\Omega = \left\langle S \times \Omega; f_n^\Omega, f_p^\Omega \right\rangle$ in S is a bipolar Ω-fuzzy bi-ideal of S, then the following assertions are valid:*

(1) $(\forall s \in [-1,0])\,(N(F_\Omega; s) \neq \varnothing \ \Rightarrow\ N(F_\Omega; s)$ *is a bi-ideal of* $S)$.
(2) $(\forall t \in [0,1])\,(P(F_\Omega; t) \neq \varnothing \ \Rightarrow\ P(F_\Omega; t)$ *is a bi-ideal of* $S)$.

Proof. Assume that $F_\Omega = \left\langle S \times \Omega; f_n^\Omega, f_p^\Omega \right\rangle$ is a bipolar Ω-fuzzy bi-ideal of S. Then it is a bipolar Ω-fuzzy sub-semigroup of S, and so the nonempty negative s-cut $N(F_\Omega; s)$ and the nonempty positive t-cut $P(F_\Omega; t)$ are sub-semigroups of S for all $(s,t) \in [-1,0] \times [0,1]$ by Lemma 3.17. Let $a \in S$. If $x, y \in N(F_\Omega; s)$, then $f_n^\Omega(x,\alpha) \le s$ and $f_n^\Omega(y,\alpha) \le s$ for all $\alpha \in \Omega$. It follows from (3.3) that

$$f_n^\Omega(xay,\alpha) \le \bigvee \left\{ f_n^\Omega(x,\alpha), f_n^\Omega(y,\alpha) \right\} \le s$$

so that $xay \in N(F_\Omega; s)$. Hence $N(F_\Omega; s)$ is a bi-ideal of S. If $x, y \in P(F_\Omega; t)$, then $f_p^\Omega(x,\alpha) \ge t$ and $f_p^\Omega(y,\alpha) \ge t$ for all $\alpha \in \Omega$. Using (3.3), we have

$$f_p^\Omega(xay,\alpha) \ge \bigwedge \left\{ f_p^\Omega(x,\alpha), f_p^\Omega(y,\alpha) \right\} \ge t$$

and so $xay \in P(F_\Omega; t)$. Therefore $P(F_\Omega; t)$ is a bi-ideal of S. $\square$

Denote by S^e a semigroup S with the identity element e. For any bipolar Ω-fuzzy set $F_\Omega = \left\langle S \times \Omega; f_n^\Omega, f_p^\Omega \right\rangle$ in S, let $G_\Omega = \left\langle S^e \times \Omega; g_n^\Omega, g_p^\Omega \right\rangle$ be a bipolar Ω-fuzzy set in S^e where g_n^Ω and g_p^Ω are defined as follows:

$$g_n^\Omega : S^e \times \Omega \to [-1,0],\ (w,\alpha) \mapsto \bigwedge \left\{ f_n^\Omega(x_2,\alpha) \mid w = x_1x_2,\ x_1, x_2 \in S^e \right\}$$

and

$$g_p^\Omega : S^e \times \Omega \to [0,1],\ (w,\alpha) \mapsto \bigvee \left\{ f_p^\Omega(x_2,\alpha) \mid w = x_1x_2,\ x_1, x_2 \in S^e \right\}.$$

Theorem 3.19. *For any bipolar Ω-fuzzy set $F_\Omega = \left\langle S \times \Omega; f_n^\Omega, f_p^\Omega \right\rangle$ in S, the bipolar Ω-fuzzy set $G_\Omega = \left\langle S^e \times \Omega; g_n^\Omega, g_p^\Omega \right\rangle$ in S^e is a bipolar Ω-fuzzy left ideal of S^e.*

Proof. Let $w \in S^e$ and $\alpha \in \Omega$. Then

$$\begin{aligned} g_n^{\Omega}(w,\alpha) &= \bigwedge \left\{ f_n^{\Omega}(x_2,\alpha) \mid w = x_1x_2,\ x_1,x_2 \in S^e \right\} \\ &\leq \bigwedge \left\{ f_n^{\Omega}(w,\alpha) \mid w = ew \right\} = f_n^{\Omega}(w,\alpha) \end{aligned}$$

and

$$\begin{aligned} g_p^{\Omega}(w,\alpha) &= \bigvee \left\{ f_p^{\Omega}(x_2,\alpha) \mid w = x_1x_2,\ x_1,x_2 \in S^e \right\} \\ &\geq \bigvee \left\{ f_p^{\Omega}(w,\alpha) \mid w = ew \right\} = f_p^{\Omega}(w,\alpha). \end{aligned}$$

Hence

$$\begin{aligned} g_n^{\Omega}(xy,\alpha) &= \bigwedge \left\{ f_n^{\Omega}(x_2,\alpha) \mid xy = x_1x_2 \right\} \\ &\leq \bigwedge \left\{ f_n^{\Omega}(z_2,\alpha) \mid xy = (xz_1)z_2,\ y = z_1z_2 \right\} \\ &\leq \bigwedge \left\{ f_n^{\Omega}(z_2,\alpha) \mid y = z_1z_2 \right\} \\ &= g_n^{\Omega}(y,\alpha) \end{aligned}$$

and

$$\begin{aligned} g_p^{\Omega}(xy,\alpha) &= \bigvee \left\{ f_p^{\Omega}(x_2,\alpha) \mid xy = x_1x_2 \right\} \\ &\geq \bigvee \left\{ f_p^{\Omega}(z_2,\alpha) \mid xy = (xz_1)z_2,\ y = z_1z_2 \right\} \\ &\geq \bigvee \left\{ f_p^{\Omega}(z_2,\alpha) \mid y = z_1z_2 \right\} \\ &= g_p^{\Omega}(y,\alpha). \end{aligned}$$

Therefore $G_\Omega = \left\langle S^e \times \Omega; g_n^{\Omega}, g_p^{\Omega} \right\rangle$ is a bipolar Ω-fuzzy left ideal of S^e. □

Definition 3.20. A bipolar Ω-fuzzy set $F_\Omega = \left\langle S \times \Omega; f_n^{\Omega}, f_p^{\Omega} \right\rangle$ in S is called a bipolar Ω-fuzzy $(1,2)$-ideal of S if it is a bipolar Ω-fuzzy sub-semigroup of S that satisfies:

$$\begin{aligned} f_n^{\Omega}(xa(yz),\alpha) &\leq \bigvee \left\{ f_n^{\Omega}(x,\alpha), f_n^{\Omega}(y,\alpha), f_n^{\Omega}(z,\alpha) \right\} \\ f_p^{\Omega}(xa(yz),\alpha) &\geq \bigwedge \left\{ f_p^{\Omega}(x,\alpha), f_p^{\Omega}(y,\alpha), f_p^{\Omega}(z,\alpha) \right\} \end{aligned} \tag{3.6}$$

for all $\alpha \in \Omega$ and $x,a,y,z \in S$.

Example 3.21. Consider a semigroup $T = \{a,b,c,x,y,z\}$ with the multiplication table which is given by Table 3. Let $\Omega = \{1,2\}$ and let $G_\Omega = \left\langle T \times \Omega; g_n^{\Omega}, g_p^{\Omega} \right\rangle$ be a bipolar Ω-fuzzy set in T defined by

$$\begin{aligned} G_\Omega = \{ &\langle (a,1); -0.7, 0.9 \rangle, \langle (a,2); -0.9, 0.8 \rangle, \langle (b,1); -0.2, 0.1 \rangle, \langle (b,2); -0.7, 0.4 \rangle, \\ &\langle (c,1); -0.4, 0.1 \rangle, \langle (c,2); -0.2, 0.6 \rangle, \langle (x,1); -0.5, 0.1 \rangle, \langle (x,2); -0.2, 0.2 \rangle, \\ &\langle (y,1); -0.1, 0.1 \rangle, \langle (y,2); -0.7, 0.3 \rangle, \langle (z,1); -0.7, 0.9 \rangle, \langle (z,2); -0.5, 0.3 \rangle \}. \end{aligned}$$

Then $G_\Omega = \left\langle T \times \Omega; g_n^{\Omega}, g_p^{\Omega} \right\rangle$ is a bipolar Ω-fuzzy $(1,2)$-ideal of T.

Theorem 3.22. *Every bipolar Ω-fuzzy bi-ideal is a bipolar Ω-fuzzy $(1,2)$-ideal.*

TABLE 3. Multiplication table

	a	b	c	x	y	z
a	a	a	a	a	a	a
b	a	a	a	a	a	a
c	a	a	a	a	a	a
x	a	a	a	a	a	b
y	a	a	a	a	b	c
z	a	a	b	a	x	a

Proof. Let $F_\Omega = \left\langle S \times \Omega; f_n^\Omega, f_p^\Omega \right\rangle$ be a bipolar Ω-fuzzy bi-ideal of S. It is sufficient to show that $F_\Omega = \left\langle S \times \Omega; f_n^\Omega, f_p^\Omega \right\rangle$ satisfies the condition (3.6). Let $\alpha \in \Omega$ and $x, a, y, z \in S$. Then

$$\begin{aligned} f_n^\Omega(xa(yz), \alpha) &= f_n^\Omega\left((xay)z, \alpha\right) \leq \bigvee \left\{ f_n^\Omega\left(xay, \alpha\right), f_n^\Omega\left(z, \alpha\right) \right\} \\ &\leq \bigvee \left\{ \bigvee \left\{ f_n^\Omega\left(x, \alpha\right), f_n^\Omega\left(y, \alpha\right) \right\}, f_n^\Omega\left(z, \alpha\right) \right\} \\ &= \bigvee \left\{ f_n^\Omega\left(x, \alpha\right), f_n^\Omega\left(y, \alpha\right), f_n^\Omega\left(z, \alpha\right) \right\} \end{aligned}$$

and

$$\begin{aligned} f_p^\Omega(xa(yz), \alpha) &= f_p^\Omega\left((xay)z, \alpha\right) \geq \bigwedge \left\{ f_p^\Omega\left(xay, \alpha\right), f_p^\Omega\left(z, \alpha\right) \right\} \\ &\geq \bigwedge \left\{ \bigwedge \left\{ f_p^\Omega\left(x, \alpha\right), f_p^\Omega\left(y, \alpha\right) \right\}, f_p^\Omega\left(z, \alpha\right) \right\} \\ &= \bigwedge \left\{ f_p^\Omega\left(x, \alpha\right), f_p^\Omega\left(y, \alpha\right), f_p^\Omega\left(z, \alpha\right) \right\}. \end{aligned}$$

Therefore $F_\Omega = \left\langle S \times \Omega; f_n^\Omega, f_p^\Omega \right\rangle$ is a bipolar Ω-fuzzy $(1,2)$-ideal of S. □

Combining Theorems 3.14 and 3.22, we have the following corollary.

Corollary 3.23. *Every bipolar Ω-fuzzy left* (resp. *right*) *ideal is a bipolar Ω-fuzzy* $(1,2)$*-ideal.*

The following example shows that the converse of Theorem 3.22 is not true.

Example 3.24. The bipolar Ω-fuzzy $(1,2)$-ideal $G_\Omega = \left\langle T \times \Omega; g_n^\Omega, g_p^\Omega \right\rangle$ in Example 3.21 is not a bipolar Ω-fuzzy bi-ideal since $g_n^\Omega(zyz, 1) > g_n^\Omega(z, 1)$.

We now consider the converse of Theorem 3.22 by adding a condition.

Theorem 3.25. *In a regular semigroup, every bipolar Ω-fuzzy* $(1,2)$*-ideal is a bipolar Ω-fuzzy bi-ideal.*

Proof. Let $F_\Omega = \left\langle S \times \Omega; f_n^\Omega, f_p^\Omega \right\rangle$ be a bipolar Ω-fuzzy $(1,2)$-ideal of a regular semigroup S. The regularity of S implies that $xa \in (xSx)S \subseteq xSx$ for all $a, x \in S$. Hence $xa = xbx$ for some $b \in S$. Thus

$$\begin{aligned} f_n^\Omega(xay, \alpha) &= f_n^\Omega\left((xbx)y, \alpha\right) = f_n^\Omega\left(xb(xy), \alpha\right) \\ &\leq \bigvee \left\{ f_n^\Omega(x, \alpha), f_n^\Omega(y, \alpha) \right\} \end{aligned}$$

and

$$f_p^\Omega(xay,\alpha) = f_p^\Omega((xbx)y,\alpha) = f_p^\Omega(xb(xy),\alpha)$$
$$\geq \bigwedge \left\{ f_p^\Omega(x,\alpha), f_p^\Omega(y,\alpha) \right\}.$$

Therefore $F_\Omega = \left\langle S\times\Omega; f_n^\Omega, f_p^\Omega \right\rangle$ is a bipolar Ω-fuzzy bi-ideal of S. □

Acknowledgements. The authors wish to thank the anonymous reviewers for their valuable suggestions.

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