Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 4, No. 2, (October 2012), pp. 243–252
ISSN 2093–9310
http://www.afmi.or.kr

Anti fuzzy near-algebras over anti fuzzy fields

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Received 25 September 2011; Revised 30 December 2011; Accepted 9 January 2012

Abstract. The notion of an anti fuzzy field and anti fuzzy near-algebra over an anti fuzzy field is introduced. Using this notion we have described some basic properties. The concepts of an anti homomorphism and anti fuzzy ideal in near-algebras are also presented.

2010 AMS Classification: 16Y30, 03E72

Keywords: Near-algebra, Anti fuzzy field, Anti fuzzy near-algebra, Anti homomorphism, Anti fuzzy ideal.

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1. Introduction

Zadeh [6] initiated the notion of a fuzzy set in 1965. Kim, Jun and Yon [4] have studied the concept of an anti fuzzy sub near-rings and anti fuzzy ideals of near-rings. Chandrasekhara Rao and Swaminathan [2] introduced the concept of an anti homomorphism in fuzzy ideals of near-rings. Brown [1] introduced the concept of near-algebras. In [5] Srinivas and Narasimha swamy have introduced the concept of fuzzy near-algebra over a fuzzy field and extended the results of fuzzy algebras over fuzzy fields obtained by Gu and Lu [3]. In this paper we introduce the concept of an anti fuzzy field and anti fuzzy near-algebra over an anti fuzzy field. We discuss an anti homomorphism and anti fuzzy ideal of near-algebras and extended the results of [4] and [2] to near-algebras.

2. Preliminaries

A *right near-algebra* Y over a field X is a linear space Y over X on which a multiplication is defined such that (i) Y forms a semigroup under multiplication, (ii) multiplication is right distributive over addition, (iii) $\lambda(ab) = (\lambda a)b$ for all $a, b \in Y$, $\lambda \in X$. A *left near-algebra* Y over a field X is a linear space Y over X on which a multiplication is defined such that (i) Y forms a semigroup under multiplication, (ii) multiplication is left distributive over addition, (iii) $\lambda(ab) = a(\lambda b)$ for all $a, b \in Y$, $\lambda \in X$.

A near-algebra Y is said to be a *zero symmetric* or *near-c-algebra*, if $n \cdot 0 = 0$ for every $n \in Y$, where 0 is the additive identity in Y. A mapping $f : Y \to Y'$ of near-algebras Y and Y' is called a *near-algebra homomorphism*, if it satisfies the following three conditions:

(i) $f(x + y) = f(x) + f(y)$,

(ii) $f(\lambda x) = \lambda f(x)$,

(iii) $f(xy) = f(x)f(y)$ for all $x, y \in Y, \lambda \in X$.

A non-empty subset I of a near-algebra Y is called a *near-algebra ideal (NA-ideal)* of Y if it satisfies the following three axioms:

(i) I is a linear subspace of the linear space Y,

(ii) $ix \in I$ for every $x \in Y, i \in I$,

(iii) $y(x + i) - yx \in I$ for every $x, y \in Y, i \in I$.

Note that, if I satisfies (i) and (ii), then I is called a right ideal of Y, and if I satisfies (i) and (iii), then I is called a left ideal of Y.

Let X be a non-empty set. Then a fuzzy subset A of X is a mapping $A : X \to L$. If A is a fuzzy subset of X, then the complement of A denoted by A^c, is a fuzzy subset in X given by $A^c(x) = 1 - A(x)$ for all $x \in X$. Throughout this paper, by a near-algebra we mean zero-symmetric right near-algebra unless otherwise specified. X stands for a field and Y stands for a near-algebra over a field X. We denote xy instead of $x \cdot y$.

A fuzzy subset F of X is called a *fuzzy field* of X, if it satisfies the following four conditions for all $x, y \in X$:

(i) $F(x + y) \geq F(x) \wedge F(y) = \min(F(x), F(y))$,

(ii) $F(-x) \geq F(x)$,

(iii) $F(xy) \geq F(x) \wedge F(y) = \min(F(x), F(y))$,

(iv) $F(x^{-1}) \geq F(x)$ for any $x(\neq 0) \in X$.

Let F be a fuzzy field of X. Then a fuzzy subset A of Y is called a *fuzzy near-algebra* of Y over a fuzzy field F of X, if it satisfies the following four conditions:

(i) $A(x + y) \geq A(x) \wedge A(y) = \min(A(x), A(y))$,

(ii) $A(\lambda x) \geq F(\lambda) \wedge A(x) = \min(F(\lambda), A(x))$,

(iii) $A(xy) \geq A(x) \wedge A(y) = \min(A(x), A(y))$ and

(iv) $F(1) \geq A(x)$ for all $x, y \in Y$ and $\lambda \in X$.

Let A be a fuzzy near-algebra of Y over a fuzzy field F of X. Then A is said to be a *fuzzy ideal* of Y, if the following two conditions hold:

(i) $A(xy) \geq A(x)$,

(ii) $A(y(x + i) - yx) \geq A(i)$ for every $x, y, i \in Y$.

Note that, A is a fuzzy right ideal of Y if it satisfies (i), and A is a fuzzy left ideal of Y if it satisfies (ii).

3. Anti fuzzy near-algebra over anti fuzzy field

In this section we define the concept of an anti fuzzy field and anti fuzzy near-algebra over an anti fuzzy field. Also w e discuss some basic results in this context.

Definition 3.1. Let F be a fuzzy subset of X. Then F is called an *anti fuzzy field* of X, if for all $x, y \in X$

(3.1.1) $F(x + y) \leq \max(F(x), F(y)) = F(x) \vee F(y)$,

(3.1.2) $F(-x) \leq F(x)$,
(3.1.3) $F(xy) \leq \max(F(x), F(y)) = F(x) \vee F(y)$,
(3.1.4) $F(x^{-1}) \leq F(x)$ for $x(\neq 0) \in X$.

An anti fuzzy field F of X is denoted by (F, X).

Definition 3.2. A fuzzy subset A of Y is called an *anti fuzzy near-algebra* of Y over an anti fuzzy field (F, X) if
(3.2.1) $A(x + y) \leq \max(A(x), A(y)) = A(x) \vee A(y)$,
(3.2.2) $A(\lambda x) \leq \max(F(\lambda), A(x)) = F(\lambda) \vee A(x)$,
(3.2.3) $A(xy) \leq \max(A(x), A(y)) = A(x) \vee A(y)$,
(3.2.4) $F(1) \leq A(x)$ for all $x, y \in Y$ and $\lambda \in X$.

An anti fuzzy near-algebra A of Y is denoted by (A, Y).

Example 3.3. Let $X = Z_3 = \{0, 1, 2\}_{\oplus_3, \otimes_3}$ and let F be a fuzzy subset of X defined by

$$F(x) = \begin{cases} 0.1 & \text{if } x = 0, \\ 0.2 & \text{otherwise.} \end{cases}$$

For any $x, y \in X$, we have $x - y \in X$. Particularly for $y \neq 0$, $xy^{-1} \in X$. Thus X is a field. We can easily see that $F(x - y) \leq \max(F(x), F(y))$ and $F(xy^{-1}) \leq \max(F(x), F(y))$. Thus (F, X) is an anti fuzzy field.

Let $Y = \{0, a, b, c\}$ be a set with two binary operations " $+$ " and " $\cdot$ " as follows:

$+$	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

$\cdot$	0	a	b	c
0	0	0	0	0
a	0	b	0	b
b	0	0	0	0
c	0	b	0	b

Also, if a scalar multiplication on Y is defined by

$$\lambda x = \begin{cases} 0 & \text{if } \lambda = 0, \\ x & \text{otherwise} \end{cases}$$

for every $x \in Y$, $\lambda \in X$. Clearly Y is a near-algebra over the field X. Let A be a fuzzy subset of Y defined by

$$A(x) = \begin{cases} 0.6 & \text{if } x = 0, \\ 0.8 & \text{otherwise.} \end{cases}$$

Let $\lambda, \mu \in X$ and $x, y \in Y$. Which implies $xy, \lambda x, \mu y, \lambda x + \mu y \in Y$. Then
(i) $A(\lambda x + \mu y) \leq \max(\max(F(\lambda), A(x)), (F(\mu), A(y)))$,
(ii) $A(xy) \leq \max(A(x), A(y))$,
(iii) $F(1) \leq A(x)$, where $1 \in X$.
Thus (A, Y) is an anti fuzzy near-algebra over the anti fuzzy field (F, X).

Theorem 3.4. *If (F, X) is an anti fuzzy field of X, then*
(i) $F(0) \leq F(x)$ *for any $x \in X$,*
(ii) $F(1) \leq F(x)$ *for every $x(\neq 0) \in X$,*
(iii) $F(0) \leq F(1)$.

Theorem 3.5. (F, X) *is an anti fuzzy field if and only if the following two conditions hold:*
(i) *for all* $x, y \in X, F(x - y) \leq \max(F(x), F(y))$ *and*
(ii) *for all* $x, y(\neq 0) \in X, F(xy^{-1}) \leq \max(F(x), F(y))$.

Proof. Suppose (F, X) is an anti fuzzy field. Then for all $x, y \in X$ we have
(i) $F(x - y) = F(x + (-y)) \leq \max(F(x), F(-y)) \leq \max(F(x), F(y))$ and
(ii) $F(xy^{-1}) \leq \max(F(x), F(y^{-1})) \leq \max(F(x), F(y))$ for every $x, y(\neq 0) \in X$.
Conversely, $F(-x) = F(0 - x) \leq \max(F(0), F(x)) \leq \max(F(x), F(x)) = F(x)$, $F(x + y) = F(x - (-y)) \leq \max(F(x), F(-y)) \leq \max(F(x), F(y))$, $F(x^{-1}) = F(1x^{-1}) \leq \max(F(1), F(x)) \leq \max(F(x), F(x)) = F(x)$ and $F(xy) = F(x(y^{-1})^{-1}) \leq \max(F(x), F(y^{-1})) \leq \max(F(x), F(y))$. Thus F is an anti fuzzy field of X. □

Theorem 3.6. *Let* X *and* Y *be two fields. Let* $f : X \to Y$ *be an onto homomorphism. If* F *is an anti fuzzy field of* X *and* G *is an anti fuzzy field of* Y, *then* $f^{-1}(G)$ *is an anti fuzzy field of* X *and* $f(F)$ *is an anti fuzzy field of* Y.

Proof. For any $x, y \in X$, we have

$$\begin{aligned}
f^{-1}(G)(x+y) &= G(f(x+y)) \\
&= G(f(x)+f(y)) \\
&\leq \max\{G(f(x)), G(f(y))\} \\
&= \max\{f^{-1}(G)(x), f^{-1}(G)(y)\}, \\
f^{-1}(G)(-x) &= G(f(-x)) \\
&= G(-f(x)) \\
&\leq G(f(x)) \\
&= f^{-1}(G(x)), \\
f^{-1}(G)(xy) &= G(f(xy)) \\
&= G(f(x)f(y)) \\
&\leq \max\{G(f(x)), G(f(y))\} \\
&= \max\{f^{-1}(G)(x), f^{-1}(G)(y)\}.
\end{aligned}$$

And for every $x(\neq 0) \in X$,

$$f^{-1}(G)(x^{-1}) = G(f(x^{-1})) = G((f(x))^{-1}) \leq G(f(x)).$$

Hence $(f^{-1}(G), X)$ is an anti fuzzy field. Similarly we can prove $(f(F), Y)$ is an anti fuzzy field. □

Theorem 3.7. *If* (A, Y) *is an anti fuzzy near-algebra over an anti fuzzy field* (F, X), *then* $F(0) \leq A(x)$ *and* $A(0) \leq A(x)$ *for every* $x \in Y$.

Theorem 3.8. (A, Y) *is an anti fuzzy near-algebra over an anti fuzzy field* (F, X) *if and only if the following three conditions hold:*
(i) $A(\lambda x + \mu y) \leq \max(\max(F(\lambda), A(x)), \max(F(\mu), A(y)))$,
(ii) $A(xy) \leq \max(A(x), A(y))$ *and*
(iii) $F(1) \leq A(x)$ *for any* $\lambda, \mu \in X$ *and* $x, y \in Y$.

Proof. Suppose (A, Y) is an anti fuzzy near-algebra over an anti fuzzy field (F, X). Then for any $\lambda, \mu \in X$ and $x, y \in Y$ we have

$$A(\lambda x + \mu y) \leq \max(A(\lambda x), A(\mu y)) \leq \max(\max(F(\lambda), A(x)), \max(F(\mu), A(y))).$$

Since A is an anti fuzzy near-algebra, then the remaining two conditions hold directly.

Conversely, suppose that the three conditions of the hypothesis hold. Then

$$\begin{aligned} A(x+y) &= A(1x+1y) \\ &\leq \max(\max(F(1),A(x)),\max(F(1),A(y))) \\ &\leq \max(\max(A(x),A(x)),\max(A(y),A(y))) \\ &\leq \max(A(x),A(y)), \end{aligned}$$

$$\begin{aligned} A(\lambda x) &= A(\lambda x+0x) \\ &\leq \max(\max(F(\lambda),A(x)),\max(F(0),A(x))) \\ &\leq \max(\max(F(\lambda),A(x)),\max(A(x),A(x))) \\ &\leq \max(\max(F(\lambda),A(x)),A(x)) \\ &= \max(F(\lambda),A(x)). \end{aligned}$$

We have $A(xy) \leq \max(A(x),A(y))$ and $F(1) \leq A(x)$.
Hence (A,Y) is an anti fuzzy near-algebra over the anti fuzzy field (F,X). □

Theorem 3.9. *Intersection of family of anti fuzzy near-algebras is an anti fuzzy near-algebra.*

Proof. Let $\{A_i\}_{i\in\Lambda}$ be a family of anti fuzzy near-algebras of Y over an anti fuzzy field (F,X). Let $A(x) = \bigcap_{i\in\Lambda} A_i(x) = \inf_{i\in\Lambda} A_i(x) = \bigwedge_{i\in\Lambda} A_i(x)$. For any $x,y \in Y$ and $\lambda,\mu \in X$ we have

$$\begin{aligned} A(\lambda x+\mu y) &= \inf_{i\in\Lambda} A_i(\lambda x+\mu y) \\ &\leq \inf_{i\in\Lambda}[\max(\max(F(\lambda),A_i(x)),\max(F(\mu),A_i(y)))] \\ &\leq \max(\max(F(\lambda),\inf_{i\in\Lambda} A_i(x)),\max(F(\mu),\inf_{i\in\Lambda} A_i(y))) \\ &= \max(\max(F(\lambda),A(x)),\max(F(\mu),A(x))), \end{aligned}$$

$$\begin{aligned} A(xy) &= \inf_{i\in\Lambda} A_i(xy) \\ &\leq \inf_{i\in\Lambda}(\max(A_i(x),A_i(y))) \\ &\leq \max(\inf_{i\in\Lambda} A_i(x),\inf_{i\in\Lambda} A_i(y)) \\ &= \max(A(x),A(y)). \end{aligned}$$

Since each A_i is an anti fuzzy near-algebra, we have

$$F(1) \leq A_i(x) \leq \inf_{i\in\Lambda} A_i(x) = A(x)$$

for every $x \in Y$ and $i \in \Lambda$. Hence (A,Y) is an anti fuzzy near-algebra over the anti fuzzy field (F,X). □

Theorem 3.10. *If $\{A_i : i \in \Lambda\}$ is a family of anti fuzzy near-algebras of Y, then so is $\bigvee_{i\in\Lambda} A_i$.*

Proof. Let $\{A_i : i \in \Lambda\}$ be a family of anti fuzzy near-algebras of Y over an anti fuzzy field (F, X). Let $x, y \in Y$ and $\lambda \in X$. Then

$$\begin{aligned}
(\bigvee_{i\in\Lambda} A_i)(x+y) &= \sup\{A_i(x+y) : i \in \Lambda\} \\
&\leq \sup\{\max(A_i(x), A_i(y)) : i \in \Lambda\} \\
&= \max\{\sup(A_i(x) : i \in \Lambda), \sup(A_i(y) : i \in \Lambda)\} \\
&= \max\{(\bigvee_{i\in\Lambda} A_i)(x), (\bigvee_{i\in\Lambda} A_i)(y)\}, \\
(\bigvee_{i\in\Lambda} A_i)(\lambda x) &= \sup\{A_i(\lambda x) : i \in \Lambda\} \\
&\leq \sup\{\max(F(\lambda), A_i(x)) : i \in \Lambda\} \\
&= \max\{sup(F(\lambda), A_i(x)) : i \in \Lambda\} \\
&= \max\{F(\lambda), \sup(A_i(x) : i \in \Lambda)\} \\
&= \max\{F(\lambda), (\bigvee_{i\in\Lambda} A_i)(x)\},
\end{aligned}$$

$$\begin{aligned}
(\bigvee_{i\in\Lambda} A_i)(xy) &= \sup\{A_i(xy) : i \in \Lambda\} \\
&\leq \sup\{\max(A_i(x), A_i(y)) : i \in \Lambda\} \\
&= \max\{\sup(A_i(x) : i \in \Lambda), \sup(A_i(y) : i \in \Lambda)\} \\
&= \max\{(\bigvee_{i\in\Lambda} A_i)(x), (\bigvee_{i\in\Lambda} A_i)(y)\}.
\end{aligned}$$

Since each A_i is an anti fuzzy near-algebra of Y, then $F(1) \leq A_i(x)$ for all $x \in Y$. This implies that $F(1) \leq (\bigvee_{i\in\Lambda} A_i)(x)$. Thus $\bigvee_{i\in\Lambda} A_i$ is an anti fuzzy near-algebra of Y. □

Theorem 3.11. *Let Y and Z be two near-algebras over a field X. Let $f : Y \to Z$ be an onto near-algebra homomorphism. If (A, Z) and (B, Y) are two anti fuzzy near-algebras over an anti fuzzy field (F, X), then $(f^{-1}(A), Y)$ and $(f(B), Z)$ are two anti fuzzy near-algebras over the anti fuzzy field (F, X).*

Proof. For any $x, y \in Y$ and $\lambda, \mu \in X$, we have

$$\begin{aligned}
f^{-1}(A)(\lambda x + \mu y) &= A(f(\lambda x + \mu y)) = A(\lambda f(x) + \mu f(y)) \\
&\leq \max(A(\lambda f(x)), A(\mu f(y))) \\
&\leq \max(\max(F(\lambda), A(f(x))), \max(F(\mu), A(f(y)))) \\
&= \max(\max(F(\lambda), f^{-1}(A)(x)), \max(F(\mu), f^{-1}(A)(y))),
\end{aligned}$$

$$\begin{aligned}
f^{-1}(A)(xy) &= A(f(xy)) = A(f(x)f(y)) \\
&\leq \max(A(f(x)), A(f(y))) \\
&= \max(f^{-1}(A)(x), f^{-1}(A)(y)).
\end{aligned}$$

Since (A, Z) is an anti fuzzy near-algebra, then we have $F(1) \leq A(f(x)) = f^{-1}(A)(x)$ for every $f(x) \in Z$, where $x \in Y$. Hence $(f^{-1}(A), Y)$ is an anti fuzzy near-algebra over the anti fuzzy field of (F, X). Similarly we can prove that $(f(B), Z)$ is an anti fuzzy near-algebra over the anti fuzzy field (F, X). □

Theorem 3.12. *Let Y be a near-algebra. Then the fuzzy subset A is an anti fuzzy near-algebra of Y over an anti fuzzy field of (F, X) if and only if A^c is a fuzzy near-algebra of Y over the anti fuzzy field of (F, X).*

Proof. Let A be an anti fuzzy near-algebra of Y. Then for any $x, y \in Y$ we have

$$\begin{array}{lcl} A^c(x+y) & = & 1 - A(x+y) \\ & \geq & 1 - \max\{A(x), A(y)\} \\ & = & \min\{1 - A(x), 1 - A(y)\} \\ & = & \min\{A^c(x), A^c(y)\}, \\ A^c(xy) & = & 1 - A(xy) \\ & \geq & 1 - \max\{A(x), A(y)\} \\ & = & \min\{1 - A(x), 1 - A(y)\} \\ & = & \min\{A^c(x), A^c(y)\}, \\ A^c(\lambda x) & = & 1 - A(\lambda x) \\ & \geq & 1 - \max\{F(\lambda), A(x)\} \\ & = & \min\{1 - F(\lambda), 1 - A(x)\} \\ & = & \min\{F^c(\lambda), A^c(x)\}, \\ F^c(1) & = & 1 - F(1) \\ & \geq & 1 - A(x) \\ & = & A^c(x). \end{array}$$

Thus A^c is a fuzzy near-algebra of Y.

Conversely, suppose that A^c is a fuzzy near-algebra of Y. Then

$$\begin{array}{lcl} A(x+y) & = & 1 - A^c(x+y) \\ & \leq & 1 - \min\{A^c(x), A^c(y)\} \\ & = & \max\{1 - A^c(x), 1 - A^c(y)\} \\ & = & \max\{A(x), A(y)\}, \\ A(xy) & = & 1 - A^c(xy) \\ & \leq & 1 - \min\{A^c(x), A^c(y)\} \\ & = & \max\{1 - A^c(x), 1 - A^c(y)\} \\ & = & \max\{A(x), A(y)\}, \\ A(\lambda x) & = & 1 - A^c(\lambda x) \\ & \leq & 1 - \min\{F^c(\lambda), A^c(x)\} \\ & = & \max\{1 - F^c(\lambda), 1 - A^c(x)\} \\ & = & \max\{F(\lambda), A(x)\}, \\ F(1) & = & 1 - F^c(1) \\ & \leq & 1 - A^c(x) \\ & = & A(x). \end{array}$$

Thus A is an anti fuzzy near-algebra of Y. □

4. Anti homomorphism, Anti fuzzy ideals of near-algebras

In this section we define anti homomorphism and anti fuzzy ideal of a near-algebra and discuss some related properties.

Definition 4.1. Let Y and Z be two near-algebras over a field X. A mapping $f : Y \to Z$ is called an *anti-homomorphism* if for all $x, y \in Y$ and $\lambda \in X$

(i) $f(x + y) = f(x) + f(y)$,
(ii) $f(\lambda x) = \lambda f(x)$ and
(iii) $f(xy) = f(y)f(x)$.

Definition 4.2. Let $f : Y \to Y'$ be a near-algebra homomorphism. Then the homomorphic image of Y is $f(Y)$, defined by $f(Y) = \{x' \in Y' : x \in Y, f(x) = x'\}$.

Theorem 4.3. *Anti homomorphic image of a right(left) near-algebra is a left(right) near-algebra.*

Proof. Let $f : Y \to Y'$ be a (right)near-algebra anti homomorphism. The homomorphic image of Y is $f(Y) = \{f(x) \in Y' : x \in Y\}$. Clearly, $f(0) = 0' \in f(Y)$. Where 0 and $0'$ are the additive identities in Y and Y' respectively. Thus $f(Y)$ is a non-empty subset of Y'. Let $x', y' \in f(Y)$. Then there exists $x, y \in Y$ such that $f(x) = x', f(y) = y'$. Which implies $x - y, xy, yx, \lambda x \in Y$, $(\lambda x)y = \lambda(xy)$, and so $f(x-y), f(xy), f(yx), f(\lambda x) \in f(Y)$. Then $x' - y' = f(x) - f(y) = f(x-y) \in f(Y)$, $x'y' = f(x)f(y) = f(yx) \in f(Y)$, $\lambda x' = \lambda f(x) = f(\lambda x) \in f(Y)$ and $\lambda(x'y') = \lambda(f(x)f(y)) = \lambda(f(yx)) = f(\lambda(yx)) = f((\lambda y)x) = f(x)f(\lambda y) = x'(\lambda f(y)) = x'(\lambda y')$. Thus if Y is a right near-algebra, then $f(Y)$ is a left near-algebra of Y'. □

Example 4.4. Consider Example 3.3 with $Y = \{0, a, b, c\}$ and

$$X = Z_3 = \{0, 1, 2\}_{\oplus_3, \otimes_3}.$$

Then it is clear that Y is a right near-algebra over the field X. Define a mapping $f : Y \to Y$ by $f(x) = 0$ for every $x \in Y$. Then

$$\begin{array}{lcl} f(x+y) & = & f(x) + f(y), \\ f(xy) & = & f(y)f(x) \text{ and} \\ f(\lambda x) & = & \lambda f(x) \text{ for every } x, y \in Y \text{ and } \lambda \in X. \end{array}$$

Thus $f : Y \to Y$ is an anti homomorphism. Now the homomorphic image of Y is $f(Y) = \{0\} \subset Y$(co-domain). Clearly $f(Y)$ forms a left near-algebra over the field X. Thus if Y(domain) is a right near-algebra, then $f(Y)$ is a left near-algebra of Y(co domain).

Definition 4.5. Let (A, Y) be an anti fuzzy near-algebra over an anti fuzzy field (F, X). Then A is called an *anti fuzzy ideal* of Y if $A(xy) \leq A(x)$ and $A(y(x+i) - yx) \leq A(i)$ for every $x, y, i \in Y$. A is an anti fuzzy right ideal of Y if $A(xy) \leq A(x)$, and A is an anti fuzzy left ideal of Y if $A(y(x+i) - yx) \leq A(i)$ for every $x, y, i \in Y$.

Example 4.6. The fuzzy subset A on a near-algebra Y defined in Example 3.3 is an anti fuzzy ideal of Y.

Proposition 4.7. *Let A be a fuzzy subset of a near-algebra Y. Then A is an anti fuzzy ideal of Y if and only if A^c is a fuzzy ideal of Y.*

Proof. Suppose that A is an anti fuzzy ideal of Y. Then from Theorem 3.12 we have that A^c is a fuzzy near-algebra of Y. And for all $x, y, i \in Y$ we have that

$$\begin{array}{lcl} A^c(xy) & = & 1 - A(xy) \\ & \geq & 1 - A(x) \\ & = & A^c(x) \text{ and} \\ A^c(y(x+i) - yx) & = & 1 - A(y(x+i) - yx) \\ & \geq & 1 - A(i) \\ & = & A^c(i). \end{array}$$

Thus A^c is a fuzzy ideal of Y. Conversely, suppose that A^c is a fuzzy ideal of Y. Then from Theorem 3.12 we have that A is an anti fuzzy near-algebra of Y. Also for all $x, y, i \in Y$ we have that

$$\begin{aligned} A(xy) &= 1 - A^c(xy) \\ &\leq 1 - A^c(x) \\ &= A(x) \text{ and} \\ A(y(x+i) - yx) &= 1 - A^c(y(x+i) - yx) \\ &\leq 1 - A^c(i) \\ &= A(i). \end{aligned}$$

Thus A is an anti fuzzy ideal of Y. This completes the proof. □

Proposition 4.8. *If $\{A_i : i \in \Lambda, \Lambda$ is the index set$\}$ is a family of anti fuzzy ideals of Y, then so is $\bigwedge_{i\in\Lambda} A_i$ (that is, intersection of family of anti fuzzy ideals is an anti fuzzy ideal).*

Proof. Let $\{A_i\}_{i\in\Lambda}$ be a family of anti fuzzy ideals of Y over an anti fuzzy field (F, X). Let $A(x) = \bigcap_{i\in\Lambda} A_i(x) = \inf_{i\in\Lambda} A_i(x) = \bigwedge_{i\in\Lambda} A_i(x)$. Then from Theorem 3.9, it is clear that (A, Y) is an anti fuzzy near-algebra over the anti fuzzy field (F, X). Now for any $x, y, z \in Y$,

$$\begin{aligned} A(xy) &= \inf\{A_i(xy) : i \in \Lambda\} \\ &\leq \inf\{A_i(x) : i \in \Lambda\} \\ &= A(x), \\ A(y(x+z) - yx) &= \inf\{A_i(y(x+z) - yx) : i \in \Lambda\} \\ &\leq \inf\{A_i(z) : i \in \Lambda\} \\ &= A(z). \end{aligned}$$

Hence A is an anti fuzzy ideal of Y. □

Proposition 4.9. *If $\{A_i : i \in \Lambda, \Lambda$ is the index set$\}$ is a family of anti fuzzy ideals of Y, then so is $\bigvee_{i\in\Lambda} A_i$.*

Proof. Let $\{A_i : i \in \Lambda\}$ be a family of anti fuzzy ideals of Y over an anti fuzzy field (F, X). Let $x, y \in Y$ and $\lambda \in X$. Then from Theorem 3.10, it is clear that $\bigvee_{i\in\Lambda} A_i$ is an anti fuzzy near-algebra of Y. Now for any $x, y, z \in Y$,

$$\begin{aligned} (\bigvee_{i\in\Lambda} A_i)(xy) &= \sup\{A_i(xy) : i \in \Lambda\} \\ &\leq \sup\{A_i(x) : i \in \Lambda\} \\ &= (\bigvee_{i\in\Lambda} A_i)(x), \\ (\bigvee_{i\in\Lambda} A_i)(y(x+z) - yx) &= \sup\{A_i(y(x+z) - yx) : i \in \Lambda\} \\ &\leq \sup\{A_i(z) : i \in \Lambda\} \\ &= (\bigvee_{i\in\Lambda} A_i)(z). \end{aligned}$$

Hence $\bigvee_{i\in\Lambda} A_i$ is an anti fuzzy ideal of Y. □

Acknowledgements. The authors would like to express their deep gratitude to Prof. Y. B. Jun for his constant encouragement. The authors also express their sincere thanks to the referee and reviewer for their valuable comments and suggestions for improving the paper.

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