

On soft Γ -semigroups

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ABSTRACT. Let U be an initial universe and E a set of parameters. A pair (F, A) is called a soft set over U if $F : E \rightarrow P(U)$ where $P(U)$ denotes the power set of U and $A \subseteq E$. In this paper, we introduce the concept of soft Γ -semigroups and soft (left, right) ideals using soft sets and investigate some properties.

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1. INTRODUCTION

In 1999, the notion of soft sets was introduced as an effective mathematical tool to deal with uncertainty by Molodtsov (see [6]). In 2003, Maji, Biswas and Roy defined binary operations on soft sets (see [5]). However, these binary operations were corrected by Ali, Feng, Lui, Min and Shabir in [3]. There are relations between soft sets and algebraic structures. In [1, 2, 4], the authors applied soft sets to semirings, rings and groups, respectively. In [8], the authors defined soft semigroups and soft ideals over a semigroup. Recently, Shabir and Ahmad introduced the concept of soft ternary semigroups as a collection of ternary subsemigroup of a ternary semigroup and defined soft (left, right, lateral, quasi, bi) ideals of a ternary semigroup (see [7]).

In [9], Sen and Saha defined a Γ -semigroup as a generalization of a semigroup. The purpose of this paper is to introduce soft Γ -semigroups, soft (left, right) ideals and investigate some properties.

Let us recall some definitions concerning Γ -semigroups. Let S and Γ be two nonempty sets. Then S is called a Γ -semigroup if there exists a mapping $S \times \Gamma \times S \rightarrow S$, written as $(x, \gamma, y) \mapsto x\gamma y$, such that $(x\gamma y)\beta z = x\gamma(y\beta z)$ for all $x, y, z \in S$ and all $\gamma, \beta \in \Gamma$. For nonempty subsets A and B of S , let

$$A\Gamma B = \{a\gamma b \mid a \in A, b \in B, \gamma \in \Gamma\}.$$

For $x \in S$, let $A\Gamma x = A\Gamma\{x\}$ and $x\Gamma A = \{x\}\Gamma A$. A nonempty subset T of S is called a Γ -subsemigroup of S if for all $x, y \in T$ and $\gamma \in \Gamma$, $x\gamma y \in T$. S is said to be commutative if for all $x, y \in S$ and $\gamma \in \Gamma$, $x\gamma y = y\gamma x$.

Let S be a Γ -semigroup. A nonempty subset A of S is called a *left (right) ideal* of S if $S\Gamma A \subseteq A$ ($A\Gamma S \subseteq A$). If A is both left and right ideal of S , then A is called an *ideal* of S . An ideal A of S is said to be *idempotent* if $A\Gamma A = A$.

Hereafter, let S be a Γ -semigroup.

2. PRELIMINARIES

Let U be an *initial universe* and E a set of *parameters*. A pair (F, A) is called a *soft set* over U if

$$F : E \rightarrow P(U)$$

where $P(U)$ denotes the power set of U and $A \subseteq E$. For two soft sets (F, A) and (G, B) over a common universe U , we say that (F, A) is a *subset* of (G, B) , denoted by $(F, A) \subseteq (G, B)$, if

- (i) $A \subseteq B$,
- (ii) $F(a) \subseteq G(a)$ for all $a \in A$.

We say that (F, A) and (G, B) are *equal* if $(F, A) \subseteq (G, B)$ and $(G, B) \subseteq (F, A)$.

The following are operations defined between soft sets.

Definition 2.1. Let (F, A) and (G, B) be soft sets over a common universe U . Let “ $(F, A)OR(G, B)$ ” be a soft set over U , denoted by $(F, A) \vee (G, B)$, defined by $(F, A) \vee (G, B) = (K, A \times B)$ where

$$K(a, b) = F(a) \cup G(b)$$

for all $(a, b) \in A \times B$.

Definition 2.2. Let (F, A) and (G, B) be soft sets over a common universe U . Let “ $(F, A)AND(G, B)$ ” be a soft set over U , denoted by $(F, A) \wedge (G, B)$, defined by $(F, A) \wedge (G, B) = (H, A \times B)$ where

$$H(a, b) = F(a) \cap G(b)$$

for all $(a, b) \in A \times B$.

Definition 2.3. Let (F, A) and (G, B) be soft sets over a common universe U .

- (1) The *extension union* of (F, A) and (G, B) , denoted by $(F, A) \cup_E (G, B)$, is the soft set (H, C) where $C = A \cup B$ and for $e \in C$,

$$H(e) = \begin{cases} F(e) & \text{if } e \in A \setminus B, \\ G(e) & \text{if } e \in B \setminus A, \\ F(e) \cup G(e) & \text{if } e \in A \cap B. \end{cases}$$

- (2) The *restricted union* of (F, A) and (G, B) , denoted by $(F, A) \cup_R (G, B)$, is the soft set (H, C) where $C = A \cap B$ and for $e \in C$, $H(e) = F(e) \cup G(e)$.
- (3) The *extension intersection* of (F, A) and (G, B) , denoted by $(F, A) \cap_E (G, B)$, is the soft set (H, C) where $C = A \cup B$ and for $e \in C$,

$$H(e) = \begin{cases} F(e) & \text{if } e \in A \setminus B, \\ G(e) & \text{if } e \in B \setminus A, \\ F(e) \cap G(e) & \text{if } e \in A \cap B. \end{cases}$$

- (4) The *restricted intersection* of (F, A) and (G, B) , denoted by $(F, A) \cap_R (G, B)$, is the soft set (H, C) where $C = A \cap B$ and for $e \in C$, $H(e) = F(e) \cap G(e)$.

Hereafter, we shall consider a soft set over S , a Γ -semigroup.

3. MAIN RESULTS

Definition 3.1. The Γ -restricted product of soft sets (F, A) and (G, B) over S , denoted by $(F, A) \bar{\Gamma} (G, B)$, is defined as a soft set

$$(K, D) = (F, A) \bar{\Gamma} (G, B)$$

where $D = A \cap B \neq \emptyset$ and $K : D \rightarrow P(S)$ such that

$$K(d) = F(d) \Gamma G(d)$$

for all $d \in D$.

Definition 3.2. A soft set (F, A) over S is called a *soft Γ -semigroup* over S if $(F, A) \bar{\Gamma} (F, A) \subseteq (F, A)$.

Example 3.3. Let $M = \{-i, 0, i\}$ and $\Gamma = A = \{-i, i\}$. We have M is a Γ -semigroup. Define $F : A \rightarrow P(M)$ by $F(i) = F(-i) = A$. Then (F, A) is a soft Γ -semigroup over M .

Example 3.4. Let M and Γ be the set of all 3×3 diagonal matrices over the set of positive integers. Then M is a Γ -semigroup under the usual matrix multiplication. Let $\mathbb{N}$ be the set of all positive integers. Define $F : \mathbb{N} \rightarrow P(M)$ by $F(n) = \{A \in M : \det(A) \geq n\}$. We obtain $(F, \mathbb{N})$ is a soft Γ -semigroup over M .

Example 3.5. For $n \in \mathbb{N}$ such that $n \geq 4$, we consider

$$\mathbb{Z}_{2n} = \{[0], [1], [2], \dots, [2n-1]\} \text{ and } \Gamma = \{[0], [n]\}.$$

It is easy to see that $\mathbb{Z}_{2n}$ is a Γ -semigroup. Take $A = \{[4a] : 0 \leq a \leq n\}$ and $B = \{[2b] : 0 \leq b \leq n\}$. Define $F : A \rightarrow P(\mathbb{Z}_{2n})$ by $F([x]) = \{[0], [2x]\}$ and defined $G : B \rightarrow P(\mathbb{Z}_{2n})$ by $G([x]) = \{[0], [2x], [4x]\}$. We have

$$F([a]) \Gamma F([a]) = \{[0]\} \subseteq F([a]) \text{ and } G([b]) \Gamma G([b]) = \{[0]\} \subseteq G([b])$$

for all $[a] \in A, [b] \in B$. Hence (F, A) and (G, B) are soft Γ -semigroups over $\mathbb{Z}_{2n}$.

Theorem 3.6. A soft set (F, A) over S is a soft Γ -semigroup over S if and only if for all $a \in A$ such that $F(a) \neq \emptyset$, $F(a)$ is a Γ -subsemigroup of S .

Proof. Assume that a soft set (F, A) over S is a soft Γ -semigroup over S . Let $a \in A$ be such that $F(a) \neq \emptyset$. We have

$$(F, A) \bar{\Gamma} (F, A) = (K, A \cap A) \text{ and } K(a) = F(a) \Gamma F(a)$$

for all $a \in A$. Since $K \subseteq F$, $K(a) \subseteq F(a)$. So, $F(a) \Gamma F(a) \subseteq F(a)$. Thus $F(a)$ is a Γ -subsemigroup of S .

Conversely, assume that $F(a)$ is a Γ -subsemigroup of S for all $a \in A$ such that $F(a) \neq \emptyset$. We have

$$(F, A) \bar{\Gamma} (F, A) = (K, A \cap A) \text{ and } K(a) = F(a) \Gamma F(a)$$

for all $a \in A$. By assumption, $K(a) \subseteq F(a)$. Thus $(K, A) \subseteq (F, A)$. Therefore, a soft set (F, A) over S is a soft Γ -semigroup over S . $\square$

Let (S, E) be the soft set over S defined by $S(e) = S$ for all $e \in S$. This is called an *absolute soft set* over S .

Definition 3.7. A soft set (F, A) over S is called a *soft l-idealistic (r-idealistic)* over S if $(S, E)\overline{\Gamma}(F, A) \subseteq (F, A)$ ($(F, A)\overline{\Gamma}(S, E) \subseteq (F, A)$).

Example 3.8. We consider $\mathbb{Z}_8 = \{[0], [1], [2], [3], [4], [5], [6], [7]\}$. Let $\Gamma = \{[1], [4]\}$, then $\mathbb{Z}_8$ is a Γ -semigroup. Take $C = \{[0], [4]\}$. Define $H : C \rightarrow P(\mathbb{Z}_8)$ by $H(c) = C$ for all $c \in C$. Then (H, C) is a soft l-idealistic over $\mathbb{Z}_8$.

Theorem 3.9. A soft set (F, A) over S is a soft l-idealistic (r-idealistic) over S if and only if for all $a \in A$ such that $F(a) \neq \emptyset$, $F(a)$ is a left (right) ideal of S .

Proof. Assume that a soft set (F, A) over S is a soft l-idealistic over S . Let $a \in A$ be such that $F(a) \neq \emptyset$. We have

$$(S, E)\overline{\Gamma}(F, A) = (K, E \cap A) \text{ and } K(a) = S(a)\Gamma F(a)$$

for all $a \in A$. Since $K \subseteq F$, $K(a) \subseteq F(a)$. Since $S\Gamma F(a) = S(a)\Gamma F(a) \subseteq F(a)$, $F(a)$ is a left ideal of S .

Conversely, assume that $F(a)$ is a left ideal of S for all $a \in A$ such that $F(a) \neq \emptyset$. We have

$$(S, E)\overline{\Gamma}(F, A) = (K, E \cap A) \text{ and } K(a) = S(a)\Gamma F(a)$$

for all $a \in A$. By assumption, $K(a) = S(a)\Gamma F(a) = S\Gamma F(a) \subseteq F(a)$. Thus $(K, A) \subseteq (F, A)$. Therefore, a soft set (F, A) over S is a soft l-idealistic over S .

That a soft set (F, A) over S is a soft r-idealistic over S if and only if for all $a \in A$ such that $F(a) \neq \emptyset$, $F(a)$ is a right ideal of S can be proved similarly. $\square$

Proposition 3.10. Let (F, A) and (G, B) be soft Γ -semigroups over S . If $A \cap B \neq \emptyset$, then $(F, A) \cap_R (G, B)$ is a soft Γ -semigroup over S .

Proof. Assume that $A \cap B \neq \emptyset$. Let $(H, C) = (F, A) \cap_R (G, B)$ where $C = A \cap B \neq \emptyset$ and $H(c) = F(c) \cap G(c)$ for all $c \in C$. To show that (H, C) is a soft Γ -semigroup over S , we have to show that $(H, C)\overline{\Gamma}(H, C) \subseteq (H, C)$. Let $(K, D) = (H, C)\overline{\Gamma}(H, C)$ where $D = C \cap C$ and $K(d) = H(d)\Gamma H(d)$ for all $d \in D$. For $d \in D$, we have

$$K(d) = H(d)\Gamma H(d) = (F(d) \cap G(d))\Gamma(F(d) \cap G(d)) \subseteq F(d) \cap G(d) = H(d)$$

Then $K \subseteq H$. Therefore, $(F, A) \cap_R (G, B)$ is a soft Γ -semigroup over S . $\square$

Proposition 3.11. Let (F, A) and (G, B) be soft Γ -semigroups over S . If $A \cap B \neq \emptyset$, then $(F, A) \cup_E (G, B)$ is a soft Γ -semigroup over S .

Proof. Assume that $A \cap B \neq \emptyset$. Let $(H, C) = (F, A) \cup_E (G, B)$ where

$$H(c) = \begin{cases} F(c) & \text{if } c \in A \setminus B, \\ G(c) & \text{if } c \in B \setminus A, \\ F(c) \cap G(c) & \text{if } c \in A \cap B. \end{cases}$$

To show that (H, C) is a soft Γ -semigroup over S , we have to show that

$$(H, C)\overline{\Gamma}(H, C) \subseteq (H, C).$$

Let $(K, D) = (H, C)\overline{\Gamma}(H, C)$ where $D = C \cap C$ and $K(d) = H(d)\Gamma H(d)$ for all $d \in D$. For $d \in D$, we have

$$K(d) = H(d)\Gamma H(d) \subseteq \begin{cases} F(d) & \text{if } d \in A \setminus B, \\ G(d) & \text{if } d \in B \setminus A, \\ F(d) \cap G(d) & \text{if } d \in A \cap B. \end{cases}$$

Then $K(d) \subseteq H(d)$. Therefore, $(F, A) \cup_E (G, B)$ is a soft Γ -semigroup over S . $\square$

Proposition 3.12. *Let (F, A) and (G, B) be soft Γ -semigroups over S . Then $(F, A) \wedge (G, B)$ is a soft Γ -semigroup over S .*

Proof. Let $(F, A) \wedge (G, B) = (H, A \times B)$ where $H(a, b) = F(a) \cap G(b)$ for all $(a, b) \in A \times B$. To show that $(H, A \times B)$ is a soft Γ -semigroup over S , let $(K, D) = (H, A \times B)\overline{\Gamma}(H, A \times B)$ where $D = (A \times B) \cap (A \times B)$ and $K(a, b) = H(a, b)\Gamma H(a, b)$ for all $(a, b) \in D$. For $(a, b) \in D$, we have

$$K(a, b) = H(a, b)\Gamma H(a, b) = (F(a) \cap G(b))\Gamma(F(a) \cap G(b)) \subseteq F(a) \cap G(b) = H(a, b)$$

Then $K \subseteq H$. Therefore, $(F, A) \wedge (G, B)$ is a soft Γ -semigroup over S . $\square$

Definition 3.13. Let (F, A) and (G, B) be soft sets over a Γ -semigroup S . Define $(F, A)\Gamma^*(G, B)$ is a soft set $(K, A \times B)$ where $K(a, b) = F(a)\Gamma G(b)$.

Proposition 3.14. *Let (F, A) and (G, B) be soft Γ -semigroups over S . If S is commutative, then $(F, A)\Gamma^*(G, B)$ is a soft Γ -semigroup over S .*

Proof. Let $(F, A)\Gamma^*(G, B) = (H, A \times B)$ where $H(a, b) = F(a)\Gamma G(b)$ for all $(a, b) \in A \times B$. To show that $(H, A \times B)$ is a soft Γ -semigroup over S , let

$$(K, D) = (H, A \times B)\overline{\Gamma}(H, A \times B)$$

where $D = (A \times B) \cap (A \times B)$ and $K(a, b) = H(a, b)\Gamma H(a, b)$ for all $(a, b) \in D$. For $(a, b) \in D$, since S is commutative, we have

$$K(a, b) = H(a, b)\Gamma H(a, b) = (F(a)\Gamma G(b))\Gamma(F(a)\Gamma G(b)) \subseteq F(a)\Gamma G(b) = H(a, b)$$

Then $K \subseteq H$. Therefore, $(F, A)\Gamma^*(G, B)$ is a soft Γ -semigroup over S . $\square$

Proposition 3.15. *If (F, A) and (G, B) are soft l-idealistic (r-idealistic) over S , then $(F, A) \cap_R (G, B)$ is a soft l-idealistic (r-idealistic) over S contained in both (F, A) and (G, B) .*

Proof. Assume that (F, A) and (G, B) are soft l-idealistic over S . Let $(H, C) = (F, A) \cap_R (G, B)$ where $C = A \cap B \neq \emptyset$ and $H(c) = F(c) \cap G(c)$ for all $c \in C$. To show that (H, C) is a soft l-idealistic over S , we have to show that $(S, E)\overline{\Gamma}(H, C) \subseteq (H, C)$. Let $(K, D) = (S, E)\overline{\Gamma}(H, C)$ where $D = S \cap C$ and $K(d) = S(d)\Gamma H(d)$ for all $d \in D$. For $d \in D$, we have

$$K(d) = S(d)\Gamma H(d) = S(d)\Gamma(F(d) \cap G(d)) \subseteq F(d) \cap G(d) = H(d)$$

Then $K \subseteq H$. Therefore, $(F, A) \cap_R (G, B)$ is a soft l -idealistic over S . Similarly, if (F, A) and (G, B) are soft r -idealistics over S , then $(F, A) \cap_R (G, B)$ is a soft r -idealistic over S . It is clear by definition that $(F, A) \cap_R (G, B)$ is contained in both (F, A) and (G, B) . $\square$

Proposition 3.16. *If (F, A) and (G, B) are soft l -idealistics (r -idealistics) over S , then $(F, A) \cup_E (G, B)$ is a soft l -idealistic (r -idealistic) over S containing both (F, A) and (G, B) .*

Proof. Straightforward. $\square$

Proposition 3.17. *If (F, A) and (G, B) are soft l -idealistics (r -idealistics) over S , then $(F, A) \vee (G, B)$ is a soft l -idealistic (r -idealistic) over S .*

Proof. Straightforward. $\square$

Proposition 3.18. *If (F, A) and (G, B) are soft l -idealistics (r -idealistics) over S , then $(F, A) \wedge (G, B)$ is a soft l -idealistic (r -idealistic) over S containing both (F, A) and (G, B) .*

Proof. Straightforward. $\square$

Definition 3.19. Let (F, A) and (G, B) be soft sets over S such that $(G, B) \subseteq (F, A)$. Then (G, B) is called a *soft Γ -subsemigroup* of (F, A) if $G(b)$ is a Γ -subsemigroup of $F(b)$ for all $b \in B$.

Definition 3.20. Let (F, A) and (G, B) be soft sets over S such that $(G, B) \subseteq (F, A)$. Then (G, B) is called a *soft ideal* of (F, A) if $G(b)$ is an ideal of $F(b)$ for all $b \in B$.

Example 3.21. Let $M = [0, 1]$ and $\Gamma = \{\frac{1}{n} : n \in \mathbb{N}\}$. For $n \in \mathbb{N}$, let $A_n = [0, \frac{1}{n}]$ and $B_n = [0, \frac{1}{2n}]$. We define $F_n : A_n \rightarrow P(M)$ by $F_n(x) = [0, x]$ for all $x \in A_n$ and $G_n : B_n \rightarrow P(M)$ by $G_n(y) = [0, \frac{y}{2}]$ for all $y \in B_n$. For $m, n \in \mathbb{N}$ such that $m \leq n$ the following hold.

- (1) (F_n, A_n) and (G_n, B_n) are soft Γ -semigroups over M .
- (2) (G_n, B_n) is a soft Γ -subsemigroup of (F_m, A_m) .
- (3) (G_n, B_n) is a soft ideal of (F_m, A_m) .
- (4) (F_n, A_n) and (G_n, B_n) are soft l -idealistics over M .

Example 3.22. We have $\mathbb{Z}_8 = \{[0], [1], [2], [3], [4], [5], [6], [7]\}$ is a Γ -semigroup where $\Gamma = \{[1], [4]\}$. Let $A = \{[0], [1], [2], [4]\}$ and $B = \{[0], [1], [4]\}$. Defined $F : A \rightarrow P(\mathbb{Z}_8)$ by $F(a) = A$ for all $a \in A$ and $G : B \rightarrow P(\mathbb{Z}_8)$ by $G(b) = B$ for all $b \in B$. Then (F, A) and (G, B) are soft Γ -semigroups over $\mathbb{Z}_8$. Moreover, (G, B) is a soft Γ -subsemigroup of (F, A) . However, (G, B) is not a soft ideal of (F, A) because $[2] = [1][1][2] \in G([1])\Gamma F([1])$ and $[2] \notin B = G([1]), G([1])\Gamma F([1]) \not\subseteq G([1])$.

Theorem 3.23. *Let (F, A) be a soft Γ -semigroup over S . Let $\{(H_i, B_i) \mid i \in I\}$ be a nonempty family of soft Γ -subsetmigroups of (F, A) .*

- (1) $\bigcap_R (H_i, B_i)$ is a soft Γ -subsetmigroup of (F, A) .
- (2) $\bigwedge_{i \in I} (H_i, B_i)$ is a soft Γ -subsetmigroup of $\bigwedge_{i \in I} (F, A)$.
- (3) $\bigcup_E (H_i, B_i)$ is a soft Γ -subsetmigroup of (F, A) if $B_i, i \in I$, are pair wise disjoint.

Proof. Straightforward. $\square$

Theorem 3.24. *Let (F, A) be a soft Γ -semigroup over S . Let $\{(H_i, B_i) \mid i \in I\}$ be a nonempty family of soft ideals of (F, A) .*

- (1) $\bigcap_R (H_i, B_i)$ is a soft ideal of (F, A) .
- (2) $\bigwedge_{i \in I} (H_i, B_i)$ is a soft ideal of $\bigwedge_{i \in I} (F, A)$.
- (3) $\bigcup_E (H_i, B_i)$ is a soft ideal of (F, A) .
- (4) $\bigvee_{i \in I} (H_i, B_i)$ is a soft ideal of $\bigvee_{i \in I} (F, A)$.

Proof. Straightforward. $\square$

Theorem 3.25. *If (G, B) is a soft ideal of (F, A) , then (G, B) is a soft Γ -subsemigroup of (F, A) .*

Proof. Let (G, B) be a soft ideal of (F, A) . Then for all $b \in B$ we have $G(b) \subseteq F(b)$ and $G(b)$ is an ideal of $F(b)$. Hence for all $b \in B$ we obtain $G(b)\Gamma G(b) \subseteq G(b)\Gamma F(b) \subseteq F(b)$. Thus (G, B) is a soft Γ -subsemigroup of (F, A) . $\square$

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