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Generalised intuitionistic fuzzy soft sets and an adjustable approach to decision making

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Abstract. The aim of this paper is to generalise the concept of intuitionistic fuzzy soft set. Relations on generalised intuitionistic fuzzy soft sets and a few of their algebraic properties are studied. Finally, an adjustable approach to generalised intuitionistic fuzzy soft sets based decision making is presented.

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1. Introduction

In 1999, Molodtsov [9] initiated the novel concept of soft set theory to deal with uncertainties which can not be handled by traditional mathematical tools. He successfully applied the soft set theory into several disciplines, such as game theory, Riemann integration, Perron integration, measure theory etc. Applications of soft set theory in real life problems are now catching momentum due to the general nature parametrization expressed by a soft set. Maji et al. [7] gave first practical application of soft sets in decision making problems. They also introduced fuzzy soft sets [5], a more generalised concept and studied some of its properties and applications. Mazumdar and Samanta [8] generalised the concept of fuzzy soft sets as introduced by Maji et al.[5].
Roy and Maji [10] presented a novel method for decision making, which involved construction of a comparison table of a fuzzy soft set in parametric sense. Kong et al. [4] investigated that the method of Roy-Maji [10] was incorrect and they presented a revised algorithm. Recently, Feng et al. [2] gave deeper insights into decision making based on fuzzy soft sets. They discussed the validity of the Roy-Maji method [10] and showed its limitations. By means of level soft sets, Feng et al. [2] presented an adjustable approach to fuzzy soft sets based decision making. Jiang

et al. [3] presented an adjustable approach to intuitionistic fuzzy soft sets based decision making.
In this paper, we generalise of the concepts of Dinda-Samanta [1] and Mazumdar-Samanta [8]. The given definition of generalised intuitionistic fuzzy soft set is more realistic as it contains a degree of preference corresponding to each parameter. Relations on generalised intuitionistic fuzzy soft sets are defined and a few of its properties are studied. Also, an adjustable approach to generalised intuitionistic fuzzy soft sets based decision making is presented.

2. Preliminaries

In this section we present some basic definition and preliminary results which will be needed in the sequel.

Definition 2.1. ([9]). Let U be an initial universe set and E be the set of parameters. Let $P(U)$ denotes the power set of U. A pair (F, E) is called a soft set over U, where F is a mapping given by $F : E \to P(U)$.

Definition 2.2. ([5]). Let U be an initial universe set and E be the set of parameters. Let $A \subset E$. A pair (F, A) is called fuzzy soft set over U where F is a mapping given by $F : A \to I^U$ and I^U denotes the collection of all fuzzy subsets of U.

Definition 2.3. ([8]). Let $U = \{x_1, x_2, \cdots, x_n\}$ be the initial universe of elements and $E = \{e_1, e_2, \cdots, e_m\}$ be the set of parameters. The pair (U, E) will be called a soft universe. Let $F : E \to I^U$ and μ be a fuzzy subset of E, *i.e.*, $\mu : E \to I = [0, 1]$, where I^U is the collection of all fuzzy subset of U. Let F_μ be a mapping $F_\mu : E \to I^U \times I$ defined as follows:
$F_\mu(e) = (F(e), \mu(e))$, where $F(e) \in I^U$. Then F_μ is called generalised fuzzy soft set over the soft universe (U, E).

Here for each parameter e_i, $F_\mu(e_i)$ indicates not only the degree of belongingness of the elements of U in $F(e_i)$ but also the degree of possibility of such belongingness which is represented by $\mu(e_i)$.

Definition 2.4. ([6]) Let U be an initial universe set and E be the set of parameters. Let IF^U denotes the collection of all intuitionistic fuzzy subsets of U. Let $A \subseteq E$. A pair $(F,\ A)$ is called intuitionistic fuzzy soft set over U, where F is a mapping given by $F :\ A \to IF^U$.

Example 2.5. Consider the following example:
Let (F, A) describes the character of the students with respect to the given parameters for finding the best student of an academic year. Let the set of students under consideration is $U = \{s_1, s_2, s_3, s_4\}$. Let $A \subseteq E$ and $A = \{r = "result", c = "conduct", g = "games\ and\ sports\ performances"\}$. Let

$F(r) = \{(s_1, 0.8, 0.1), (s_2, 0.9, 0.1), (s_3, 0.8, 0.1), (s_4, 0.7, 0.2)\}$
$F(c) = \{(s_1, 0.6, 0.3), (s_2, 0.65, 0.2), (s_3, 0.7, 0.2), (s_4, 0.65, 0.2)\}$
$F(g) = \{(s_1, 0.8, 0.2), (s_2, 0.5, 0.3), (s_3, 0.5, 0.4), (s_4, 0.7, 0.2)\}$

Then the sub-family $\{F(r), F(c), F(g)\}$ of IF^U is an intuitionistic fuzzy soft set over U.

	r	c	g
s_1	(0.8,0.1)	(0.6,0.3)	(0.8,0.2)
s_2	(0.9,0.1)	(0.65,0.2)	(0.5,0.3)
s_3	(0.8,0.1)	(0.7,0.2)	(0.5,0.4)
s_4	(0.7,0.2)	(0.65,0.2)	(0.7,0.2)

Table 1. Tabular representation of the intuitionistic fuzzy soft set

	r	c	g
s_1	1	0	1
s_2	1	1	0
s_3	1	1	0
s_4	0	1	1

Table 2. Tabular representation of mid level soft set (see Example 2 of [3])

Definition 2.6. ([6]). Intersection of two intuitionistic fuzzy soft sets (F, A) and (G, B) over a common universe U is the intuitionistic fuzzy soft set (H, C) where $C = A \cap B$, and $\forall\, e \in C,\ H(e) = F(e) \cap G(e)$. We write $(F, A)\tilde{\cap}(G, B) = (H, C)$.

Definition 2.7. ([6]). Union of two intuitionistic fuzzy soft sets (F, A) and (G, B) over a common universe U is the intuitionistic fuzzy soft set (H, C) where $C = A \cup B$, and $\forall\, e \in C$,

$$\begin{aligned} H(e) &= F(e), && if\ e \in A - B \\ &= G(e), && if\ e \in B - A \\ &= F(e) \cup G(e), && if\ e \in A \cap B \end{aligned}$$

We write $(F, A)\tilde{\cup}(G, B) = (H, C)$.

Definition 2.8. ([6]). For two intuitionistic fuzzy soft sets (F, A) and (G, B) over a common universe U, we say that (F, A) is an intuitionistic fuzzy soft subset of (G, B) if

(i) $A \subset B$, and

(ii) $\forall\, e \in A,\ F(e)$ is an intuitionistic fuzzy subset of $G(e)$.

It is denoted by $(F, A)\tilde{\subset}(G, B)$.

Definition 2.9. ([11]). A binary operation $* : [0, 1] \times [0, 1] \to [0, 1]$ is continuous t-norm if $*$ satisfies the following conditions :

(i) $*$ is commutative and associative,

(ii) $*$ is continuous,

(iii) $a * 1 = a \quad \forall\, a \in [0, 1]$,

(iv) $a * b \leq c * d$ whenever $a \leq c, b \leq d$ and $a, b, c, d \in [0, 1]$.

A few examples of continuous t-norm are $a * b = ab,\ a * b = \min\{a, b\},\ a * b = \max\{a + b - 1, 0\}$.

Definition 2.10. ([11]). A binary operation $\diamond : [0,1] \times [0,1] \to [0,1]$ is continuous t-conorm if $\diamond$ satisfies the following conditions:
(i) $\diamond$ is commutative and associative,
(ii) $\diamond$ is continuous,
(iii) $a \diamond 0 = a \quad \forall\ a \in [0,1]$,
(iv) $a \diamond b \leq c \diamond d$ whenever $a \leq c$, $b \leq d$ and $a, b, c, d \in [0,1]$.

A few examples of continuous t-conorm are $a \diamond b = a + b - ab$, $a \diamond b = \max\{a, b\}$, $a \diamond b = \min\{a + b, 1\}$.

3. Generalised intuitionistic fuzzy soft sets

Throughout the paper, unless otherwise stated explicitly, U be the initial universe set and E be the set of parameters. We consider $A, B, C \subseteq E$ and α, β, δ are fuzzy subset of A, B, C respectively.

Definition 3.1. Let $\mathcal{F} : A \to IF^U$ and α be a fuzzy subset of A. Then $\mathcal{F}_\alpha : A \to IF^U \times [0,1]$ is a function defined as follows:

$$\mathcal{F}_\alpha(a) = (\mathcal{F}(a),\, \alpha(a)) = \big(\{(x,\, \mu_{\mathcal{F}(a)}(x),\, \nu_{\mathcal{F}(a)}(x)) : x \in U\},\, \alpha(a)\big)$$

where μ, ν denotes the degree of membership and degree of non-membership respectively. Then $\mathcal{F}_\alpha$ is called a generalised intuitionistic fuzzy soft set over (U, E).

Here for each parameter e_i, $\mathcal{F}_\alpha(e_i)$ indicates not only degree of belongingness of the elements of U in $\mathcal{F}(e_i)$ but also degree of preference of such belongingness which is represented by $\alpha(e_i)$.

Example 3.2. Let $U = \{s_1, s_2, s_3, s_4\}$ be the set of students under consideration for the best student of an academic year with respect to the given parameters $A \subseteq E$ and $A = \{r =$"result"$, c =$"conduct"$, g =$"games and sports performances"$\}$. Let $\alpha : A \to [0,1]$ be given as follows: $\alpha(r) = 0.7,\ \alpha(c) = 0.5,\ \alpha(g) = 0.6$
We define $\mathcal{F}_\alpha$ as follows:
$\mathcal{F}_\alpha(r) = (\{(s_1, 0.8, 0.1), (s_2, 0.9, 0.05), (s_3, 0.85, 0.1), (s_4, 0.75, 0.2)\},\ 0.7)$
$\mathcal{F}_\alpha(c) = (\{(s_1, 0.6, 0.3), (s_2, 0.65, 0.2), (s_3, 0.7, 0.2), (s_4, 0.65, 0.2)\},\ 0.5)$
$\mathcal{F}_\alpha(g) = (\{(s_1, 0.75, 0.2), (s_2, 0.5, 0.3), (s_3, 0.5, 0.4), (s_4, 0.7, 0.2)\},\ 0.6)$
Then $\mathcal{F}_\alpha$ is an generalised intuitionistic fuzzy soft set.

Definition 3.3. Let $\mathcal{F}_\alpha$ and $\mathcal{G}_\beta$ be two generalised intuitionistic fuzzy soft set over (U, E). Now $\mathcal{F}_\alpha$ is called a generalised intuitionistic fuzzy soft subset of $\mathcal{G}_\beta$ if
(i) α is a fuzzy subset of β,
(ii) $A \subseteq B$,
(iii) $\forall a \in A$, $\mathcal{F}(a)$ is an intuitionistic fuzzy subset of $\mathcal{G}(a)$ i.e., $\mu_{\mathcal{F}(a)}(x) \leq \mu_{\mathcal{G}(a)}(x)$ and $\nu_{\mathcal{F}(a)}(x) \geq \nu_{\mathcal{G}(a)}(x) \quad \forall x \in U$ and $a \in A$.
It is denoted by $\mathcal{F}_\alpha \tilde{\subseteq} \mathcal{G}_\beta$.

Example 3.4. Let $\mathcal{G}_\beta$ be a generalised intuitionistic fuzzy soft set defined as follows:
$\mathcal{G}_\beta(r) = (\{(s_1, 0.85, 0.05), (s_2, 0.9, 0.025), (s_3, 0.9, 0.1), (s_4, 0.8, 0.1)\},\ 0.75)$
$\mathcal{G}_\beta(c) = (\{(s_1, 0.7, 0.2), (s_2, 0.7, 0.15), (s_3, 0.75, 0.2), (s_4, 0.65, 0.15)\},\ 0.6)$
$\mathcal{G}_\beta(g) = (\{(s_1, 0.8, 0.2), (s_2, 0.6, 0.3), (s_3, 0.7, 0.2), (s_4, 0.7, 0.1)\},\ 0.65)$
and consider the generalised intuitionistic fuzzy soft set $\mathcal{F}_\alpha$ given in Example 3.2. Then $\mathcal{F}_\alpha$ is a generalised intuitionistic fuzzy soft subset of $\mathcal{G}_\beta$.

Definition 3.5. The intersection of two generalised intuitionistic fuzzy soft sets $\mathcal{F}_\alpha$ and $\mathcal{G}_\beta$ is denoted by $\mathcal{F}_\alpha \tilde{\cap} \mathcal{G}_\beta$ and defined by a generalised intuitionistic fuzzy soft set $\mathcal{H}_\delta : A \cap B \to IF^U \times [0,1]$ such that for each $e \in A \cap B$

$$\mathcal{H}_\delta(e) = \big(\{(x,\, \mu_{\mathcal{H}(e)}(x),\, \nu_{\mathcal{H}(e)}(x)) : x \in U\},\ \delta(e)\big)$$

where $\mu_{\mathcal{H}(e)}(x) = \mu_{\mathcal{F}(e)}(x) * \mu_{\mathcal{G}(e)}(x)$, $\nu_{\mathcal{H}(e)}(x) = \nu_{\mathcal{F}(e)}(x) \diamond \nu_{\mathcal{G}(e)}(x)$, $\delta(e) = \alpha(e) * \beta(e)$.

Definition 3.6. The union of two generalised intuitionistic fuzzy soft sets $\mathcal{F}_\alpha$ and $\mathcal{G}_\beta$ is denoted by $\mathcal{F}_\alpha \tilde{\cup} \mathcal{G}_\beta$ and defined by a generalised intuitionistic fuzzy soft set $\mathcal{H}_\delta : A \cup B \to IF^U \times [0,1]$ such that for each $e \in A \cup B$

$$\begin{aligned}\mathcal{H}_\delta(e) &= \big(\{(x,\, \mu_{\mathcal{F}(e)}(x),\, \nu_{\mathcal{F}(e)}(x)) : x \in U\},\ \alpha(e)\big) \quad if\ e \in A - B\\ &= \big(\{(x,\, \mu_{\mathcal{G}(e)}(x),\, \nu_{\mathcal{G}(e)}(x)) : x \in U\},\ \beta(e)\big) \quad if\ e \in B - A\\ &= \big(\{(x,\, \mu_{\mathcal{H}(e)}(x),\, \nu_{\mathcal{H}(e)}(x)) : x \in U\},\ \delta(e)\big) \quad if\ e \in A \cap B\end{aligned}$$

where $\mu_{\mathcal{H}(e)}(x) = \mu_{\mathcal{F}(e)}(x) \diamond \mu_{\mathcal{G}(e)}(x)$, $\nu_{\mathcal{H}(e)}(x) = \nu_{\mathcal{F}(e)}(x) * \nu_{\mathcal{G}(e)}(x)$, $\delta(e) = \alpha(e) \diamond \beta(e)$.

Example 3.7. Let us consider the generalised intuitionistic fuzzy soft sets $\mathcal{F}_\alpha$ and $\mathcal{G}_\beta$ defined in example 3.2 and 3.4 respectively. Let us consider the t-norm $*$ and the t-conorm $\diamond$ as follows: $a * b = ab$ and $a \diamond b = a + b - ab$. Then
$(\mathcal{F}_\alpha \tilde{\cup} \mathcal{G}_\beta)(r) = (\{(s_1, 0.97, 0.005), (s_2, 0.99, 0.00125), (s_3, 0.985, 0.01),$
$(s_4, 0.95, 0.02)\},\ 0.68\,)$
$(\mathcal{F}_\alpha \tilde{\cup} \mathcal{G}_\beta)(c) = (\{(s_1, 0.88, 0.06), (s_2, 0.895, 0.03), (s_3, 0.925, 0.04),$
$(s_4, 0.8775, 0.1625)\},\ 0.1625\,)$
$(\mathcal{F}_\alpha \tilde{\cup} \mathcal{G}_\beta)(g) = (\{(s_1, 0.95, 0.04), (s_2, 0.8, 0.09), (s_3, 0.85, 0.08),$
$(s_4, 0.91, 0.02)\},\ 0.86\,)$.
Since $\{r, c, g\} \in A \cap B$,
$(\mathcal{F}_\alpha \tilde{\cap} \mathcal{G}_\beta)(r) = (\{(s_1, 0.68, 0.145), (s_2, 0.81, 0.07375), (s_3, 0.765, 0.19),$
$(s_4, 0.6, 0.28)\},\ 0.12\,)$
$(\mathcal{F}_\alpha \tilde{\cap} \mathcal{G}_\beta)(c) = (\{(s_1, 0.42, 0.44), (s_2, 0.455, 0.32), (s_3, 0.525, 0.36),$
$(s_4, 0.4225, 0.7375)\},\ 0.375\,)$
$(\mathcal{F}_\alpha \tilde{\cap} \mathcal{G}_\beta)(g) = (\{(s_1, 0.6, 0.36), (s_2, 0.3, 0.5), (s_3, 0.35, 0.52),$
$(s_4, 0.49, 0.28)\},\ 0.39\,)$.

Theorem 3.8. *Let $\mathcal{F}_\alpha$, $\mathcal{G}_\beta$ and $\mathcal{H}_\delta$ be any three generalised intuitionistic fuzzy soft sets over (U, E). Then the following holds:*
(i) $\mathcal{F}_\alpha \tilde{\cup} \mathcal{G}_\beta = \mathcal{G}_\beta \tilde{\cup} \mathcal{F}_\alpha$.
(ii) $\mathcal{F}_\alpha \tilde{\cap} \mathcal{G}_\beta = \mathcal{G}_\beta \tilde{\cap} \mathcal{F}_\alpha$.
(iii) $\mathcal{F}_\alpha \tilde{\cup} (\mathcal{G}_\beta \tilde{\cup} \mathcal{H}_\delta) = (\mathcal{F}_\alpha \tilde{\cup} \mathcal{G}_\beta) \tilde{\cup} \mathcal{H}_\delta$.
(iv) $\mathcal{F}_\alpha \tilde{\cap} (\mathcal{G}_\beta \tilde{\cap} \mathcal{H}_\delta) = (\mathcal{F}_\alpha \tilde{\cap} \mathcal{G}_\beta) \tilde{\cap} \mathcal{H}_\delta$.

Proof. Since the t-norm function and t-conorm functions are commutative and associative, therefore the theorem follows.

Remark 3.9. Let $\mathcal{F}_\alpha$, $\mathcal{G}_\beta$ and $\mathcal{H}_\delta$ be any three generalised intuitionistic fuzzy soft sets over (U, E). If we consider $a * b = \min\{a,\, b\}$ and $a \diamond b = \max\{a,\, b\}$ then the following holds:

(i) $\mathcal{F}_\alpha \tilde{\cap} (\mathcal{G}_\beta \tilde{\cup} \mathcal{H}_\delta) = (\mathcal{F}_\alpha \tilde{\cap} \mathcal{G}_\beta) \tilde{\cup} (\mathcal{F}_\alpha \tilde{\cap} \mathcal{H}_\delta)$
(ii) $\mathcal{F}_\alpha \tilde{\cup} (\mathcal{G}_\beta \tilde{\cap} \mathcal{H}_\delta) = (\mathcal{F}_\alpha \tilde{\cup} \mathcal{G}_\beta) \tilde{\cap} (\mathcal{F}_\alpha \tilde{\cup} \mathcal{H}_\delta)$.
But in general above relations do not hold.

4. Relations on generalised intuitionistic fuzzy soft sets

Definition 4.1. Let $\mathcal{F}_\alpha$ and $\mathcal{G}_\beta$ be two generalised intuitionistic fuzzy soft set over (U, E). Then generalised intuitionistic fuzzy soft relation (in short GIFSR) R from $\mathcal{F}_\alpha$ to $\mathcal{G}_\beta$ is a function $R : A \times B \to IF^U \times [0,1]$ satisfying

$$R(a,b) \tilde{\subseteq} \mathcal{F}_\alpha(a) \tilde{\cap} \mathcal{G}_\beta(b) \quad \forall (a,b) \in A \times B.$$

Definition 4.2. Let R be a GIFSR from $\mathcal{F}_\alpha$ to $\mathcal{G}_\beta$. Then R^{-1} is defined as follows:

$$R^{-1}(b,a) = R(a,b), \quad \forall\, (a,b) \in A \times B.$$

Note 4.1. If R is a GIFSR from $\mathcal{F}_\alpha$ to $\mathcal{G}_\beta$ then R^{-1} is a GIFSR from $\mathcal{G}_\beta$ to $\mathcal{F}_\alpha$.

Proposition 4.3. *If R_1 and R_2 are GIFSR from $\mathcal{F}_\alpha$ to $\mathcal{G}_\beta$,*
(i) $\left(R_1^{-1}\right)^{-1} = R_1$.
(ii) $R_1 \subseteq R_2 \;\Rightarrow\; R_1^{-1} \subseteq R_2^{-1}$.

Proof. Let $(a,b) \in A \times B$.
(i) $\left(R_1^{-1}\right)^{-1}(a,b) = R_1^{-1}(b,a) = R_1(a,b)$. Hence $\left(R_1^{-1}\right)^{-1} = R_1$.
(ii) $R_1(a,b) \subseteq R_2(a,b) \;\Rightarrow\; \left(R_1^{-1}\right)^{-1}(a,b) \subseteq \left(R_2^{-1}\right)^{-1}(a,b) \Rightarrow R_1^{-1}(b,a) \subseteq R_2^{-1}(b,a)$. Hence $R_1^{-1} \subseteq R_2^{-1}$.

Definition 4.4. The composition $\circ$ of two GIFSR R_1 and R_2 is defined by

$$(R_1 \circ R_2)(a,c) = \tilde{\bigcup_{b\in B}} \left(R_1(a,b) \tilde{\cap} R_2(b,c)\right), \quad \forall\, (a,c) \in\; A \times C$$

where R_1 is a GIFSR from $\mathcal{F}_\alpha$ to $\mathcal{G}_\beta$ and R_2 is a GIFSR from $\mathcal{G}_\beta$ to $\mathcal{H}_\delta$.

Theorem 4.5. *Let R_1 be a GIFSR from $\mathcal{F}_\alpha$ to $\mathcal{G}_\beta$ and R_2 be a relation $\mathcal{G}_\beta$ to $\mathcal{H}_\delta$. Then $R_1 \circ R_2$ is a GIFSR from $\mathcal{F}_\alpha$ to $\mathcal{H}_\delta$.*

Proof. By definition
$R_1(a,b) \subseteq \mathcal{F}_\alpha(a) \tilde{\cap} \mathcal{G}_\beta(b)$
$= \{\big(\{x,\, \mu_{\mathcal{F}(a)}(x) * \mu_{\mathcal{G}(b)}(x),\, \nu_{\mathcal{F}(a)}(x) \diamond \nu_{\mathcal{G}(b)}(x)\},\ \alpha(a) * \beta(b)\big) \,:\, x \in U\}$,
$\forall (a,b) \in A \times B$.
$R_2(b,c) \subseteq \mathcal{G}_\beta(b) \tilde{\cap} \mathcal{H}_\delta(c)$
$= \{\big(\{x,\, \mu_{\mathcal{G}(b)}(x) * \mu_{\mathcal{H}(c)}(x),\, \nu_{\mathcal{G}(b)}(x) \diamond \nu_{\mathcal{H}(c)}(x)\},\ \beta(b) * \delta(c)\big) \,:\, x \in U\}$,
$\forall (b,c) \in B \times C$. Therefore,
$(R_1 \circ R_2)(a,c) = \tilde{\bigcup}_{b\in B} \left(R_1(a,b) \tilde{\cap} R_2(b,c)\right)$
$= \tilde{\bigcup}_{b\in B} \{(\{x,\ (\mu_{\mathcal{F}(a)}(x) * \mu_{\mathcal{G}(b)}(x)) \;*\; (\mu_{\mathcal{G}(b)}(x) * \mu_{\mathcal{H}(c)}(x)),\ (\nu_{\mathcal{F}(a)}(x) \diamond \nu_{\mathcal{G}(b)}(x)) \;\diamond$ $(\nu_{\mathcal{G}(b)}(x) \diamond \nu_{\mathcal{H}(c)}(x))\},\ (\alpha(a) * \beta(b)) \,*\, (\beta(b) * \delta(c))) \,:\, x \in U\}$, $\forall (a,c) \in A \times C$. Now
$(\mu_{\mathcal{F}(a)}(x) * \mu_{\mathcal{G}(b)}(x)) \;*\; (\mu_{\mathcal{G}(b)}(x) * \mu_{\mathcal{H}(c)}(x))$
$= \mu_{\mathcal{F}(a)}(x) * \mu_{\mathcal{G}(b)}(x) * \mu_{\mathcal{H}(c)}(x)$
$\leq \mu_{\mathcal{F}(a)}(x) * 1 * \mu_{\mathcal{H}(c)}(x)$
$= \mu_{\mathcal{F}(a)}(x) * \mu_{\mathcal{H}(c)}(x)$
and $(\nu_{\mathcal{F}(a)}(x) \diamond \nu_{\mathcal{G}(b)}(x)) \;\diamond\; (\nu_{\mathcal{G}(b)}(x) \diamond \nu_{\mathcal{H}(c)}(x))$

$= \nu_{\mathcal{F}(a)}(x) \diamond \nu_{\mathcal{G}(b)}(x) \diamond \nu_{\mathcal{H}(c)}(x)$
$\geq \nu_{\mathcal{F}(a)}(x) \diamond 0 \diamond \nu_{\mathcal{H}(c)}(x)$
$= \nu_{\mathcal{F}(a)}(x) \diamond \nu_{\mathcal{H}(c)}(x)$.
Also, $(\alpha(a) * \beta(b)) * (\beta(b) * \delta(c)) = \alpha(a) * \beta(b) * \delta(c) \leq \alpha(a) * 1 * \delta(c) = \alpha(a) * \delta(c)$.
Hence $\tilde{\bigcup}_{b \in B} (R_1(a,b) \tilde{\cap} R_2(b,c)) \subseteq \mathcal{F}_\alpha \tilde{\cap} \mathcal{H}_\delta$.
Thus $R_1 \circ R_2$ is a GIFSR from $\mathcal{F}_\alpha$ to $\mathcal{H}_\delta$.

Proposition 4.6. $(R_1 \circ R_2)^{-1} = R_2^{-1} \circ R_1^{-1}$ *where R_1 is a IFSR from $\mathcal{F}_\alpha$ to $\mathcal{G}_\beta$ and R_2 are GIFSR from $\mathcal{G}_\beta$ to $\mathcal{H}_\delta$.*

Proof. Let $a \in A,\ b \in B,\ c \in C$.
$(R_1 \circ R_2)^{-1}(c,a) = (R_1 \circ R_2)(a,c) = \tilde{\bigcup}_{b \in B} (R_1(a,b) \tilde{\cap} R_2(b,c))$
$= \tilde{\bigcup}_{b \in B} (R_2(b,c) \tilde{\cap} R_1(a,b)) = \tilde{\bigcup}_{b \in B} \left(R_2^{-1}(c,b) \tilde{\cap} R_1^{-1}(b,a)\right) = (R_2^{-1} \circ R_1^{-1})(c,a)$.
Hence $(R_1 \circ R_2)^{-1} = R_2^{-1} \circ R_1^{-1}$.

5. An application of generalised intuitionistic fuzzy soft set in decision making

There are several application of generalised intuitionistic fuzzy soft set theory to deal with uncertainties from our different kinds of daily life problems. Here we present such an application for solving a socialistic decision making problem. Suppose there are six men in the universe U and $U = \{b_1, b_2, b_3, b_4, b_5, b_6\}$ and the parameter set $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9\}$, where each $e_i,\ 1 \leq i \leq 9$ indicates an attributes of $b_j,\ 1 \leq j \leq 6$.
e_1 stands for "education qualification". e_2 stands for "hard working". e_3 stands for "responsible". e_4 stands for "government employee". e_5 stands for "non-government employee". e_6 stands for "businessman". e_7 stands for "family status". e_8 stands for "spiritual and ideal". e_9 stands for "handsome".
Suppose an woman Miss. Y wishes to marry a man on the basis of some criteria listed above. Our aim is to find out the most appropriate partner for Miss. Y.
Suppose the wishing parameters for Miss. Y is $A \subseteq E$ where $A = \{e_3, e_4, e_7, e_9\}$.
Let the preference of the criterions for Miss. Y be described by the fuzzy subset $\alpha : A \to [0,1]$ of A, as follows:
$\alpha(e_3) = 0.4,\ \ \alpha(e_4) = 0.6,\ \ \alpha(e_7) = 0.6,\ \ \alpha(e_9) = 0.8$.
Consider the generalised intuitionistic fuzzy soft sets $\mathcal{F}_\alpha$ as a collection of intuitionistic fuzzy approximation as below:
$\mathcal{F}_\alpha(e_3) = (\{(b_1, 0.3, 0.3), (b_2, 0.5, 0.3), (b_3, 0.3, 0.2), (b_4, 0.6, 0.3), (b_5, 0.4, 0.3), (b_6, 0.3, 0.4)\},\ 0.4\,)$
$\mathcal{F}_\alpha(e_4) = (\{(b_1, 0, 0.8), (b_2, 1, 0), (b_3, 0.8, 0.02), (b_4, 0, 0.24), (b_5, 0, 0.2), (b_6, 0, 0.06)\},\ 0.6\,)$
$\mathcal{F}_\alpha(e_7) = (\{(b_1, 0.6, 0.3), (b_2, 0.5, 0.4), (b_3, 0.6, 0.3), (b_4, 0.7, 0.2), (b_5, 0.7, 0.28), (b_6, 0.8, 0.02)\},\ 0.5\,)$
$\mathcal{F}_\alpha(e_9) = (\{(b_1, 0.5, 0.3), (b_2, 0.4, 0.3), (b_3, 0.6, 0.4), (b_4, 0.5, 0.3), (b_5, 0.6, 0.3), (b_6, 0.7, 0.2)\},\ 0.8\,)$

Algorithm:
(i) Input a weighted intuitionistic fuzzy soft set in tabular form.

(ii) Choose mid-level soft set (or, input a threshold intuitionistic fuzzy set or, give a threshold value pair or, choose top-bottom-level decision rule or, choose top-top-level decision rule or, choose bottom-bottom-level decision rule) for decision making.
(iii) Compute the level soft set with respect to the threshold intuitionistic fuzzy set (or, the threshold-level soft set or, the mid-level soft set or, the top-bottom-level soft set or, the top-top-level soft set or, the bottom-bottom-level soft set) in tabular form.
(iv) Compute the weighted choice value α'_i of $b_i \ \forall\, i$.
(v) If the maximum score occurs in k-th row then Miss. Y will marry to b_k.
(vi) If k has more than one value then one of b_k may be chosen.

	e_3, $\alpha(e_3) = 0.4$	e_4, $\alpha(e_4) = 0.6$	e_7, $\alpha(e_7) = 0.5$	e_9, $\alpha(e_9) = 0.8$
b_1	(0.3,0.3)	(0,0.8)	(0.6,0.3)	(0.5,0.3)
b_2	(0.5,0.3)	(1,0)	(0.5,0.4)	(0.4,0.3)
b_3	(0.3,0.2)	(0.8,0.02)	(0.6,0.3)	(0.6,0.4)
b_4	(0.6,0.3)	(0,0.24)	(0.7,0.2)	(0.5,0.3)
b_5	(0.4,0.3)	(0,0.2)	(0.7,0.28)	(0.6,0.3)
b_6	(0.3,0.4)	(0,0.06)	(0.8,0.02)	(0.7,0.2)

TABLE 3. Tabular representation of the generalised intuitionistic fuzzy soft set $\mathcal{F}_\alpha$.

Here we deal with the problem by mid-level decision rule. It is clear that the mid-threshold of (F_α, A) is $\{(e_3, 0.4, 0.3), (e_4, 0.3, 0.22), (e_7, 0.65, 0.25), (e_9, 0.55, 0.3)\}$.

	e_3, $\alpha(e_3) = 0.4$	e_4, $\alpha(e_4) = 0.6$	e_7, $\alpha(e_7) = 0.5$	e_9, $\alpha(e_9) = 0.8$	α_i
b_1	0	0	0	0	0.0
b_2	1	1	0	0	1.0
b_3	0	1	0	0	0.6
b_4	1	0	1	0	0.9
b_5	1	0	0	1	1.2
b_6	0	0	1	1	1.3

TABLE 4. Tabular representation of mid-level set with degree of preferences.

Clearly the maximum score is scored by the man b_6.
Decision: Miss. Y will marry to b_6. In case, if she does not want to marry b_6 due to certain reasons, her second choice will be b_5.

CONCLUSION

In this paper, we have introduced the weighted intuitionistic fuzzy soft sets and intuitionistic fuzzy soft relations with respect to preference. An application of this theory to solve a socialistic problem in a different approach has been investigated. It is expected that the approach will be useful to handle several realistic uncertain problems and give more perfect results.

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