

## Applications of intuitionistic fuzzy $\alpha\psi$ -closed sets

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**ABSTRACT.** In this paper, we introduce the notion of intuitionistic fuzzy  $\alpha\psi$ - $T_{1/2}$  spaces in intuitionistic fuzzy topological spaces and investigate some of the properties of intuitionistic fuzzy  $\alpha\psi$ -closed sets. Further, we introduce and study the concept of intuitionistic fuzzy  $\alpha\psi$ -irresolute mapping.

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### 1. INTRODUCTION

**F**uzzy set (FS), proposed by Zadeh [11] in 1965, as a framework to encounter uncertainty, vagueness and partial truth, represents a degree of membership for each member of the universe of discourse to a subset of it. By adding the degree of non-membership to FS, Atanassov proposed intuitionistic fuzzy set (IFS) in 1983 [1] which looks more accurately to uncertainty quantification and provides the opportunity to precisely model the problem based on the existing knowledge and observations. Later on fuzzy topology was introduced by Chang [3] in 1986. After this, there have been several generalizations of notions of fuzzy sets and fuzzy topology. In last few years various concepts in fuzzy were extended to intuitionistic fuzzy sets. In 1997, Coker [4] introduced the concept of intuitionistic fuzzy topological space. Recently R. Devi et al [6] introduced the notion of intuitionistic fuzzy  $\alpha\psi$ -closed set in topological space. We introduce intuitionistic fuzzy  $\alpha\psi$ - $T_{1/2}$  spaces to study the applications of intuitionistic fuzzy  $\alpha\psi$ -closed set and obtained some characterizations and several preservation theorems of such spaces. Moreover, we introduce intuitionistic fuzzy  $\alpha\psi$ -irresolute mapping and study some of its basic properties.

## 2. PRELIMINARIES

Throughout this paper, by  $(X, \tau)$  or simply by  $X$  we will denote the Coker's intuitionistic fuzzy topological space (briefly, IFTS). For a subset  $A$  of a space  $(X, \tau)$ ,  $cl(A)$ ,  $int(A)$  and  $\bar{A}$  denote the closure of  $A$ , the interior of  $A$  and the compliment of  $A$  respectively. Each intuitionistic fuzzy set (briefly, IFS) which belongs to  $(X, \tau)$  is called an intuitionistic fuzzy open set (briefly, IFOS) in  $X$ . The complement  $\bar{A}$  of an IFOS  $A$  in  $X$  is called an intuitionistic fuzzy closed set (IFCS) in  $X$ .

We introduce some basic notions and results that are used in the sequel.

**Definition 2.1** ([1]). Let  $X$  be a nonempty fixed set and  $I$  be the closed interval  $[0, 1]$ . An intuitionistic fuzzy set (IFS)  $A$  is an object of the following form

$$A = \{\langle x, \mu_A(x), \nu_A(x) \rangle : x \in X\}$$

where the mappings  $\mu_A : X \rightarrow I$  and  $\nu_A : X \rightarrow I$  denote the degree of membership (namely  $\mu_A(x)$ ) and the degree of nonmembership (namely  $\nu_A(x)$ ) for each element  $x \in X$  to the set  $A$ , respectively, and  $0 \leq \mu_A(x) + \nu_A(x) \leq 1$  for each  $x \in X$ . Obviously, every fuzzy set  $A$  on a nonempty set  $X$  is an IFS of the following form

$$A = \{\langle x, \mu_A(x), 1 - \mu_A(x) \rangle : x \in X\}.$$

**Definition 2.2** ([1]). Let  $A$  and  $B$  are IFSs of the form  $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle : x \in X\}$  and  $B = \{\langle x, \mu_B(x), \nu_B(x) \rangle : x \in X\}$ . Then

- (i)  $A \subseteq B$  if and only if  $\mu_A(x) \leq \mu_B(x)$  and  $\nu_A(x) \geq \nu_B(x)$ ;
- (ii)  $\bar{A} = \{\langle x, \nu_A(x), \mu_A(x) \rangle : x \in X\}$ ;
- (iii)  $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle : x \in X\}$ ;
- (iv)  $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle : x \in X\}$ .

We will use the notation  $A = \langle x, \mu_A, \nu_A \rangle$  instead of  $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle : x \in X\}$ . A constant fuzzy set  $\alpha$  taking value  $\alpha \in [0, 1]$  will be denoted by  $\underline{\alpha}$ . The IFSs  $0_{\sim}$  and  $1_{\sim}$  are defined by  $0_{\sim} = \{\langle x, \underline{0}, \underline{1} \rangle : x \in X\}$  and  $1_{\sim} = \{\langle x, \underline{1}, \underline{0} \rangle : x \in X\}$ .

Let  $\alpha, \beta \in [0, 1]$  such that  $\alpha + \beta \leq 1$ . An intuitionistic fuzzy point (IFP)  $p_{(\alpha, \beta)}$  is intuitionistic fuzzy set defined by

$$p_{(\alpha, \beta)}(x) = \begin{cases} (\alpha, \beta)(x) & \text{if } x = p \\ (0, 1) & \text{otherwise.} \end{cases}$$

Let  $f$  be a mapping from an ordinary set  $X$  into an ordinary set  $Y$ , If  $B = \{\langle y, \mu_B(y), \nu_B(y) \rangle : y \in Y\}$  is an IFS in  $Y$ , then the inverse image of  $B$  under  $f$  is an IFS defined by

$$f^{-1}(B) = \{\langle x, f^{-1}(\mu_B)(x), f^{-1}(\nu_B)(x) \rangle : x \in X\}$$

The image of IFS  $A = \{\langle y, \mu_A(y), \nu_A(y) \rangle : y \in Y\}$  under  $f$  is an IFS defined by  $f(A) = \{\langle y, f(\mu_A)(y), f(\nu_A)(y) \rangle : y \in Y\}$  where

$$f(\mu_A)(y) = \begin{cases} \sup_{x \in f^{-1}(y)} \mu_A(x), & \text{if } f^{-1}(y) \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

$$f(\nu_A)(y) = \begin{cases} \inf_{x \in f^{-1}(y)} \nu_A(x), & \text{if } f^{-1}(y) \neq \emptyset \\ 1, & \text{otherwise} \end{cases}$$

for each  $y \in Y$ .

**Definition 2.3** ([4]). An intuitionistic fuzzy topology (IFT) in Coker's sense on a nonempty set  $X$  is a family  $\tau$  of IFSs in  $X$  satisfying the following axioms:

- (T1)  $0_\sim, 1_\sim \in \tau$ ;
- (T2)  $G_1 \cap G_2 \in \tau$  for any  $G_1, G_2 \in \tau$ ;
- (T3)  $\cup G_i \in \tau$  for any arbitrary family  $\{G_i : i \in J\} \subseteq \tau$ .

**Definition 2.4** ([4]). Let  $A$  be an IFS in IFTS  $X$ . Then

$\text{int}(A) = \cup\{G : G \text{ is an IFOS in } X \text{ and } G \subseteq A\}$  is called an intuitionistic fuzzy interior of  $A$ ;

$\text{cl}(A) = \cap\{G : G \text{ is an IFCS in } X \text{ and } G \supseteq A\}$  is called an intuitionistic fuzzy closure of  $A$ .

**Definition 2.5** ([8]). Let  $p_{(\alpha, \beta)}$  be an IFP in IFTS  $X$ . An IFS  $A$  in  $X$  is called an intuitionistic fuzzy neighborhood (IFN) of  $p_{(\alpha, \beta)}$  if there exists an IFOS  $B$  in  $X$  such that  $p_{(\alpha, \beta)} \in B \subseteq A$ .

**Definition 2.6.** A subset  $A$  of an intuitionistic fuzzy space  $(X, \tau)$  is called

- (i) an intuitionistic fuzzy pre-open set [8] if  $A \subseteq \text{int}(\text{cl}(A))$  and an intuitionistic fuzzy pre-closed set if  $\text{cl}(\text{int}(A)) \subseteq A$ ;
- (ii) an intuitionistic fuzzy semi-open set [7] if  $A \subseteq \text{cl}(\text{int}(A))$  and an intuitionistic fuzzy semi-closed set if  $\text{int}(\text{cl}(A)) \subseteq A$ ;
- (iii) an intuitionistic fuzzy  $\alpha$ -open set [2] if  $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$  and an intuitionistic fuzzy  $\alpha$ -closed set if  $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$ .
- (iv) an intuitionistic fuzzy regular-open set [8] if  $\text{int}(\text{cl}(A)) = A$  and an intuitionistic fuzzy regular-closed set if  $A = \text{cl}(\text{int}(A))$ ;

The pre-closure (resp. semi-closure,  $\alpha$ -closure) of a subset  $A$  of an intuitionistic fuzzy space  $(X, \tau)$  is the intersection of all pre-closed (resp. semi-closed,  $\alpha$ -closed) sets that contain  $A$  and is denoted by  $IFpcl(A)$  (resp.  $IFscl(A)$ ,  $IF\alpha cl(A)$ ).

**Definition 2.7.** A subset  $A$  of an intuitionistic fuzzy topological space  $(X, \tau)$  is called

- (i) an intuitionistic fuzzy generalized closed set [10] (briefly, IFGCS) if  $\text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is an IFOS;
- (ii) an intuitionistic fuzzy regular generalized closed set [10] (briefly, IFRGCS) if  $\text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is an IFROS;
- (iii) an intuitionistic fuzzy semi-generalized closed set [9] (briefly, IFSGCS) if  $\text{scl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is semi-open in  $(X, \tau)$ ;
- (iv) an intuitionistic fuzzy  $\psi$ -closed set [6] (briefly, IF $\psi$ CS) if  $\text{scl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $IFsg$ -open in  $(X, \tau)$ .
- (v) an intuitionistic fuzzy  $\alpha\psi$ -closed set [6] (briefly,  $IF\alpha\psi$ CS) if  $\psi\text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $IF\alpha$ -open in  $(X, \tau)$ .

**Definition 2.8** ([5]). An IFTS  $(X, \tau)$  is said to be intuitionistic fuzzy  $T_{1/2}$  space if every intuitionistic fuzzy  $g$ -closed set in  $X$  is intuitionistic fuzzy closed in  $X$ .

**Definition 2.9** ([10]). An IFTS  $(X, \tau)$  is said to be intuitionistic fuzzy regular- $T_{1/2}$  space if every intuitionistic fuzzy  $rg$ -closed set in  $X$  is intuitionistic fuzzy regular closed in  $X$ .

**Definition 2.10.** A mapping  $f : X \rightarrow Y$  from an IFTS  $X$  into an IFTS  $Y$  is called an

- (i) an intuitionistic fuzzy  $\alpha\psi$ -continuous mapping [6] if  $f^{-1}(A)$  is an  $\text{IF}\alpha\psi\text{CS}$  in  $(X, \tau)$  for every  $\text{IFCS}$   $A$  in  $(Y, \sigma)$ .
- (ii) an intuitionistic fuzzy  $\psi$ -irresolute mapping if  $f^{-1}(A)$  is an  $\text{IF}\psi\text{CS}$  in  $(X, \tau)$  for every  $\text{IF}\psi\text{CS}$   $A$  in  $(Y, \sigma)$ .
- (iii) an intuitionistic fuzzy  $\alpha\psi$ -closed (resp. open) mapping if  $f(A)$  is an  $\text{IF}\alpha\psi\text{CS}$  (resp.  $\text{IF}\alpha\psi\text{OS}$ ) in  $(Y, \sigma)$  for every  $\text{IF}\alpha\psi\text{CS}$  (resp.  $\text{IF}\alpha\psi\text{OS}$ )  $A$  in  $(X, \tau)$ .

### 3. APPLICATIONS OF INTUITIONISTIC FUZZY $\alpha\psi$ -CLOSED SETS

In this section, we introduce intuitionistic fuzzy  $\alpha\psi$ - $T_{1/2}$  space, which utilizes intuitionistic fuzzy  $\alpha\psi$ -closed sets and its characterizations are proved.

**Definition 3.1.** An IFTS  $(X, \tau)$  is called an intuitionistic fuzzy  $\alpha\psi$ - $T_{1/2}$  space, if every  $\text{IF}\alpha\psi\text{CS}$  is an  $\text{IF}\psi\text{CS}$ .

**Theorem 3.2.** An IFTS  $(X, \tau)$  is an intuitionistic fuzzy  $\alpha\psi$ - $T_{1/2}$  space if and only if  $\text{IF}\psi\text{OS}(X) = \text{IF}\alpha\psi\text{OS}(X)$ .

*Proof.* Let  $A$  be an  $\text{IF}\alpha\psi\text{OS}$  in  $X$ , then  $\overline{A}$  is an  $\text{IF}\alpha\psi\text{CS}$  in  $X$ . By hypothesis,  $\overline{A}$  is an  $\text{IF}\psi\text{CS}$  of  $X$  and therefore  $A$  is an  $\text{IF}\psi\text{OS}$  of  $X$ . Hence  $\text{IF}\psi\text{OS}(X) = \text{IF}\alpha\psi\text{OS}(X)$ .

Conversely, let  $A$  be an  $\text{IF}\alpha\psi\text{CS}$  in  $X$ , then  $\overline{A}$  is an  $\text{IF}\alpha\psi\text{OS}$  of  $X$ . By our assumption  $\overline{A}$  is an  $\text{IF}\psi\text{OS}$  in  $X$ , which in turn implies  $A$  is an  $\text{IF}\psi\text{CS}$  in  $X$ . Hence  $(X, \tau)$  is an intuitionistic fuzzy  $\alpha\psi$ - $T_{1/2}$  space.  $\square$

**Theorem 3.3.** For an IFTS  $(X, \tau)$  the following conditions are equivalent:

- (i)  $(X, \tau)$  is an intuitionistic fuzzy  $\alpha\psi$ - $T_{1/2}$  space.
- (ii) Every singleton set of  $X$  is either an  $\text{IF}\psi\text{CS}$  or  $\text{IF}\psi\text{OS}$ .

*Proof.* (i)  $\Rightarrow$  (ii). Assume that  $(X, \tau)$  is an intuitionistic fuzzy  $\alpha\psi$ - $T_{1/2}$  space. Suppose that  $\{x\}$  is not an  $\text{IF}\psi\text{CS}$  for some  $x \in X$ . Then  $X - \{x\}$  is not  $\text{IF}\psi\text{OS}$  and hence  $X$  is the only  $\text{IF}\psi\text{OS}$  containing  $X - \{x\}$ . Therefore,  $X - \{x\}$  is  $\text{IF}\alpha\psi\text{CS}$  in  $(X, \tau)$ . Since  $(X, \tau)$  is an intuitionistic fuzzy  $\alpha\psi$ - $T_{1/2}$  space, then  $X - \{x\}$  is  $\text{IF}\psi\text{CS}$  or equivalently  $\{x\}$  is an  $\text{IF}\psi\text{OS}$ .

(ii)  $\Rightarrow$  (i). Assume that every singleton set of  $X$  is either  $\text{IF}\psi\text{CS}$  or  $\text{IF}\psi\text{OS}$ . Let  $A$  be an  $\text{IF}\alpha\psi\text{CS}$  of  $(X, \tau)$ . Let  $x \in X$ . We show that  $x \in A$  in two cases.

Case (i): Assume that  $\{x\}$  is  $\text{IF}\psi\text{CS}$ . If  $x \in A$ , then  $x \in \psi cl(A) - A$ . Now  $\psi cl(A) - A$  contains a non-empty  $\text{IF}\psi\text{CS}$ . Since  $A$  is  $\text{IF}\alpha\psi\text{CS}$ , ([6], by Theorem 3.6), we arrived to a contradiction. Hence  $x \in A$ .

Case (ii): Assume that  $\{x\}$  is  $\text{IF}\psi\text{OS}$ . Since  $x \in \psi cl(A)$ , then  $\{x\} \cap A \neq \phi$ . So  $x \in A$ . Thus in any case  $x \in A$ . So  $\psi cl(A) \subseteq A$ . Therefore  $A = \psi cl(A)$  or equivalently  $A$  is an  $\text{IF}\psi\text{CS}$ . Thus every  $\text{IF}\alpha\psi\text{CS}$  is  $\text{IF}\psi\text{CS}$ . Hence  $(X, \tau)$  is an intuitionistic fuzzy  $\alpha\psi$ - $T_{1/2}$  space.  $\square$

**Theorem 3.4.** Every intuitionistic fuzzy  $T_{1/2}$  space is an intuitionistic fuzzy  $\alpha\psi$ - $T_{1/2}$  space.

*Proof.* It is obvious.  $\square$

The following example supports that an intuitionistic fuzzy  $\alpha\psi$ - $T_{1/2}$  space need not be an intuitionistic fuzzy  $T_{1/2}$  space.

**Example 3.5.** Let  $X = \{a, b\}$  and let  $A = \langle x, (\frac{a}{0.3}, \frac{b}{0.1}), (\frac{a}{0.2}, \frac{b}{0.4}) \rangle$ . Then  $\tau = \{0_\sim, A, 1_\sim\}$  be an IFT on  $X$ . Clearly  $\{X, \tau\}$  is an intuitionistic fuzzy  $\alpha\psi$ - $T_{1/2}$  space, but not an intuitionistic fuzzy  $T_{1/2}$  space.

**Theorem 3.6.** Every intuitionistic fuzzy regular  $T_{1/2}$  space is an intuitionistic fuzzy  $\alpha\psi$ - $T_{1/2}$  space.

*Proof.* In ([5].Remark.5.3), it has been proved that every intuitionistic fuzzy regular  $T_{1/2}$  space is an intuitionistic fuzzy  $T_{1/2}$  space. By Theorem 3.3., every intuitionistic fuzzy  $T_{1/2}$  space is an intuitionistic fuzzy  $\alpha\psi$ - $T_{1/2}$  space. Hence every intuitionistic fuzzy regular  $T_{1/2}$  space is an intuitionistic fuzzy  $\alpha\psi$ - $T_{1/2}$  space.  $\square$

The following example shows that the converse of the above theorem is not true.

**Example 3.7.** Let  $X = \{a, b\}$  and let  $A = \langle x, (\frac{a}{0.3}, \frac{b}{0.7}), (\frac{a}{0.6}, \frac{b}{0.3}) \rangle$ . Then  $\tau = \{0_\sim, A, 1_\sim\}$  be an IFT on  $X$ . Clearly  $\{X, \tau\}$  is an intuitionistic fuzzy  $\alpha\psi$ - $T_{1/2}$  space, but not an intuitionistic fuzzy regular  $T_{1/2}$  space.

#### 4. ON INTUITIONISTIC FUZZY $\alpha\psi$ -IRRESOLUTE MAPPING

**Definition 4.1.** A mapping  $f : X \rightarrow Y$  from an IFTS  $X$  into an IFTS  $Y$  is called an intuitionistic fuzzy  $\alpha\psi$ -irresolute mapping if  $f^{-1}(A)$  is an IF $\alpha\psi$ CS in  $(X, \tau)$  for every IF $\alpha\psi$ CS  $A$  in  $(Y, \sigma)$ .

If  $f : X \rightarrow Y$  is an intuitionistic fuzzy  $\alpha\psi$ -irresolute, then  $f$  is an IF $\alpha\psi$ -continuous mapping but not conversely.

Let  $f$  be an IF $\alpha\psi$ -irresolute mapping. Let  $A$  be any IFCS in  $Y$ . Since every IFCS is an IF $\alpha\psi$ CS,  $A$  is an IF $\alpha\psi$ CS in  $Y$ . By hypothesis,  $f^{-1}(A)$  is an IF $\alpha\psi$ CS in  $X$ . Hence  $f$  is an IF $\alpha\psi$ -continuous mapping.

**Theorem 4.2.** Let  $f : X \rightarrow Y$  be an intuitionistic fuzzy  $\alpha\psi$ -irresolute, then  $f$  is an intuitionistic fuzzy  $\psi$ -irresolute mapping if  $X$  is an intuitionistic fuzzy  $\alpha\psi$ - $T_{1/2}$  space.

*Proof.* Let  $A$  be an IF $\psi$ CS in  $Y$ . Then  $A$  is an IF $\alpha\psi$ CS in  $Y$ . Therefore,  $f^{-1}(A)$  is an IF $\alpha\psi$ CS in  $X$ , by hypothesis. Since  $X$  is an intuitionistic fuzzy  $\alpha\psi$ - $T_{1/2}$  space,  $f^{-1}(A)$  is an IF $\psi$ CS in  $X$ . Hence  $f$  is an intuitionistic fuzzy  $\psi$ -irresolute mapping.  $\square$

**Theorem 4.3.** Let  $f : X \rightarrow Y$  be a mapping from an IFTS  $X$  into an IFTS  $Y$ . Then the following statements are equivalent:

- (i)  $f$  is intuitionistic fuzzy  $\alpha\psi$ -irresolute mapping;
- (ii)  $f^{-1}(B)$  is an IF $\alpha\psi$ CS in  $X$  for each IF $\alpha\psi$ CS  $B$  in  $Y$ ;
- (iii)  $\alpha\psi cl(f^{-1}(B)) \subseteq f^{-1}(\alpha\psi cl(B))$  for each IFS  $B$  of  $Y$ ;
- (iv)  $f^{-1}(\alpha\psi int(B)) \subseteq \alpha\psi int(f^{-1}(B))$  for each IFS  $B$  of  $Y$ .

*Proof.* (i)  $\Rightarrow$  (ii) It can be proved by using the complement and the definition.

(ii)  $\Rightarrow$  (iii) Let  $B$  be any IFS in  $Y$ . From  $B \subseteq \alpha\psi cl(B)$  follows that  $f^{-1}(B) \subseteq f^{-1}(\alpha\psi cl(B))$ . Since  $\alpha\psi cl(B)$  is an IF $\alpha\psi$ CS in  $Y$ , according to the assumption,

we have that  $f^{-1}(\alpha\psi cl(B))$  is an IF $\alpha\psi$ CS in  $X$ . Therefore,  $\alpha\psi cl(f^{-1}(B)) \subseteq f^{-1}\alpha\psi cl(B)$ .

(iii)  $\Rightarrow$  (iv) It can be proved by using the complement.

(iv)  $\Rightarrow$  (i) Let  $B$  be any IF $\alpha\psi$ OS in  $Y$ . Then  $\alpha\psi int(B) = f^{-1}\alpha\psi int(B) \subseteq \alpha\psi int(f^{-1}(B))$ , so  $f^{-1}(B)$  is an IF $\alpha\psi$ OS in  $X$ . Hence  $f$  is an intuitionistic fuzzy  $\alpha\psi$ -irresolute mapping.  $\square$

**Theorem 4.4.** *A mapping  $f : X \rightarrow Y$  from an IFTS  $X$  into an IFTS  $Y$  is intuitionistic fuzzy  $\alpha\psi$ -irresolute mapping if and only if for each IFP  $p_{(\alpha,\beta)}$  in  $X$  and IF $\alpha\psi$ OS  $B$  in  $Y$  such that  $f(p_{(\alpha,\beta)}) \in B$  there exists an IF $\alpha\psi$ OS  $A$  in  $X$  such that  $p_{(\alpha,\beta)} \in A$  and  $f(A) \subseteq B$ .*

*Proof.* Let  $f$  be any intuitionistic fuzzy  $\alpha\psi$ -irresolute mapping,  $p_{(\alpha,\beta)}$  an IFP in  $X$  and  $B$  in  $Y$  such that  $f(p_{(\alpha,\beta)}) \in B$ . Then  $p_{(\alpha,\beta)} \in f^{-1}(B) = \alpha\psi int(f^{-1}(B))$ . We put  $A = \alpha\psi int(f^{-1}(B))$ . Then  $A$  is an IF $\alpha\psi$ OS in  $X$  which containing IFP  $p_{(\alpha,\beta)}$  and  $f(A) = f(\alpha\psi int(f^{-1}(B))) \subseteq f(f^{-1}(B)) \subseteq B$ .

Conversely, let  $B$  be any IF $\alpha\psi$ OS in  $Y$  and  $p_{(\alpha,\beta)}$  an IFP in  $X$  such that  $p_{(\alpha,\beta)} \in f^{-1}(B)$ . According to the assumption there exists an IF $\alpha\psi$ OS  $A$  in  $X$  such that  $p_{(\alpha,\beta)} \in A$  and  $f(A) \subseteq B$ . Therefore  $p_{(\alpha,\beta)} \in A \subseteq f^{-1}(B)$  and  $p_{(\alpha,\beta)} \in A = \alpha\psi int(A) \subseteq \alpha\psi int(f^{-1}(B))$ . Since  $p_{(\alpha,\beta)}$  is an arbitrary IFP and  $f^{-1}(B)$  is the union of all IFP containing  $f^{-1}(B)$ , we obtain that  $f^{-1}(B) = \alpha\psi int(f^{-1}(B))$ , so  $f$  is an intuitionistic fuzzy  $\alpha\psi$ -irresolute mapping.  $\square$

**Corollary 4.5.** *A mapping  $f : X \rightarrow Y$  from an IFTS  $X$  into an IFTS  $Y$  is intuitionistic fuzzy  $\alpha\psi$ -irresolute mapping if and only if for each IFP  $p_{(\alpha,\beta)}$  in  $X$  and IF $\alpha\psi$ OS  $B$  in  $Y$  such that  $f(p_{(\alpha,\beta)}) \in B$  there exists an IF $\alpha\psi$ OS  $A$  in  $X$  such that  $p_{(\alpha,\beta)} \in A$  and  $A \subseteq f^{-1}(B)$ .*

**Theorem 4.6.** *Let  $f : X \rightarrow Y$  and  $g : Y \rightarrow Z$  are mappings, where  $X, Y$  and  $Z$  are IFTS.*

- (i) *If  $f$  and  $g$  are intuitionistic fuzzy  $\alpha\psi$ -irresolute mapping, then  $g \circ f$  is an intuitionistic fuzzy  $\alpha\psi$ -irresolute mapping.*
- (ii) *If  $f$  is intuitionistic fuzzy  $\alpha\psi$ -irresolute mapping and  $g$  is an intuitionistic fuzzy  $\alpha\psi$ -continuous mapping, then  $g \circ f$  is an intuitionistic fuzzy  $\alpha\psi$ -continuous mapping.*
- (iii) *If  $f$  is intuitionistic fuzzy  $\alpha\psi$ -irresolute mapping and  $g$  is an intuitionistic fuzzy continuous mapping, then  $g \circ f$  is an intuitionistic fuzzy  $\alpha\psi$ -continuous mapping.*
- (iv) *If  $g \circ f$  intuitionistic fuzzy  $\alpha\psi$ -irresolute mapping and  $g$  is an injective intuitionistic fuzzy  $\alpha\psi$ -open mapping, then  $f$  is an intuitionistic fuzzy  $\alpha\psi$ -continuous mapping.*
- (v) *If  $g \circ f$  is an intuitionistic fuzzy  $\alpha\psi$ -open ( $\alpha\psi$ -closed) mapping and  $g$  is an injective intuitionistic fuzzy  $\alpha\psi$ -irresolute mapping, then  $f$  is an intuitionistic fuzzy  $\alpha\psi$ -open ( $\alpha\psi$ -closed) mapping.*

*Proof.* It follows from the relations  $(g \circ f)(c) = f^{-1}(g^{-1}(c))$ , for each IFS  $c$  in  $Z$ ,  $f^{-1}(B) = (gf)^{-1}g(B)$ , for each IFS  $B$  in  $Y$  and  $g$  is injective and  $f(A) = g^{-1}(gf)A$ , for each IFS  $A$  in  $X$  and  $g$  is injective.  $\square$

**Theorem 4.7.** *Let  $X, X_1$  and  $X_2$  are IFTSs and  $p_i : X_1 \times X_2 \rightarrow X_i (i = 1, 2)$  are the projections of  $X_1 \times X_2$  onto  $X_i$ . If  $f : X \rightarrow X_1 \times X_2$  is intuitionistic fuzzy  $\alpha\psi$ -irresolute, then  $p_i f$  are intuitionistic fuzzy  $\alpha\psi$ -continuous mapping.*

*Proof.* It follows from the fact that  $p_i (i = 1, 2)$  are intuitionistic fuzzy continuous mappings.  $\square$

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