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Intersectional soft BCK/BCI-ideals

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Abstract. The notions of intersectional soft BCK/BCI-algebras and intersectional soft BCK/BCI-ideals in BCK/BCI-algebras based on subalgebras are introduced, and their properties are investigated. Relations between an intersectional soft BCK/BCI-algebra and an intersectional soft BCK/BCI-ideal is provided. The concept of ω-support of a soft set is also introduced, and conditions for a ω-support to be a BCK/BCI-ideal are considered.

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1. Introduction

Various problems in system identification involve characteristics which are essentially non-probabilistic in nature [20]. In response to this situation Zadeh [21] introduced *fuzzy set theory* as an alternative to probability theory. Uncertainty is an attribute of information. In order to suggest a more general framework, the approach to uncertainty is outlined by Zadeh [22]. To solve complicated problem in economics, engineering, and environment, we can't successfully use classical methods because of various uncertainties typical for those problems. There are three theories: theory of probability, theory of fuzzy sets, and the interval mathematics which we can consider as mathematical tools for dealing with uncertainties. But all these theories have their own difficulties. Uncertainties can't be handled using traditional mathematical tools but may be dealt with using a wide range of existing theories such as probability theory, theory of (intuitionistic) fuzzy sets, theory of vague sets, theory of interval mathematics, and theory of rough sets. However, all of these theories have their own difficulties which are pointed out in [18]. Maji et al. [16] and Molodtsov [18] suggested that one reason for these difficulties may be due to the inadequacy of the parametrization tool of the theory. To overcome these difficulties, Molodtsov [18] introduced the concept of soft set as a new mathematical tool for dealing with uncertainties that is free from the difficulties that have troubled the usual theoretical approaches. Molodtsov pointed out several directions for the applications of soft sets. At present, works on the soft set theory are progressing rapidly. Maji et al. [16] described the application of soft set theory to a decision making problem. Maji et al. [15] also studied several operations on the theory of soft sets. Chen et al. [5] presented a new definition of soft set parametrization reduction, and compared this definition to the related concept of attributes reduction in rough set theory. The algebraic structure of set theories dealing with uncertainties has been studied by some authors. Çağman et al. [4] introduced fuzzy parameterized (FP) soft sets and their related properties. They proposed a decision making method based on FP-soft set theory, and provided an example which shows that the method can be successfully applied to the problems that contain uncertainties. Feng [6] considered the application of soft rough approximations in multicriteria group decision making problems. Aktaş and Çağman [2] studied the basic concepts of soft set theory, and compared soft sets to fuzzy and rough sets, providing examples to clarify their differences. They also discussed the notion of soft groups. After than, many algebraic properties of soft sets are studied (see [1, 3, 7, 9, 10, 12, 13, 11, 14, 19, 23]).

In this paper, we introduce the notion of intersectional soft BCK/BCI-algebras and intersectional soft BCK/BCI-ideals in BCK/BCI-algebras based on subalgebras, and investigate their properties. We discuss relations between an intersectional soft BCK/BCI-algebra and an intersectional soft BCK/BCI-ideal. We also introduce the notion of ω-support of a soft set, and provide conditions for a ω-support to be a BCK/BCI-ideal.

2. Preliminaries

A BCK/BCI-algebra is an important class of logical algebras introduced by K. Iséki and was extensively investigated by several researchers.

An algebra $(X;*,0)$ of type $(2,0)$ is called a *BCI-algebra* if it satisfies the following conditions:

(I) $(\forall x,y,z\in X)$ $(((x*y)*(x*z))*(z*y)=0)$,
(II) $(\forall x,y\in X)$ $((x*(x*y))*y=0)$,
(III) $(\forall x\in X)$ $(x*x=0)$,
(IV) $(\forall x,y\in X)$ $(x*y=0, y*x=0 \Rightarrow x=y)$.

If a BCI-algebra X satisfies the following identity:

(V) $(\forall x\in X)$ $(0*x=0)$,

then X is called a *BCK-algebra*. Any BCK/BCI-algebra X satisfies the following axioms:

(a1) $(\forall x\in X)$ $(x*0=x)$,
(a2) $(\forall x,y,z\in X)$ $(x\le y \Rightarrow x*z\le y*z,\ z*y\le z*x)$,
(a3) $(\forall x,y,z\in X)$ $((x*y)*z=(x*z)*y)$,
(a4) $(\forall x,y,z\in X)$ $((x*z)*(y*z)\le x*y)$

where $x\le y$ if and only if $x*y=0$. A nonempty subset S of a BCK/BCI-algebra X is called a *BCK/BCI-subalgebra* of X if $x*y\in S$ for all $x,y\in S$. A subset A of a BCK/BCI-algebra X is called a *BCK/BCI-ideal* of X if it satisfies:

(b1) $0 \in A$.
(b2) $(\forall x \in X)(\forall y \in A)(x * y \in A \;\Rightarrow\; x \in A)$.

We refer the reader to the books [8, 17] for further information regarding BCK/BCI-algebras.

3. Intersectional soft BCK/BCI-ideals

In what follows, let U and E denote an initial universe set and a set of parameters, respectively. Molodtsov [18] defined the soft set in the following way:

Definition 3.1 ([18])**.** A pair $(\mathscr{F}, E)$ is called a *soft set* over U if $\mathscr{F}$ is a mapping given by

$$\mathscr{F} : E \to \mathscr{P}(U).$$

Definition 3.2. Given a non-empty subset A of E, a soft set $(\mathscr{F}, E)$ over U satisfying the following condition:

$$\mathscr{F}(x) = \varnothing \text{ for all } x \notin A \tag{3.1}$$

is called an A-*soft set* over U and is denoted by $\mathscr{F}_A$, that is, an A-soft set $\mathscr{F}_A$ over U is a function $\mathscr{F}_A : E \to \mathscr{P}(U)$ such that $\mathscr{F}_A(x) = \varnothing$ for all $x \notin A$.

Note that an E-soft set over U is a soft set over U.

Definition 3.3. Let $E = X$ be a BCK/BCI-algebra. Given a subalgebra A of E, let $\mathscr{F}_A$ be an A-soft set over U. Then $\mathscr{F}_A$ is called an *intersectional* A*-soft* BCK/BCI*-algebra* over U if it satisfies the following condition:

$$(\forall x, y \in A)\left(\mathscr{F}_A(x) \cap \mathscr{F}_A(y) \subseteq \mathscr{F}_A(x * y)\right). \tag{3.2}$$

An intersectional A-soft BCK/BCI-algebra over U with $A = E$ is called an *intersectional soft* BCK/BCI*-algebra* over U.

Definition 3.4. Let $E = X$ be a BCK/BCI-algebra. Given a subalgebra A of E, an A-soft set $\mathscr{F}_A$ over U is called an *intersectional* A*-soft* BCK/BCI*-ideal* over U if it satisfies:

(c1) $(\forall x \in A)\left(\mathscr{F}_A(x) \subseteq \mathscr{F}_A(0)\right)$,
(c2) $(\forall x, y \in A)\left(\mathscr{F}_A(x * y) \cap \mathscr{F}_A(y) \subseteq \mathscr{F}_A(x)\right)$.

An intersectional A-soft BCK/BCI-ideal over U with $A = E$ is called an *intersectional soft* BCK/BCI*-ideal* over U.

Example 3.5. Let $U = \mathbb{Z}$ be the initial universe set and let $E = \{0, 1, a, b, c\}$ be a BCI-algebra with the following Cayley table:

$*$	0	1	a	b	c
0	0	0	c	b	a
1	1	0	c	b	a
a	a	a	0	c	b
b	b	b	a	0	c
c	c	c	b	a	0

For a subalgebra $A = \{0, a, b, c\}$ of E, let $\mathscr{F}_A$ be an A-soft set over $\mathbb{Z}$ defined by

$$\mathscr{F}_A(x) = \begin{cases} \mathbb{Z} & \text{if } o(x) = 1, \\ \{k \in \mathbb{Z} \mid -r \leq k \leq r\} & \text{if } o(x) = r \neq 1 \end{cases}$$

where $o(x) = \min\{n \in \mathbb{N} \mid 0 * x^n = 0\}$. Then $\mathscr{F}_A(0) = \mathscr{F}_A(b) = \mathbb{Z}$ and $\mathscr{F}_A(a) = \mathscr{F}_A(c) = \{-2, -1, 0, 1, 2\}$, which satisfy two conditions (c1) and (c2). Therefore $\mathscr{F}_A$ is an intersectional A-soft BCI-ideal over $\mathbb{Z}$.

If we define an A-soft set $\mathscr{G}_A$ over $\mathbb{Z}$ by $\mathscr{G}_A(0) = \mathbb{Z}$, $\mathscr{G}_A(1) = \varnothing$, $\mathscr{G}_A(a) = 2\mathbb{Z}$, $\mathscr{G}_A(b) = 3\mathbb{Z}$ and $\mathscr{G}_A(c) = 4\mathbb{Z}$, then $\mathscr{G}_A$ is not an intersectional A-soft BCI-ideal over $\mathbb{Z}$ since $\mathscr{G}_A(b * c) \cap \mathscr{G}_A(c) = \mathscr{G}_A(c) \cap \mathscr{G}_A(c) = 4\mathbb{Z} \nsubseteq 3\mathbb{Z} = \mathscr{G}_A(b)$.

Example 3.6. Let $U = \mathbb{N}$ be the initial universe set. Take $E = \mathbb{N}$ and define a binary operation $*$ on E as follows:

$$(\forall a, b \in E)\left(a * b = \tfrac{a}{(a,b)}\right)$$

where (a, b) is the greatest common divisor of a and b. Then $(E; *, 1)$ is a BCK-algebra (see [8]). For a subalgebra $A = \{1, 2, 3, 4, 5\}$, let $\mathscr{F}_A$ be an A-soft set over U defined by $\mathscr{F}_A(1) = \mathbb{N}$, $\mathscr{F}_A(2) = 2\mathbb{N}$, $\mathscr{F}_A(3) = 4\mathbb{N}$, $\mathscr{F}_A(4) = 8\mathbb{N}$ and $\mathscr{F}_A(5) = 12\mathbb{N}$. Then $\mathscr{F}_A$ is an intersectional A-soft BCK-ideal over U.

Proposition 3.7. *Let $E = X$ be a BCK/BCI-algebra. Given a subalgebra A of E, every intersectional A-soft BCK/BCI-ideal $\mathscr{F}_A$ over U satisfies the following condition:*

(1) $(\forall x, y \in A)\,(x \leq y \;\Rightarrow\; \mathscr{F}_A(y) \subseteq \mathscr{F}_A(x))$.
(2) $(\forall x, y, z \in A)\,(x * y \leq z \;\Rightarrow\; \mathscr{F}_A(y) \cap \mathscr{F}_A(z) \subseteq \mathscr{F}_A(x))$.

Proof. (1) Let $x, y \in A$ be such that $x \leq y$. Then $x * y = 0 \in A$, and so

$$\mathscr{F}_A(y) = \mathscr{F}_A(0) \cap \mathscr{F}_A(y) = \mathscr{F}_A(x * y) \cap \mathscr{F}_A(y) \subseteq \mathscr{F}_A(x)$$

by (c1) and (c2).

(2) Let $x, y, z \in A$ be such that $x * y \leq z$. Then

$$\mathscr{F}_A(z) = \mathscr{F}_A(0) \cap \mathscr{F}_A(z) = \mathscr{F}_A((x * y) * z) \cap \mathscr{F}_A(z) \subseteq \mathscr{F}_A(x * y).$$

It follows that $\mathscr{F}_A(y) \cap \mathscr{F}_A(z) \subseteq \mathscr{F}_A(x * y) \cap \mathscr{F}_A(y) \subseteq \mathscr{F}_A(x)$. □

Corollary 3.8. *Let $E = X$ be a BCK/BCI-algebra. Then every intersectional soft BCK/BCI-ideal $(\mathscr{F}, E)$ over U satisfies the following condition:*

(1) *$\mathscr{F}(y) \subseteq \mathscr{F}(x)$ for all $x, y \in E$ with $x \leq y$.*
(2) *$\mathscr{F}(y) \cap \mathscr{F}(z) \subseteq \mathscr{F}(x)$ for all $x, y, z \in E$ with $x * y \leq z$.*

The following corollary is easily proved by induction.

Corollary 3.9. *Let $E = X$ be a BCK/BCI-algebra. Given a subalgebra A of E, every intersectional A-soft BCK/BCI-ideal $\mathscr{F}_A$ over U satisfies the following condition:*

$$(\cdots(x * a_1) * \cdots) * a_n = 0 \;\Rightarrow\; \bigcap_{k=1}^{n} \mathscr{F}_A(a_k) \subseteq \mathscr{F}_A(x) \tag{3.3}$$

for all $x, a_1, a_2, \cdots, a_n \in E$.

Proposition 3.10. *Let $E = X$ be a BCK/BCI-algebra and let A be a subalgebra of E. For any intersectional A-soft BCK/BCI-ideal $\mathscr{F}_A$ over U, the following assertions are equivalent:*

(1) $(\forall x, y \in A)\left(\mathscr{F}_A((x*y)*y) \subseteq \mathscr{F}_A(x*y)\right).$
(2) $(\forall x, y, z \in A)\left(\mathscr{F}_A((x*y)*z) \subseteq \mathscr{F}_A((x*z)*(y*z))\right).$

Proof. Assume that (1) is valid and let $x, y, z \in A$. Since

$$((x*(y*z))*z)*z = ((x*z)*(y*z))*z \leq (x*y)*z,$$

it follows from Proposition Proposition 3.7(1), (1) and (a3) that

$$\begin{aligned}\mathscr{F}_A((x*y)*z) &\subseteq \mathscr{F}_A(((x*(y*z))*z)*z)\\ &\subseteq \mathscr{F}_A((x*(y*z))*z)\\ &= \mathscr{F}_A((x*z)*(y*z)).\end{aligned}$$

Conversely, suppose that (2) holds. If we use z instead of y in (2), then

$$\mathscr{F}_A((x*z)*z) \subseteq \mathscr{F}_A((x*z)*(z*z)) = \mathscr{F}_A((x*z)*0) = \mathscr{F}_A(x*z)$$

by (III) and (a1). This proves (1). □

Corollary 3.11. *Let $E = X$ be a BCK/BCI-algebra. For any intersectional soft BCK/BCI-ideal $(\mathscr{F}, E)$ over U, the following assertions are equivalent:*

(1) $\mathscr{F}((x*y)*y) \subseteq \mathscr{F}(x*y)$ *for all* $x, y \in E$.
(2) $\mathscr{F}((x*y)*z) \subseteq \mathscr{F}((x*z)*(y*z))$ *for all* $x, y, z \in E$.

Theorem 3.12. *Let $E = X$ be a BCK-algebra. Given a subalgebra A of E, every intersectional A-soft BCK-ideal over U is an intersectional A-soft BCK-algebra over U.*

Proof. Let $\mathscr{F}_A$ be an intersectional A-soft BCK-ideal over U. Then

$$\begin{aligned}\mathscr{F}_A(x*y) &\supseteq \mathscr{F}_A((x*y)*x) \cap \mathscr{F}_A(x) = \mathscr{F}_A((x*x)*y) \cap \mathscr{F}_A(x)\\ &= \mathscr{F}_A(0*y) \cap \mathscr{F}_A(x) = \mathscr{F}_A(0) \cap \mathscr{F}_A(x) \supseteq \mathscr{F}_A(x) \cap \mathscr{F}_A(y)\end{aligned}$$

by (c2), (a3), (III), (V) and (c1). Hence $\mathscr{F}_A$ is an intersectional A-soft BCK-algebra over U. □

Corollary 3.13. *Let $E = X$ be a BCK-algebra. Then every intersectional soft BCK-ideal over U is an intersectional soft BCK-algebra over U.*

The following example shows that the converse of Corollary 3.13 is not true.

Example 3.14. Let $U = \mathbb{N}$ be the initial universe set and let $E = \{0, a, b, c, d\}$ be a BCK-algebra with the following Cayley table:

$*$	0	a	b	c	d
0	0	0	0	0	0
a	a	0	0	0	0
b	b	b	0	0	0
c	c	c	c	0	0
d	d	c	c	a	0

Let $(\mathscr{F}, E)$ be a soft set in which $\mathscr{F}$ is defined by $\mathscr{F}(0) = \mathbb{N}$, $\mathscr{F}(a) = 4\mathbb{N}$, $\mathscr{F}(b) = 2\mathbb{N}$, $\mathscr{F}(c) = 3\mathbb{N}$ and $\mathscr{F}(d) = 8\mathbb{N}$. Then $(\mathscr{F}, E)$ is an intersectional soft BCK-algebra over U, but not an intersectional soft BCK-ideal over U since $\mathscr{F}(d*b) \cap \mathscr{F}(b) = 3\mathbb{N} \cap 2\mathbb{N} = 6\mathbb{N} \nsubseteq 8\mathbb{N} = \mathscr{F}(d)$.

Definition 3.15. For a soft set $(\mathscr{F}, E)$ over U and $\omega \in E$, consider a set

$$\omega\mathscr{F}(E) := \{x \in E \mid \mathscr{F}(\omega) \subseteq \mathscr{F}(x)\}.$$

We say that $\omega\mathscr{F}(E)$ is the ω-*support* of $(\mathscr{F}, E)$.

Theorem 3.16. *Let $E = X$ be a BCK/BCI-algebra. If a soft set $(\mathscr{F}, E)$ over U is an intersectional soft BCK/BCI-ideal over U, then the ω-support of $(\mathscr{F}, E)$ is a BCK/BCI-ideal of E for all $\omega \in E$.*

Proof. Obviously, $0 \in \omega\mathscr{F}(E)$. Let $x, y \in E$ be such that $x * y \in \omega\mathscr{F}(E)$ and $y \in \omega\mathscr{F}(E)$. Then $\mathscr{F}(\omega) \subseteq \mathscr{F}(x * y)$ and $\mathscr{F}(\omega) \subseteq \mathscr{F}(y)$, which imply that

$$\mathscr{F}(\omega) \subseteq \mathscr{F}(x * y) \cap \mathscr{F}(y) \subseteq \mathscr{F}(x).$$

Hence $x \in \omega\mathscr{F}(E)$, which proves that $\omega\mathscr{F}(E)$ is a BCK/BCI-ideal of E. □

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