Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 4, No. 1, (July 2012), pp. 99- 107
ISSN 2093–9310
http://www.afmi.or.kr

Application of α-cuts for interval data in DEA with fuzzy coefficients

Sohrab Kordrostami, Alireza Amirteimoori,
Shahla Kazemi, Ali Ebrahimnejad

Received 19 August 2011; Accepted 27 October 2011

Abstract. In this paper, a new method for solving DEA model with interval data involving fuzzy parameters in constraints is proposed. To do this, we use the concept of α-cuts to evaluate efficiency scores. Finally, we compare our approach with Despotis et al.'s method.

2010 AMS Classification: 90Bxx, 90Cxx.

Keywords: Data Envelopment Analysis, Interval Data, Fuzzy Coefficients, Linear Semi-Infinite Programming.

Corresponding Author: Sohrab Kordrostami (krostami@guilan.ac.ir)

1. Introduction

Data envelopment analysis (DEA) is a non-parametric method for evaluating the relative efficiency of decision-making units (DMUs) on the basis of multiple inputs and outputs. The original DEA models assume that inputs and outputs are measured by exact values on a ratio scale. Recently, Cooper et al. [1] studied how to deal with imprecise data such as bounded data, ordinal data and ratio bounded data in DEA. The resulting DEA model was called Imprecise DEA (IDEA). They proposed some methods to convert the non-linear model to a linear one. Despotis and Smirlis [2] converted a non-linear DEA model to an LP equivalent by transforming only on the variables. In addition, Fang et al. [3], studied a linear programming problem with fuzzy coefficients in coefficient matrix and right hand side vector. They have shown that such problems can be reduced to linear semi-infinite programming problem. A cutting plane algorithm has been proposed for solving a linear programming problem with fuzzy coefficients in term of linear semi -infinite programming. In this paper, we focused on DEA with fuzzy coefficients in constraints with infinite α-Cuts [6].

The rest of the paper is organized as follows. In Section 2, a DEA model for dealing with interval data has been formulated. Section 3 presents a linear semi-infinite programming problem. In Section 4, we present a DEA model with interval data with fuzzy coefficients in constraints using infinite α-Cuts. Conclusions are given in Section 5.

2. A DEA model with interval data

Assume that there are n DMUs to be evaluated, that each produces s outputs by using m inputs. DMU_j consumes amounts $X_j = \{x_{ij}\}$of inputs $(i = 1, 2, ..., m)$ and produces amounts $Y_j = \{y_{rj}\}$of outputs$(r = 1, 2, ..., s)$. Without loss of generality, we assume that all the input and output data are known to lie within bounded intervals, i.e. $x_{ij} \in [x^L_{ij}, x^U_{ij}]$ and $y_{rj} \in [y^L_{rj}, y^U_{rj}]$ and assume strictly positive. The following CCR DEA model evaluates DMU_j:

$$\begin{array}{c} \max \quad h_{j_0} = \sum_{r=1}^{s} u_r y_{rj_0} \\ \text{s.t.} \quad \sum_{i=1}^{m} v_i x_{ij_0} = 1, \\ \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \ j = 1, ..., n, \\ u_r, v_i \geq \varepsilon, \text{ for all} r, i. \end{array} \tag{2.1}$$

In this model $u_1, ..., u_s$ and $v_1, ..., v_m$ are weights for outputs and inputs, respectively.

To transform the above model (2.1) into an equivalent linear programming; the following transformations have been applied to the variables x_{ij} and y_{rj}:

$$x_{ij} = x_l + s_{ij}(x^U_{ij} - x^L_{ij}), \ i = 1, ..., m; j = 1, ..., n, \qquad 0 \leq s_{ij} \leq 1,$$

$$y_{rj} = y_l + t_{rj}(y^U_{rj} - y^L_{rj}), \ r = 1, ..., s; j = 1, ..., n, \qquad 0 \leq t_{rj} \leq 1,$$

Using these transformations, model (2.1) is transformed into the following linear programming:

$$\begin{array}{c} \max \quad h_{j_0} = \sum_{r=1}^{s} u_r y^L_{rj_0} + p_{rj_0}(y^U_{rj_0} - y^L_{rj_0}) \\ \text{s.t.} \quad \sum_{i=1}^{m} v_i x^L_{ij_0} + q_{ij_0}(x^U_{ij_0} - x^L_{ij_0}) = 1, \\ \\ \sum_{r=1}^{s} u_r y^L_{rj} + p_{rj_0}(y^U_{rj} - y^L_{rj}) - \sum_{i=1}^{m} v_i x^L_{ij} + q_{ij}(x^U_{ij} - x^L_{ij}) \leq 0, \ j = 1, ..., n, \\ p_{rj} - u_r \leq 0, r = 1, ..., s; j = 1, ..., n, \\ q_{ij} - v_i \leq 0, i = 1, ..., m; j = 1, ..., n, \\ u_r, v_i \geq \varepsilon, \text{ for all } r, i, \\ p_{rj} \geq 0, q_{ij} \geq 0, \text{ for all } i, r, j. \end{array} \tag{2.2}$$

With $p_{rj} = u_r t_{rj}$ and $q_{ij} = v_i s_{ij}$, where the new variables q_{ij} and p_{rj}meet the conditions $0 \leq q_{ij} \leq v_i$ and $0 \leq p_{rj} \leq u_r$ [3, 6].

The following model provides an upper bound of the efficiency scores for unitj_o:

$$\begin{array}{c} \max \ \sum_{r=1}^{s} u_r y^U_{rj_0} \\ \text{s.t.} \ \sum_{i=1}^{m} v_i x^L_{ij_0} = 1, \\ \sum_{r=1}^{s} u_r y^U_{rj_0} - \sum_{i=1}^{m} v_i x^L_{ij_0} \leq 0, \\ \sum_{r=1}^{s} u_r y^L_{rj} - \sum_{i=1}^{m} v_i x^U_{ij} \leq 0, \ j = 1, ..., n, j \neq 0, \\ u_r, v_i \geq \varepsilon, \text{ for all } r, i, \end{array} \tag{2.3}$$

In this manner, the efficiency score is attained by unit j_o the model (2.3) say h_{jo}, that $h^*_{jo} = h^U_{jo}$ [2]. The model below provides a lower bound of the efficiency scores for unit j_o say h_{jo}, that $h^*_{jo} = h^L_{jo}$[2]:

$$
\begin{array}{c}
\max \ \sum_{r=1}^{s} u_r y^L_{rj_0} \\
\text{s.t.} \ \sum_{i=1}^{m} v_i x^U_{ij_0} = 1, \\
\sum_{r=1}^{s} u_r y^L_{rj_0} - \sum_{i=1}^{m} v_i x^U_{ij_0} \leq 0, \\
\sum_{r=1}^{s} u_r y^U_{rj} - \sum_{i=1}^{m} v_i x^L_{ij} \leq 0, \ j = 1, ..., n, j \neq 0, \\
u_r, v_i \geq \varepsilon, \ \text{for all } r, i,
\end{array}
\tag{2.4}
$$

Now we are in a possition to find the DEA efficiency with interval data by a numerical example. Here, 5 units with 2 inputs and 2 outputs are taken into consideration in a way that input-output data are known to lie within bounded intervals, and the efficiency scores are obtained by applying models (2.3) and (2.4) [2, 5].

Table 1. Efficiency scores for interval data.

DMU J	Inputs X_{1j}		Inputs X_{2j}		outputs Y_{1j}		outputs Y_{2j}		Efficiencies h^L_j	Efficiencies h^*_j
1	12	15	0.21	0.48	138	144	21	22	0.224	1
2	10	17	0.1	0.7	143	159	28	35	0.227	1
3	4	12	0.16	0.35	157	198	21	29	0.823	1
4	19	22	0.12	0.19	158	181	21	25	0.445	0.907
5	14	15	0.06	0.09	157	161	28	40	1	1

3. Liner semi-infinite programming problem (LSIP)

In this section, linear programming problem with fuzzy coefficients in both A and b has been considered:

$$
\begin{array}{c}
\min \ \sum_{j=1}^{n} C_j x_j \\
\text{s.t.} \ \sum_{j=1}^{n} \widetilde{a}_{ij} x_j \geq_\alpha \widetilde{b}_i, \forall \alpha \in [0,1], \ i = 1, ..., m, \\
x_j \geq 0, \ j = 1, ..., n.
\end{array}
\tag{3.1}
$$

It is shown that the model (3.1) can be reduced to a linear semi-infinite programming (LSIP) problem [3, 6].

$$
\begin{array}{c}
\min \ \sum_{j=1}^{n} C_j x_j \\
\text{s.t.} \ \begin{pmatrix} f_{11}(t) & \dots & f_{1n}(t) \\ \vdots & \ddots & \vdots \\ f_{m1}(t) & \dots & f_{mn}(t) \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \geq \begin{pmatrix} b_1(t) \\ \vdots \\ b_m(t) \end{pmatrix} \quad \forall t \in T \\
x_j \geq 0, \ j = 1, ..., n.
\end{array}
\tag{3.2}
$$

where T is a compact metric space, $f_{ij}(t)$ and $b_i(t)$, $(i = 1, \ldots, m, j = 1, \ldots, n)$ are real-valued continuous functions on T and

$$f_{ij}(t) = \begin{cases} L_{\widetilde{A}_{ij}(t)} & i = 1, ..., m \\ R_{\widetilde{A}_{ij}(t)} & i = m+1, ..., 2m \end{cases}$$

$$b_i(t) = \begin{cases} L_{\widetilde{B}_{ij}(t)} & i = 1, ..., m \\ R_{\widetilde{B}_{ij}(t)} & i = m+1, ..., 2m \end{cases}$$

In the above relations $\widetilde{A} = [L_{\widetilde{A}}(\alpha), R_{\widetilde{A}}(\alpha)]$, $\widetilde{B} = [L_{\widetilde{B}}(\alpha), R_{\widetilde{B}}(\alpha)]$ and $L_{\widetilde{N}}(\alpha) \cong \min\{x \in R \mid \mu_{\widetilde{N}}(x) \geq \alpha\}$, $R_{\widetilde{N}}(\alpha) \cong \max\{x \in R \mid \mu_{\widetilde{N}}(x) \geq \alpha\}$; $\widetilde{N}$ is a fuzzy set defined on R with a membership function $\mu_{\widetilde{N}}$ and $\widetilde{N} \cong \{x \in R \mid \mu_{\widetilde{N}}(x) \geq \alpha\}$ for all $\alpha \in [0, 1]$.

Problem (3.2) is a linear semi-infinite programming problem with n variables and infinitely many constraints. In this paper, the feasible region and the optimal objective value are denoted by FP and $v(LSIP)$, respectively. Dual model (3.2) is used to solve the mentioned problem, and according to the Strong Duality we conclude that $v(LSIP) = v(DLSIP)$.

Let T be a compact metric space, $c(T)$ be the space of all real-valued continuous functions on T, $M(T)$ be the space of bounded regular borel measures on T, $C^+(T) \cong \{h \in C(T) \mid h(t) \geq 0, \forall t \in T\}$ and $M^+(T) \cong \{M \in M(T) \mid \mu(B) \geq 0, \forall B \in B(T)\}$, where $B(T)$ is the set of all Borel set in T. The dual problem of $LSIP$ (3.2) denoted by $DLSIP$ could be considered as follows:

$$\begin{array}{rl} \max & \sum_{i=1}^{m} \int_{t\in T} b_i(t) d\mu_i \\ \text{s.t.} & \sum_{i=1}^{m} \int_{t\in T} f_{ij}(t) d\mu_i \leq c_j, \quad j = 1, ..., n, \\ & \mu_i \in M^+(T), \quad i = 1, ..., m. \end{array} \tag{3.3}$$

Let FD and $v(DLSIP)$ be the feasible region and optimal objective value of $DLSIP$, then we have the follwoing theorems [3].

Theorem 3.1. *Assume that $FD \neq \emptyset$ and $-\infty < v(DLSIP) < \infty$. If there exists $\mu^0 = (\mu_1^0, \mu_2^0, ..., \mu_m^0) \in (M^+(T))^m$ such that $\sum_{i=1}^{m} \int_T f_{ij}(T) d\mu_i^0 < c_j, j = 1, ..., n$. Then $FD \neq \emptyset$ and $v(LSIP) = v(DLSIP)$.*

Theorem 3.2. *Assume that v(LSIP) =v(DLSIP), then $x^* \in FP$ solves (LSIP) and $\mu^* \in FD$ solves (DLSIP) if and if $\sum_{j=1}^{n} f_{ij}(t) x_j^* - b_i(t) = 0, \forall t \in \sup(\mu_i^*), (i = 1, ..., m)$ and $c_j - \sum_{i=1}^{m} \int_{t\in T} f_{ij}(t) d\mu_i^* = 0$, for all $j \in \{k \mid x_k^* \neq 0\}$.*

Theorem 3.3. *If FP is bounded, then $LSIP$ has an optimal solution which is an extreme point of FP.*

Definition 3.4 ([3])**.** Let E and F be real linear spaces, and $A : E \to F$ a linear operator. Consider the following linear program (LP):

$$\begin{array}{rl} \min & (C^*, X) \\ \text{s.t.} & AX = b, \\ & x \in p. \end{array} \tag{3.4}$$

Where C^* is a linear functional in $E, b \in F$, and P is a positive convex cone in E. For $x^0 \in p$, $B(X^0) = \{x \in E \mid x^0 \pm \lambda x \in p, \lambda > 0\}$. In this case x^0 is an extreme point of the feasible region for (LP) if and only if $B(X^0) \cap N(A) = 0$ Where 0 denotes the zero vector and $N(A) = \{x \in E \mid AX = 0\}$ is the null space of A.

There are many semi-infinite programming algorithms for solving linear semi-infinite programming problems. Based on a recent review [4], the cutting plane approach is an effective one for such applications. We can easily design an iterative algorithm which adds m constraints at each time until an optimal solution is identified.

At the k^{th} iteration, given $T_k = \{t^1, t^2, ..., t^k\}$, where $t^k = (t_1^k, t_2^k, ..., t_m^k) \in T^m$, $k \geq 1$, the following linear programming problem (LP^k) would be considered:

$$\min \ \sum_{j=1}^{n} C_j x_j$$
$$\text{(3.5)} \qquad \text{s.t.} \begin{pmatrix} f_{11}(t_1^1) & \dots & f_{1n}(t_1^1) \\ \vdots & \ddots & \vdots \\ f_{m1}(t_m^1) & \dots & f_{mn}(t_m^1) \\ \vdots & \ddots & \vdots \\ f_{11}(t_1^k) & \dots & f_{1n}(t_1^k) \\ \vdots & \ddots & \vdots \\ f_{m1}(t_m^k) & \dots & f_{mn}(t_m^k) \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \geq \begin{pmatrix} b_1(t_1^1) \\ \vdots \\ b_m(t_m^1) \\ \vdots \\ b_1(t_1^k) \\ \vdots \\ b_m(t_m^k) \end{pmatrix}$$
$$x_j \geq 0, \ j = 1, ..., n.$$

Let F^k be the feasible region of (LP^k) and $X^k = (x_1^k, ..., x_n^k)$ is an optimal solution of (LP^k). We define the "constraint violation functions" as follows:

$$V_i^{k+1} \cong \sum_{j=1}^{n} f_{ij}(t) x_j^k - b_i(t) \quad \forall t \in T, i = 1, ..., m$$

Since $f_{ij}(t)$ and $b_i(t)$ are continuous over T and also T is compact, the function $V_i^{k+1}(t)$ achieves its minimum over T, for $i = 1, ..., m$.

A cutting plane algorithm for solving (LSIP)[3]:

Step 1. Set $k = 1, V_i^{k+1}(t_i^{k+1}) \geq 0$. Choose any $t_i^1 \in T$; set $T_1 = \{t^1\}$.
Step 2. Solve problem (3.5) and obtain an optimal solution x^k.
Step 3. Find a minimize t_i^{k+1} of $V_i^{k+1}(t)$ over T, for $i = 1, ..., m$.
Step 4. If $V_i^{k+1}(t_i^{k+1}) \geq 0$, for $i = 1, ..., m$, then stop; x^k is an optimal solution of $LSIP$. Otherwise; set $T_{K+1} = T_k \cup \{t^{k+1}\}$ and $k + 1 \to k$; go to Step 1.

Theorem 3.5 ([3]). *Let $\{x^k\}$ be a sequence generated by the above algorithm. If there exists an $M > 0$ such that $\|x^k\| \leq M$for each k, then there is a subsequence of $\{x^k\}$ which converges to an optimal solution of $LSIP$.*

4. The suggested method

To solve DEA problems with interval data as a linear semi- infinite programming problem the cutting plane approach was used. Please consider 5 units with 2 inputs and 2 outputs in a way that inputs and outputs are placed within bounded intervals.

The membership function for each input and output is calculated separately. For example for DMU_1 given in Table 1, we have the following membership function:

Table 2. Fuzzy data for DMU_1.

X_{11}	X_{21}	Y_{11}	Y_{21}
[21 22]	[12 15]	[0.21 0.48]	[138 144]
21.5	13.5	0.345	141

$$\mu_{\widetilde{13.5}}(x) = \begin{cases} \frac{x-12}{1.5} & 12 \le x \le 13.5, \\ \frac{15-x}{1.5} & 13.5 \le x \le 15, \end{cases} \qquad \mu_{\widetilde{0.345}}(x) = \begin{cases} \frac{x-0.21}{0.135} & 0.21 \le x \le 0.345, \\ \frac{0.48-x}{0.135} & 0.345 \le x \le 0.48, \end{cases}$$

$$\mu_{\widetilde{141}}(x) = \begin{cases} \frac{x-138}{3} & 138 \le x \le 141, \\ \frac{144-x}{3} & 141 \le x \le 144, \end{cases} \qquad \mu_{\widetilde{21.5}}(x) = \begin{cases} \frac{x-21}{0.5} & 21 \le x \le 21.5, \\ \frac{22-x}{0.5} & 21.5 \le x \le 22. \end{cases}$$

Table 3. Fuzzy Data for all $DMUs$.

DMU	inputs	inputs	outputs	outputs
j	I_1	I_2	O_1	O_2
1	$\widetilde{13.5}$	$\widetilde{0.345}$	$\widetilde{141}$	$\widetilde{21.5}$
2	$\widetilde{13.5}$	$\widetilde{0.4}$	$\widetilde{1541}$	$\widetilde{31.5}$
3	$\widetilde{8}$	$\widetilde{0.255}$	$\widetilde{177.5}$	$\widetilde{25}$
4	$\widetilde{20.5}$	$\widetilde{0.155}$	$\widetilde{169.5}$	$\widetilde{23}$
5	$\widetilde{14.5}$	$\widetilde{0.075}$	$\widetilde{159}$	$\widetilde{34}$

Consider the following DEA model:

$$\begin{array}{c} \max \ \sum_{r=1}^{s} u_r y_{rj_0} \\ \text{s.t.} \ \sum_{i=1}^{m} v_i x_{ij_0} = 1, \\ \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \ j = 1, ..., n, \\ u_r, v_i \ge 0, \ \text{for all} r, i. \end{array} \tag{4.1}$$

The above model becomes as follows for DMU_5:

$$\begin{array}{c} \max \ \ 157u_1 + 28u_2 \\ \text{s.t.} \ \widetilde{14.5}v_1 + \widetilde{0.075}v_2 = 1, \\ \widetilde{141}u_1 + \widetilde{21.5}u_2 - \widetilde{13.5}v_1 - \widetilde{0.345}v_2 \le 0, \\ \widetilde{151}u_1 + \widetilde{31.5}u_2 - \widetilde{13.5}v_1 - \widetilde{0.4}v_2 \le 0, \\ \widetilde{177}u_1 + \widetilde{25}u_2 - \widetilde{8}v_1 - \widetilde{0.255}v_2 \le 0, \\ \widetilde{169}u_1 + \widetilde{23}u_2 - \widetilde{20}v_1 - \widetilde{0.155}v_2 \le 0, \\ \widetilde{159}u_1 + \widetilde{34}u_2 - \widetilde{14.5}v_1 - \widetilde{0.075}v_2 \le 0, \\ u_1, u_2, v_1, v_2, \ge 0. \end{array} \tag{4.2}$$

Including membership function, we would have:

(4.3)

$$\begin{aligned}&\max \quad 157u_1+28u_2\\ &\text{s.t.} \begin{pmatrix} 0.5t_5^1+14 & 0.015t_5^1+0.06\\ 15-0.5t_{10}^1 & 0.09-0.015t_{10}^1\end{pmatrix}\begin{pmatrix} v_1\\ v_2\end{pmatrix}=\begin{pmatrix}1\\1\end{pmatrix},\end{aligned}$$

$$\begin{pmatrix} 1.5t_1^1+12 & 0.135t_1^1+0.21 & 3t_1^1+138 & 0.5t_1^1+21\\ 3.5t_2^1+10 & 0.3t_2^1+0.1 & 8t_2^1+143 & 3.5t_2^1+28\\ 4t_3^1+4 & 0.095t_3^1+0.16 & 20.5t_3^1+157 & 4t_3^1+21\\ 1.5t_4^1+19 & 0.035t_4^1+0.12 & 11.5t_4^1+158 & 2t_4^1+21\\ 0.5t_5^1+14 & 0.015t_5^1+0.06 & 2t_5^1+157 & 6t_5^1+28\\ 15-1.5t_6^1 & 0.48-0.135t_6^1 & 144-3t_6^1 & 22-0.5t_6^1\\ 17-3.5t_7^1 & 0.7-0.3t_7^1 & 159-8t_7^1 & 35-3.5t_7^1\\ 12-4t_8^1 & 0.35-0.095t_8^1 & 198-20.5t_8^1 & 29-4t_8^1\\ 22-1.5t_9^1 & 0.19-0.035t_9^1 & 181-11.5t_9^1 & 25-2t_9^1\\ 15-0.5t_{10}^1 & 0.09-0.015t_{10}^1 & 161-2t_{10}^1 & 40-6t_{10}^1 \end{pmatrix}\begin{pmatrix} v_1\\ v_2\\ u_1\\ u_2\end{pmatrix}$$

$$\leq \begin{pmatrix}0\\0\\ \vdots\\ 0\\ 0\end{pmatrix}.$$

Given any $t_i \in [\alpha, 1]$, say $\alpha = 0.6$ in this example and an arbitrary starting point, say $t^1 = (t_1^1, t_2^1, t_3^1, ..., t_{10}^1)$=$(0.6, 0.63, 0.61, 0.66, 1, 0.61, 0.63, 0.69, 0.91, 1)$ and by substituting t^1 into the model (4.3), we will reach the following solutions:

$$v = (v_1, v_2) = (0.069, 0), \quad u = (u_1, u_2) = (0, 0.0189)$$

Also, in this case the optimal objective function is equal to 0.5305.

Given any $t_i \in [0.3, 1]$, say$\alpha = 0.3$ in this example for DMU_3 and an arbitrary starting point, say $t^2 = (t_1^2, t_2^2, t_3^2, ..., t_{10}^2)$=$(0.3, 0.42, 1, 0.6, 0.61, 0.63, 0.69, 1, 0.8, 0.9)$ and by substituting t^2 within the model (4.3), we will reach the solution based on the following transformation:

$$\begin{aligned}&\max \quad 157u_1+21u_2\\ &\text{s.t.} \begin{pmatrix} 4t_3^2+4 & 0.095t_3^2+0.16\\ 12-4t_8^2 & 0.35-0.095t_8^2\end{pmatrix}\begin{pmatrix} v_1\\ v_2\end{pmatrix}=\begin{pmatrix}1\\1\end{pmatrix}.\end{aligned}$$

In this case we have $v = (v_1, v_2) = (0.125, 0), u = (u_1, u_2) = (0.0056, 0)$. Also, the optimal value of the objective function is equal to 0.8845.

In addition, using $\alpha = 0.3$, $t_i \in [0.3, 1]$, we would begin with $t^3 = (t_1^3, t_2^3, t_3^3, ..., t_{10}^3) = (0.3, 0.42, 0.5, 1, 0.61, 0.63, 0.69, 0.7, 1, 0.9)$ for DMU_4. Substituting t^3 into the model (4.3), we obtain the optimal solution using the following transformation:

$$\begin{aligned}&\max \quad 158u_1+21u_2\\ &\text{s.t.} \begin{pmatrix} 1.5t_4^3+19 & 0.035t_4^3+0.12\\ 22-1.5t_9^3 & 0.19-0.035t_9^3\end{pmatrix}\begin{pmatrix} v_1\\ v_2\end{pmatrix}=\begin{pmatrix}1\\1\end{pmatrix}.\end{aligned}$$

This gives the optimal solution as $v = (v_1, v_2) = (0.0322, 2.1907), u = (u_1, u_2) = (0.0039, 0)$. Also, the optimal value of the objective function is equal to 0.611. Moreover, with $\alpha = 0.2$,$t_i \in [0.2, 1]$, we would start with $t^4 = (t_1^4, t_2^4, t_3^4, ..., t_{10}^4) = (0.2, 1, 0.3, 1, 0.42, 0.61, 0.69, 1, 0.8, 0.85, 0.9)$ for DMU_2. Thus, substituting t^4 into the model (4.3), gives the following transformation:

$$\max \quad 143u_1 + 28u_2$$
$$\text{s.t.} \quad \begin{pmatrix} 3.5t_2^4 + 10 & 0.035t_2^4 + 0.1 \\ 17 - 3.5t_7^4 & 0.7 - 0.3t_7^4 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

This gives the optimal solytaion as $v = (v_1, v_2) = (0.0394, 1.1702), u = (u_1, u_2) = (0, 0.0191)$. In addition, the optimal value of the objective function is equal to 0.5359. Finally, with $\alpha = 0.3$, $t_i \in [0.3, 1]$, we would start with $t^5 = (t_1^5, t_2^5, t_3^5, ..., t_{10}^5) = (1, 0.42, 0.5, 0.6, 0.61, 1, 0.63, 0.69, 0.7, 0.9)$ for DMU_1. Substituting t^5 into the model (4.3), gives the following transformation:

$$\max \quad 138u_1 + 21u_2$$
$$\text{s.t.} \quad \begin{pmatrix} 1.5t_1^4 + 12 & 0.135t_1^4 + 0.21 \\ 15 - 1.5t_6^4 & 0.48 - 0.135t_6^4 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

This gives the optimal solution as $v = (v_1, v_2) = (0.0463, 1.0866), u = (u_1, u_2) = (0, 0.0219)$. Also, the optimal value of objective function is equal to 0.459.

In table 4 our suggested method has been compared with Despotis's method. Considering the fact that the coefficients in constraints are fuzzicated in our suggested method, the optimal solutions within this method have been closed together.

Table 4. Comparing Model (2.2) and Model (4.3).

DMU	Desposit		The suggested method	v_1	v_2	u_1	u_2
j	h_j^l	h_j^*					
1	0.224	1	0.459	0.0463	1.0866	0	0.0219
2	0.227	1	0.5359	0.0394	1.1702	0	0.0191
3	0.823	1	0.8845	0.125	0	0.0056	0
4	0.445	0.907	0.611	0.0322	2.1907	0.0039	0
5	1	1	0.5305	0.069	0	0	0.0189

5. Conclusions

In this paper, a linear programming with fuzzy coefficients in A and b considered and then this problem has been reduced to a semi-infinite linear programming and it has been solved using a cutting plane algorithm. We applied it to DEA model with interval data, giving a new model. Finally our suggested method has been compared with Despotis's Method.

Acknowledgements. The authors would like to thank the anonymous reviewers and honorable Editor of the journal for accepting the paper.

References

[1] W. W. Copper, K. S. Park and G. Yu, IDEA and AR-IDEA: Models for dealing with imprecise data in DEA, Management Sci. 45 (1999) 597–607.

[2] D. K. Despoti and Y. G. Smirlis, Data envelopment analysis with imprecise data, European J. Oper. Res. 140 (2002) 24–36.

[3] C. S. Fang, F. C. Hu, H. F. Wang and S. Y. Wu, Linear programming with fuzzy coefficients in constraints, Comput. Math. Appl. 37 (1999) 63–76.

[4] A. Hettich and K. Kortanek, Semi-infinite programming: Theory, method and applications, SIAM Review 35 (1993) 380–429.

[5] C. Kao, Interval efficiency measures in data envelopment analysis with imprecise data, European J. Oper. Res. 174 (2006) 1087-1089.

[6] H. W. Lu, G. H. Huang and L. He, Development of an interval-valued fuzzy linear-programming method based on infinite α-cuts for water resources management, Environ. Model. Softw. 25 (2010) 354–361.

Sohrab Kordrostami (krostami@guilan.ac.ir)
Department of Mathematics, Lahijan Branch, Islamic Azad University, Lahijan, Iran.

Alireza Amirteimoori (teimoori@guilan.ac.ir)
Department of Mathematics, Rasht Branch, Islamic Azad University, Rasht, Iran.

Shahla Kazemi (sh_k500@yahoo.com)
Department of Mathematics, Lahijan Branch, Islamic Azad University, Lahijan, Iran.

Ali Ebrahimnejad (a.ebrahimnejad@qaemshahriau.ac.ir)
Department of Mathematics, Qaemshahr Branch, Islamic Azad University, Qaemshahr, Iran.