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Anti fuzzy ideals of Γ-near-rings

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Abstract. In this paper we introduce the notion of an anti fuzzy ideals of a Γ-near-ring, anti fuzzy ideals of a Γ-residue class near-ring, anti level subset of fuzzy set and complement of anti fuzzy ideals. Further, we discuss the relation between anti fuzzy ideals of Γ-near-rings and anti level subset of a fuzzy sets. Also we prove that if μ is an anti fuzzy ideal of a Γ-near-ring M then $M_\mu = \{x \in M : \mu(x) = \mu(0)\}$ is an ideal of M.

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1. Introduction

After the introduction of fuzzy set by Zadeh [14] in 1965, the researchers in mathematics were trying to introduce and study the concept of fuzzyness in different mathematical systems under study. Γ-near-rings were defined by Bh. Satyanarayana [11]. The ideal theory in Γ-near-rings was studied by Bh. Satynarayana [11] and G. L. Booth [2]. Fuzzy ideals of rings were introduced by W. Liu [9] and it has been studied by several authors [4, 5, 7, 10, 13]. R. Biswas [1] introduced the concept of anti fuzzy subgroups and K. H. Kim and Y. B. Jun studied the notion of anti fuzzy ideals in near-rings in [8]. In [3] S. K. Datta introduced the concept of anti fuzzy bi-ideals in rings. T. S. Ravisankar and U. S. Shukla in [12], Y. B. Jun, M. Sapanci and M. A. Ozturk in [6] studied the structure of Γ-near-rings. Anti fuzzy ideals of Γ-rings were studied by Min Zhou, D. Xiang and J. Zhan in [15]. In this paper we introduce the concept of anti fuzzy ideals of Γ-near-ring, f-invariant anti fuzzy ideals and anti level subset of a fuzzy set. We examine some related properties and study the relation between anti fuzzy ideal of a Γ-near-ring and anti level subset of a fuzzy set. The proofs are almost similar to anti fuzzy ideals in near-rings [8] and anti fuzzy ideals of Γ-rings [15].

2. Preliminaries

A non-empty set N with two binary operations " + " (addition) and " $\cdot$ " (multiplication) is called a near-ring, if it satisfies the following axioms:
(i) $(N, +)$ is a group,
(ii) $(N, \cdot)$ is a semigroup,
(iii) $(x + y) \cdot z = x \cdot z + y \cdot z$
for all $x, y, z \in N$.

Precisely speaking it is a right near-ring, because it satisfies the right distributive law. We will use the word "near-ring" to mean "right near-ring". We denote xy instead of $x \cdot y$. Moreover, a near-ring N is said to be a zero-symmetric if $r \cdot 0 = 0$ for all $r \in N$, where 0 is the additive identity in N.

Definition 2.1. Let (M, +) be a group and Γ be a non empty set. Then M is said to be a Γ - near-ring, if there exist a mapping $M \times \Gamma \times M \to M$ (The image of (x, α, y) is denoted by $x\alpha y$) satisfying the following conditions:
(i) $(x + y)\alpha z = x\alpha z + y\alpha z$,
(ii) $(x\alpha y)\beta z = x\alpha(y\beta z)$ for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$.

Definition 2.2. Let M be a Γ-near-ring. A normal subgroup $(I, +)$ of $(M, +)$ is called
(i) a left ideal if $x\alpha(y + i) - x\alpha y \in I$ for all $x, y \in M, \alpha \in \Gamma, i \in I$,
(ii) a right ideal if $i\alpha x \in I$ for all $x \in M, \alpha \in \Gamma, i \in I$,
(iii) an ideal if it is both a left ideal and a right ideal of M.

A Γ-near-ring M is said to be a zero-symmetric if $a\alpha 0 = 0$ for all $a \in M$ and $\alpha \in \Gamma$, where 0 is the additive identity in M.

Definition 2.3. Let A be a non-empty set. A fuzzy subset of A is a function $\mu : A \to [0, 1]$. For any $t \in [0, 1]$, the set $\mu_t = \{x \in A : \mu(x) \geq t\}$ is called level subset of μ. For any $t \in [0, 1]$ the set $\mu_t = \{x \in A : \mu(x) \leq t\}$ is called anti level subset of μ.

Definition 2.4. Let M be a Γ-near-ring and μ be a fuzzy subset of M. Then the complement of μ is denoted by μ^c and is defined by $\mu^c(x) = 1 - \mu(x)$ for any $x \in M$.

Definition 2.5. Let M and N be Γ-near-rings. A map $f : M \to N$ is called a Γ-near-ring homomorphism, if $f(x + y) = f(x) + f(y)$ and $f(x\alpha y) = f(x)\alpha f(y)$ for all $x, y \in M$ and $\alpha \in \Gamma$.

Definition 2.6. Let μ be a fuzzy set of a Γ-near-ring M and f be a function defined on M, then the fuzzy set ν in $f(M)$ is defined by

$$\nu(y) = \inf_{x \in f^{-1}(y)} \mu(x)$$

for all $y \in f(M)$ is called the image of μ under f.
Similarly, if ν is a fuzzy set in $f(M)$, then $\mu = \nu o f$ in M (that is, the fuzzy set defined by $\mu(x) = \nu(f(x))$) for all $x \in M$ is called the pre-image of ν under f.

Definition 2.7 ([7])**.** For a family of fuzzy sets $\{\mu_i : i \in \wedge\}$ in a Γ-near-ring M, the union $\bigvee_{i \in \wedge} \mu_i$ of $\{\mu_i : i \in \wedge\}$ is defined by

$$\left(\bigvee_{i \in \wedge} \mu_i\right)(x) = \sup\{\mu_i(x) : i \in \wedge\}$$

for each $x \in M$.

Definition 2.8. Let μ be a fuzzy set of a non empty set M. Then μ has inf property, if for any subset N of M there exists $n_0 \in N$ such that

$$\mu(n_0) = \inf_{n \in N} \mu(n).$$

Definition 2.9. Let M be a Γ-near-ring. For an endomorphism f of M and fuzzy set μ in M, we define a new fuzzy set μ^f in M by $\mu^f(x) = \mu(f(x))$ for all $x \in M$.

Definition 2.10. Let M and N be any two sets and let $f : M \to N$ be any function. A fuzzy subset μ of M is called f-invariant, if $f(x) = f(y)$ implies $\mu(x) = \mu(y)$ for all $x, y \in M$.

Definition 2.11. A fuzzy set μ in a Γ-near-ring M is called a fuzzy left (resp. right) ideal of M, if

(i) μ is a fuzzy normal subgroup with respect to the addition,

(ii) $\mu(u\alpha(x + v) - u\alpha v) \geq \mu(x)(resp.\ \ \mu(x\alpha u) \geq \mu(x))$

for all $x, u, v \in M$ and $\alpha \in \Gamma$.

The condition (i) means that, μ satisfies

$$\mu(x - y) \geq \min\{\mu(x), \mu(y)\} \text{ and } \mu(y + x - y) \geq \mu(x)$$

for all $x, y \in M$.

Definition 2.12. A fuzzy ideal μ of a Γ-near-ring M is said to be normal, if there exists $a \in M$ such that $\mu(a) = 1$. We note that μ is normal of a Γ-near-ring M if and only if $\mu(1) = 1$.

3. Anti fuzzy ideals

Definition 3.1. A fuzzy set μ in a Γ-near-ring M is called an anti fuzzy left (resp. right) ideal of M, if

(i) $\mu(x - y) \leq \max\{\mu(x), \mu(y)\}$,

(ii) $\mu(y + x - y) \leq \mu(x)$ for all $x, y \in M$,

(iii) $\mu(u\alpha(x + v) - u\alpha v) \leq \mu(x)(resp.\ \mu(x\alpha u) \leq \mu(x))$

for all $x, u, v \in M$ and $\alpha \in \Gamma$.

A fuzzy set μ in a Γ-near-ring M is called an anti fuzzy ideal of M, if μ is both an anti fuzzy left ideal and an anti fuzzy right ideal of M.

Example 3.2. Let $M = \{0, a, b, c\}$ and $\Gamma = \{0_\Gamma, 1\}$. Define a binary operation "+" on M and a mapping $M \times \Gamma \times M \to M$ by the following tables:

$+$	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

0_Γ	0	a	b	c
0	0	0	0	0
a	0	0	0	0
b	0	0	0	0
c	0	0	0	0

1	0	a	b	c
0	0	0	0	0
a	0	a	a	a
b	0	b	b	b
c	0	c	c	c

Clearly, $(M,+)$ is a group and (i) $(x+y)\gamma z = x\gamma z + y\gamma z$ for every $x,y,z \in M$, $\gamma \in \Gamma$, (ii) $(x\gamma y)\omega z = x\gamma(y\omega z)$ for every $x,y,z \in M$ and $\gamma,\omega \in \Gamma$. So M is a Γ-near-ring. Define a fuzzy set $\mu : M \to [0,1]$ by $\mu(0) < \mu(a) = \mu(b) = \mu(c)$. The routine calculation shows that μ is an anti fuzzy ideal of M.

Example 3.3. Let M be a Γ-near-ring. Define a fuzzy set $\mu : M \to [0,1]$ by $\mu(x) = 0.6$ for every $x \in M$. Then μ is an anti fuzzy ideal of M.

Theorem 3.4. *Let M be a Γ-near-ring and μ be an anti fuzzy left (resp. right) ideal of M. Then the set $M_\mu = \{x \in M : \mu(x) = \mu(0)\}$ is a left (resp. right) ideal of M.*

Proof. Let μ be an anti fuzzy left ideal of M. Let $M_\mu = \{x \in M : \mu(x) = \mu(0)\}$. If $x, y \in M_\mu$, then $\mu(x) = \mu(0)$ and $\mu(y) = \mu(0)$. Then $\mu(x-y) \le \max\{\mu(x), \mu(y)\}$, which implies that $\mu(x-y) \le \max\{\mu(0), \mu(0)\} = \mu(0)$. Hence $x - y \in M_\mu$. Now for every $y \in M$ and $x \in M_\mu$, we have $\mu(y+x-y) \le \mu(x) = \mu(0)$. This implies that $y + x - y \in M_\mu$. Therefore M_μ is a normal subgroup of M with respect to addition. Let $x \in M_\mu, \alpha \in \Gamma$ and $u, v \in M$. Then $\mu(u\alpha(x+v) - u\alpha v) \le \mu(x) = \mu(0)$, and thus $u\alpha(x+v) - u\alpha v \in M_\mu$. Hence M_μ is left ideal of M. □

Theorem 3.5. *Let μ be a fuzzy set in a Γ-near-ring M. If μ is an anti fuzzy left (resp. right) ideal of M, then each anti level subset μ_t, $t \in Im(\mu)$ is a left (resp. right) ideal of M.*

Proof. Let μ be an anti fuzzy left ideal of M. Then the anti level subset $\mu_t = \{x \in M : \mu(x) \le t\}, t \in Im(\mu)$. Let $x, y \in \mu_t$. Then $\mu(x) \le t, \mu(y) \le t$. Thus $\mu(x-y) \le \max\{\mu(x), \mu(y)\} \le t$, and so $x - y \in \mu_t$. Let $y \in M$ and $x \in \mu_t$, then $\mu(y+x-y) \le \mu(x) \le t$ which implies that $y+x-y \in \mu_t$. Therefore μ_t is a normal subgroup of M with respect to addition. Let $x \in \mu_t, \alpha \in \Gamma$ and $u, v \in M$, then $\mu(u\alpha(x+v) - u\alpha v) \le \mu(x) \le t$. This implies $u\alpha(x+v) - u\alpha v \in \mu_t$. Hence μ_t is a fuzzy left ideal of M. □

Theorem 3.6. *If $\{\mu_i : i \in \wedge\}$ is a family of anti fuzzy ideals of a Γ-near-ring M then so is $\bigvee\limits_{i\in I} \mu_i$.*

Proof. Let $\{\mu_i : i \in \wedge\}$ be a family of anti fuzzy ideals of M and let $x, y \in M$. Then

$$\begin{aligned}
\left(\bigvee_{i\in\wedge} \mu_i\right)(x-y) &= \sup\{\mu_i(x-y) : i \in \wedge\} \\
&\le \sup\{\max\ \mu_i(x), \mu_i(y) : i \in \wedge\} \\
&= \max\{\sup\{\mu_i(x) : i \in \wedge\}, \sup\{\mu_i(y) : i \in \wedge\}\} \\
&= \max\left\{\left(\bigvee_{i\in\wedge} \mu_i\right)(x), \left(\bigvee_{i\in\wedge} \mu_i\right)(y)\right\}
\end{aligned}$$

and

$$\begin{aligned}\left(\bigvee_{i\in\wedge}\mu_i\right)(y+x-y) &= \sup\{\mu_i(y+x-y) : i\in\wedge\}\\ &\leq \sup\{\mu_i(x) : i\in\wedge\}\\ &= \left(\bigvee_{i\in\wedge}\mu_i\right)(x).\end{aligned}$$

Now, let $x, u, v \in M$ and $\alpha \in \Gamma$. Then

$$\begin{aligned}\left(\bigvee_{i\in\wedge}\mu_i\right)(u\alpha(x+v)-u\alpha v) &= \sup\{\mu_i(u\alpha(x+v)-u\alpha v) : i\in\wedge\}\\ &\leq \sup\{\mu_i(x) : i\in\wedge\}\\ &= \left(\bigvee_{i\in\wedge}\mu_i\right)(x),\end{aligned}$$

and

$$\begin{aligned}\left(\bigvee_{i\in\wedge}\mu_i\right)(x\alpha u) &= \sup\{\mu_i(x\alpha u) : i\in\wedge\}\\ &\leq \sup\{\mu_i(x) : i\in\wedge\}\\ &= \left(\bigvee_{i\in\wedge}\mu_i\right)(x).\end{aligned}$$

This completes the proof. □

Theorem 3.7. *Intersection of a non empty collection of anti fuzzy left (resp. right) ideals of a Γ-near-ring M is an anti fuzzy left (resp. right) ideal of M.*

Proof. Let M be a Γ-near-ring. Let $\{\mu_i : i \in I\}$ be the family of anti fuzzy left (resp. right) ideals of M and let $x, y \in M$. Then we have,

$$\begin{aligned}\left(\bigcap_{i\in I}\mu_i\right)(x-y) &= \inf_{i\in I}[\mu_i(x-y)]\\ &\leq \inf_{i\in I}[\max\{\mu_i(x),\mu_i(y)\}]\\ &= \max[\inf_{i\in I}\mu_i(x), \inf_{i\in I}\mu_i(y)]\\ &= \max\left[\left(\bigcap_{i\in I}\mu_i\right)(x), \left(\bigcap_{i\in I}\mu_i\right)(y)\right],\end{aligned}$$

$$\begin{aligned}\left(\bigcap_{i\in I}\mu_i\right)(y+x-y) &= \inf_{i\in I}[\mu_i(y+x-y)]\\ &\leq \inf_{i\in I}\mu_i(x) = \left(\bigcap_{i\in I}\mu_i\right)(x).\end{aligned}$$

Let $x, u, v \in M$ and $\alpha \in \Gamma$. Then

$$\begin{array}{rcl}\left(\bigcap_{i\in I}\mu_i\right)(u\alpha(x+v)-u\alpha v) & = & \inf_{i\in I}[\mu_i(u\alpha(x+v)-u\alpha v)] \\ & \leq & \inf_{i\in I}\mu_i(x) = \left(\bigcap_{i\in I}\mu_i\right)(x).\end{array}$$

This completes the proof. □

Theorem 3.8. *Let M be a Γ-near-ring. Then a fuzzy set μ is an anti fuzzy ideal of M if and only if μ^c is a fuzzy ideal of M.*

Proof. Let $x, y \in M$ and μ be an anti fuzzy ideal of M, then we have

$$\begin{array}{rcl}\mu^c(x-y) & = & 1-\mu(x-y) \\ & \geq & 1-\max\{\mu(x),\mu(y)\} \\ & = & \min\{1-\mu(x), 1-\mu(y)\} \\ & = & \min\{\mu^c(x),\mu^c(y)\},\end{array}$$

and

$$\mu^c(y+x-y) = 1-\mu(y+x-y) \geq 1-\mu(x) = \mu^c(x).$$

Let $x, u, v \in M$ and $\alpha \in \Gamma$, then we have

$$\begin{array}{rcl}\mu^c(u\alpha(x+v)-u\alpha v) & = & 1-\mu(u\alpha(x+v)-u\alpha v) \\ & \geq & 1-\mu(x) = \mu^c(x),\end{array}$$

and $\mu^c(x\alpha u) = 1-\mu(x\alpha u) \geq 1-\mu(x) = \mu^c(x)$. Hence μ^c is a fuzzy ideal of M. Similarly converse can be proved. □

Theorem 3.9. *A Γ-near-ring homomorphic pre-image of an anti fuzzy ideal is an anti fuzzy ideal.*

Proof. Let M and N be Γ-near-rings. Let $f : M \to N$ be a Γ-near-ring homomorphism, ν be an anti fuzzy ideal of N and μ be the pre-image of ν under f. Let $x, y, u, v \in M$ and $\gamma \in \Gamma$, then we have

$$\mu(x-y) = \nu(f(x-y)) = \nu(f(x)-f(y)) \leq \max\{\nu f(x), \nu(f(y))\} = \max\{\mu(x),\mu(y)\},$$

$$\mu(y+x-y) = \nu(f(y+x-y)) = \nu(f(y)+f(x)-f(y)) \leq \nu(f(x)) = \mu(x),$$

$$\begin{array}{rcl}\mu(u\alpha(x+v)-u\alpha v) & = & \nu(f(u\alpha(x+v)-u\alpha v)) \\ & = & \nu(f(u)\alpha f(x+v)-f(u)\alpha f(v)) \\ & = & \nu(f(u)\alpha(f(x)+f(v))-f(u)\alpha f(v)) \\ & \leq & \nu(f(x)) = \mu(x),\end{array}$$

and $\mu(x\alpha u) = \nu(f(x\alpha u)) = \nu(f(x)\alpha f(u)) \leq \nu(f(x)) = \mu(x)$. Hence μ is an anti fuzzy ideal of M. □

Theorem 3.10. *A Γ-homomorphic image of an anti fuzzy left (resp. right) ideal which has the inf property is an anti fuzzy left (resp. right) ideal.*

Proof. Let M and N be Γ - near-rings. Let $f : M \to N$ be a Γ-near-ring homomorphism, μ be an anti fuzzy left ideal of M with the inf property and ν be the image of μ under f. Let $f(x), f(y) \in f(M)$, $x_0 \in f^{-1}(f(x)), y_0 \in f^{-1}(f(y)), u_0 \in f^{-1}(f(u))$ and $v_0 \in f^{-1}(f(v))$ be such that $\mu(x_0) = \inf_{n\in f^{-1}(f(x))} \mu(n)$, $\mu(y_0) = \inf_{n\in f^{-1}(f(y))} \mu(n)$. Then for any $x, y, u, v \in M$ and $\alpha \in \Gamma$, we have

$$\begin{aligned}
\nu(f(x) - f(y)) &= \inf_{z\in f^{-1}(f(x)-f(y))} \mu(z) \\
&\le \mu(x_0 - y_0) \\
&\le \max\{\mu(x_0), \mu(y_0)\} \\
&= \max\left\{\inf_{n\in f^{-1}(f(x))} \mu(n), \inf_{n\in f^{-1}(f(y))} \mu(n)\right\} \\
&= \max\{\nu(f(x)), \nu(f(y))\},
\end{aligned}$$

$$\begin{aligned}
\nu(f(y) + f(x) - f(y)) &= \inf_{z\in f^{-1}(f(y)+f(x)-f(y))} \mu(z) \\
&\le \mu(y_0 + x_0 - y_0) \\
&\le \mu(x_0) \\
&= \inf_{n\in f^{-1}(f(x))} \mu(n) \\
&= \nu(f(x)),
\end{aligned}$$

and

$$\begin{aligned}
\nu\{f(u)\alpha(f(x) + f(v)] - f(u)\alpha f(v)\} &= \inf_{z\in f^{-1}\{f(u)\alpha(f(x)+f(v)]-f(u)\alpha f(v)\}} \mu(z) \\
&\le \mu(u_0\alpha(x_0 + v_0) - u_0\alpha v_0) \\
&\le \mu(x_0) \\
&= \inf_{n\in f^{-1}(f(x)} \mu(n) \\
&= v(f(x)).
\end{aligned}$$

Hence Γ-homomorphic image of an anti fuzzy left ideal which has the inf property is an anti fuzzy left ideal. $\square$

Theorem 3.11. *Let μ be an anti fuzzy left (resp. right) ideal of a Γ-near-ring M and μ^+ be a fuzzy set in M given by $\mu^+(x) = \mu(x) + 1 - \mu(1)$ for all $x \in M$. Then μ^+ is an anti fuzzy left (resp. right) ideal of M.*

Proof. Let μ be an anti fuzzy left ideal of a Γ-near-ring M. For all $x, y, u, v \in M$ and $\alpha \in \Gamma$, we have

$$\begin{aligned}
\mu^+(x - y) &= \mu(x - y) + 1 - \mu(1) \\
&\le \max\{\mu(x), \mu(y)\} + 1 - \mu(1) \\
&= \max\{\mu(x) + 1 - \mu(1), \mu(y) + 1 - \mu(1)\} \\
&= \max\{\mu^+(x), \mu^+(y)\},
\end{aligned}$$

$$\mu^+(y + x - y) = \mu(y + x - y) + 1 - \mu(1) \le \mu(x) + 1 - \mu(1) = \mu^+(x),$$

and

$$\begin{aligned}\mu^+(u\alpha(x+v)-u\alpha v) &= \mu(u\alpha(x+v)-u\alpha v)+1-\mu(1)\\ &\leq \mu(x)+1-\mu(1)\\ &= \mu^+(x).\end{aligned}$$

Hence μ^+ is an anti fuzzy left ideal of a Γ-near-ring M. □

Theorem 3.12. *Let M be a Γ-near-ring. Then a fuzzy set μ is a normal anti fuzzy left (resp. right) ideal of a Γ-near-ring M if and only if $\mu^+ = \mu$.*

Proof. Sufficient is direct. To prove necessary part. Suppose μ is normal anti fuzzy left (resp. right) ideal of M. Then $\mu^+(x) = \mu(x)+1-\mu(1) = \mu(x)+1-1 = \mu(x)$ for all $x \in M$. Hence $\mu^+ = \mu$. □

Theorem 3.13. *Let μ be an anti fuzzy left (resp. right) ideal of a Γ-near-ring M. Then $(\mu^+)^+ = \mu^+$.*

Proof. For any $x \in M$, we have $(\mu^+)^+(x) = \mu^+(x)+1-\mu^+(1) = \mu(x)+1-\mu(1) = \mu^+(x)$. Hence $(\mu^+)^+ = \mu^+$. □

Theorem 3.14. *Let μ be an anti fuzzy left (resp. right) ideal of a Γ-near-ring M and $\phi : [0, \mu(0)] \to [0,1]$ be an increasing function. Let μ_ϕ be a fuzzy set in M defined by $\mu_\phi(x) = \phi(\mu(x))$ for all $x \in M$. Then μ_ϕ is an anti fuzzy left (resp. right) ideal of M.*

Proof. Let $x, y, u, v \in M$ and $\alpha \in \Gamma$. Then

$$\begin{aligned}\mu_\phi(x-y) &= \phi(\mu(x-y))\\ &\leq \phi(\max(\mu(x), \mu(y))\\ &= \max\{\phi(\mu(x)), \phi(\mu(y))\}\\ &= \max\{\mu_\phi(x), \mu_\phi(y)\},\end{aligned}$$

$$\mu_\phi(y+x-y) = \phi(\mu(y+x-y)) \leq \phi(\mu(x)) = \mu_\phi(x),$$

and $\mu_\phi(u\alpha(x+v)-u\alpha v) = \phi(u\alpha(x+v)-u\alpha v)) \leq \phi(\mu(x)) = \mu_\phi(x)$. Hence μ_ϕ is an anti fuzzy left ideal of M. □

Theorem 3.15. *Let M be a Γ-near-ring. Let f be an endomorphism of M. If μ is an anti fuzzy left (resp. right) ideal of M, then so is μ^f.*

Proof. Let $x, y, u, v \in M$ and $\alpha \in \Gamma$. Then

$$\begin{aligned}\mu^f(x-y) &= \mu(f(x-y)) = \mu(f(x)-f(y))\\ &\leq \max\{\mu(f(x)), \mu(f(y))\}\\ &= \max\{\mu^f(x), \mu^f(y)\},\end{aligned}$$

$$\mu^f(y+x-y) = \mu(f(y+x-y)) = \mu(f(y)+f(x)-f(y)) \leq \mu(f(x)) = \mu^f(x)$$

and

$$\begin{aligned}
\mu^f(u\alpha(x+v)-u\alpha v) &= \mu(f(u\alpha(x+v)-u\alpha v)) \\
&= \mu(f(u)\alpha(f(x)+f(v))-f(u)\alpha f(v))) \\
&= \mu((f(u)\alpha f(x)+f(u)\alpha f(v))-f(u)\alpha f(v)) \\
&\leq \mu(f(x))=\mu^f(x).
\end{aligned}$$

Hence μ^f is an anti fuzzy left ideal of M. □

Theorem 3.16. *Let I be an ideal of a Γ-near-ring M and μ be an anti fuzzy left (resp. right) ideal of M. Then the fuzzy set $\bar{\mu}$ of M/I defined by*

$$\bar{\mu}(a+I)=\inf_{x\in I}\mu(a+x)$$

is an anti fuzzy left (resp. right) ideal of the Γ-residue class near-ring M/I of M with respect to I.

Proof. Clearly $\bar{\mu}$ is well defined. Let $x+I, y+I \in M/I$ and $\alpha\in\Gamma$. Then we have

$$\begin{aligned}
\bar{\mu}\{(x+I)-(y+I)\} &= \bar{\mu}\{(x-y)+I\}=\inf_{z\in I}\mu[(x-y)+z] \\
&= \inf_{z=u-v\in I}[\mu(x-y)+(u-v)] \\
&= \inf_{u,v\in I}\mu[(x+u)-(y+v)] \\
&\leq \max\{\inf_{u,v\in I}\{\mu(x+u),\mu(y+v)\}\} \\
&= \max\{\inf_{u\in I}\mu(x+u),\inf_{v\in I}\mu(y+v)\} \\
&= \max\{\bar{\mu}(x+I),\bar{\mu}(y+I)\},
\end{aligned}$$

and

$$\begin{aligned}
\bar{\mu}[(y+x-y)+I] &= \bar{\mu}[((y+x)-y)+I] \\
&= \inf_{z\in I}\mu[((y+x)-y)+z] \\
&= \inf_{z=u+v-w\in I}\mu[(y+x)-y+(u+v-w)] \\
&= \inf_{u,v,w\in I}\mu[(y+u)+(x+v)-(y+w)] \\
&\leq \inf_{v\in I}\mu(x+v) \\
&= \bar{\mu}(x+I).
\end{aligned}$$

Let $x,u,v\in M$ and $\alpha\in\Gamma$. Then

$$\begin{aligned}
\bar{\mu}[(u\alpha((x+v)+I)-(u\alpha v+I)] &= \bar{\mu}[(u\alpha(x+v)-u\alpha v)+I] \\
&= \inf_{z\in I}\mu[(u\alpha(x+v)-u\alpha v)+z] \\
&\leq \inf_{z\in I}\mu[x+z] \\
&= \bar{\mu}[x+I].
\end{aligned}$$

Hence $\bar{\mu}$ is an anti fuzzy left ideal of M. □

Theorem 3.17. *Let I be an ideal of a Γ-near-ring M. If $\bar{\mu}$ with $\bar{\mu}(a+I) = \mu(a)$, where $a \in M$, is an anti fuzzy left (resp. right) ideal of M/I, then μ is an anti fuzzy left (resp. right) ideal of M.*

Proof. Let I be an ideal of a Γ-near-ring M and $\bar{\mu}$ be an anti fuzzy left ideal of M/I. Let $x, y, u, v \in M$ and $\alpha \in \Gamma$, then we have

$$\begin{aligned} \mu(x-y) &= \bar{\mu}((x-y)+I) = \bar{\mu}((x+I)-(y+I)) \\ &\leq \max\{\bar{\mu}(x+I), \bar{\mu}(y+I)\} \\ &= \max\{\mu(x), \mu(y)\}, \end{aligned}$$

$$\begin{aligned} \mu(y+x-y) &= \bar{\mu}((y+x-y)+I) \\ &= \bar{\mu}[(y+I)+(x+I)-(y+I)] \\ &\leq \bar{\mu}(x+I) = \mu(x) \end{aligned}$$

and

$$\begin{aligned} \mu[u\alpha(x+v)-u\alpha v] &= \bar{\mu}[(u\alpha(x+v)-u\alpha v)+I] \\ &= \bar{\mu}[(u\alpha(x+v)+I)-(u\alpha v+I)] \\ &= \bar{\mu}[(u+I)\alpha[(x+I)+(v+I)]-(u+I)\alpha(v+I)] \\ &\leq \bar{\mu}(x+I) = \mu(x). \end{aligned}$$

Hence μ is an anti fuzzy left ideal of M. □

Lemma 3.18 ([13])**.** *Let M and N be Γ-near-rings and $f : M \to N$ be a homomorphism. Let μ be f-invariant fuzzy ideal of M. If $x = f(a)$ then $f(\mu)(x) = \mu(a)$ for all $a \in M$.*

Proof. Since μ is f-invariant, we have that $\mu(t) = \mu(a)$ for all $t, a \in M$. Let $f(\mu)$ be the image of μ under f. Then

$$\begin{aligned} f(\mu)(x) &= \sup_{t \in f^{-1}(x)} \mu(t), \quad \text{if} \quad f^{-1}(x) \neq \phi \\ &= \sup_{f(t)=x} \mu(t) \qquad (\textit{since } x = f(a)) \\ &= \sup_{f(t)=f(a)} \mu(t) \\ &= \sup_{f(t)=f(a)} \mu(a) = \mu(a). \end{aligned}$$

Hence $f(\mu)(x) = \mu(a)$ for all $a \in M$. □

Theorem 3.19. *Let $f : M \to N$ be an epimorphism of Γ-near-rings M and N. If μ is f-invariant anti fuzzy left (resp. right) ideal of M, then $f(\mu)$ is an anti fuzzy left (resp. right) ideal of N.*

Proof. Let $a, b \in N$, then there exist $x, y \in M$ such that $f(x) = a, f(y) = b$. Suppose μ is f-invariant anti fuzzy ideal of M, then by Lemma 3.18, we have

$$\begin{aligned} f(\mu)(a-b) &= f(\mu)(f(x)-f(y)) \\ &= f(\mu)(f(x-y)) \\ &= \mu(x-y) \\ &\leq \max\{\mu(x), \mu(y)\} \\ &= \max\{f(\mu)(a), f(\mu)(b)\} \end{aligned}$$

and

$$\begin{aligned} f(\mu)(b+a-b) &= f(\mu)(f(y)+f(x)-f(y)) \\ &= f(\mu)(f(y+x-y)) \\ &= \mu(y+x-y) \\ &\leq \mu(x) = f(\mu)(a). \end{aligned}$$

Let $a, r, s \in N$ and $\alpha \in \Gamma$. Then there exists $x, u, v \in M$ such that $f(x) = a$, $f(u) = r$ and $f(v) = s$. Now

$$\begin{aligned} f(\mu)[r\alpha(a+s) - r\alpha s] &= f(\mu)[f(u)\alpha(f(x)+f(v)) - f(u)\alpha f(v)] \\ &= f(\mu)[f(u\alpha(x+v) - u\alpha v)] \\ &= \mu[u\alpha(x+v) - u\alpha v] \\ &\leq \mu(x) = f(\mu)(a). \end{aligned}$$

Hence $f(\mu)$ is an anti fuzzy left ideal of N. □

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