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Generalized semi-pre connectedness in intuitionistic fuzzy topological spaces

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Abstract. In this paper we have introduced the intuitionistic fuzzy generalized semi-pre connected space, intuitionistic fuzzy generalized semi-pre super connected space and intuitionistic fuzzy generalized semi-pre extremally disconnected space. We investigated some of their properties. Also we characterized the intuitionistic fuzzy generalized semi-pre super connected space.

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1. Introduction

Zadeh [10] introduced the notion of fuzzy sets. Fuzzy topological space was introduced by Chang [2]. After that there have been a number of generalizations of this fundamental concept. Atanassov [1] introduced the notion of intuitionistic fuzzy sets. Using the notion of intuitionistic fuzzy sets, Çoker [3] introduced the notion of intuitionistic fuzzy topological space. Connectedness in intuitionistic fuzzy special topological spaces was introduced by Ösçağ and Çoker [7]. Jun and song [4] discussed intuitionistic fuzzy semi-pre opennes and intuitionitic fuzzy semi-pre continuity.

In this paper we have introduced intuitionistic fuzzy generalized semi-pre connected space, intuitionistic fuzzy generalized semi-pre super connected space and intuitionistic fuzzy generalized semi-pre extremally disconnected space. We investigated some of their properties. Also we characterized the intuitionistic fuzzy generalized semi-pre super connected space.

2. Preliminaries

Definition 2.1 ([1])**.** An intuitionistic fuzzy set (IFS for short) A in X is an object having the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$$

where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

Definition 2.2 ([1])**.** Let A and B be IFSs of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle : x \in X\}$. Then

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$;
(b) $A = B$ if and only if and only if $A \subseteq B$ and $B \subseteq A$;
(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}$;
(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle : x \in X\}$;
(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle : x \in X\}$.

The intuitionistic fuzzy sets $0_\sim = \{\langle x, 0, 1\rangle : x \in X\}$ and $1_\sim = \{\langle x, 1, 0\rangle : x \in X\}$ are respectively the empty set and the whole set of X.

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A\rangle$ instead of $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$.

Definition 2.3 ([3])**.** An intuitionistic fuzzy topology (IFT for short) on X is a family τ of IFSs in X satisfying the following axioms:

(i) $0_\sim, 1_\sim \in \tau$;
(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$;
(iii) $\cup G_i \in \tau$ for any family $\{G_i : i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS for short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS for short) in X. The complement A^c of an IFOS A in IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS for short) in X.

Definition 2.4 ([3])**.** An IFS $A = \langle x, \mu_A, \nu_A\rangle$ in an IFTS (X, τ) is said to be

(i) intuitionistic fuzzy semi-pre closed set (IFSPCS for short) if there exists an intuitionistic fuzzy pre closed set (IFPCS for short) B such that $\mathrm{int}(B) \subseteq A \subseteq B$;

(ii) intuitionistic fuzzy semi-pre open set (IFSPOS for short) if there exists an intuitionistic fuzzy pre open set (IFPOS for short) B such that $B \subseteq A \subseteq \mathrm{cl}(B)$.

Note that an IFS A is an IFSPCS if and only if $\mathrm{int}(\mathrm{cl}(\mathrm{int}(A))) \subseteq A$ [5].

Definition 2.5 ([5])**.** Let A be an IFS in an IFTS (X, τ). Then the semi-pre interior and the semi-pre closure of A are defined as

$$\mathrm{spint}(A) = \cup\{G : G \text{ is an IFSPOS in } X \text{ and } G \subseteq A\};$$
$$\mathrm{spcl}(A) = \cap\{K : K \text{ is an IFSPCS in } X \text{ and } A \subseteq K\}.$$

Note that for any IFS A in (X, τ), we have $\mathrm{spcl}(A^c) = [\mathrm{spint}(A)]^c$ and $\mathrm{spint}(A^c) = [\mathrm{spcl}(A)]^c$ [5].

Definition 2.6 ([8]). An IFS A is an intuitionistic fuzzy generalized closed set (IFGCS for short) if $\mathrm{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS. The complement of an IFGCS is called an intuitionistic fuzzy generalized open set (IFGOS for short).

Definition 2.7 ([5]). An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy generalized semi-pre closed set (IFGSPCS for short) if $\mathrm{spcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, τ).

Every IFCS, IFSPCS is an IFGSPCS but the separate converses may not be true in general [5].

Definition 2.8 ([5]). The complement A^c of an IFGSPCS A in an IFTS (X, τ) is called an intuitionistic fuzzy generalized semi-pre open set (IFGSPOS for short) in X.

Every IFOS, IFSPOS is an IFGSPOS but the separate converses may not be true in general [5].

Definition 2.9 ([5]). Let A be an IFS in an IFTS (X, τ). Then the generalized semi-pre interior and the generalized semi-pre closure of A are defined as

$$\mathrm{gspint}(A) = \cup\{G : G \text{ is an IFGSPOS in } X \text{ and } G \subseteq A\};$$
$$\mathrm{gspcl}(A) = \cap\{K : K \text{ is an IFGSPCS in } X \text{ and } A \subseteq K\}.$$

Note that for any IFS A in an IFTS (X, τ), we have $\mathrm{gspcl}(A^c) = [\mathrm{gspint}(A)]^c$ and $\mathrm{gspint}(A^c) = [\mathrm{gspcl}(A)]^c$ [5].

Definition 2.10 ([5]). If every IFGSPCS in an IFTS (X, τ) is IFSPCS in (X, τ), then the space can be called as an intuitionistic fuzzy semi-pre $T_{1/2}$ space (IFSP $T_{1/2}$ space for short).

Definition 2.11 ([6]). A mapping $f : (X, \tau) \to (Y, \sigma)$ is called an intuitionistic fuzzy generalized semi-pre continuous (IFGSP continuous for short) mapping if $f^{-1}(V)$ is an IFGSPCS in (X, τ) for every IFCS V of (Y, σ).

Definition 2.12 ([6]). A mapping $f : (X, \tau) \to (Y, \sigma)$ is called an intuitionistic fuzzy generalized semi-pre irresolute (IFGSP irresolute for short) mapping if $f^{-1}(V)$ is an IFGSPCS in (X, τ) for every IFGSPCS V of (Y, σ).

Definition 2.13 ([8]). Two IFSs A and B in X are said to be q-coincident ($A\mathrm{q}B$ for short) if and only if there exists an element $x \in X$ such that $\mu_A(x) > \nu_A(x)$ or $\nu_A(x) < \mu_B(x)$.

Definition 2.14 ([8]). Two IFSs A and B in X are said to be not q-coincident ($A\mathrm{q}^cB$ for short) if and only if $A \subseteq B^c$.

Definition 2.15 ([8]). An IFTS (X, τ) is said to be an IF $T_{1/2}$ space if every IFGCS in (X, τ) is an IFCS in (X, τ).

Definition 2.16 ([9]). An IFTS (X, τ) is said to be an intuitionistic fuzzy C_5-connected (IF C_5-connected for short) space if the only IFSs which are both intuitionistic fuzzy open and intuitionistic fuzzy closed are $0_\sim$ and $1_\sim$.

Definition 2.17 ([9]). An IFTS (X, τ) is said to be an intuitionistic fuzzy GO-connected (IFGO-connected for short) space if the only IFSs which are both intuitionistic fuzzy generalized open and intuitionistic fuzzy generalized closed are $0_\sim$ and $1_\sim$.

3. Intuitionistic fuzzy generalized semi-pre connected spaces

In this section we introduce intuitionistic fuzzy generalized semi-pre connected space and intuitionistic fuzzy generalized semi-pre super connected space. We investigate some of their properties. Also we provide a characterization theorem for an intuitionistic fuzzy generalized semi-pre super connected space.

Definition 3.1. An IFTS (X, τ) is said to be an intuitionistic fuzzy generalized semi-pre connected space if the only IFSs which are both intuitionistic fuzzy generalized semi-pre open and intuitionistic fuzzy generalized semi-pre closed are $0_\sim$ and $1_\sim$.

Example 3.2. Let $X = \{a, b\}$ and $\tau = \{0_\sim, M, 1_\sim\}$ be an IFT on X, where $M = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$. Then (X, τ) is an intuitionistic fuzzy generalized semi-pre connected space.

Theorem 3.3. *Every intuitionistic fuzzy generalized semi-pre connected space is an intuitionistic fuzzy C_5-connected space but not conversely.*

Proof. Let (X, τ) be an intuitionistic fuzzy generalized semi-pre connected space. Suppose (X, τ) is not an intuitionistic fuzzy C_5-connected space, then there exists a proper IFS A which both intuitionistic fuzzy open and intuitionistic fuzzy closed in (X, τ). That is A is both intuitionistic fuzzy generalized semi-pre open and intuitionistic fuzzy generalized semi-pre closed in (X, τ). This implies that (X, τ) is not an intuitionistic fuzzy generalized semi-pre connected space. This is a contradiction. Therefore (X, τ) is an intuitionistic fuzzy C_5-connected space. □

Example 3.4. Let $X = \{a, b\}$ and $\tau = \{0_\sim, M, 1_\sim\}$ be an IFT on X, where $M = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$. Then (X, τ) is an intuitionistic fuzzy C_5-connected space but not an intuitionistic fuzzy generalized semi-pre connected space, since the IFS M in τ is both intuitionistic fuzzy generalized semi-pre open and intuitionistic fuzzy generalized semi-pre closed in (X, τ).

Theorem 3.5. *Every intuitionistic fuzzy generalized semi-pre connected space is an intuitionistic fuzzy GO-connected space but not conversely.*

Proof. Let (X, τ) be an intuitionistic fuzzy generalized semi-pre connected space. Suppose (X, τ) is not an intuitionistic fuzzy GO-connected space, then there exists a proper IFS A which both intuitionistic fuzzy g-open and intuitionistic fuzzy g-closed in (X, τ). That is A is both intuitionistic fuzzy generalized semi-pre open and intuitionistic fuzzy generalized semi-pre closed in (X, τ). This implies that (X, τ) is not an intuitionistic fuzzy generalized semi-pre connected space. This is a contradiction. Therefore (X, τ) is an intuitionistic fuzzy GO-connected space. □

Example 3.6. In Example 3.4, (X, τ) is an intuitionistic fuzzy GO-connected space but not an intuitionistic fuzzy generalized semi-pre connected space.

The relation among various types of intuitionistic fuzzy connectedness is given in the following diagram.

$$\begin{tikzcd} & \text{IFGO-connected space} \arrow[dd] \\ \text{IFGSP connected space} \arrow[ur] \arrow[dr] & \\ & \text{IF}C_5\text{-connected space} \end{tikzcd}$$

The reverse implications are not true in general in the above diagram.

Theorem 3.7. *An IFTS (X, τ) is an intuitionistic fuzzy generalized semi-pre connected space if and only if there exist no non-zero intuitionistic fuzzy generalized semi-pre open sets A and B in (X, τ) such that $A = B^c$.*

Proof. Necessity: Let A and B be two intuitionistic fuzzy generalized semi-pre open sets in (X, τ) such that $A \neq 0_\sim \neq B$ and $A = B^c$. Therefore B^c is an intuitionistic fuzzy generalized semi-pre closed set. Since $A \neq 0_\sim$, $B \neq 1_\sim$. This implies B is a proper IFS which is both intuitionistic fuzzy generalized semi-pre open and intuitionistic fuzzy generalized semi-pre closed in (X, τ). Hence (X, τ) is not an intuitionistic fuzzy generalized semi-pre connected space. But this is a contradiction to our hypothesis. Thus there exist no non-zero intuitionistic fuzzy semi-pre open sets A and B in (X, τ) such that $A = B^c$.

Sufficiency: Let A be both intuitionistic fuzzy generalized semi-pre open and intuitionistic fuzzy generalized semi-pre closed in (X, τ) such that $1_\sim \neq A \neq 0_\sim$. Now let $B = A^c$. Then B is an intuitionistic fuzzy generalized semi-pre open set and $B \neq 1_\sim$. This implies $B = A^c \neq 0_\sim$, which is a contradiction to our hypothesis. Therefore (X, τ) is an intuitionistic fuzzy generalized semi-pre connected space. □

Theorem 3.8. *An IFTS (X, τ) is an intuitionistic fuzzy generalized semi-pre connected space if and only if there exist no non-zero intuitionistic fuzzy generalized semi-pre open sets A and B in (X, τ) such that $A = B^c$, $B = (\mathrm{spcl}(A))^c$ and $A = (\mathrm{spcl}(B))^c$.*

Proof. Necessity: Assume that there exist IFSs A and B such that $A \neq 0_\sim \neq B$, $B = A^c$, $B = (\mathrm{spcl}(A))^c$ and $A = (\mathrm{spcl}(B))^c$. Since $(\mathrm{spcl}(A))^c$ and $(\mathrm{spcl}(B))^c$ are intuitionistic fuzzy generalized semi-pre open sets in (X, τ), A and B are intuitionistic fuzzy generalized semi-pre open sets in (X, τ). This implies (X, τ) is not an intuitionistic fuzzy generalized semi-pre connected space, which is a contradiction. Therefore there exist no non-zero intuitionistic fuzzy generalized semi-pre open sets A and B in (X, τ) such that $A = B^c$, $B = (\mathrm{spcl}(A))^c$ and $A = (\mathrm{spcl}(B))^c$.

Sufficiency: Let A be both intuitionistic fuzzy generalized semi-pre open and intuitionistic fuzzy generalized semi-pre closed in (X, τ) such that $1_\sim \neq A \neq 0_\sim$. Now by taking $B = A^c$, we obtain a contradiction to our hypothesis. Hence (X, τ) is an intuitionistic fuzzy generalized semi-pre connected space. □

Definition 3.9. An IFTS (X, τ) is said to be an intuitionistic fuzzy semi-pre $T^*_{1/2}$ space if every intuitionistic fuzzy generalized semi-pre closed set is an intuitionistic fuzzy closed set in (X, τ).

Example 3.10. In Example 3.2, the IFTS (X, τ) is an intuitionistic fuzzy semi-pre $T^*_{1/2}$ space.

Remark 3.11. Every intuitionistic fuzzy semi-pre $T^*_{1/2}$ space is an intuitionistic fuzzy semi-pre $T_{1/2}$ space but not conversely.

Proof. Let (X, τ) be an intuitionistic fuzzy semi-pre $T^*_{1/2}$ space. Let A be an intuitionistic fuzzy generalized semi-pre closed set in (X, τ). By hypothesis A is an intuitionistic fuzzy closed set. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy semi-pre closed set, A is an intuitionistic fuzzy semi-pre closed set in (X, τ). Hence (X, τ) is an intuitionistic fuzzy semi-pre $T_{1/2}$ space. □

Example 3.12. In Example 3.4, (X, τ) is an intuitionistic fuzzy generalized semi-pre $T_{1/2}$ space, but not an intuitionistic fuzzy semi-pre $T^*_{1/2}$ space, since the IFS M is an intuitionistic fuzzy generalized semi-pre closed set in (X, τ) but not an intuitionistic fuzzy closed set in (X, τ), since $\mathrm{cl}(M) = M^c \neq M$.

Remark 3.13. Every intuitionistic fuzzy semi-pre $T^*_{1/2}$ space is an intuitionistic fuzzy $T_{1/2}$ space but not conversely.

Proof. Let (X, τ) be an intuitionistic fuzzy semi-pre $T^*_{1/2}$ space. Let A be an intuitionistic fuzzy generalized closed set in (X, τ). Since every intuitionistic fuzzy generalized closed set is an intuitionistic fuzzy generalized semi-pre closed set, A is an intuitionistic fuzzy generalized semi-pre closed set in (X, τ). By hypothesis, A is an intuitionistic fuzzy closed set. Hence (X, τ) is an intuitionistic fuzzy $T_{1/2}$ space. □

Example 3.14. In Example 3.4, the IFTS (X, τ) is an intuitionistic fuzzy $T_{1/2}$ space, but not an intuitionistic fuzzy semi-pre $T^*_{1/2}$ space, since the IFS M is an intuitionistic fuzzy generalized semi-pre closed set in (X, τ) but not an intuitionistic fuzzy closed set in (X, τ), since $\mathrm{cl}(M) = M^c \neq M$.

Theorem 3.15. *Let (X, τ) be an intuitionistic fuzzy semi-pre $T^*_{1/2}$ space, then the following are equivalent.*

(i) (X, τ) *is an intuitionistic fuzzy generalized semi-pre connected space.*
(ii) (X, τ) *is an intuitionistic fuzzy GO-connected space.*
(iii) (X, τ) *is an intuitionistic fuzzy C_5-connected space.*

Proof. (i)⇒(ii) is obvious from Theorem 3.5.

(ii)⇒(iii) is obvious from [8].

(iii)⇒(i) Let (X, τ) be an intuitionistic fuzzy C_5-connected space. Suppose (X, τ) is not an intuitionistic fuzzy generalized semi-pre connected space, then there exists a proper IFS A in (X, τ) which is both intuitionistic fuzzy generalized semi-pre open and intuitionistic fuzzy generalized semi-pre closed in (X, τ). But since (X, τ) is an intuitionistic fuzzy semi-pre $T^*_{1/2}$ space, A is both intuitionistic fuzzy open and intuitionistic fuzzy closed in (X, τ). This implies that (X, τ) is not an intuitionistic fuzzy C_5-connected space, which is a contradiction to our hypothesis. Therefore (X, τ) is intuitionistic fuzzy generalized semi-pre connected space. □

Theorem 3.16. *If $f : (X, \tau) \to (Y, \sigma)$ is an intuitionistic fuzzy semi-pre continuous surjection and (X, τ) is an intuitionistic fuzzy generalized semi-pre connected space, then (Y, σ) is an intuitionistic fuzzy C_5-connected space.*

Proof. Let (X, τ) be an intuitionistic fuzzy generalized semi-pre connected space. Suppose (Y, σ) is not an intuitionistic fuzzy C_5-connected space, then there exists a proper IFS A which is both intuitionistic fuzzy open and intuitionistic fuzzy closed in (Y, σ). Since f is an intuitionistic fuzzy semi-pre continuous mapping, $f^{-1}(A)$ is both intuitionistic fuzzy generalized semi-pre open and intuitionistic fuzzy generalized semi-pre closed in (X, τ). But this is a contradiction to hypothesis. Hence (Y, σ) is an intuitionistic fuzzy C_5-connected space. □

Theorem 3.17. *If $f : (X, \tau) \to (Y, \sigma)$ is an intuitionistic fuzzy semi-pre irresolute surjection and (X, τ) is an intuitionistic fuzzy generalized semi-pre connected space, then (Y, σ) is also an intuitionistic fuzzy generalized semi-pre connected space.*

Proof. Suppose (Y, σ) is not an intuitionistic fuzzy generalized semi-pre connected space, then there exists a proper IFS A which is both intuitionistic fuzzy generalized semi-pre open and intuitionistic fuzzy generalized semi-pre closed in (Y, σ). Since f is an intuitionistic fuzzy semi-pre irresolute mapping, $f^{-1}(A)$ is both intuitionistic fuzzy generalized semi-pre open and intuitionistic fuzzy generalized semi-pre closed in (X, τ). But this is a contradiction to hypothesis. Hence (Y, σ) is an intuitionistic fuzzy generalized semi-pre connected space. □

Definition 3.18. An IFTS (X, τ) is called intuitionistic fuzzy C_5-connected between two IFSs A and B if there is no intuitionistic fuzzy open set E in (X, τ) such that $A \subseteq E$ and $E\mathrm{q}^c B$.

Definition 3.19. An IFTS (X, τ) is called intuitionistic fuzzy generalized semi-pre connected between two IFSs A and B if there is no intuitionistic fuzzy generalized semi-pre open set E in (X, τ) such that $A \subseteq E$ and $E\mathrm{q}^c B$.

Example 3.20. Let $X = \{a, b\}$ and $\tau = \{0_\sim, M, 1_\sim\}$ be an IFT on X, where $M = \langle x, (0.5_a, 0.3_b), (0.5_a, 0.1_b)\rangle$. Then (X, τ) is an intuitionistic fuzzy generalized semi-pre connected between the two IFSs $A = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.3_b)\rangle$ and $B = \langle x, (0.5_a, 0.2_b), (0.5_a, 0.5_b)\rangle$.

Theorem 3.21. *If an IFTS (X, τ) is intuitionistic fuzzy generalized semi-pre connected between two IFSs A and B, then it is intuitionistic fuzzy C_5-connected between two IFSs A and B but the converse my not be true in general.*

Proof. Suppose (X, τ) is not intuitionistic fuzzy C_5-connected between A and B, then there exists an intuitionistic fuzzy open set E in (X, τ) such that $A \subseteq E$ and $E\mathrm{q}^c B$. Since every intuitionistic fuzzy open set is intuitionistic fuzzy generalized semi-pre open set, there exists an intuitionistic fuzzy generalized semi-pre open set E in (X, τ) such that $A \subseteq E$ and $E\mathrm{q}^c B$. This implies (X, τ) is not intuitionistic fuzzy generalized semi-pre connected between A and B, a contradiction to our hypothesis. Therefore (X, τ) is intuitionistic fuzzy C_5-connected between A and B. □

Example 3.22. Let $X = \{a, b\}$ and $\tau = \{0_\sim, M, 1_\sim\}$ be an IFT on X, where $M = \langle x, (0.4_a, 0.3_b), (0.2_a, 0.3_b)\rangle$. Then (X, τ) is an intuitionistic fuzzy C_5-connected between the IFSs $A = \langle x, (0.4_a, 0.4_b), (0.6_a, 0.6_b)\rangle$ and $B = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$. But (X, τ) is not an intuitionistic fuzzy generalized semi-pre connected between A and B, since the IFS $E = \langle x, (0.4_a, 0.4_b), (0.5_a, 0.5_b)\rangle$ is an intuitionistic fuzzy generalized semi-pre open set such that $A \subseteq E$ and $E \subseteq B^c$.

Theorem 3.23. *An IFTS (X, τ) is intuitionistic fuzzy generalized semi-pre connected between two IFSs A and B if and only if there is no intuitionistic fuzzy generalized semi-pre open and intuitionistic fuzzy generalized semi-pre closed set E in (X, τ) such that $A \subseteq E \subseteq B^c$.*

Proof. Necessity: Let (X, τ) be intuitionistic fuzzy generalized semi-pre connected between two IFSs A and B. Suppose that there exists an intuitionistic fuzzy generalized semi-pre open and intuitionistic fuzzy generalized semi-pre closed set E in (X, τ) such that $A \subseteq E \subseteq B^c$, then $E\mathrm{q}^cB$ and $A \subseteq E$. This implies (X, τ) is not intuitionistic fuzzy generalized semi-pre connected between A and B, by Definition 3.19. A contradiction to our hypothesis. Therefore there is no intuitionistic fuzzy generalized semi-pre open and intuitionistic fuzzy generalized semi-pre closed set E in (X, τ) such that $A \subseteq E \subseteq B^c$.

Sufficiency: Suppose that (X, τ) is not intuitionistic fuzzy generalized semi-pre connected between A and B. Then there exists an intuitionistic fuzzy generalized semi-pre open set E in (X, τ) such that $A \subseteq E$ and $E\mathrm{q}^cB$. This implies that there is no intuitionistic fuzzy generalized semi-pre open set E in (X, τ) such that $A \subseteq E \subseteq B^c$. But this is a contradiction to our hypothesis. Hence (X, τ) is intuitionistic fuzzy generalized semi-pre connected between A and B. □

Theorem 3.24. *If an IFTS (X, τ) is intuitionistic fuzzy generalized semi-pre connected between two IFSs A and B, $A \subseteq A_1$ and $B \subseteq B_1$, then (X, τ) is intuitionistic fuzzy generalized semi-pre connected between A_1 and B_1.*

Proof. Suppose that (X, τ) is not intuitionistic fuzzy generalized semi-pre connected between A_1 and B_1, then by Definition 3.19, there exists an intuitionistic fuzzy generalized semi-pre open set E in (X, τ) such that $A_1 \subseteq E$ and $E\mathrm{q}^cB_1$. This implies $E \subseteq B_1^c$ and $A_1 \subseteq E$ implies $A \subseteq A_1 \subseteq E$. That is $A \subseteq E$. Now let us prove that $E \subseteq B^c$, that is let us prove $E\mathrm{q}^cB$. Suppose that $E\mathrm{q}B$, then by Definition 2.12, there exists an element x in X such that $\mu_E(x) > \nu_B(x)$ and $\nu_E(x) < \mu_B(x)$. Therefore $\mu_E(x) > \nu_B(x) > \nu_{B_1}(x)$ and $\nu_E(x) < \mu_B(x) < \mu_{B_1}(x)$, since $B \subseteq B_1$. Thus $E\mathrm{q}B_1$. But $E \subseteq B_1$. That is $E\mathrm{q}^cB_1$, which is a contradiction. Therefore $E\mathrm{q}^cB$. That is $E \subseteq B^c$. Hence (X, τ) is not intuitionistic fuzzy generalized semi-pre connected between A and B, which is a contradiction to our hypothesis. Thus (X, τ) is intuitionistic fuzzy generalized semi-pre connected between A_1 and B_1. □

Theorem 3.25. *Let (X, τ) be an IFTS and A and B be IFSs in (X, τ). If $A\mathrm{q}B$, then (X, τ) is intuitionistic fuzzy generalized semi-pre connected between A and B.*

Proof. Suppose (X, τ) is not intuitionistic fuzzy generalized semi-pre connected between A and B. Then there exists an intuitionistic fuzzy generalized semi-pre open set E in (X, τ) such that $A \subseteq E$ and $E \subseteq B^c$. This implies that $A \subseteq B^c$. That is

$A\text{q}^c B$. But this is a contradiction to our hypothesis. Therefore (X, τ) is intuitionistic fuzzy generalized semi-pre connected between A and B. □

Remark 3.26. The converse of the above theorem may not be true in general.

Example 3.27. In Example 3.20, (X, τ) is intuitionistic fuzzy generalized semi-pre connected between the IFSs A and B but not q-coincident with B, since $\mu_A(x) < \nu_B(x)$ and $\mu_B(x) < \nu_A(x)$.

Definition 3.28. An intuitionistic fuzzy generalized semi-pre open set A is called an intuitionistic fuzzy regular generalized semi-pre open set if $A = \text{gspint}(\text{gspcl}(A))$. The complement of an intuitionistic fuzzy regular generalized semi-pre open set is called an an intuitionistic fuzzy regular generalized semi-pre closed set.

Definition 3.29. An IFTS (X, τ) is called an intuitionistic fuzzy generalized semi-pre super connected space if there exists no intuitionistic fuzzy regular generalized semi-pre open set in (X, τ).

Theorem 3.30. *Let (X, τ) be an IFTS, then the following are equivalent.*
(i) *(X, τ) is an intuitionistic fuzzy generalized semi-pre super connected space.*
(ii) *For every non-zero intuitionistic fuzzy regular generalized semi-pre open set A, $\text{gspcl}(A) = 1_\sim$.*
(iii) *For every intuitionistic fuzzy regular generalized semi-pre closed set A with $A \neq 1_\sim$, $\text{gspint}(A) = 0_\sim$.*
(iv) *There exists no intuitionistic fuzzy regular generalized semi-pre open sets A and B in (X, τ) such that $A \neq 0_\sim \neq B$, $A \subseteq B^c$.*
(v) *There exists no intuitionistic fuzzy regular generalized semi-pre open sets A and B in (X, τ) such that $A \neq 0_\sim \neq B$, $B = (\text{gspcl}(A))^c$, $A = (\text{gspcl}(B))^c$.*
(vi) *There exists no intuitionistic fuzzy regular generalized semi-pre closed sets A and B in (X, τ) such that $A \neq 1_\sim \neq B$, $B = (\text{gspint}(A))^c$, $A = (\text{gspint}(B))^c$.*

Proof. (i)⇒(ii) Assume that there exists an intuitionistic fuzzy regular generalized semi-pre open set A in (X, τ) such that $A \neq 0_\sim$ and $\text{gspcl}(A) \neq 1_\sim$. Now let $B = \text{gspint}(\text{gspcl}(A))^c$. Then B is a proper intuitionistic fuzzy regular generalized semi-pre open set in (X, τ). But this is a contradiction to the fact that (X, τ) is an intuitionistic fuzzy generalized semi-pre super connected space. Therefore $\text{gspcl}(A) = 1_\sim$.

(ii)⇒(iii) Let $A \neq 1_\sim$ be an intuitionistic fuzzy regular generalized semi-pre closed set in (X, τ). If $B = A^c$, then B is an intuitionistic fuzzy regular generalized semi-pre open set in (X, τ) with $B \neq 0_\sim$. Hence $\text{gspcl}(B) = 1_\sim$. This implies $(\text{gspcl}(B))^c = 0_\sim$. That is $\text{gspint}(B^c) = 0_\sim$. Hence $\text{gspint}(A) = 0_\sim$.

(iii)⇒(iv) Let A and B be two intuitionistic fuzzy regular generalized semi-pre open sets in (X, τ) such that $A \neq 0_\sim \neq B$, $A \subseteq B^c$. Since B^c is an intuitionistic fuzzy regular generalized semi-pre closed set in (X, τ) and $B \neq 0_\sim$ implies $B^c \neq 1_\sim$, $B^c = \text{gspcl}(\text{gspint}(B^c))$ and we have $\text{gspint}(B^c) = 0_\sim$. But $A \subseteq B^c$. Therefore $0_\sim \neq A = \text{gspint}(\text{gspcl}(A)) \subset \text{gspint}(\text{gspcl}(B^c)) = \text{gspint}(\text{gspcl}(\text{gspcl}(\text{gspnit}(B^c)))) = \text{gspint}(\text{gspcl}(\text{gspint}(B^c))) = \text{gspnit}(B^c) = 0_\sim$. A contradiction arises. Therefore (iv) is true.

(iv)$\Rightarrow$(i) Let $0_\sim \neq A \neq 1_\sim$ be an intuitionistic fuzzy regular generalized semi-pre open set in (X, τ). If we take $B = (\text{gspcl}(A))^c$, then B is an intuitionistic fuzzy regular generalized semi-pre open set, since $\text{gspint}(\text{gspcl}(B)) = \text{gspint}(\text{gspcl}(\text{gspcl}(A))^c) = \text{gspint}(\text{gspint}(\text{gspcl}(A)))^c = \text{gspint}(A^c) = (\text{gspcl}(A))^c = B$. Also we get $B \neq 0_\sim$, since otherwise, we have $B = 0_\sim$ and this implies $(\text{gspcl}(A))^c = 0_\sim$. This is $\text{gspcl}(A) = 1_\sim$. Hence $A = \text{gspint}(\text{gspcl}(A)) = \text{gspint}(1_\sim) = 1_\sim$. This is $A = 1_\sim$, which is a contradiction. Therefore $B \neq 0_\sim$ and $A \subseteq B^c$. But this is a contradiction to (iv). Therefore (X, τ) is an intuitionistic fuzzy generalized semi-pre super connected space.

(i)$\Rightarrow$(v) Let A and B be two intuitionistic fuzzy regular generalized semi-pre open sets in (X, τ) such that $A \neq 0_\sim \neq B$, $B = (\text{gspcl}(A))^c$ and $A = (\text{gspcl}(B))^c$. Now we have $\text{gspint}(\text{gspcl}(A)) = \text{gspint}(B^c) = (\text{gspcl}(B))^c = A$, $A \neq 0_\sim$ and $A \neq 1_\sim$, since if $A = 1_\sim$, then $1_\sim = (\text{gspcl}(B))^c \Rightarrow \text{gspcl}(B) = 0_\sim \Rightarrow B = 0_\sim$. But $B \neq 0_\sim$. Therefore $A \neq 1_\sim \Rightarrow A$ is proper intuitionistic fuzzy regular generalized semi-pre open set in (X, τ), which is a contradiction to (i). Hence (v) is true.

(v)$\Rightarrow$(i) Let A be an intuitionistic fuzzy regular generalized semi-pre open set in (X, τ) such that $A = \text{gspnit}(\text{gspcl}(A))$ and $0_\sim \neq A \neq 1_\sim$. Now take $B = (\text{gspcl}(A))^c$. In this case we get $B \not 0_\sim$ and B is intuitionistic fuzzy regular generalized semi-pre open set in (X, τ), $B = (\text{gspcl}(A))^c$ and $(\text{gspcl}(B))^c = (\text{gspcl}(\text{gspcl}(A))^c)^c = \text{gspint}(\text{gspcl}(A)^c)^c = \text{gspint}(\text{gspcl}(A)) = A$. But this is a contradiction to (v). Therefore (X, τ) is an intuitionistic fuzzy generalized semi-pre super connected space.

(v)$\Rightarrow$(vi) Let A and B be two intuitionistic fuzzy regular generalized semi-pre closed sets in (X, τ) such that $A \neq 1_\sim \neq B$, $B = (\text{gspint}(A))^c$ and $A = (\text{gspint}(B))^c$. Taking $C = A^c$ and $D = B^c$, C and D become intuitionistic fuzzy regular generalized semi-pre open sets in (X, τ) with $C \neq 0_\sim \neq D$, $D = (\text{gspcl}(C))^c$ and $C = (\text{gspcl}(D))^c$, which is a contradiction to (v). Hence (vi) is true.

(vi)$\Rightarrow$(v) can be easily proved by the similar way as in (v)$\Rightarrow$(vi). $\square$

Definition 3.31. An IFTS (X, τ) is said to be an intuitionistic fuzzy generalized semi-pre extremally disconnected if the generalized semi-pre closure of every intuitionistic fuzzy generalized semi-pre open set in (X, τ) is an intuitionistic fuzzy generalized semi-pre open set.

Theorem 3.32. *Let (X, τ) be an intuitionistic fuzzy semi-pre $T_{1/2}$ space, then the following are equivalent.*

(i) *(X, τ) is an intuitionistic fuzzy generalized semi-pre extremally disconnected space.*

(ii) *For each intuitionistic fuzzy generalized semi-pre closed set A, $\text{gspint}(A)$ is an intuitionistic fuzzy generalized semi-pre closed set.*

(iii) *For each intuitionistic fuzzy generalized semi-pre open set A, $\text{gspcl}(A) = (\text{gspcl}(\text{gspcl}(A))^c)^c$.*

(iv) *For each intuitionistic fuzzy generalized semi-pre open sets A and B with $\text{gspcl}(A) = B^c$, $\text{gspcl}(A) = (\text{gspcl}(B))^c$.*

Proof. (i)$\Rightarrow$(ii) Let A be any intuitionistic fuzzy generalized semi-pre closed set. Then A^c is an intuitionistic fuzzy generalized semi-pre open set. So (i) implies that $\text{gspcl}(A^c) = (\text{gspint}(A))^c$ is an intuitionistic fuzzy generalized semi-pre open set. Thus $\text{gspcl}(A)$ is an intuitionistic fuzzy generalized semi-pre closed set in (X, τ).

(ii)⇒(iii) Let A be an intuitionistic fuzzy generalized semi-pre open set. Then we have $\mathrm{gspcl}(\mathrm{gspcl}(A))^c = \mathrm{gspcl}(\mathrm{gspint}(A^c))$. Therefore $(\mathrm{gspcl}(\mathrm{gspcl}(A))^c)^c = (\mathrm{gspcl}(\mathrm{gspint}(A^c)))^c$. Since A is an intuitionistic fuzzy generalized semi-pre open set, A^c is an intuitionistic fuzzy generalized semi-pre closed set. So by (ii) $\mathrm{gspint}(A^c)$ is an intuitionistic fuzzy generalized semi-pre closed set. That is $\mathrm{gspcl}(\mathrm{gspint}(A^c)) = \mathrm{gspint}(A^c)$. Hence $(\mathrm{gspcl}(\mathrm{gspint}(A^c)))^c = (\mathrm{gspint}(A^c))^c = \mathrm{gspcl}(A)$.

(iii)⇒(iv) Let A and B be any two intuitionistic fuzzy generalized semi-pre open sets in (X, τ) such that $\mathrm{gspcl}(A) = B^c$. (iii) implies $\mathrm{gspcl}(A) = (\mathrm{gspcl}(\mathrm{gspcl}(A))^c)^c = (\mathrm{gspcl}(B^c)^c)^c = (\mathrm{gspcl}(B))^c$.

(iv)⇒(i) Let A be any intuitionistic fuzzy generalized semi-pre open set in (X, τ). Put $B = (\mathrm{gspcl}(A))^c$. Then $\mathrm{gspcl}(A) = B^c$. Hence by (iv), $\mathrm{gspcl}(A) = (\mathrm{gspcl}(B))^c$. Since $\mathrm{gspcl}(B)$ is an intuitionistic fuzzy generalized semi-pre closed set as the space is an intuitionistic fuzzy semi-pre $T_{1/2}$ space, it follows that $\mathrm{gspcl}(A)$ is an intuitionistic fuzzy generalized semi-pre open set. This implies that (X, τ) is an intuitionistic fuzzy generalized semi-pre extremally disconnected space. □

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