Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 3, No. 2, (April 2012), pp. 203- 211
ISSN 2093–9310
http://www.afmi.or.kr

Rough prime bi-ideals and rough fuzzy prime bi-ideals in semigroups

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Received 9 April 2011; Revised 24 July 2011; Accepted 11 August 2011

Abstract. In this paper, we introduced the notion of rough prime bi-ideals and rough fuzzy prime bi-ideals of semigroups. We proved that the lower and the upper approximation of a prime bi-ideal is a prime bi-ideal and we also proved that a fuzzy subset f of a semigroup S is a fuzzy prime bi-ideal of S iff $f_\lambda \neq \varnothing$ ($f^s_\lambda \neq \varnothing$) is a prime bi-ideal of S for every $\lambda \in [0,1]$ and also proved that if f is a fuzzy prime bi-ideal of a semigroup S and for a complete congruence on S, f is a rough fuzzy prime bi-ideal of S.

2010 AMS Classification: 18B40, 20M12

Keywords: Prime bi-ideals, Rough prime bi-ideals, Rough fuzzy prime bi-ideals.

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1. Introduction

The notion of a rough set was originally proposed by Pawlak [13, 14] as a formal tool for modeling and processing incomplete information in information systems. The theory of rough set is an extension of set theory. The equivalence classes are the building blocks for the construction of the lower and upper approximations. The lower approximation of a given set is the union of all equivalence classes which are subsets of the set, and the upper approximation is the union of all equivalence classes which have a nonempty intersection with the set. Some authors have studied the algebraic properties of rough sets. Aslam et al. [1], Biswas and Nanda [3], Chinram [4], Davvaz [5], Jun [7], Kuroki and mordeson [8], Kuroki [9], Petchkhaew and Chinram [15] and Xiao et al. [18], applied roughness in different algebraic structures.

A fuzzy subset f of a set S is a function from S to a closed interval $[0,1]$, this concept of a fuzzy set was introduced by Zadeh [19], in 1965. Rosenfeld [16], was the first who studied fuzzy sets in the structure of groups. Kuroki [10, 11], studied the fuzzy ideals and fuzzy bi-ideals in semigroups. Banerjee [2], gave the concept of roughness of a fuzzy set. Also see [6].

The notion of prime bi-ideals of groupoids was studied by Lee [12]. Further, many authors studied the prime bi-ideals in different structures. Also Shabir et al. [17], studied prime bi-ideals of semigroups.

This paper concerns the relationship between rough sets and fuzzy sets in semigroups. In this paper, we study ρ-lower and ρ-upper rough prime bi-ideals and ρ-lower and ρ-upper rough fuzzy prime bi-ideals in semigroups.

2. Preliminaries

Let S be a semigroup. A nonempty subset T of S is called a subsemigroup of S if $ab \in T$, for all $a, b \in T$. A nonempty subset L of S is called a left ideal of S if $SL \subseteq L$ and a nonempty subset R of S is called a right ideal of S if $RS \subseteq R$. A nonempty subset I of S is called an ideal of S if I is both a left and a right ideal of S. A subsemigroup B of S is called a bi-ideal of S if $BSB \subseteq B$.

Let S denote a semigroup unless otherwise specified. Let ρ be a congruence relation on S, that is, ρ is an equivalence relation on S such that

$$(a, b) \in \rho \text{ implies } (ax, bx) \in \rho \text{ and } (xa, xb) \in \rho$$

for all $x \in S$. If ρ is a congruence relation on S, then for every $x \in S$, $[x]_\rho$ stands for the congruence class of x with respect to ρ. A congruence ρ on S is called complete if $[a]_\rho[b]_\rho = [ab]_\rho$ for all $a, b \in S$.

Let A be a nonempty subset of a semigroup S and ρ be a congruence relation on S. Then the sets

$$\rho_-(A) = \left\{x \in S : [x]_\rho \subseteq A\right\} \quad \text{and} \quad \rho^-(A) = \left\{x \in S : [x]_\rho \cap A \neq \varnothing\right\}$$

are called ρ-lower and ρ-upper approximations of A respectively. For a nonempty subset A of S,

$$\rho(A) = (\rho_-(A), \rho^-(A))$$

is called a rough set with respect to ρ if $\rho_-(A) \neq \rho^-(A)$.

A subset A of a semigroup S is called a ρ-upper [ρ-lower] rough bi-ideal of S if $\rho^-(A)$ [$\rho_-(A)$] is a bi-ideal of S.

The following theorems are proved in [9].

Theorem 2.1. *Let ρ be a complete congruence relation on a semigroup S. If A is a bi-ideal of S, then $\rho_-(A)$ is, if it is nonempty, a bi-ideal of S.*

Theorem 2.2. *Let ρ be a congruence relation on a semigroup S. If A is a bi-ideal of S, then it is a ρ-upper rough bi-ideal of S.*

A function f from S to the unit interval $[0, 1]$ is called a fuzzy subset of S. A fuzzy subset f of a semigroup S is called a fuzzy subsemigroup of S if $f(xy) \geq f(x) \wedge f(y)$ for all $x, y \in S$. A fuzzy subsemigroup f of a semigroup S is called a fuzzy bi-ideal of S if $f(xay) \geq f(x) \wedge f(y)$ for all $x, y, a \in S$. Let f be a fuzzy subset of S, $\lambda \in [0, 1]$. Then the sets

$$f_\lambda = \{x \in S : f(x) \geq \lambda\} \quad \text{and} \quad f^s_\lambda = \{x \in S : f(x) > \lambda\}$$

are called, respectively, λ-levelset and λ-strong levelset of the fuzzy set f.

In [10], the following theorem is proved.

Theorem 2.3. *Let f be a fuzzy subset of a semigroup S. Then f is a fuzzy bi-ideal of S iff f_λ, f_λ^s are, if they are nonempty, bi-ideals of S for every $\lambda \in [0,1]$.*

3. Rough prime bi-ideals in semigroups

A bi-ideal B of a semigroup S is said to be a prime bi-ideal of S, if for $x, y \in S$, $xay \in B$ implies $x \in B$ or $y \in B$ for all $a \in S$. Let ρ be a congruence relation on a semigroup S. Then a subset A of S is called a ρ-lower rough prime bi-ideal of S if $\rho_-(A)$ is a prime bi-ideal of S. A ρ-upper rough prime bi-ideal of S is defined analogously. A is called a rough prime bi-ideal of S if A is a ρ-lower and a ρ-upper rough prime bi-ideal of S.

Theorem 3.1. *Let ρ be a complete congruence relation on a semigroup S and A be a prime bi-ideal of S. Then $\rho_-(A)$ is, if it is nonempty, a prime bi-ideal of S.*

Proof. Since A is a bi-ideal of S, by Theorem 2.1, we know that $\rho_-(A)$ is a bi-ideal of S. Let a be any arbitrary element of S, then for

$$xay \in \rho_-(A) \quad \text{for some} \quad x, y \in S,$$

then

$$[x]_\rho [a]_\rho [y]_\rho = [xay]_\rho \subseteq A.$$

We suppose that $\rho_-(A)$ is not a prime bi-ideal, then there exist $x, y \in S$ and an arbitrary element $a \in S$, such that $xay \in \rho_-(A)$ but $x \notin \rho_-(A)$ and $y \notin \rho_-(A)$. Thus

$$[x]_\rho \nsubseteq A \quad \text{and} \quad [y]_\rho \nsubseteq A,$$

then there exist

$$x' \in [x]_\rho\ ,\ x' \notin A \quad \text{and} \quad y' \in [y]_\rho\ ,\ y' \notin A.$$

Thus

$$x'ay' \in [x]_\rho [a]_\rho [y]_\rho \subseteq A.$$

Since A is a prime bi-ideal, we have $x' \in A$ or $y' \in A$. It contradicts the supposition. This means that $\rho_-(A)$ is, if it is nonempty, a prime bi-ideal of S. □

The following example shows that the ρ-upper approximation of a prime bi-ideal is not a prime bi-ideal in general.

Example 3.2. Let $S = \{0, 1, 2, 3\}$ be a semigroup. Define a binary operation "·" in S as follows:

$\cdot$	0	1	2	3
0	0	2	2	3
1	2	1	2	3
2	2	2	2	3
3	3	3	3	3

Let ρ be a congruence relation on S such that ρ-congruence classes are the subsets $\{0\}$, $\{1\}$, $\{2, 3\}$, then for $A = \{3\} \subseteq S$, $\rho^-(A) = \{2, 3\}$. It is clear that A is a prime bi-ideal of S. The set $\rho^-(A)$ is not a prime bi-ideal for $0 \cdot 2 \cdot 1 = 2 \in \rho^-(A)$ but $0 \notin \rho^-(A)$ and $1 \notin \rho^-(A)$.

Theorem 3.3. *Let ρ be a complete congruence relation on a semigroup S. If A is a prime bi-ideal of S, then A is a ρ-upper rough prime bi-ideal of S.*

Proof. Since A is a bi-ideal of S, then by Theorem 2.2, $\rho^-(A)$ is bi-ideal of S. Let a be any arbitrary element of S, then for

$$xay \in \rho^-(A) \quad \text{for some} \quad x, y \in S,$$

then

$$[x]_\rho[a]_\rho[y]_\rho \cap A = [xay]_\rho \cap A \neq \varnothing.$$

Thus there exist $x' \in [x]_\rho$, $a' \in [a]_\rho$ and $y' \in [y]_\rho$ such that $x'a'y' \in A$. Since A is a prime bi-ideal, we have $x' \in A$ or $y' \in A$. Thus

$$x' \in [x]_\rho \cap A \quad \text{or} \quad y' \in [y]_\rho \cap A$$

so

$$[x]_\rho \cap A \neq \varnothing \quad \text{or} \quad [y]_\rho \cap A \neq \varnothing,$$

and so $x \in \rho^-(A)$ or $y \in \rho^-(A)$. Therefore $\rho^-(A)$ is a prime-bi-ideal of S. □

We call A a rough prime bi-ideal of S if it is both a ρ-lower and a ρ-upper rough prime bi-ideal of S. From the above, we know that a prime bi-ideal is a rough prime bi-ideal with respect to a complete congruence relation on a semigroup. The following example shows that the converse does not hold in general.

Example 3.4. Let $S = \{0, 1, 2, 3\}$ be a semigroup. Define a binary operation "$\cdot$" in S as follows:

$\cdot$	0	1	2	3
0	0	0	0	3
1	0	0	1	3
2	0	1	2	3
3	3	3	3	3

Let ρ be a complete congruence relation on S such that ρ-congruence classes are the subsets $\{0, 1, 2\}$, $\{3\}$, then for $A = \{0, 3\} \subseteq S$, $\rho^-(A) = \{0, 1, 2, 3\}$, and $\rho_-(A) = \{3\}$. It is clear that $\rho^-(A)$, $\rho_-(A)$ are prime bi-ideals of S. The bi-ideal A is not a prime bi-ideal for $1 \cdot 0 \cdot 2 = 0 \in A$ but $1 \notin A$ and $2 \notin A$.

4. Rough prime bi-ideals in the quotient semigroups

Let ρ be a congruence relation on a semigroup S and A be a subset of S. The ρ-lower and the ρ-upper approximations can be presented in an equivalent form as shown below

$$\rho_-(A)/\rho = \left\{[x]_\rho \in S/\rho : [x]_\rho \subseteq A\right\} \text{ and } \rho^-(A)/\rho = \left\{[x]_\rho \in S/\rho : [x]_\rho \cap A \neq \varnothing\right\}.$$

The following two theorems are proved in [9].

Theorem 4.1. *Let ρ be a congruence relation on a semigroup S. If A is a bi-ideal of S. Then $\rho_-(A)/\rho$ is, if it is nonempty, a bi-ideal of S/ρ.*

Theorem 4.2. *Let ρ be a congruence relation on a semigroup S. If A is a bi-ideal of S. Then $\rho^-(A)/\rho$ is a bi-ideal of S/ρ.*

Theorem 4.3. *Let ρ be a complete congruence relation on a semigroup S. If A is a ρ-lower rough prime bi-ideal of S, then $\rho_-(A)/\rho$ is a prime bi-ideal of S/ρ.*

Proof. By Theorem 4.1, we know that $\rho_-(A)/\rho$ is a bi-ideal of S/ρ. Suppose for any $a \in S$,

$$[x]_\rho[a]_\rho[y]_\rho \in \rho_-(A)/\rho \text{ for some } [x]_\rho, [y]_\rho \in S/\rho$$

such that

$$[xay]_\rho \in \rho_-(A)/\rho \text{ for some } [x]_\rho, [y]_\rho \in S/\rho$$

then $[xay]_\rho \subseteq A$. Thus $xay \in \rho_-(A)$. Since A is a ρ-lower rough prime bi-ideal of S, that is $\rho_-(A)$ is a prime bi-ideal, we have

$$x \in \rho_-(A) \quad \text{or} \quad y \in \rho_-(A)$$

so $[x]_\rho \subseteq A$ or $[y]_\rho \subseteq A$. Hence

$$[x]_\rho \in \rho_-(A)/\rho \quad \text{or} \quad [y]_\rho \in \rho_-(A)/\rho.$$

Therefore $\rho_-(A)/\rho$ is a prime bi-ideal of S/ρ. □

Theorem 4.4. *Let ρ be a complete congruence relation on a semigroup S. If A is a ρ-upper rough prime bi-ideal of S, then $\rho^-(A)/\rho$ is a prime bi-ideal of S/ρ.*

Proof. By Theorem 4.2, we know that $\rho^-(A)/\rho$ is a bi-ideal of S/ρ. Suppose for any $a \in S$,

$$[x]_\rho[a]_\rho[y]_\rho \in \rho^-(A)/\rho \text{ for some } [x]_\rho, [y]_\rho \in S/\rho$$

such that

$$[xay]_\rho \in \rho^-(A)/\rho \text{ for some } [x]_\rho, [y]_\rho \in S/\rho$$

then $[xay]_\rho \cap A \neq \varnothing$. Thus $xay \in \rho^-(A)$. Since A is an upper rough prime bi-ideal of S, that is $\rho^-(A)$ is a prime bi-ideal, we have

$$x \in \rho^-(A) \quad \text{or} \quad y \in \rho^-(A)$$

so $[x]_\rho \cap A \neq \varnothing$ or $[y]_\rho \cap A \neq \varnothing$. Hence

$$[x]_\rho \in \rho^-(A)/\rho \quad \text{or} \quad [y]_\rho \in \rho^-(A)/\rho.$$

Therefore $\rho^-(A)/\rho$ is a prime bi-ideal of S/ρ. This completes the proof. □

5. Rough fuzzy prime bi-ideals in semigroups

Let f be a fuzzy subset of S. Let $\rho^-(f)(x)$ and $\rho_-(f)(x)$ be fuzzy subsets of S defined by

$$\rho^-(f)(x) = \bigvee_{a \in [x]_\rho} f(a) \text{ and } \rho_-(f)(x) = \bigwedge_{a \in [x]_\rho} f(a)$$

are called, respectively, the ρ-upper and ρ-lower approximations of the fuzzy set f. $\rho(f) = (\rho_-(f), \rho^-(f))$ is called a rough fuzzy set with respect to ρ if $\rho_-(f) \neq \rho^-(f)$.

Theorem 5.1. *Let ρ be a complete congruence relation on a semigroup S. Let f be a fuzzy subset of S. If f is a fuzzy subsemigroup of S. Then*
(1) *$\rho^-(f)$ is a fuzzy subsemigroup of S.*
(2) *$\rho_-(f)$ is, if it is nonempty, a fuzzy subsemigroup of S.*

Proof. (1) Assume f is a fuzzy subsemigroup of S. Let $x, y \in S$. Then $f(xy) \geq f(x) \wedge f(y)$. We have

$$\begin{aligned}\rho^-(f)(xy) &= \bigvee_{s\in[xy]_\rho} f(s) = \bigvee_{s\in[x]_\rho[y]_\rho} f(s)\\ &= \bigvee_{p\in[x]_\rho,\ q\in[y]_\rho} f(pq) \geq \left(\bigvee_{p\in[x]_\rho} f(p)\right) \wedge \left(\bigvee_{q\in[y]_\rho} f(q)\right)\\ &= \rho^-(f)(x) \wedge \rho^-(f)(y).\end{aligned}$$

Then $\rho^-(f)(xy) \geq \rho^-(f)(x) \wedge \rho^-(f)(y)$. Therefore $\rho^-(f)$ is a fuzzy subsemigroup of S.

(2) Assume f is a fuzzy subsemigroup of S. Let $x, y \in S$. Then $f(xy) \geq f(x) \wedge f(y)$. We have

$$\begin{aligned}\rho_-(f)(xy) &= \bigwedge_{s\in[xy]_\rho} f(s) = \bigwedge_{s\in[x]_\rho[y]_\rho} f(s)\\ &= \bigwedge_{p\in[x]_\rho,\ q\in[y]_\rho} f(pq) \geq \left(\bigwedge_{p\in[x]_\rho} f(p)\right) \wedge \left(\bigwedge_{q\in[y]_\rho} f(q)\right)\\ &= \rho_-(f)(x) \wedge \rho_-(f)(y).\end{aligned}$$

Then $\rho_-(f)(xy) \geq \rho_-(f)(x) \wedge \rho_-(f)(y)$. Therefore $\rho_-(f)$ is a fuzzy subsemigroup of S. □

Theorem 5.2. *Let ρ be a complete congruence relation on a semigroup S. Let f be a fuzzy subset of S. If f is a fuzzy bi-ideal of S. Then*
(1) *$\rho^-(f)$ is a fuzzy bi-ideal of S.*
(2) *$\rho_-(f)$ is, if it is nonempty, a fuzzy bi-ideal of S.*

Proof. (1) Assume f is a fuzzy bi-ideal of S. Let $x, a, y \in S$. Then $f(xay) \geq f(x) \wedge f(y)$. We have

$$\begin{aligned}\rho^-(f)(xay) &= \bigvee_{s\in[xay]_\rho} f(s) = \bigvee_{s\in[x]_\rho[a]_\rho[y]_\rho} f(s)\\ &= \bigvee_{p\in[x]_\rho,\ q\in[a]_\rho,\ r\in[y]_\rho} f(pqr) \geq \left(\bigvee_{p\in[x]_\rho} f(p)\right) \wedge \left(\bigvee_{r\in[y]_\rho} f(r)\right)\\ &= \rho^-(f)(x) \wedge \rho^-(f)(y).\end{aligned}$$

Then $\rho^-(f)(xay) \geq \rho^-(f)(x) \wedge \rho^-(f)(y)$. Therefore from this and Theorem 5.1(1), we obtain that $\rho^-(f)$ is a fuzzy bi-ideal of S.

(2) Assume f is a fuzzy bi-ideal of S. Let $x, a, y \in S$. Then $f(xay) \geq f(x) \wedge f(y)$. We have

$$\begin{aligned}
\rho_-(f)(xay) &= \bigwedge_{s \in [xay]_\rho} f(s) = \bigwedge_{s \in [x]_\rho [a]_\rho [y]_\rho} f(s) \\
&= \bigwedge_{p \in [x]_\rho,\ q \in [a]_\rho,\ r \in [y]_\rho} f(pqr) \geq \left(\bigwedge_{p \in [x]_\rho} f(p) \right) \wedge \left(\bigwedge_{r \in [y]_\rho} f(r) \right) \\
&= \rho_-(f)(x) \wedge \rho_-(f)(y).
\end{aligned}$$

Then $\rho_-(f)(xay) \geq \rho_-(f)(x) \wedge \rho_-(f)(y)$. Therefore from this and Theorem 5.1(2), we obtain that $\rho_-(f)$ is, if it is nonempty, a fuzzy bi-ideal of S. $\square$

A fuzzy bi-ideal f of a semigroup S is called a fuzzy prime bi-ideal of S if $f(xay) = f(x)$ or $f(xay) = f(y)$ for all $x, y, a \in S$.

Theorem 5.3. *Let f be a fuzzy subset of a semigroup S. Then f is a fuzzy prime bi-ideal of S iff $f_\lambda \neq \varnothing$ is a prime bi-ideal of S for every $\lambda \in [0, 1]$.*

Proof. Assume f is a fuzzy prime bi-ideal of S. Then f is a fuzzy bi-ideal of S. Assume $f_\lambda \neq \varnothing$. By Theorem 2.3, f_λ is a bi-ideal of S. Let $x, y, a \in S$ such that $xay \in f_\lambda$. Since f is a fuzzy prime bi-ideal of S, $f(xay) = f(x)$ or $f(xay) = f(y)$. This implies $x \in f_\lambda$ or $y \in f_\lambda$. Therefore f_λ is a prime bi-ideal of S.

Conversely, assume for all $\lambda \in [0, 1]$, if $f_\lambda \neq \varnothing$, then f_λ is a prime bi-ideal of S. Let $x, y, a \in S$. By Theorem 2.3, f is a fuzzy bi-ideal of S. This implies $f(xay) \geq f(x)$ and $f(xay) \geq f(y)$. Let $\lambda = f(xay)$. Thus $xay \in f_\lambda$. Since f_λ is a prime bi-ideal of S, $x \in f_\lambda$ or $y \in f_\lambda$. This implies that $f(x) \geq \lambda = f(xay)$ or $f(y) \geq \lambda = f(xay)$. Hence $f(xay) = f(x)$ or $f(xay) = f(y)$. Hence f is a fuzzy prime bi-ideal of S. $\square$

Theorem 5.4. *Let f be a fuzzy subset of a semigroup S. Then f is a fuzzy prime bi-ideal of S iff $f^s_\lambda \neq \varnothing$ is a prime bi-ideal of S for every $\lambda \in [0, 1]$.*

Proof. Assume f is a fuzzy prime bi-ideal of S. Then f is a fuzzy bi-ideal of S. Assume $f^s_\lambda \neq \varnothing$. By Theorem 2.3, f^s_λ is a bi-ideal of S. Let $x, y, a \in S$ such that $xay \in f^s_\lambda$. Then $f(xay) > \lambda$. Since f is a fuzzy prime bi-ideal of S, $f(xay) = f(x)$ or $f(xay) = f(y)$. This implies that $f(x) > \lambda$ or $f(y) > \lambda$. hence $x \in f^s_\lambda$ or $y \in f^s_\lambda$. Therefore f^s_λ is a prime bi-ideal of S.

Conversely, assume for all $\lambda \in [0, 1]$, if $f^s_\lambda \neq \varnothing$, then f^s_λ is a prime bi-ideal of S. Let $x, y, a \in S$. By Theorem 2.3, f is a fuzzy bi-ideal of S. This implies $f(xay) \geq f(x)$ and $f(xay) \geq f(y)$. We have $xay \in f^s_\lambda$ for all $\lambda < f(xay)$. Since f^s_λ is a prime bi-ideal of S for all $\lambda < f(xay)$, $x \in f^s_\lambda$ or $y \in f^s_\lambda$ for all $\lambda < f(xay)$. This implies that $f(x) > \lambda$ or $f(y) > \lambda$ for all $\lambda < f(xay)$. Then $f(x) \geq f(xay)$ or $f(y) \geq f(xay)$. Hence $f(xay) = f(x)$ or $f(xay) = f(y)$. Hence f is a fuzzy prime bi-ideal of S. $\square$

Let ρ be a congruence on a semigroup S. A fuzzy subset f of S is called a ρ-lower rough fuzzy prime bi-ideal of S if $\rho_-(f)$ is a fuzzy prime bi-ideal of S. A ρ-upper rough fuzzy prime bi-ideal of S is defined analogously. We call f a rough fuzzy prime bi-ideal of S if it is both a ρ-lower and a ρ-upper rough fuzzy prime bi-ideal of S.

Lemma 5.5. *Let ρ be a congruence relation on a semigroup S. If f is a fuzzy subset of S and $\lambda \in [0,1]$, then*
(i) $(\rho_-(f))_\lambda = \rho_-(f_\lambda)$,
(ii) $(\rho^-(f))^s_\lambda = \rho^-(f^s_\lambda)$.

Proof. The proof of this theorem can be seen in [18]. □

Theorem 5.6. *Let f be a fuzzy prime bi-ideal of a semigroup S and ρ be a complete congruence on S. Then f is a rough fuzzy prime bi-ideal of S.*

Proof. Let f be a fuzzy prime bi-ideal of a semigroup S and ρ a complete congruence on S. By Theorem 5.3, for all $\lambda \in [0,1]$, if $f_\lambda \neq \varnothing$, then f_λ is a prime bi-ideal of S. By Theorem 3.1, for all $\lambda \in [0,1]$, if $\rho_-(f_\lambda) \neq \varnothing$, then $\rho_-(f_\lambda)$ is a prime bi-ideal of S. From this and Lemma 5.5(i), for all $\lambda \in [0,1]$, if $(\rho_-(f))_\lambda \neq \varnothing$, $(\rho_-(f))_\lambda$ is a prime bi-ideal of S. By Theorem 5.3, $\rho_-(f)$ is a fuzzy prime bi-ideal of S. Hence f is a ρ-lower rough fuzzy prime bi-ideal of S. Similarly, f is a ρ-upper rough fuzzy prime bi-ideal of S. Therefore f is a rough fuzzy prime bi-ideal of S. □

Theorem 5.7. *Let ρ be a congruence on a semigroup S. Then f is a ρ-lower rough fuzzy prime bi-ideal if and only if for all $\lambda \in [0,1]$, if $\rho_-(f_\lambda) \neq \varnothing$, then f_λ is a ρ-lower rough prime bi-ideal of S.*

Proof. By Theorem 5.3 and Lemma 5.5(i), we can obtain the conclusion easily. □

Theorem 5.8. *Let ρ be a congruence on a semigroup S. Then f is a ρ-upper rough fuzzy prime bi-ideal if and only if for all $\lambda \in [0,1]$, if $f^s_\lambda \neq \varnothing$, then f^s_λ is a ρ-upper rough prime bi-ideal of S.*

Proof. By Theorem 5.4 and Lemma 5.5(ii), we can obtain the conclusion easily. □

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