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Several types of (r, s)-fuzzy compactness defined by an (r, s)-fuzzy regular semiopen sets

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Abstract. The aim of this paper is to use the notion of (r, s)-fuzzy regular semiopen sets to introduce Rs-compactness in double fuzzy topological spaces. The notions of (r, s)-fuzzy Rs-compactness, (r, s)-fuzzy almost Rs-compactness and (r, s)-fuzzy weakly Rs-compactness have been studied, some properties of these compactness are studied.

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Keywords: Double fuzzy topology, (r, s)-fuzzy regular semiopen, (r, s)-fuzzy Rs-compact, (r, s)-fuzzy almost Rs-compact, (r, s)-fuzzy weakly Rs-compact.

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1. Introduction and preliminaries

The concept of fuzzy sets was initially investigated by Zadeh [14] as a new way to represent vagueness in every day life. Subsequently, it was developed extensively by many authors and used in various fields. After the introduction of fuzzy sets Chang [4] was the first to introduce the concept of topology of fuzzy subsets. In his definition of topology of fuzzy subsets, fuzziness in the concept of openness of a fuzzy subset was absent. In 1980, Höhle [8] defined a topology as a fuzzy subset of a powerset. This was followed in 1985 by the independent and parallel generalization of Kubiak [9] and Šostak [13], papers in which a topology was a fuzzy subset of a powerset of fuzzy subsets. On the other hand, various generalizations of the concept of fuzzy set have been done by many authors. In [1, 2, 3], Atanassove introduced the idea of intuitionistic fuzzy sets. In [5, 6], Çoker introduced the idea of the topology of intuitionistic fuzzy sets. In [11, 12], Samanta and Mondal introduced the notions of intuitionistic gradation of openness as a generalization of fuzzy topological spaces.

The term " intuitionistic " is still used in literature until 2005, when Garcia and Rodabaugh [7] pointed out that the term " intuitionistic fuzzy set " must be replaced by double fuzzy sets.

In [10], some kinds of compactness are introduced like (r,s)-fuzzy compactness, (r,s)-fuzzy almost compact and (r,s)-fuzzy near compactness. In this paper, we aim to introduce new kinds of compactness for double fuzzy topological spaces named by (r,s)-fuzzy Rs-compactness, (r,s)-fuzzy almost Rs-compactness and (r,s)-fuzzy weakly Rs-compactness. Some properties of this concept is given and the relationships between them and the other kinds of compactness are studied.

Throughout this paper, let X be a nonempty set and I is the closed unit interval $[0,1]$. $I_0=(0,1]$ and $I_1=[0,1)$. The family of all fuzzy sets on X denoted by I^X. A function $f: I^X \to I^Y$ and its inverse $f^{-1}: I^Y \to I^X$ are defined by $f(\lambda)(y)=\bigvee_{f(x)=y}\lambda(x)$ and $f^{-1}(\nu)(x)=\nu(f(x))$, for each $\lambda\in I^X$, $\nu\in I^Y$ and $x\in X$, respectively. Notions and notations not described in this paper are standard and usual.

2. Double fuzzy topological spaces

Definition 2.1 ([11, 12])**.** A double fuzzy topology is the functions $\mathcal{T}$, $\mathcal{T}^*: I^X\to I$, which satisfies the following conditions:

(1) $\mathcal{T}(\lambda)+\mathcal{T}^*(\lambda)\le 1$, for each $\lambda\in I^X$,
(2) $\mathcal{T}(\lambda_1\wedge\lambda_2)\ge\mathcal{T}(\lambda_1)\wedge\mathcal{T}(\lambda_2)$ and $\mathcal{T}^*(\lambda_1\wedge\lambda_2)\le\mathcal{T}^*(\lambda_1)\vee\mathcal{T}^*(\lambda_2)$, for each $\lambda_1,\ \lambda_2\in I^X$,
(3) $\mathcal{T}(\bigvee_{i\in\Delta}\lambda_i)\ge\bigwedge_{i\in\Delta}\mathcal{T}(\lambda_i)$ and $\mathcal{T}^*(\bigvee_{i\in\Delta}\lambda_i)\le\bigvee_{i\in\Delta}\mathcal{T}^*(\lambda_i)$, for each $\{\lambda_i|i\in\Delta\}\subset I^X$.

The triplet $(X,\mathcal{T},\mathcal{T}^*)$ is called a double fuzzy topological space. $\mathcal{T}(\lambda)$ and $\mathcal{T}^*(\lambda)$ may be interpreted as gradation of openness and gradation of nonopenness for $\lambda\in I^X$, respectively. A function $f: X\to Y$ is said to be a double fuzzy continuous if $\mathcal{T}_1(f^{-1}(\nu))\ge\mathcal{T}_2(\nu)$ and $\mathcal{T}^*{}_1(f^{-1}(\nu))\le\mathcal{T}^*{}_2(\nu)$, for each $\nu\in I^Y$.

Definition 2.2 ([6])**.** Let $(X,\mathcal{T},\mathcal{T}^*)$ be a double fuzzy topological space. Then double fuzzy closure operator and double fuzzy interior operator of $\lambda\in I^X$ are defined by

$$\mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\lambda,r,s)=\bigwedge\{\mu\in I^X|\quad \lambda\le\mu,\ \mathcal{T}(\underline{1}-\mu)\ge r,\mathcal{T}^*(\underline{1}-\mu)\le s\},$$

$$\mathcal{I}_{\mathcal{T},\mathcal{T}^*}(\lambda,r,s)=\bigvee\{\gamma\in I^X|\quad \gamma\le\lambda,\ \mathcal{T}(\gamma)\ge r,\mathcal{T}^*(\gamma)\le s\}.$$

Where $r\in I_0$ and $s\in I_1$ with $r+s\le 1$.

Theorem 2.3 ([6])**.** *A double fuzzy closure operator and double fuzzy interior operator satisfy the following properties:*

(1) $\lambda\le\mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\lambda,r,s)$,
(2) $\mathcal{I}_{\mathcal{T},\mathcal{T}^*}(\lambda,r,s)\le\lambda$,
(3) $\lambda\le\mu$ *and* $r_1\le r_2,\ s_1\ge s_2$ *implies* $\mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\lambda,r_1,s_1)\le\mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\mu,r_2,s_2)$,
(4) $\lambda\le\mu$ *and* $r_1\le r_2,\ s_1\ge s_2$ *implies* $\mathcal{I}_{\mathcal{T},\mathcal{T}^*}(\lambda,r_2,s_2)\le\mathcal{I}_{\mathcal{T},\mathcal{T}^*}(\mu,r_1,s_1)$,
(5) $\mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\lambda,r,s),r,s)=\mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\lambda,r,s)$,
(6) $\mathcal{I}_{\mathcal{T},\mathcal{T}^*}(\mathcal{I}_{\mathcal{T},\mathcal{T}^*}(\lambda,r,s),r,s)=\mathcal{I}_{\mathcal{T},\mathcal{T}^*}(\lambda,r,s)$,
(7) $\mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\lambda\vee\mu,r,s)=\mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\lambda,r,s)\vee\mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\mu,r,s)$,
(8) $\mathcal{I}_{\mathcal{T},\mathcal{T}^*}(\lambda\wedge\mu,r,s)=\mathcal{I}_{\mathcal{T},\mathcal{T}^*}(\lambda,r,s)\wedge\mathcal{I}_{\mathcal{T},\mathcal{T}^*}(\mu,r,s)$,
(9) $\underline{1}-\mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\underline{1}-\lambda,r,s)=\mathcal{I}_{\mathcal{T},\mathcal{T}^*}(\lambda,r,s)$ *and* $\underline{1}-\mathcal{I}_{\mathcal{T},\mathcal{T}^*}(\underline{1}-\lambda,r,s)=\mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\lambda,r,s)$.

Definition 2.4. Let $(X, \mathcal{T}, \mathcal{T}^*)$ be a double fuzzy topological space, $\lambda \in I^X$, $r \in I_0$, and $s \in I_1$. Then λ is called:

(1) (r, s)-fuzzy semiopen [10] if there exists $\mu \in I^X$ with $\mathcal{T}(\mu) \geq r$, $\mathcal{T}^*(\mu) \leq s$ such that $\mu \leq \lambda \leq \mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\mu, r, s)$.

(2) (r, s)-fuzzy regular open [10] if $\lambda = \mathcal{I}_{\mathcal{T},\mathcal{T}^*}(\mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\lambda, r, s), r, s)$.

(3) (r, s)-fuzzy regular closed [10] if $\lambda = \mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\mathcal{I}_{\mathcal{T},\mathcal{T}^*}(\lambda, r, s), r, s)$.

(4) (r, s)-fuzzy regular semiopen if there exists an (r, s)-fuzzy regular open set μ such that $\mu \leq \lambda \leq \mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\mu, r, s)$.

Remark 2.5. (a) λ is (r, s)-fuzzy regular open if and only if $\underline{1} - \lambda$ is (r, s)-fuzzy regular closed [10].

(b) λ is an (r, s)-fuzzy regular open set $\Rightarrow$ an (r, s)-fuzzy regular semiopen set $\Rightarrow$ an (r, s)-fuzzy semiopen set.

None of the converses need be true in general, as shown by the following example.

Example 2.6. (1) Let $X = \{a, b, c\}$ and $\mu_1, \mu_2, \mu_3 \in I^X$ defined as follows:

$$\mu_1 = \{\langle a, 0.3\rangle, \langle b, 0.3\rangle, \langle c, 0.3\rangle\},$$
$$\mu_2 = \{\langle a, 0.4\rangle, \langle b, 0.3\rangle, \langle c, 0.3\rangle\},$$
$$\mu_3 = \{\langle a, 0.6\rangle, \langle b, 0.4\rangle, \langle c, 0.3\rangle\}.$$

We define a double fuzzy topology $\mathcal{T}$, $\mathcal{T}^* : I^X \to I$ as follows:

$$\mathcal{T}(\lambda) = \begin{cases} 1, & \text{if } \lambda \in \{\underline{1}, \underline{0}\}; \\ 0.6, & \text{if } \lambda = \mu_1; \\ 0, & \text{otherwise.} \end{cases} \quad , \quad \mathcal{T}^*(\lambda) = \begin{cases} 0, & \text{if } \lambda \in \{\underline{1}, \underline{0}\}; \\ 0.3, & \text{if } \lambda = \mu_1; \\ 1, & \text{otherwise.} \end{cases}$$

Let $r = 0.6, s = 0.3$; then in $(X, \mathcal{T}, \mathcal{T}^*)$, μ_1 is $(0.6, 0.3)$-fuzzy regular open and $\mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\mu_1, 0.6, 0.3) = \mu_3$. Thus $\mu_1 \leq \mu_2 \leq \mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\mu_1, 0.6, 0.3)$, i.e., μ_2 is $(0.6, 0.3)$-fuzzy regular semiopen. But μ_2 is neither an $(0.6, 0.3)$-fuzzy open set nor an $(0.6, 0.3)$-fuzzy regular open set in $(X, \mathcal{T}, \mathcal{T}^*)$.

(2) Let $X = \{a, b, c\}$ and $\mu_1, \mu_2 \in I^X$ defined as follows:

$$\mu_1 = \{\langle a, 0.3\rangle, \langle b, 0.3\rangle, \langle c, 0.3\rangle\},$$
$$\mu_2 = \{\langle a, 0.4\rangle, \langle b, 0.3\rangle, \langle c, 0.3\rangle\}.$$

We define a double fuzzy topology $\mathcal{T}$, $\mathcal{T}^* : I^X \to I$ as follows:

$$\mathcal{T}(\lambda) = \begin{cases} 1, & \text{if } \lambda \in \{\underline{1}, \underline{0}\}; \\ 0.6, & \text{if } \lambda = \mu_2; \\ 0, & \text{otherwise.} \end{cases} \quad , \quad \mathcal{T}^*(\lambda) = \begin{cases} 0, & \text{if } \lambda \in \{\underline{1}, \underline{0}\}; \\ 0.3, & \text{if } \lambda = \mu_2; \\ 1, & \text{otherwise.} \end{cases}$$

Let $r = 0.6, s = 0.3$; then in $(X, \mathcal{T}, \mathcal{T}^*)$, μ_2 is $(0.6, 0.3)$-fuzzy open but is not $(0.6, 0.3)$-fuzzy regular semiopen.

Theorem 2.7. *Let $(X, \mathcal{T}, \mathcal{T}^*)$ be a double fuzzy topological space, $\lambda \in I^X$, $r \in I_0$ and $s \in I_1$. If λ is (r, s)-fuzzy regular semiopen, then*

(1) *$\underline{1} - \lambda$ is (r, s)-fuzzy regular semiopen.*

(2) $\mathcal{I}_{\mathcal{T},\mathcal{T}^*}(\lambda, r, s) = \mathcal{I}_{\mathcal{T},\mathcal{T}^*}(\mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\lambda, r, s), r, s)$.

(3) $\mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\lambda, r, s) = \mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\mathcal{I}_{\mathcal{T},\mathcal{T}^*}(\lambda, r, s))$.

Proof. (1) Let λ be an (r,s)-fuzzy regular semiopen set. Then there exists an (r,s)-fuzzy regular open set μ such that $\mu \leq \lambda \leq \mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\mu,r,s)$. Since $\underline{1}-\mu$ is (r,s)-fuzzy regular closed, $\mathcal{I}_{\mathcal{T},\mathcal{T}^*}(\underline{1}-\mu,r,s)$ is an (r,s)-fuzzy regular open set such that

$$\mathcal{I}_{\mathcal{T},\mathcal{T}^*}(\underline{1}-\mu,r,s) \leq \underline{1}-\lambda \leq \underline{1}-\mu = \mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\mathcal{I}_{\mathcal{T},\mathcal{T}^*}(\underline{1}-\mu,r,s),r,s).$$

Thus $\underline{1}-\lambda$ is (r,s)-fuzzy regular semiopen.

(2) Let λ be an (r,s)-fuzzy regular semiopen set. Then there exists an (r,s)-fuzzy regular open set μ such that $\mu \leq \lambda \leq \mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\mu,r,s)$. Hence $\mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\lambda,r,s) = \mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\mu,r,s)$. Since $\mathcal{I}_{\mathcal{T},\mathcal{T}^*}(\mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\lambda,r,s),r,s) = \mu$,

$$\begin{aligned}\mathcal{I}_{\mathcal{T},\mathcal{T}^*}(\mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\lambda,r,s),r,s) &= \mu \leq \mathcal{I}_{\mathcal{T},\mathcal{T}^*}(\lambda,r,s)\\ &\leq \mathcal{I}_{\mathcal{T},\mathcal{T}^*}(\mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\mu,r,s),r,s)\\ &= \mathcal{I}_{\mathcal{T},\mathcal{T}^*}(\mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\lambda,r,s),r,s).\end{aligned}$$

Thus $\mathcal{I}_{\mathcal{T},\mathcal{T}^*}(\lambda,r,s) = \mathcal{I}_{\mathcal{T},\mathcal{T}^*}(\mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\lambda,r,s),r,s)$.

(3) Similar to (2). □

Definition 2.8. Let $(X,\mathcal{T}_1,\mathcal{T}^*{}_1)$ and $(Y,\mathcal{T}_2,\mathcal{T}^*{}_2)$ be two double fuzzy topological spaces. A function $f : X \to Y$ is called

1. a double fuzzy weakly continuous function [10] if $\mathcal{T}_2(\nu) \geq r$, $\mathcal{T}^*{}_2(\nu) \leq s \Rightarrow \mathcal{T}_1(f^{-1}(\nu)) \geq r$, $\mathcal{T}^*{}_1(f^{-1}(\nu)) \leq s$, for each $r \in I_0$, $s \in I_1$ and $\nu \in I^Y$.
2. a double fuzzy weakly open if $f(\lambda) \leq \mathcal{I}_{\mathcal{T}_2,\mathcal{T}^*{}_2}(f(\mathcal{C}_{\mathcal{T}_1,\mathcal{T}^*{}_1}(\lambda,r,s)),r,s)$, for each $\lambda \in I^X$, $r \in I_0$, $s \in I_1$ such that $\mathcal{T}_1(\lambda) \geq r$, $\mathcal{T}^*{}_1(\lambda) \leq s$.

Theorem 2.9. *Let $(X,\mathcal{T}_1,\mathcal{T}^*{}_1)$ and $(Y,\mathcal{T}_2,\mathcal{T}^*{}_2)$ be two double fuzzy topological spaces and $f : X \to Y$. Then the following are equivalent:*

1. *f is double fuzzy weakly continuous.*
2. *$f(\mathcal{C}_{\mathcal{T}_1,\mathcal{T}^*{}_1}(\lambda,r,s)) \leq \mathcal{C}_{\mathcal{T}_2,\mathcal{T}^*{}_2}(f(\lambda),r,s)$, for each $\lambda \in I^X$, $r \in I_0$, $s \in I_1$.*
3. *$\mathcal{C}_{\mathcal{T}_2,\mathcal{T}^*{}_1}(f^{-1}(\nu),r,s) \leq f^{-1}(\mathcal{C}_{\mathcal{T}_2,\mathcal{T}^*{}_2}(\nu,r,s))$, for each $\nu \in I^Y$, $r \in I_0$, $s \in I_1$.*
4. *$f^{-1}(\mathcal{I}_{\mathcal{T}_2,\mathcal{T}^*{}_2}(\nu,r,s)) \leq \mathcal{I}_{\mathcal{T}_1,\mathcal{T}^*{}_1}(f^{-1}(\nu),r,s)$, for each $\nu \in I^Y$, $r \in I_0$, $s \in I_1$.*

Proof. (1) $\Rightarrow$ (2): Trivial.

(2) $\Rightarrow$ (3): For every $\nu \in I^Y$, $r \in I_0$, $s \in I_1$, from (2) we have

$$f(\mathcal{C}_{\mathcal{T}_1,\mathcal{T}^*{}_1}(f^{-1}(\nu),r,s)) \leq \mathcal{C}_{\mathcal{T}_2,\mathcal{T}^*{}_2}(f(f^{-1}(\nu)),r,s) \leq \mathcal{C}_{\mathcal{T}_2,\mathcal{T}^*{}_2}(\nu,r,s).$$

Hence $\mathcal{C}_{\mathcal{T}_1,\mathcal{T}^*{}_1}(f^{-1}(\nu),r,s) \leq f^{-1}(\mathcal{C}_{\mathcal{T}_2,\mathcal{T}^*{}_2}(\nu,r,s))$.

(3) $\Rightarrow$ (4): For every $\nu \in I^Y$, $r \in I_0$, $s \in I_1$, from (3)we have

$$\begin{aligned}\underline{1}-\mathcal{I}_{\mathcal{T}_1,\mathcal{T}^*{}_1}(f^{-1}(\nu),r,s) &= \mathcal{C}_{\mathcal{T}_1,\mathcal{T}^*{}_1}(\underline{1}-f^{-1}(\nu),r,s) = \mathcal{C}_{\mathcal{T}_1,\mathcal{T}^*{}_1}(f^{-1}(\underline{1}-\nu),r,s)\\ &\leq f^{-1}(\mathcal{C}_{\mathcal{T}_2,\mathcal{T}^*{}_2}(\underline{1}-\nu,r,s)) = f^{-1}(\underline{1}-\mathcal{I}_{\mathcal{T}_2,\mathcal{T}^*{}_2}(\nu,r,s))\\ &= \underline{1}-f^{-1}(\mathcal{I}_{\mathcal{T}_2,\mathcal{T}^*{}_2}(\nu,r,s))\end{aligned}$$

Hence $f^{-1}(\mathcal{I}_{\mathcal{T}_2,\mathcal{T}^*{}_2}(\nu,r,s)) \leq \mathcal{I}_{\mathcal{T}_1,\mathcal{T}^*{}_1}(f^{-1}(\nu),r,s)$.

(4) $\Rightarrow$ (1): Let $\mathcal{T}_2(\nu) \geq r$, $\mathcal{T}^*{}_2(\nu) \leq s$ for $\nu \in I^Y$, $r \in I_0$, $s \in I_1$. Then $\nu = \mathcal{I}_{\mathcal{T}_2,\mathcal{T}^*{}_2}(\nu,r,s)$. From (4) we have

$$f^{-1}(\nu) = f^{-1}(\mathcal{I}_{\mathcal{T}_2,\mathcal{T}^*{}_2}(\nu,r,s)) \leq \mathcal{I}_{\mathcal{T}_1,\mathcal{T}^*{}_1}(f^{-1}(\nu),r,s).$$

Thus $f^{-1}(\nu) = \mathcal{I}_{\mathcal{T}_1,\mathcal{T}^*{}_1}(f^{-1}(\nu),r,s)$. Hence $\mathcal{T}_1(f^{-1}(\nu)) \geq r$, $\mathcal{T}^*{}_1(f^{-1}(\nu)) \leq s$. Thus f is a double fuzzy weakly continuous function. □

Theorem 2.10. *Let $(X, \mathcal{T}_1, \mathcal{T}^*{}_1)$ and $(Y, \mathcal{T}_2, \mathcal{T}^*{}_2)$ be two double fuzzy topological spaces and $f : X \to Y$ is double fuzzy weakly open and double fuzzy weakly continuous, then $f^{-1}(\nu)$ is an (r,s)-fuzzy regular open (resp. (r,s)-fuzzy regular closed) set for every (r,s)-fuzzy regular open set $\nu \in I^Y$; $r \in I_0$, $s \in I_1$.*

Proof. Let ν be an (r,s)-fuzzy regular open set in Y, we have $\mathcal{T}_2(\nu) \geq r$, $\mathcal{T}^*{}_2(\nu) \leq s$. Since f is double fuzzy weakly continuous, $\mathcal{T}_1(f^{-1}(\nu)) \geq r$, $\mathcal{T}^*{}_1(f^{-1}(\nu)) \leq s$. Hence

$$f^{-1}(\nu) = \mathcal{I}_{\mathcal{T}_1,\mathcal{T}^*{}_1}(f^{-1}(\nu), r, s) \leq \mathcal{I}_{\mathcal{T}_1,\mathcal{T}^*{}_1}(\mathcal{C}_{\mathcal{T}_1,\mathcal{T}^*{}_1}(f^{-1}(\nu), r, s), r, s).$$

Since f is double fuzzy weakly open,

$$f(\mathcal{I}_{\mathcal{T}_1,\mathcal{T}^*{}_1}(\mathcal{C}_{\mathcal{T}_1,\mathcal{T}^*{}_1}(f^{-1}(\nu), r, s), r, s)) \leq \mathcal{I}_{\mathcal{T}_2,\mathcal{T}^*{}_2}(f(\mathcal{C}_{\mathcal{T}_1,\mathcal{T}^*{}_1}(f^{-1}(\nu), r, s)), r, s).$$

Since f is double fuzzy weakly continuous,

$$\begin{aligned} \mathcal{I}_{\mathcal{T}_2,\mathcal{T}^*{}_2}(f(\mathcal{C}_{\mathcal{T}_1,\mathcal{T}^*{}_1}(f^{-1}(\nu), r, s)), r, s) &\leq \mathcal{I}_{\mathcal{T}_2,\mathcal{T}^*{}_2}(ff^{-1}(\mathcal{C}_{\mathcal{T}_2,\mathcal{T}^*{}_2}(\nu, r, s), r, s)) \\ &\leq \mathcal{I}_{\mathcal{T}_2,\mathcal{T}^*{}_2}(\mathcal{C}_{\mathcal{T}_2,\mathcal{T}^*{}_2}(\nu, r, s), r, s) = \nu. \end{aligned}$$

Hence

$$\mathcal{I}_{\mathcal{T}_1,\mathcal{T}^*{}_1}(\mathcal{C}_{\mathcal{T}_1,\mathcal{T}^*{}_1}(f^{-1}(\nu), r, s), r, s) \leq f^{-1}(\nu).$$

Thus $f^{-1}(\nu)$ is (r,s)-fuzzy regular open. An (r,s)-regular closed case will be trivial by using the previous result. □

Definition 2.11 ([10]). A double fuzzy topological space $(X, \mathcal{T}, \mathcal{T}^*)$ is called an (r,s)-fuzzy compact((r,s)-fuzzy nearly compact and (r,s)-fuzzy almost compact) if and only if for every family $\{\lambda_i : i \in J\}$ in $\{\lambda : \lambda \in I^X, \mathcal{T}(\lambda) \geq r,\ \mathcal{T}^*(\lambda) \leq s\}$ such that $\bigvee_{i\in J} \lambda_i = \underline{1}$, there exists a finite subset J_0 of J such that $\bigvee_{i\in J_0} \lambda_i = \underline{1}$ (resp. $\bigvee_{i\in J_0} \mathcal{I}_{\mathcal{T},\mathcal{T}^*}(\mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\lambda_i, r, s), r, s) = \underline{1}$ and $\bigvee_{i\in J_0} \mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\lambda_i, r, s) = \underline{1}$).

Definition 2.12. A double fuzzy topological space $(X, \mathcal{T}, \mathcal{T}^*)$ is called an (r,s)-fuzzy S-closed if and only if for every an (r,s)-fuzzy semiopen cover $\{\lambda_i | i \in J\}$ of X, there exists a finite subset J_0 of J such that

$$\bigvee_{i\in J_0} \mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\lambda_i, r, s) = \underline{1}.$$

3. (r,s)-fuzzy Rs-compactness in double fuzzy topological spaces

Definition 3.1. A double fuzzy topological space $(X, \mathcal{T}, \mathcal{T}^*)$ is called

(a) (r,s)-fuzzy Rs-compact if for every (r,s)-fuzzy regular semiopen cover $\{\lambda_i : i \in J\}$ of X, there exists a finite subset J_0 of J such that $\bigvee_{i\in J_0} \lambda_i = \underline{1}$.

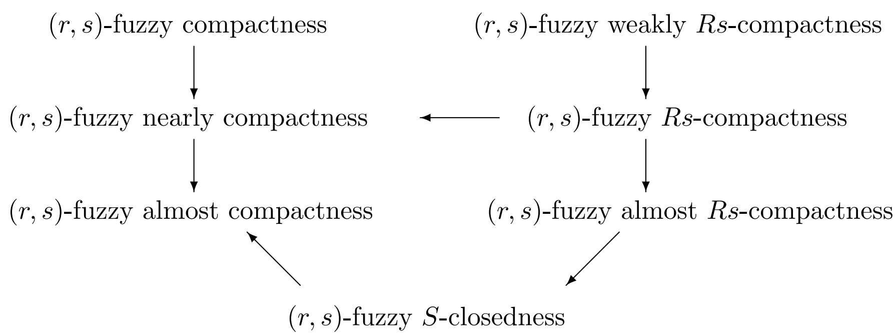
(b) (r,s)-fuzzy weakly Rs-compact if for every (r,s)-fuzzy regular semiopen cover $\{\lambda_i : i \in J\}$ of X, there exists a finite subset J_0 of J such that

$$\bigvee_{i\in J_0} \mathcal{I}_{\mathcal{T},\mathcal{T}^*}(\lambda_i, r, s) = \underline{1}.$$

(c) (r,s)-fuzzy almost Rs-compact if for every (r,s)-fuzzy regular semiopen cover $\{\lambda_i : i \in J\}$ of X, there exists a finite subset J_0 of J such that

$$\bigvee_{i\in J_0} \mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\lambda_i, r, s) = \underline{1}.$$

Remark 3.2. From the above definition and Definition 2.12, it is clear that the following implications are true for each $r \in I_0$ and $s \in I_1$ with $r + s \leq 1$:



From Theorem 2.7, we have the following theorem:

Theorem 3.3. *A double fuzzy topological space $(X, \mathcal{T}, \mathcal{T}^*)$ is (r, s)-fuzzy Rs-compact if and only if for each family $\{\lambda_i | i \in J\}$ of (r, s)-fuzzy regular semiopen sets of X such that $\bigwedge_{i \in J} \lambda_i = \underline{0}$, there exists a finite subset J_0 of J such that*

$$\bigwedge_{i \in J_0} \lambda_i = \underline{0}.$$

Theorem 3.4. *A double fuzzy topological space $(X, \mathcal{T}, \mathcal{T}^*)$ is (r, s)-fuzzy weakly Rs-compact if and only if for each family $\{\lambda_i | i \in J\}$ of (r, s)-fuzzy regular semiopen sets of X such that $\bigwedge_{i \in J} \lambda_i = \underline{0}$, there exists a finite subset J_0 of J such that*

$$\bigwedge_{i \in J_0} \mathcal{C}_{\mathcal{T}, \mathcal{T}^*}(\lambda_i, r, s) = \underline{0}.$$

Proof. Suppose that $(X, \mathcal{T}, \mathcal{T}^*)$ is (r, s)-fuzzy weakly Rs-compact. Let $\{\lambda_i | i \in J\}$ be a family of (r, s)-fuzzy regular semiopen sets of X such that $\bigwedge_{i \in J} \lambda_i = \underline{0}$. Then by Theorem 2.7, $\{\underline{1} - \lambda_i | i \in J\}$ is a family of (r, s)-fuzzy regular semiopen sets of X such that $\bigvee_{i \in J} \underline{1} - \lambda_i = \underline{1} - (\bigwedge_{i \in J} \lambda_i) = \underline{1}$. Since $(X, \mathcal{T}, \mathcal{T}^*)$ is (r, s)-fuzzy weakly Rs-compact, there exists a finite subset J_0 of J such that $\bigvee_{i \in J_0} \mathcal{I}_{\mathcal{T}, \mathcal{T}^*}(\underline{1} - \lambda_i, r, s) = \underline{1}$. Hence $\bigwedge_{i \in J_0} \mathcal{C}_{\mathcal{T}, \mathcal{T}^*}(\lambda_i, r, s) = \underline{1} - (\bigvee_{i \in J_0} \mathcal{I}_{\mathcal{T}, \mathcal{T}^*}(\underline{1} - \lambda_i, r, s)) = \underline{0}$.

Converse follows by reversing the previous arguments. □

Theorem 3.5. *Let $(X, \mathcal{T}, \mathcal{T}^*)$ be a double fuzzy topological space. Then the following are equivalent:*

(1) *$(X, \mathcal{T}, \mathcal{T}^*)$ is (r, s)-fuzzy weakly Rs-compact.*
(2) *For each family $\{\lambda_i | i \in J\}$ of (r, s)-fuzzy regular open sets of X such that $\bigwedge_{i \in J} \lambda_i = \underline{0}$, there exists a finite subset J_0 of J such that*

$$\bigwedge_{i \in J_0} \mathcal{C}_{\mathcal{T}, \mathcal{T}^*}(\lambda_i, r, s) = \underline{0}.$$

(3) *For each (r, s)-fuzzy regular closed cover $\{\lambda_i | i \in J\}$ of X, there exists a finite subset J_0 of J such that $\bigvee_{i \in J_0} \mathcal{I}_{\mathcal{T}, \mathcal{T}^*}(\lambda_i, r, s) = \underline{1}$.*

Proof. (1) ⇒ (2): Trivial.

(2) ⇒ (1): Let $\{\lambda_i | i \in J\}$ be a family of (r,s)-fuzzy regular semiopen sets of X such that $\bigwedge_{i\in J} \lambda_i = \underline{0}$. Since λ_i is an (r,s)-fuzzy regular semiopen set for each $i \in J$, $\mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\lambda_i, r, s) = \mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\mathcal{I}_{\mathcal{T},\mathcal{T}^*}(\lambda_i, r, s), r, s)$ for each $i \in J$. Since $\{\mathcal{I}_{\mathcal{T},\mathcal{T}^*}(\lambda_i, r, s) | i \in J\}$ is a family of (r,s)-fuzzy regular open sets of X such that $\bigwedge_{i\in J} \mathcal{I}_{\mathcal{T},\mathcal{T}^*}(\lambda_i, r, s) = \underline{0}$, by (2) there exists a finite subset J_0 of J such that

$$\bigwedge_{i\in J_0} \mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\lambda_i, r, s) = \bigwedge_{i\in J_0} \mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\mathcal{I}_{\mathcal{T},\mathcal{T}^*}(\lambda_i, r, s), r, s) = \underline{0}.$$

Thus $(X, \mathcal{T}, \mathcal{T}^*)$ is (r,s)-fuzzy weakly Rs-compact.

(2) ⇔ (3): It is obvious. □

Theorem 3.6. *Let $(X, \mathcal{T}_1, \mathcal{T}^*{}_1)$ and $(Y, \mathcal{T}_2, \mathcal{T}^*{}_2)$ be two double fuzzy topological spaces and let $f : X \to Y$ be surjective, double fuzzy weakly open and double fuzzy weakly continuous function. If $(X, \mathcal{T}_1, \mathcal{T}^*{}_1)$ is (r,s)-fuzzy weakly Rs-compact, then so is $(Y, \mathcal{T}_2, \mathcal{T}^*{}_2)$.*

Proof. Let $\{\nu_i | i \in J\}$ be an (r,s)-fuzzy regular closed cover over Y. By Theorem 2.10. $\{f^{-1}(\nu_i) | i \in J\}$ is an (r,s)-fuzzy regular closed cover of X. Since X is (r,s)-fuzzy weakly Rs-compact, by Theorem 3.3 there exists a finite subset J_0 of J such that $\bigvee_{i\in J_0} \mathcal{I}_{\mathcal{T}_1,\mathcal{T}^*{}_1}(f^{-1}(\nu_i), r, s) = \underline{1}$. From the surjectivity and double fuzzy weakly openness of f, we have

$$\begin{aligned}
\underline{1} &= f(\bigvee_{i\in J_0} (\mathcal{I}_{\mathcal{T}_1,\mathcal{T}^*{}_1}(f^{-1}(\nu_i), r, s))) = \bigvee_{i\in J_0} f(\mathcal{I}_{\mathcal{T}_1,\mathcal{T}^*{}_1}(f^{-1}(\nu_i), r, s)) \\
&\le \bigvee_{i\in J_0} \mathcal{I}_{\mathcal{T}_2,\mathcal{T}^*{}_2}(f(\mathcal{C}_{\mathcal{T}_1,\mathcal{T}^*{}_1}(\mathcal{I}_{\mathcal{T}_1,\mathcal{T}^*{}_1}(f^{-1}(\nu_i), r, s), r, s)), r, s) \\
&= \bigvee_{i\in J_0} \mathcal{I}_{\mathcal{T}_2,\mathcal{T}^*{}_2}(f(f^{-1}(\nu_i)), r, s) = \bigvee_{i\in J_0} \mathcal{I}_{\mathcal{T}_2,\mathcal{T}^*{}_2}(\nu_i, r, s).
\end{aligned}$$

Hence $\bigvee_{i\in J_0} \mathcal{I}_{\mathcal{T}_2,\mathcal{T}^*{}_2}(\nu_i, r, s) = \underline{1}$, and thus $(Y, \mathcal{T}_2, \mathcal{T}^*{}_2)$ is (r,s)-fuzzy nearly Rs-compact. □

Theorem 3.7. *A double fuzzy topological space $(X, \mathcal{T}, \mathcal{T}^*)$ is (r,s)-fuzzy almost Rs-compact if and only if for each family $\{\lambda_i | i \in J\}$ of (r,s)-fuzzy regular semiopen sets of X such that $\bigwedge_{i\in J} \lambda_i = \underline{0}$, there exists a finite subset J_0 of J such that*

$$\bigwedge_{i\in J_0} \mathcal{I}_{\mathcal{T},\mathcal{T}^*}(\lambda_i, r, s) = \underline{0}.$$

Proof. Let $(X, \mathcal{T}, \mathcal{T}^*)$ be (r,s)-fuzzy almost Rs-compact and let $\{\lambda_i | i \in J\}$ be a family of (r,s)-fuzzy regular semiopen sets of X such that $\bigwedge_{i\in J} \lambda_i = \underline{0}$. Then $\{\underline{1} - \lambda_i | i \in J\}$ is a family of (r,s)-fuzzy regular semiopen sets of X such that $\bigvee_{i\in J} \underline{1} - \lambda_i = \underline{1} - (\bigwedge_{i\in J} \lambda_i) = \underline{1}$. Since $(X, \mathcal{T}, \mathcal{T}^*)$ is (r,s)-fuzzy almost Rs-compact, there exists a finite subset J_0 of J such that $\bigvee_{i\in J_0} \mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\underline{1} - \lambda_i, r, s) = \underline{1}$. Hence

$$\bigwedge_{i\in J_0} \mathcal{I}_{\mathcal{T},\mathcal{T}^*}(\lambda_i, r, s) = \underline{1} - (\bigvee_{i\in J_0} \mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\underline{1} - \lambda_i, r, s)) = \underline{0}.$$

The converse can be proved similarly. □

Theorem 3.8. *Let $(X, \mathcal{T}, \mathcal{T}^*)$ be a double fuzzy topological space. Then the following statements are equivalent:*

(1) *$(X, \mathcal{T}, \mathcal{T}^*)$ is (r, s)-fuzzy almost Rs-compact.*
(2) *For each family $\{\lambda_i | i \in J\}$ of (r, s)-fuzzy regular open sets of X such that $\bigwedge_{i \in J} \lambda_i = \underline{0}$, there exists a finite subset J_0 of J such that $\bigwedge_{i \in J_0} \lambda_i = \underline{0}$.*
(3) *For each (r, s)-fuzzy regular closed cover $\{\lambda_i | i \in J\}$ of X, there exists a finite subset J_0 of J such that $\bigvee_{i \in J_0} \lambda_i = \underline{1}$.*

Proof. Straightforward. □

Theorem 3.9. *A double fuzzy topological space $(X, \mathcal{T}, \mathcal{T}^*)$ is (r, s)-fuzzy almost Rs-compact if and only if $(X, \mathcal{T}, \mathcal{T}^*)$ is (r, s)-fuzzy S-closed.*

Proof. Let $(X, \mathcal{T}, \mathcal{T}^*)$ be (r, s)-fuzzy S-closed. Since every (r, s)-fuzzy regular semiopen is (r, s)-fuzzy semiopen, $(X, \mathcal{T}, \mathcal{T}^*)$ is (r, s)-fuzzy almost Rs-compact.

Conversely, suppose that $(X, \mathcal{T}, \mathcal{T}^*)$ is (r, s)-fuzzy almost Rs-compact and let $\{\lambda_i | i \in J\}$ be an (r, s)-fuzzy semiopen cover of X. Then there exists $\mu_i \in I^X$ with $\mathcal{T}(\mu_i) \geq r$, $\mathcal{T}^*(\mu_i) \leq s$ such that $\mu_i \leq \lambda_i \leq \mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\mu_i, r, s)$, for each $i \in J$. We can easily show that $\mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\mu_i, r, s)$ is an (r, s)-fuzzy regular closed set for each $i \in J$. Since $\mu_i \leq \lambda_i \leq \mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\lambda_i, r, s)$ for each $i \in J$,

$$\mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\mu_i, r, s) \leq \mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\lambda_i, r, s) \leq \mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\mu_i, r, s), r, s)$$

for each $i \in J$. Thus $\mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\lambda_i, r, s) = \mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\mu_i, r, s)$ for each $i \in J$. Thus $\{\mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\lambda_i, r, s) | i \in J\}$ is an (r, s)-fuzzy regular closed cover of X. Since $(X, \mathcal{T}, \mathcal{T}^*)$ is (r, s)-fuzzy almost Rs-compact, there exists a finite subset J_0 of J such that $\bigvee_{i \in J_0} \mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\lambda_i, r, s) = \underline{1}$. Hence $(X, \mathcal{T}, \mathcal{T}^*)$ is (r, s)-fuzzy S-closed. □

Theorem 3.10. *A double fuzzy topological space $(X, \mathcal{T}, \mathcal{T}^*)$ is an (r, s)-fuzzy weakly Rs-compact if and only if for every an (r, s)-semiopen cover $\{\lambda_i | i \in J\}$ of X, there exists a finite subset J_0 of J such that*

$$\bigvee_{i \in J_0} \mathcal{I}_{\mathcal{T},\mathcal{T}^*}(\mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\lambda_i, r, s) r, s) = \underline{1}.$$

Proof. Similar to Theorem 3.9. □

Theorem 3.11. *Let $(X, \mathcal{T}_1, {\mathcal{T}^*}_1)$ and $(Y, \mathcal{T}_2, {\mathcal{T}^*}_2)$ ba two double fuzzy topological spaces and let $f : X \to Y$ be a surjective, double fuzzy weakly open and double fuzzy weakly continuous function. If $(X, \mathcal{T}_1, {\mathcal{T}^*}_1)$ is (r, s)-fuzzy almost Rs-compact, then so is $(Y, \mathcal{T}_2, {\mathcal{T}^*}_2)$.*

Proof. Let $\{\nu_i | i \in J\}$ be an (r, s)-fuzzy regular closed cover of Y. By Theorem 2.10, $\{f^{-1}(\nu_i) | i \in J\}$ is an (r, s)-fuzzy regular closed cover of X. Since $(X, \mathcal{T}, \mathcal{T}^*)$ is (r, s)-fuzzy almost Rs-compact, by Theorem 2.10, there exists a finite subset J_0 of J such that $\bigvee_{i \in J_0} f^{-1}(\nu_i) = \underline{1}$. From the surjectivity of f we have

$$\underline{1} = f(\bigvee_{i \in J_0} f^{-1}(\nu_i)) = \bigvee_{i \in J_0} f(f^{-1}(\nu_i)) = \bigvee_{i \in J_0} \nu_i.$$

Hence $\bigvee_{i \in J_0} \nu_i = \underline{1}$. Thus $(Y, \mathcal{T}_2, {\mathcal{T}^*}_2)$ is (r, s)-fuzzy almost Rs-compact. □

Definition 3.12. A double fuzzy topological space $(X, \mathcal{T}, \mathcal{T}^*)$ is called (r, s)-fuzzy extremally disconnected if $\mathcal{T}(\mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\lambda, r, s)) \geq r$ and $\mathcal{T}^*(\mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\lambda, r, s)) \leq s$ for every $\lambda \in I^X$ with $\mathcal{T}(\lambda) \geq r$, $\mathcal{T}^*(\lambda) \leq s$.

Theorem 3.13. *Let $(X, \mathcal{T}_1, \mathcal{T}^*{}_1)$ and $(Y, \mathcal{T}_2, \mathcal{T}^*{}_2)$ be two double fuzzy topological spaces, and let $f : X \to Y$ be a surjective, double fuzzy weakly open and double fuzzy weakly continuous function. If $(X, \mathcal{T}_1, \mathcal{T}^*{}_1)$ is (r, s)-fuzzy extremally disconnected, then so is $(Y, \mathcal{T}_2, \mathcal{T}^*{}_2)$.*

Proof. Let $\nu \in I^Y$ with $\mathcal{T}_2(\nu) \geq r$, $\mathcal{T}^*{}_2(\nu) \leq s$. Then $\nu = \mathcal{I}_{\mathcal{T}_2,\mathcal{T}^*{}_2}(\nu, r, s)$. Hence $\mathcal{C}_{\mathcal{T}_2,\mathcal{T}^*{}_2}(\nu, r, s)$ is (r, s)-fuzzy regular closed. By Theorem 2.10, $f^{-1}(\mathcal{C}_{\mathcal{T}_2,\mathcal{T}^*{}_2}(\nu, r, s))$ is (r, s)-fuzzy regular closed, i.e.,

$$f^{-1}(\mathcal{C}_{\mathcal{T}_2,\mathcal{T}^*{}_2}(\nu, r, s)) = \mathcal{C}_{\mathcal{T}_1,\mathcal{T}^*{}_1}(\mathcal{I}_{\mathcal{T}_1,\mathcal{T}^*{}_1}(f^{-1}(\mathcal{C}_{\mathcal{T}_2,\mathcal{T}^*{}_2}(\nu, r, s))), r, s), r, s).$$

Since $(X, \mathcal{T}_1, \mathcal{T}^*{}_1)$ is (r, s)-fuzzy extremally disconnected and

$$\mathcal{T}_1(\mathcal{I}_{\mathcal{T}_1,\mathcal{T}^*{}_1}(f^{-1}(\mathcal{C}_{\mathcal{T}_2,\mathcal{T}^*{}_2}(\nu, r, s)), r, s)) \geq r,$$
$$\mathcal{T}^*{}_1(\mathcal{I}_{\mathcal{T}_1,\mathcal{T}^*{}_1}(f^{-1}(\mathcal{C}_{\mathcal{T}_2,\mathcal{T}^*{}_2}(\nu, r, s)), r, s)) \leq s,$$

and

$$\mathcal{T}_1(\mathcal{C}_{\mathcal{T}_1,\mathcal{T}^*{}_1}(\mathcal{I}_{\mathcal{T}_1,\mathcal{T}^*{}_1}(f^{-1}(\mathcal{C}_{\mathcal{T}_2,\mathcal{T}^*{}_2}(\nu, r, s)), r, s), r, s)) \geq r,$$
$$\mathcal{T}^*{}_1(\mathcal{C}_{\mathcal{T}_1,\mathcal{T}^*{}_1}(\mathcal{I}_{\mathcal{T}_1,\mathcal{T}^*{}_1}(f^{-1}(\mathcal{C}_{\mathcal{T}_2,\mathcal{T}^*{}_2}(\nu, r, s)), r, s), r, s)) \leq s.$$

From the surjectivity and double fuzzy weakly openness of $f^{\to}$ we have

$$\begin{aligned}
\mathcal{C}_{\mathcal{T}_2,\mathcal{T}^*{}_2}(\nu, r, s) &= f(f^{-1}(\mathcal{C}_{\mathcal{T}_2,\mathcal{T}^*{}_2}(\nu, r, s))) \\
&= f(\mathcal{C}_{\mathcal{T}_1,\mathcal{T}^*{}_1}(\mathcal{I}_{\mathcal{T}_1,\mathcal{T}^*{}_1}(f^{-1}(\mathcal{C}_{\mathcal{T}_2,\mathcal{T}^*{}_2}(\nu, r, s)), r, s), r, s)) \\
&\leq \mathcal{I}_{\mathcal{T}_2,\mathcal{T}^*{}_2}(f(\mathcal{C}_{\mathcal{T}_1,\mathcal{T}^*{}_1}(\mathcal{I}_{\mathcal{T}_1,\mathcal{T}^*{}_1}(f^{-1}(\mathcal{C}_{\mathcal{T}_2,\mathcal{T}^*{}_2}(\nu, r, s)), r, s), r, s)), r, s) \\
&= \mathcal{I}_{\mathcal{T}_2,\mathcal{T}^*{}_2}(f(\mathcal{C}_{\mathcal{T}_1,\mathcal{T}^*{}_1}(f^{-1}(\mathcal{C}_{\mathcal{T}_2,\mathcal{T}^*{}_2}(\nu, r, s)), r, s)), r, s) \\
&= \mathcal{I}_{\mathcal{T}_2,\mathcal{T}^*{}_2}(f(f^{-1}(\mathcal{C}_{\mathcal{T}_2,\mathcal{T}^*{}_2}(\nu, r, s))), r, s) \\
&= \mathcal{I}_{\mathcal{T}_2,\mathcal{T}^*{}_2}(\mathcal{C}_{\mathcal{T}_2,\mathcal{T}^*{}_2}(\nu, r, s), r, s).
\end{aligned}$$

Hence $\mathcal{C}_{\mathcal{T}_2,\mathcal{T}^*{}_2}(\nu, r, s) = \mathcal{I}_{\mathcal{T}_2,\mathcal{T}^*{}_2}(\mathcal{C}_{\mathcal{T}_2,\mathcal{T}^*{}_2}(\nu, r, s), r, s)$ and so

$$\mathcal{T}_2(\mathcal{C}_{\mathcal{T}_2,\mathcal{T}^*{}_2}(\nu, r, s)) \geq r,\ \mathcal{T}^*{}_2(\mathcal{C}_{\mathcal{T}_2,\mathcal{T}^*{}_2}(\nu, r, s)) \leq s.$$

Thus $(Y, \mathcal{T}_2, \mathcal{T}^*{}_2)$ is (r, s)-fuzzy extremally disconnected. □

Theorem 3.14. *Let a double fuzzy topological space $(X, \mathcal{T}, \mathcal{T}^*)$ be (r, s)-fuzzy extremally disconnected. If $\lambda \in I^X$ is (r, s)-fuzzy regular semiopen, then*

$$\mathcal{I}_{\mathcal{T},\mathcal{T}^*}(\lambda, r, s) = \lambda = \mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\lambda, r, s).$$

Proof. Let λ be an (r, s)-fuzzy regular semiopen set. Then there exists an (r, s)-fuzzy regular open set μ such that $\mu \leq \lambda \leq \mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\mu, r, s)$. Since X is (r, s)-fuzzy extremally disconnected, $\mu = \mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\mu, r, s)$. And we get $\mu = \mathcal{I}_{\mathcal{T},\mathcal{T}^*}(\mu, r, s)$ since μ is an (r, s)-fuzzy regular open set. Thus we have the following:

$$\begin{aligned}
\mu &= \mathcal{I}_{\mathcal{T},\mathcal{T}^*}(\mu, r, s) \leq \mathcal{I}_{\mathcal{T},\mathcal{T}^*}(\lambda, r, s) \leq \lambda \\
&\leq \mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\lambda, r, s) \leq \mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\mu, r, s) = \mu.
\end{aligned}$$

Hence $\mathcal{I}_{\mathcal{T},\mathcal{T}^*}(\lambda, r, s) = \lambda = \mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\lambda, r, s)$. □

From the above theorem, we get the following:

Theorem 3.15. *Let a double fuzzy topological space $(X, \mathcal{T}, \mathcal{T}^*)$ be (r, s)-fuzzy extremally disconnected. Then the following are equivalent:*

(1) *$(X, \mathcal{T}, \mathcal{T}^*)$ is (r, s)-fuzzy weakly Rs-compact.*
(2) *$(X, \mathcal{T}, \mathcal{T}^*)$ is (r, s)-fuzzy Rs-compact.*
(3) *$(X, \mathcal{T}, \mathcal{T}^*)$ is (r, s)-fuzzy almost Rs-compact.*

Theorem 3.16. *For an (r, s)-fuzzy extremally disconnected double fuzzy topological space, the following are true:*

(1) *(r, s)-fuzzy compactness implies (r, s)-fuzzy weakly Rs-compactness.*
(2) *(r, s)-fuzzy nearly compactness implies (r, s)-fuzzy Rs-compactness.*
(3) *(r, s)-fuzzy almost compactness implies (r, s)-fuzzy almost Rs-compactness.*

Proof. (2) Let $(X, \mathcal{T}, \mathcal{T}^*)$ be an (r, s)-extremally disconnected and (r, s)-fuzzy nearly compact space, let $\{\lambda_i | i \in J\}$ be an (r, s)-fuzzy regular semiopen cover of X. Then there exists an (r, s)-fuzzy regular open set μ_i such that $\mu_i \le \lambda_i \le \mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\mu_i, r, s)$ for each $i \in J$. Since $(X, \mathcal{T}, \mathcal{T}^*)$ is (r, s)-fuzzy extremally disconnected and $\mu_i = \mathcal{I}_{\mathcal{T},\mathcal{T}^*}(\mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\mu_i, r, s), r, s)$ for each $i \in J$, $\lambda_i = \mathcal{I}_{\mathcal{T},\mathcal{T}^*}(\lambda_i, r, s)$ for each $i \in J$. Thus we get $\lambda_i = \mathcal{I}_{\mathcal{T},\mathcal{T}^*}(\mathcal{C}_{\mathcal{T},\mathcal{T}^*}(\lambda_i, r, s), r, s)$ for each $i \in J$ from Theorem 2.7. Hence $(X, \mathcal{T}, \mathcal{T}^*)$ is (r, s)-fuzzy Rs-compact since X is (r, s)-fuzzy nearly compact.

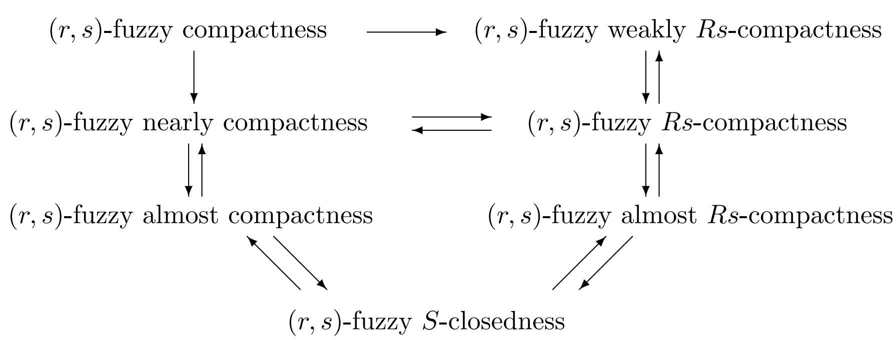
(1) and (3) are similar to (2). □

Corollary 3.17. *If a double fuzzy topological space $(X, \mathcal{T}, \mathcal{T}^*)$ is (r, s)-fuzzy extremally disconnected, then the following are equivalent:*

(1) *(r, s)-fuzzy nearly compactness.*
(2) *(r, s)-fuzzy almost compactness.*
(3) *(r, s)-fuzzy S-closeness.*

Proof. We get the results from Theorems 3.9, 3.15 and 3.16. □

Remark 3.18. If a given double fuzzy topological space is (r, s)-fuzzy extremally disconnected, we get the following implications for each $r \in I_0$, $s \in I_1$ with $r \le s'$:



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