Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 2, No. 2, (October 2011), pp. 239- 257
ISSN 2093–9310
http://www.afmi.or.kr

Optimization of fuzzy integrated vendor-buyer inventory models

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Received 19 April 2011; Revised 9 June 2011; Accepted 10 June 2011

Abstract. Trade Credit is an important service in modern business operation. Therefore to incorporate the concept of vendor-buyer integration and ordersize, dependent trade credit, we present a stylized model to determine the optimal strategy for an integrate vendor-buyer inventory system under the condition of trade credit. This paper develops an approach to determine the optimum economic order quantity and total annual integrated cost for both vendor and buyer under the fuzzy arithmetical operations of function principle are proposed. A full fuzzy model is developed where the input parameters annual demand, production rate, set up cost, holding cost, purchase cost, transportation cost, order processing cost, carrying cost are fuzzy trapezoidal numbers. The optimal policy for the fuzzy production inventory model is determined using the algorithm of extension of the Lagrangean method for solving inequality constraint problem and graded mean integration method is used for defuzzifying the fuzzy total annual integrated cost. A numerical example is used to show the feasibility of the proposed integration models.

2010 AMS Classification: 03E72

Keywords: Fuzzy inventory, Buyer Vendor Inventory Model, Function Principle, Graded Mean Integration Representation, The Annual Integrated Total Cost.

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1. Introduction

In this paper, we consider the situation that a vendor and a buyer can invest in reducing the buyer's ordering cost to decrease their joint total cost. We consider a model to determine an optimal integrated vendor-buyer inventory policy under conditions of order processing time reduction and permissible delay in payments. The total annual cost function of the model possesses some kinds of convexities. The vendor and buyer usually establish a long term production purchasing agreement before any action is taken and then work together towards maximizing their mutual benefits. This implies that the optimal contract quantity and number of deliveries must be determined at the outset of the contract based on their integrated total cost function.

Trade credit plays an important role in finance. Suppliers offer trade credit to retailers to encourage sales, promote market share and reduce on landstock levels. On the otherhand retailers can gain capital materials and service without any payment during the tradecredit period. Hence both supplier and the retailer can take advantage of the trade credit policy. Teng [34] indicated that the trade credit produces two benefits to the supplier.

(i) It should attract new customers who consider it to be a type of price reduction.
(ii) It should cause a reduction in sales outstanding.

Since some established customers will pay more promptly in order to take advantage of tradecredit more frequently. Trade credit intended to link financing, marketing as well as operations concerns. The joint inventory models, payment for the quantity ordered is made when the buyer receives the ordered quantity. . This is not true in today's business transactions since vendors frequently allow credit for some fixed time period for settling the payment for the goods and don't change any interest to the buyer on the amount owed during this credit period. Buyer's don't have to pay the vendor immediately after receiving the goods. They can delay their payment until the end of the allowed period.

Based on this phenomenon, Goyal [16] established a single item inventory model under permissible delay in payments. Chand and Ward [5] modeled the cost of funds tied up in inventory consistent with the assumptions of the classical EOQ model. Gupta [18] showed that when the delay is infinite, any policy with positive and finite order quantity will be optimal.

Chung [13] developed an alternative approach to find the economic order quantity under the condition of permissible delay in payments. Chang [6] extended this issue with linear trend demand. Hwang and Shin [23] modeled an inventory system for retailer's pricing and lotsizing policy for exponentially deteriorating products under the condition of permissible delay in payments.

Huang [21] modified the model by two levels of trade credit. Huang [20] incorporated Huang's model with Teng's model by considering limitation of the retailer's storage space to reflect real life conditions. Since the integrated inventory model would be necessary to incorporate the tradecredit.

This paper incorporates the permissible delay in payments and order processing cost reduction into the integrated inventory model. In the real world the parameter and variables in inventory model may be almost uncertain datum. Park [29] used fuzzy set concept to treat the inventory model with fuzzy inventory cost under arithmetic operation of extension principle. Integrated inventory model for single supplier and a single customer was first introduced by Goyal [16]. The joint economic lotsize model for a single vendor and a single customer was introduced by Banerjee [2]. Goyal modified Banerjee model on the assumption that vendor may possible product a lotsize that may supply an integer number of orders to the buyer.

Recently fuzzy concepts have introduced in EOQ models. First time Mahata [28] investigated the joint economic lotsize model for both buyer and vendor in fuzzy sense.

In this paper we consider integrated inventory model with a single vendor and single buyer for a single product with fuzzy input parameters. Here demand and cost are represented as a trapezoidal fuzzy number. Chen's [8] function principle is proposed for arithmetic operation of fuzzy number and Lagrangean method is used for optimization. Graded mean integration is used for defuzzifying the annual integrated total cost for both vendor and buyer with order processing cost reduction and permissible delay in payments.

In Section 2, we deal with crisp integrated inventory model with different situation. Section 3 discuss with basic concepts of function principle, Lagrangean method, graded mean integration representation method. Section 4 deals with fuzzy inventory model with crisp production quantity and fuzzy production quantity , Section 5 presents a numerical example illustrates the solution procedure demonstrating that the developed model. Last session concluded the discussion of the proposed work.

2. An integrated inventory model with order processing cost reduction and permissible delay in payments

This section examines the cost implications of integrating the lot sizing policies by determining a common economic policy using the total cost for both parties. Figure 1 shows the behavior of inventory levels for both the vendor and the buyer based on the notations and assumptions.

2.1. Notations and Assumptions.

The following notations and assumptions are used throughout to develop the integrated inventory model.

2.1.1. Notations

Q	Lotsize per production run
D	Annual demand
R	Production rate, $R > D$
S_v	Set up cost per production run for the vendor
h_v	Unit stock-holding cost per item per year for the vendor
h_b	Unit stock-holding cost per item per year excluding interest charges for the buyer
P	Unit purchase price
n	Total number of shipments in a batch from the vendor to the buyer, a positive integer
q	Size of each shipment from the vendor to the buyer
F	Fixed transportation cost per shipment
U	Order processing cost per unit time for the buyer
L_0	Original order-processing time per shipment
t	Permissible delay in settling accounts
I	Carrying cost per dollar per year

2.1.2. Assumptions

1. The integrated inventory model only deals with a single vendor and single buyer for a single product.
2. The demand for the item is constant overtime.
3. Shortages are not allowed.
4. The Lead time L has mutually independent components.
5. Since the vendor allows the buyer a delay in payment, the cost for improving the annual order processing cost are assigned to the buyer.
6. The time period is infinite.

2.2. **Mathematical Model.** This section examines the cost implications of integrating the lotsizing policies by determining a common economic policy using the total cost for both parties. Figure 1 shows the behavior of inventory levels for both the vendor and the buyer based on the above notations and assumptions. The annual integrated total cost for the both the vendor and the buyer consist of (1) the vendor's total annual cost, (2) the buyer's total annual cost.

(1) The accumulation and depletion processes of the vendor's inventory for each production cycle are shown in Figure 1 according to Woo et al. [36]. The vendor's holding cost per production cycle is equal to the unit holding cost times the value of accumulated inventory (bold-lined area) minus the depleted inventory (shaded area). Therefore the vendor's holding cost per year is given by

$$\text{Holding cost per year} = \frac{(\text{Bold Lined Area - Shaded Area}) \times h_v}{\text{Cycle Time}}$$

$$= \frac{\left\{\left[Q\left(\frac{Q/n}{R} + (n-1)\frac{Q}{nD}\right) - \frac{Q(Q/R)}{2}\right] - \frac{Q}{nD}\left[\frac{Q}{n} + \frac{2Q}{n} + \cdots + \frac{(n-1)Q}{n}\right\}\right\} h_v}{Q/D}$$

$$= \frac{(n-2)Q}{2n}\left(1 - \tfrac{D}{R}\right) h_v + \frac{Q}{2n} h_v.$$

After adding the setup cost, the vendor's total annual cost is given by

$$TC_v(n,Q) = \tfrac{DS_v}{Q} + \tfrac{(n-2)Q}{2n}\left(1 - \tfrac{D}{R}\right) h_v + \tfrac{Q}{2n} h_v. \tag{2.1}$$

(2) The buyer's annual inventory cost consists of the cost of placing orders, holding cost excluding interest charges and the cost of interest charges for the items kept in stock during the permissible settlement period. Chand and Ward [5] assumed that the delay of t periods in making the payment to the supplier is equivalent to a price discount. Therefore if p is the price per unit with permissible delay of t periods, then the effective price per unit is

$$\hat{P} = \frac{P}{1 + It}.$$

After adding the transportation cost, order processing cost, carrying cost and interest charges the buyer's annual cost is

$$TC_b(n, Q, L_0) = n(F + UL_0)\tfrac{D}{Q} + \tfrac{Q}{2n}\left[h_b + \tfrac{PI}{1+It}\right] + R(L_0).$$

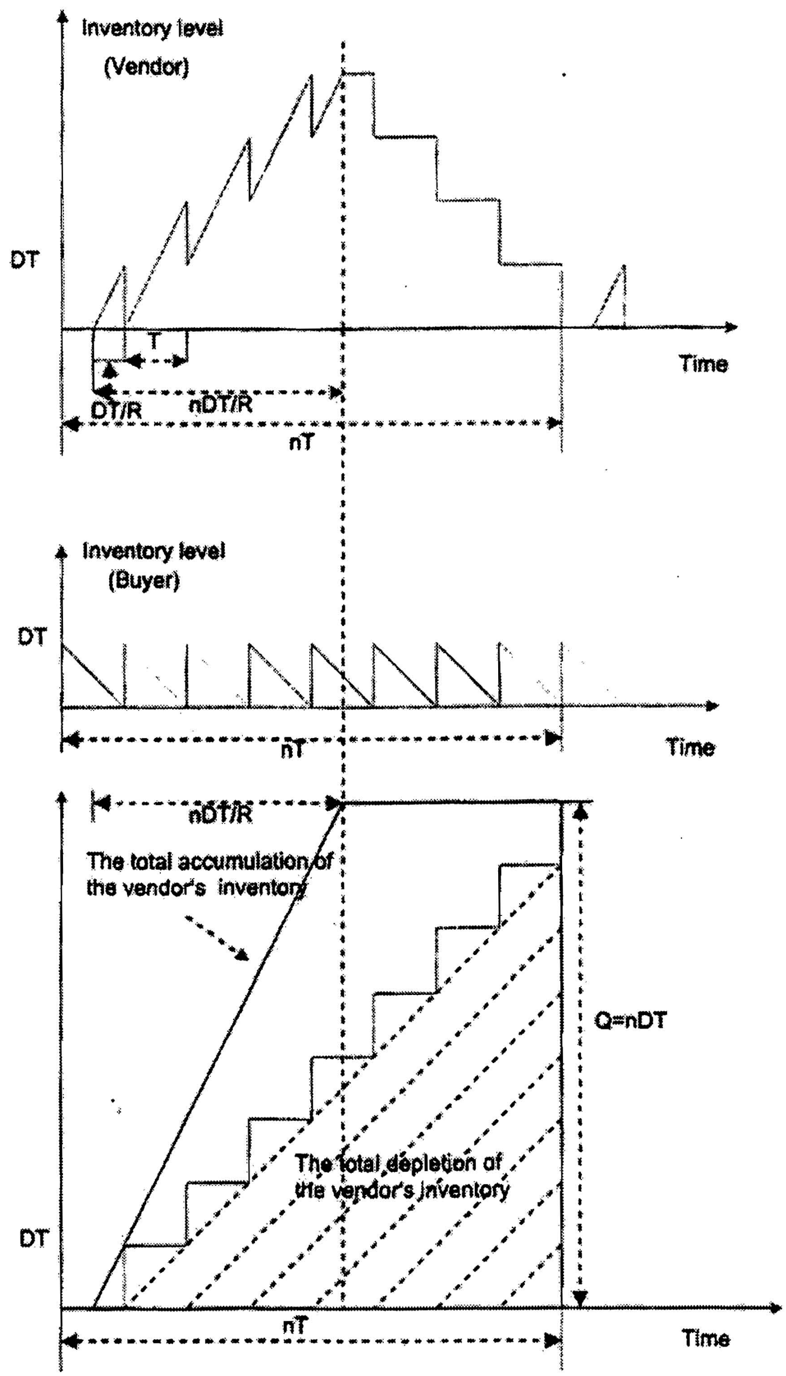

FIGURE 1. Time weighted inventory for vendor and buyer

The annual integrated total cost for both the vendor and buyer is

$$JTC(n,Q,L_0) = TC_v(n,Q) + TC_b(n,Q,L_0)$$

$$= \frac{D\left[S_v+n(F+UL_0)\right]}{Q} + \frac{Q}{2n}\left[(n-2)\left(1-\frac{D}{R}\right)h_v + h_v + h_b + \frac{PI}{1+It}\right] + R(L_0).$$

The objective is to find the optimal shortage quantity and optimal order quantity which minimize the annual integrated total cost.

The necessary condition for minimum $\frac{\partial JTC}{\partial Q} = 0$

At a particular value of n, let $JTC(Q) = JTC(n,Q,L_0)$

$$Q^* = \sqrt{\frac{2nD\left[S_v + n(F+UL_0)\right]}{(n-2)\left(1-\frac{D}{R}\right)h_v + h_v + h_b + \frac{PI}{1+It}}}.$$

3. The fuzzy arithmetical operations under function principle

Function principle is proposed to be as the fuzzy arithmetical operations by trapezoidal fuzzy numbers. We define some fuzzy arithmetical operations under Function Principle as follows :

Suppose $\tilde{A} = (a_1,a_2,a_3,a_4)$ and $\tilde{B} = (b_1,b_2,b_3,b_4)$ are two trapezoidal fuzzy numbers. Then

(1) The addition of $\tilde{A}$ and $\tilde{B}$ is

$$\tilde{A} \oplus \tilde{B} = (a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4)$$

where $a_1,a_2,a_3,a_4,b_1,b_2,b_3$ and b_4 are any real numbers.

(2) The multiplication of $\tilde{A}$ and $\tilde{B}$ is

$$\tilde{A} \otimes \tilde{B} = (C_1,C_2,C_3,C_4)$$

where $T = \{a_1b_1, a_1b_4, a_4b_1, a_4b_4\}$, $T_1 = \{a_2b_2, a_2b_3, a_3b_2, a_3b_3\}$, $C_1 = \min T$, $C_2 = \min T_1$, $C_3 = \max T$, $C_4 = \max T_1$.

If $a_1,a_2,a_3,a_4,b_1,b_2,b_3$ and b_4 are all zero positive real numbers then

$$\tilde{A} \otimes \tilde{B} = (a_1b_1, a_2b_2, a_3b_3, a_4b_4).$$

(3) $-\tilde{B} = (-b_4,-b_3,-b_2,-b_1)$ and the subtraction of $\tilde{A}$ and $\tilde{B}$ is

$$\tilde{A} \ominus \tilde{B} = (a_1-b_4, a_2-b_3, a_3-b_2, a_4-b_1)$$

where $a_1,a_2,a_3,a_4,b_1,b_2,b_3$ and b_4 are any real numbers.

(4) $\frac{1}{\tilde{B}} = \tilde{B}^{-1} = \left(\frac{1}{b_4},\frac{1}{b_3},\frac{1}{b_2},\frac{1}{b_1}\right)$ where b_1,b_2,b_3 and b_4 are positive real numbers. If $a_1,a_2,a_3,a_4,b_1,b_2,b_3$ and b_4 are nonzero positive numbers, then the division of $\tilde{A}$ and $\tilde{B}$ is

$$\tilde{A} \oslash \tilde{B} = \left(\frac{a_1}{b_4},\frac{a_2}{b_3},\frac{a_3}{b_2},\frac{a_4}{b_1}\right).$$

(5) For any $\alpha \in \mathbb{R}$,

(i) If $\alpha \geq 0$, then $\alpha \otimes \tilde{A} = (\alpha a_1, \alpha a_2, \alpha a_3, \alpha a_4)$,

(ii) If $\alpha < 0$, then $\alpha \otimes \tilde{A} = (\alpha a_4, \alpha a_3, \alpha a_2, \alpha a_1)$.

3.1. **Extension of the Lagrangean method.** Taha [32] discussed how to solve the optimum solution of nonlinear programming problem with equality constraints by using Lagrangean Method, and showed how the Lagrangean method may be extended to solve inequality constraints. The general idea of extending the Lagrangean procedure is that if the unconstrained optimum the problem does not satisfy all constraints, the constrained optimum must occur at a boundary point of the solution space. Suppose that the problem is given by

Minimize $y = f(x)$
Sub to $g_i(x) \geq 0$, $i = 1, 2, \cdots, m$.

The nonnegativity constraints $x \geq 0$ if any are included in the m constraints. Then the procedure of the Extension of the Lagrangean method involves the following steps.

Step 1: Solve the unconstrained problem

Min $y = f(x)$
If the resulting optimum satisfies all the constraints, stop because all constraints are redundant. Otherwise set $K = 1$ and go to step 2.

Step 2: : Activate any K constraints (i,e., convert them into equality) and optimize $f(x)$ subject to the K active constraints by the Lagrangean method. If the resulting solution is feasible with respect to the remaining constraints and repeat the step. If all sets of active constraints taken K at a time are considered without encountering a feasible solution, go to step 3.

Step 3:

If $K = m$, stop; no feasible solution exists.
Otherwise set $K = K + 1$ and go to step 2.

3.2. **Graded mean integration representation method.** Chen and Hsieh [11] introduced Graded mean Integration Representation Method based on the integral value of graded mean h-level of generalized fuzzy number for defuzzifying generalized fuzzy number. Here, we fist define generalized fuzzy number as follows:

Suppose $\tilde{A}$ is a generalized fuzzy number as shown in Figure 2.

It is described as any fuzzy subset of the real line $\mathbb{R}$, whose membership function $\mu_{\tilde{A}}$ satisfies the following conditions.

1. $\mu_{\tilde{A}}(x)$ is a continuous mapping from $\mathbb{R}$ to $[0, 1]$,
2. $\mu_{\tilde{A}}(x) = 0$, $-\infty < x \leq a_1$,
3. $\mu_{\tilde{A}}(x) = L(x)$ is strictly increasing on $[a_1, a_2]$,
4. $\mu_{\tilde{A}}(x) = W_A$, $a_2 \leq x \leq a_3$,
5. $\mu_{\tilde{A}}(x) = R(x)$ is strictly decreasing on $[a_3, a_4]$,
6. $\mu_{\tilde{A}}(x) = 0$, $a_4 \leq x < \infty$,

where $0 < W_A \leq 1$ and a_1, a_2, a_3 and a_4 are real numbers. This type of generalized fuzzy numbers are denoted as

$$\tilde{A} = (a_1, a_2, a_3, a_4; w_A)_{LR}.$$

When $w_A = 1$, it can be formed as $\tilde{A} = (a_1, a_2, a_3, a_4)_{LR}$. Second, by Graded Mean Integration Representation Method, L^{-1} and R^{-1} are the inverse functions of L and R respectively and the graded mean h-level value of generalized fuzzy number

$\tilde{A} = (a_1, a_2, a_3, a_4; w_A)_{LR}$ is given by $\frac{h}{2}\left(L^{-1}(h) + R^{-1}(h)\right)$ (see Figure 2). Then the graded Mean Integration Representation of $P(\tilde{A})$ with grade w_A, where

$$P(\tilde{A}) = \frac{\int_0^{\omega_A} \frac{h}{2}\left(L^{-1}(h) + R^{-1}(h)\right) dh}{\int_0^{w_A} h dh} \tag{3.1}$$

where $0 < h \leq w_A$ and $0 < w_A \leq 1$.

Throughout this paper, we only use popular trapezoidal fuzzy number as the type of all fuzzy parameters in our proposed fuzzy production inventorymodels. Let $\tilde{B}$ be a trapezoidal fuzzy number and be denoted as $\tilde{B} = (b_1, b_2, b_3, b_4)$. Then we can get the Graded Mean Integration Representation of by the formula (3.1) as

$$\begin{aligned} P(\tilde{B}) &= \frac{\int_0^1 \frac{h}{2}\left[(b_1 + b_4) + h(b_2 - b_1 - b_4 + b_3)\right] dh}{\int_0^1 h dh} \\ &= \frac{b_1 + 2b_2 + 2b_3 + b_4}{6}. \end{aligned} \tag{3.2}$$

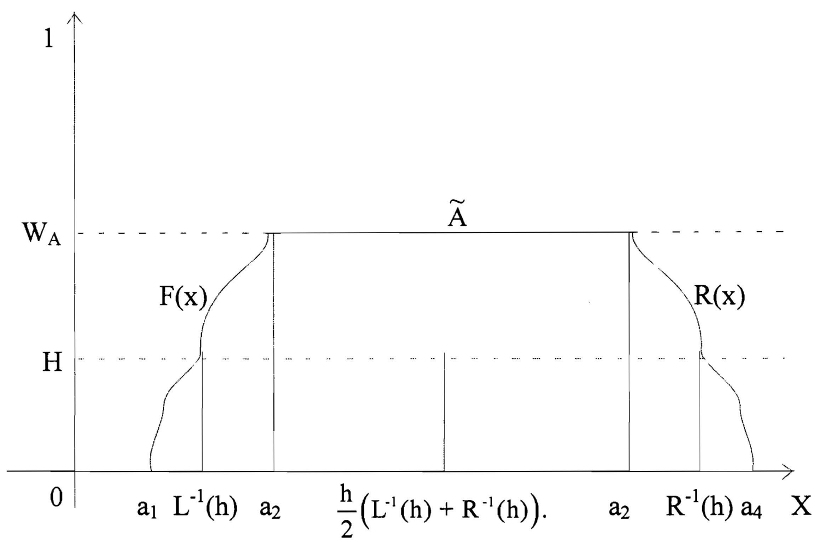

FIGURE 2. The graded mean h-level value of generalized fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4 : w_A)_{LR}$

4. Fuzzy integrated inventory model for crisp production quantity

Throughout this paper, we use of the following variables in order to simplify the treatment of an integrated inventory models.

Let $\tilde{D}$, $\tilde{R}$, $\tilde{S}_v$, $\tilde{h}_v$, $\tilde{h}_b$, $\tilde{P}$, $\tilde{F}$, $\tilde{U}$ and $\tilde{I}$ be fuzzy parameters. We introduce an integrated inventory model with fuzzy parameters for crisp production quantity $JTC(n, Q, L_0)$ as follows.

The annual integrated total inventory cost for both the vendor and buyer

$$
\begin{aligned}
&J\tilde{T}C(n,Q,L_0)\\
&=\Big\{\tfrac{D_1[Sv_1+n(F_1+U_1L_0)]}{Q}+\tfrac{Q}{2n}\left[(n-2)\left(1-\tfrac{D_4}{R_1}\right)h_{v_1}+h_{v_1}+h_{b_1}+\tfrac{P_1I_1}{1+I_4t}\right],\\
&\quad\tfrac{D_2[Sv_2+n(F_2+U_2L_0)]}{Q}+\tfrac{Q}{2n}\left[(n-2)\left(1-\tfrac{D_3}{R_2}\right)h_{v_2}+h_{v_2}+h_{b_2}+\tfrac{P_2I_2}{1+I_2t}\right],\\
&\quad\tfrac{D_3[Sv_3+n(F_3+U_3L_0)]}{Q}+\tfrac{Q}{2n}\left[(n-2)\left(1-\tfrac{D_2}{R_3}\right)h_{v_3}+h_{v_3}+h_{b_3}+\tfrac{P_3I_3}{1+I_3t}\right],\\
&\quad\tfrac{D_4[Sv_4+n(F_4+U_4L_0)]}{Q}+\tfrac{Q}{2n}\left[(n-2)\left(1-\tfrac{D_1}{R_4}\right)h_{v_4}+h_{v_4}+h_{b_4}+\tfrac{P_4I_4}{1+I_1t}\right]\Big\}
\end{aligned}
$$

$$
\begin{aligned}
J\tilde{T}C(n,Q,L_0)=\Big[\tilde{D}\otimes\Big[\tilde{S}_v\oplus n\left(\tilde{F}\oplus\tilde{U}\otimes L_0\right)\Big]\oslash\tilde{Q}\Big]\oplus\\
\tilde{Q}\oslash 2\otimes n\Big[(n-2)\otimes(1-\tilde{D}\oslash\tilde{R})\otimes h_v\Big]\oplus\\
h_v\oplus h_b\oplus\Big[P\otimes\tilde{I}\oslash(1\oplus\tilde{I}\otimes t)\Big]
\end{aligned}
\tag{4.1}
$$

where $\oslash$, $\otimes$ and $\oplus$ are the fuzzy arithmetical operators under Function Principle. Suppose

$\tilde{D}=(D_1,D_2,D_3,D_4)$
$\tilde{S}_v=(S_{v_1},S_{v_2},S_{v_3},S_{v_4})$
$\tilde{F}=(F_1,F_2,F_3,F_4)$
$\tilde{U}=(U_1,U_2,U_3,U_4)$
$\tilde{Q}=(Q_1,Q_2,Q_3,Q_4)$
$\tilde{R}=(R_1,R_2,R_3,R_4)$
$\tilde{h}_v=(h_{v_1},h_{v_2},h_{v_3},h_{v_4})$
$\tilde{h}_b=(h_{b_1},h_{b_2},h_{b_3},h_{b_4})$
$\tilde{P}=(P_1,P_2,P_3,P_4)$
$\tilde{I}=(I_1,I_2,I_3,I_4)$

are nonnegative trapezoidal fuzzy numbers. Then we solve the optimal production quantity of formula (4.1) as the following steps.

Second, we defuzzify the fuzzy total production inventory for the vendor and buyer cost by formula (3.2). Graded mean integration representation of $J\tilde{T}C(n,Q,L_0)$ is

$$
\begin{aligned}
&P(J\tilde{T}C(n,Q,L_0))\\
&=\tfrac{1}{6}\Big\{\tfrac{D_1[Sv_1+n(F_1+U_1L_0)]}{Q}+\tfrac{Q}{2n}\left[(n-2)\left(1-\tfrac{D_4}{R_1}\right)h_{v_1}+h_{v_1}+h_{b_1}+\tfrac{P_1I_1}{1+I_4t}\right]+\\
&\quad\tfrac{D_2[Sv_2+n(F_2+U_2L_0)]}{Q}+\tfrac{Q}{2n}\left[(n-2)\left(1-\tfrac{D_3}{R_2}\right)h_{v_2}+h_{v_2}+h_{b_2}+\tfrac{P_2I_2}{1+I_3t}\right]+\\
&\quad\tfrac{D_3[Sv_3+n(F_3+U_3L_0)]}{Q}+\tfrac{Q}{2n}\left[(n-2)\left(1-\tfrac{D_2}{R_3}\right)h_{v_3}+h_{v_3}+h_{b_3}+\tfrac{P_3I_3}{1+I_2t}\right]+\\
&\quad\tfrac{D_4[Sv_4+n(F_4+U_4L_0)]}{Q}+\tfrac{Q}{2n}\left[(n-2)\left(1-\tfrac{D_1}{R_4}\right)h_{v_4}+h_{v_4}+h_{b_4}+\tfrac{P_4I_4}{1+I_1t}\right]\Big\}.
\end{aligned}
$$

Third, we can get the optimal production quantity Q^* when $P(J\tilde{T}C(n,Q,L_0))$ is minimization.

In order to find the minimization of $P(J\tilde{T}C(n,Q,L_0))$, the derivative of $P(J\tilde{T}C(n,Q,L_0))$ with Q is

$$\frac{\partial P\left(J\tilde{T}C(n,Q,L_0)\right)}{\partial Q} = 0.$$

We find the optimal production quantity

$$Q = Q^* = \sqrt{\frac{\begin{pmatrix} 2n\Big[D_1(Sv_1+n(F_1+U_1L_0))+2D_2(Sv_2+n(F_2+U_2L_0))+ \\ 2D_3(Sv_3+n(F_3+U_3L_0))+D_4(Sv_4+n(F_4+U_4L_0))\Big] \end{pmatrix}}{\begin{pmatrix} (n-2)\Big[\left(1-\frac{D_4}{R_1}\right)hv_1+2\left(1-\frac{D_3}{R_2}\right)hv_2+2\left(1-\frac{D_2}{R_3}\right)hv_3+\left(1-\frac{D_1}{R_4}\right)hv_4\Big]+ \\ (h_{v_1}+2h_{v_2}+2h_{v_3}+h_{v_4})+(h_{b_1}+2h_{b_2}+2h_{b_3}+h_{b_4})+ \\ \frac{P_1I_1}{1+I_4t}+\frac{2P_2I_2}{1+I_3t}+\frac{2P_3I_3}{1+I_2t}+\frac{P_4I_4}{1+I_1t} \end{pmatrix}}}. \tag{4.2}$$

4.1. **Fuzzy Integrated Inventory Model for fuzzy production quantity.** In this section, we introduce an integrated inventory model by changing the crisp production quantity into fuzzy production quantity.

Suppose fuzzy production quantity $\tilde{Q}$ be a trapezoidal fuzzy number $\tilde{Q} = (Q_1, Q_2, Q_3, Q_4)$ with $0 < Q_1 \leq Q_2 \leq Q_3 \leq Q_4$. Thus we can get the fuzzy total production inventory cost

$$\begin{aligned}
&P(J\tilde{T}C_1(n,Q,L_0)) \\
&= \Big\{ \tfrac{D_1[Sv_1+n(F_1+U_1L_0)]}{Q_4} + \tfrac{Q_1}{2n}\left[(n-2)\left(1-\tfrac{D_4}{R_1}\right)h_{v_1} + h_{v_1} + h_{b_1} + \tfrac{P_1I_1}{1+I_4t}\right], \\
&\quad 2\tfrac{D_2[Sv_2+n(F_2+U_2L_0)]}{Q_3} + \tfrac{2Q_2}{2n}\left[(n-2)\left(1-\tfrac{D_3}{R_2}\right)h_{v_2} + h_{v_2} + h_{b_2} + \tfrac{P_2I_2}{1+I_3t}\right], \\
&\quad \tfrac{2D_3[Sv_3+n(F_3+U_3L_0)]}{Q_2} + \tfrac{2Q_3}{2n}\left[(n-2)\left(1-\tfrac{D_2}{R_3}\right)h_{v_3} + h_{v_3} + h_{b_3} + \tfrac{P_3I_3}{1+I_2t}\right], \\
&\quad \tfrac{D_4[Sv_4+n(F_4+U_4L_0)]}{Q_1} + \tfrac{Q_4}{2n}\left[(n-2)\left(1-\tfrac{D_1}{R_4}\right)h_{v_4} + h_{v_4} + h_{b_4} + \tfrac{P_4I_4}{1+I_1t}\right]\Big\}.
\end{aligned}$$

We can obtain the Graded Mean Integration Representation of

$$P(J\tilde{T}C_1(n,Q,L_0))$$

by formula (3.2) as

$$
\begin{aligned}
P(J\tilde{T}C_1(n,Q,L_0)) = \tfrac{1}{6}\Big\{ & \tfrac{D_1[Sv_1+n(F_1+U_1L_0)]}{Q_4} + \\
& \tfrac{Q_1}{2n}\left[(n-2)\left(1-\tfrac{D_4}{R_1}\right)h_{v_1}+h_{v_1}+h_{b_1}+\tfrac{P_1I_1}{1+I_4t}\right] + \\
& \tfrac{2D_2[Sv_2+n(F_2+U_2L_0)]}{Q_3} + \\
& \tfrac{2Q_2}{2n}\left[(n-2)\left(1-\tfrac{D_3}{R_2}\right)h_{v_2}+h_{v_2}+h_{b_2}+\tfrac{P_2I_2}{1+I_3t}\right] + \\
& \tfrac{2D_3[Sv_3+n(F_3+U_3L_0)]}{Q_2} + \\
& \tfrac{2Q_3}{2n}\left[(n-2)\left(1-\tfrac{D_2}{R_3}\right)h_{v_3}+h_{v_3}+h_{b_3}+\tfrac{P_3I_3}{1+I_2t}\right] + \\
& \tfrac{D_4[Sv_4+n(F_4+U_4L_0)]}{Q_1} + \\
& \tfrac{Q_4}{2n}\left[(n-2)\left(1-\tfrac{D_1}{R_4}\right)h_{v_4}+h_{v_4}+h_{b_4}+\tfrac{P_4I_4}{1+I_1t}\right]\Big\}
\end{aligned}
\tag{4.3}
$$

with $0 < Q_1 \le Q_2 \le Q_3 \le Q_4$. It will not change the meaning of formula (4.3) if we replace inequality conditions $0 < Q_1 \le Q_2 \le Q_3 \le Q_4$ into the following inequality $Q_2 - Q_1 \ge 0$, $Q_3 - Q_2 \ge 0$, $Q_4 - Q_3 \ge 0$, $Q_1 > 0$. In the following steps, extension of the Lagrangean method is used to find the solutions of Q_1, Q_2, Q_3, Q_4 to minimize $P(J\tilde{T}C_1(n,Q,L_0))$ in formula (4.3).

Step 1 : Solve the unconstraint problem. Consider min $P(J\tilde{T}C_1(n,Q,L_0))$

To find the min $P(J\tilde{T}C_1(n,Q,L_0))$, we have to find the derivative of

$$P(J\tilde{T}C_1(n,Q,L_0))$$

with respect to Q_1, Q_2, Q_3, Q_4.

$$\frac{\partial P}{\partial Q_1} = \frac{1}{6}\left\{\frac{1}{2n}\left[(n-2)\left(1-\frac{D_4}{R_1}\right)h_{v_1}+h_{v_1}+h_{b_1}+\frac{P_1I_1}{1+I_4t}\right] - \frac{D_4(Sv_4+n(F_4+U_4L_0))}{Q_1^2}\right\}$$

$$\frac{\partial P}{\partial Q_2} = \frac{1}{6}\left\{\frac{2}{2n}\left[(n-2)\left(1-\frac{D_3}{R_2}\right)h_{v_2}+h_{v_2}+h_{b_2}+\frac{P_2I_2}{1+I_3t}\right] - \frac{2D_3(Sv_3+n(F_3+U_3L_0))}{Q_2^2}\right\}$$

$$\frac{\partial P}{\partial Q_3} = \frac{1}{6}\left\{\frac{2}{2n}\left[(n-2)\left(1-\frac{D_2}{R_3}\right)h_{v_3}+h_{v_3}+h_{b_3}+\frac{P_3I_3}{1+I_2t}\right] - \frac{2D_2(Sv_2+n(F_2+U_2L_0))}{Q_3^2}\right\}$$

$$\frac{\partial P}{\partial Q_4} = \frac{1}{6}\left\{\frac{1}{2n}\left[(n-2)\left(1-\frac{D_1}{R_4}\right)h_{v_4}+h_{v_4}+h_{b_4}+\frac{P_4I_4}{1+I_1t}\right] - \frac{D_1(Sv_1+n(F_1+U_1L_0))}{Q_4^2}\right\}.$$

Let all the above results partial derivatives equal to zero and solve Q_1, Q_2, Q_3, Q_4.

Let $\frac{\partial P}{\partial Q_1} = 0$. Then

$$Q_1 = \sqrt{\frac{2nD_4(Sv_4+n(F_4+U_4L_0))}{(n-2)\left(1-\frac{D_4}{R_1}\right)h_{v_1}+h_{v_1}+h_{b_1}+\frac{P_1I_1}{1+I_4t}}}.$$

Let $\frac{\partial P}{\partial Q_2} = 0$. Then

$$Q_2 = \sqrt{\frac{4nD_3(Sv_3+n(F_3+U_3L_0))}{2\left[(n-2)\left(1-\frac{D_3}{R_2}\right)h_{v_2}+h_{v_2}+h_{b_2}+\frac{P_2I_2}{1+I_3t}\right]}}.$$

Let $\frac{\partial P}{\partial Q_3} = 0$. Then

$$Q_3 = \sqrt{\frac{4nD_2(Sv_2+n(F_2+U_2L_0))}{2\left[(n-2)\left(1-\frac{D_2}{R_3}\right)h_{v_3}+h_{v_3}+h_{b_3}+\frac{P_3I_3}{1+I_2t}\right]}}.$$

Let $\frac{\partial P}{\partial Q_4} = 0$. Then

$$Q_4 = \sqrt{\frac{2nD_1(Sv_1+n(F_1+U_1L_0))}{(n-2)\left(1-\frac{D_1}{R_4}\right)h_{v_4}+h_{v_4}+h_{b_4}+\frac{P_4I_4}{1+I_1t}}}.$$

Because the above show that $Q_1 > Q_2 > Q_3 > Q_4$, it does not satisfy the constraint $0 < Q_1 \le Q_2 \le Q_3 \le Q_4$. Therefore set $K = 1$ and go to Step 2.

Step 2 : Convert the inequality constraint $Q_2 - Q_1 \ge 0$ into equality constraint $Q_2 - Q_1 = 0$ and optimize $P(J\tilde{T}C_1(n, Q, L_0))$ subject to $Q_2 - Q_1 = 0$ by the Lagrangean Method. We have Lagrangean function as

$$L(Q_1, Q_2, Q_3, Q_4, \lambda) = P(J\tilde{T}C_1(n, Q, L_0)) - \lambda(Q_2 - Q_1).$$

Taking the partial derivatives of $L(Q_1, Q_2, Q_3, Q_4, \lambda)$ with respect to Q_1, Q_2, Q_3, Q_4 and λ to find the minimization of $L(Q_1, Q_2, Q_3, Q_4, \lambda)$. Let all the partial derivatives equal to zero and solve Q_1, Q_2, Q_3 and Q_4. Then we get,

$$\frac{\partial L}{\partial Q_1} = \left\{\frac{1}{2n}\left[(n-2)\left(1-\frac{D_4}{R_1}\right)h_{v_1} + h_{v_1} + h_{b_1} + \frac{P_1I_1}{1+I_4t}\right] - \frac{D_4(Sv_4+n(F_4+U_4L_0))}{Q_1^2}\right\}\frac{1}{6} + \lambda$$

implies that

$$-\frac{1}{2n}\left[(n-2)\left(1-\frac{D_4}{R_1}\right)h_{v_1} + h_{v_1} + h_{b_1} + \frac{P_1I_1}{1+I_4t}\right] + \frac{D_4(Sv_4+n(F_4+U_4L_0))}{Q_1^2} = 6\lambda,$$

$$\frac{\partial L}{\partial Q_2} = \frac{1}{6}\Big[-\frac{2D_3(Sv_3+n(F_3+U_3L_0))}{Q_2^2}+ \frac{2}{2n}\left[(n-2)\left(1-\frac{D_3}{R_2}\right)h_{v_2}+h_{v_2}+h_{b_2}+\frac{P_2I_2}{1+I_3t}\right]\Big]-\lambda=0,$$

$$\frac{\partial L}{\partial Q_3} = \frac{1}{6}\Big[-\frac{2D_2(Sv_2+n(F_2+U_2L_0))}{Q_3^2}+ \frac{2}{2n}\left[(n-2)\left(1-\frac{D_2}{R_3}\right)h_{v_3}+h_{v_3}+h_{b_3}+\frac{P_3I_3}{1+I_2t}\right]\Big]=0,$$

$$\frac{\partial L}{\partial Q_4} = \frac{1}{6}\Big[-\frac{D_1(Sv_1+n(F_1+U_1L_0))}{Q_4^2}+ \frac{1}{2n}\left[(n-2)\left(1-\frac{D_1}{R_4}\right)h_{v_4}+h_{v_4}+h_{b_4}+\frac{P_4I_4}{1+I_1t}\right]\Big]=0,$$

$$\frac{\partial P}{\partial \lambda} = -(Q_2-Q_1),$$

$$Q_1 = Q_2 = \sqrt{\frac{2n\left[D_4(Sv_4+n(F_4+U_4L_0))+2D_3(Sv_3+n(F_3+U_3L_0))\right]}{\begin{pmatrix}(n-2)\left(1-\frac{D_4}{R_1}\right)hv_1+h_{v_1}+h_{b_1}+\frac{P_1I_1}{1+I_4t}+\\ 2(n-2)\left(1-\frac{D_3}{R_2}\right)hv_2+2h_{v_2}+2h_{b_2}+\frac{2P_2I_2}{1+I_3t}\end{pmatrix}}}$$

$$Q_3 = \sqrt{\frac{4nD_2\left(Sv_2+n(F_2+U_2L_0)\right)}{2(n-2)\left(1-\frac{D_2}{R_3}\right)hv_3+2h_{v_3}+2h_{b_3}+\frac{2P_3I_3}{1+I_2t}}}$$

$$Q_4 = \sqrt{\frac{2nD_1\left(Sv_1+n(F_1+U_1L_0)\right)}{(n-2)\left(1-\frac{D_1}{R_4}\right)hv_4+h_{v_4}+h_{b_4}+\frac{P_4I_4}{1+I_1t}}}.$$

Because the above results show that $Q_3 > Q_4$, it does not satisfy the constraint $0 < Q_1 \le Q_2 \le Q_3 \le Q_4$. Therefore it is not a local optimum. Similarly we can get the same result if we select any other one inequality constraint to be equality constraint, therefore set $K = 2$ and go to Step 3.

Step 3 : Convert the inequality constraints $Q_2 - Q_1 \ge 0$, $Q_3 - Q_2 \ge 0$, into equality constraints $Q_2 - Q_1 = 0$ and $Q_3 - Q_2 = 0$. We optimize $P(J\tilde{T}C_1(n, Q, L_0))$. Subject to $Q_2 - Q_1 = 0$ and $Q_3 - Q_2 = 0$ by the Lagrangean Method. Then the Lagrangean method is

$$L(Q_1, Q_2, Q_3, Q_4, \lambda_1, \lambda_2) = P(J\tilde{T}C_1(n, Q, L_0)) - \lambda_1(Q_2 - Q_1) - \lambda_2(Q_3 - Q_2).$$

In order to find the minimization of $L(Q_1, Q_2, Q_3, Q_4, \lambda_1, \lambda_2)$, we take the partial derivatives of $L(Q_1, Q_2, Q_3, Q_4, \lambda_1, \lambda_2)$ with respect to Q_1, Q_2, Q_3, Q_4, λ_1, λ_2 and let all the partial derivatives equal to zero and solve Q_1, Q_2, Q_3 and Q_4.

$$\frac{\partial L}{\partial Q_1} = \frac{1}{6}\Big[\frac{1}{2n}\Big[(n-2)\left(1-\frac{D_4}{R_1}\right)h_{v_1}+h_{v_1}+h_{b_1}+\frac{P_1I_1}{1+I_4t}\Big] - \frac{D_4(Sv_4+n(F_4+U_4L_0))}{Q_1^2}\Big]+\lambda_1=0,$$

$$\frac{\partial L}{\partial Q_2} = \frac{1}{6}\Big[\frac{2}{2n}\Big[(n-2)\left(1-\frac{D_3}{R_2}\right)h_{v_2}+h_{v_2}+h_{b_2}+\frac{P_2I_2}{1+I_3t}\Big] - \frac{2D_3(Sv_3+n(F_3+U_3L_0))}{Q_2^2}\Big]+\lambda_2-\lambda_1=0,$$

$$\frac{\partial L}{\partial Q_3} = \frac{1}{6}\Big[\frac{2}{2n}\Big[(n-2)\left(1-\frac{D_2}{R_3}\right)h_{v_3}+h_{v_3}+h_{b_3}+\frac{P_3I_3}{1+I_2t}\Big] - \frac{2D_2(Sv_2+n(F_2+U_2L_0))}{Q_3^2}\Big]-\lambda_2=0,$$

$$\frac{\partial L}{\partial Q_4} = \frac{1}{6}\Big[\frac{1}{2n}\Big[(n-2)\left(1-\frac{D_1}{R_4}\right)h_{v_4}+h_{v_4}+h_{b_4}+\frac{P_4I_4}{1+I_1t}\Big] - \frac{D_1(Sv_1+n(F_1+U_1L_0))}{Q_4^2}\Big]=0,$$

$$\frac{\partial L}{\partial \lambda_1} = -(Q_2-Q_1),\quad \frac{\partial L}{\partial \lambda_2} = -(Q_3-Q_2),$$

$$Q_1=Q_2=Q_3 = \sqrt{\frac{\left(2n\Big[2D_2(Sv_2+n(F_2+U_2L_0))+2D_3(Sv_3+n(F_3+U_3L_0))+D_4(Sv_4+n(F_4+U_4L_0))\Big]\right)}{\left(\begin{array}{c}2\Big[(n-2)\left(1-\frac{D_3}{R_2}\right)h_{v_2}+h_{v_2}+h_{b_2}+\frac{P_2I_2}{1+I_3t}\Big]+\\ 2\Big[(n-2)\left(1-\frac{D_2}{R_3}\right)h_{v_3}+h_{v_3}+h_{b_3}+\frac{P_3I_3}{1+I_2t}\Big]+\\ \Big[(n-2)\left(1-\frac{D_4}{R_1}\right)h_{v_1}+h_{v_1}+h_{b_1}+\frac{P_1I_1}{1+I_4t}\Big]\end{array}\right)}}$$

$$Q_4 = \sqrt{\frac{2nD_1\left(Sv_1+n(F_1+U_1L_0)\right)}{(n-2)\left(1-\frac{D_1}{R_4}\right)hv_4+2h_{v_4}+2h_{b_4}+\frac{2P_4I_4}{1+I_1t}}}.$$

The above results $Q_1 > Q_4$, does not satisfy the constraint $0 < Q_1 \le Q_2 \le Q_3 \le Q_4$. Therefore it is not a local optimum. Similarly we can get the same result if we select any other two inequality constraints to be equality constraint, therefore set $K = 3$ and go to Step 4.

Step 4 : Convert the inequality constraints $Q_2 - Q_1 \ge 0$, $Q_3 - Q_2 \ge 0$ and $Q_4 - Q_3 \ge 0$ into equality constraints $Q_2 - Q_1 = 0$, $Q_3 - Q_2 = 0$ and $Q_4 - Q_3 = 0$. We optimize $P\left(J\tilde{T}C_1(n,Q,L_0)\right)$. Subject to $Q_2 - Q_1 = 0$, $Q_3 - Q_2 = 0$ and $Q_4 - Q_3 = 0$ by the Lagrangean Method. The Lagrangean function is given by

$$L(Q_1,Q_2,Q_3,Q_4,\lambda_1,\lambda_2,\lambda_3) = P\left(J\tilde{T}C_1(n,Q,L_0)\right) - \lambda_1(Q_2-Q_1) - \lambda_2(Q_3-Q_2) - \lambda_3(Q_4-Q_3).$$

In order to find the minimization of $L(Q_1, Q_2, Q_3, Q_4, \lambda_1, \lambda_2, \lambda_3)$, we take the partial derivatives of $L(Q_1, Q_2, Q_3, Q_4, \lambda_1, \lambda_2, \lambda_3)$ with respect to Q_1, Q_2, Q_3, Q_4, λ_1, λ_2, λ_3) and let all the partial derivatives equal to zero and solve Q_1, Q_2, Q_3 and Q_4. Then we get

$$\frac{\partial L}{\partial Q_1} = \frac{1}{6}\left[\frac{1}{2n}\left[(n-2)\left(1-\frac{D_4}{R_1}\right)h_{v_1} + h_{v_1} + h_{b_1} + \frac{P_1 I_1}{1+I_4 t}\right] - \frac{D_4(Sv_4 + n(F_4 + U_4 L_0))}{Q_1^2}\right] + \lambda_1 = 0,$$

$$\frac{\partial L}{\partial Q_2} = \frac{1}{6}\left[\frac{2}{2n}\left[(n-2)\left(1-\frac{D_3}{R_2}\right)h_{v_2} + h_{v_2} + h_{b_2} + \frac{P_2 I_2}{1+I_3 t}\right] - \frac{2D_3(Sv_3 + n(F_3 + U_3 L_0))}{Q_2^2}\right] - \lambda_1 + \lambda_2 = 0,$$

$$\frac{\partial L}{\partial Q_3} = \frac{1}{6}\left[\frac{2}{2n}\left[(n-2)\left(1-\frac{D_2}{R_3}\right)h_{v_3} + h_{v_3} + h_{b_3} + \frac{P_3 I_3}{1+I_2 t}\right] - \frac{2D_2(Sv_2 + n(F_2 + U_2 L_0))}{Q_3^2}\right] - \lambda_2 + \lambda_3 = 0,$$

$$\frac{\partial L}{\partial Q_4} = \frac{1}{6}\left[\frac{1}{2n}\left[(n-2)\left(1-\frac{D_1}{R_4}\right)h_{v_4} + h_{v_4} + h_{b_4} + \frac{P_4 I_4}{1+I_1 t}\right] - \frac{D_1(Sv_1 + n(F_1 + U_1 L_0))}{Q_4^2}\right] - \lambda_3 = 0,$$

$$\frac{\partial L}{\partial \lambda_1} = -(Q_2 - Q_1),\ \frac{\partial L}{\partial \lambda_2} = -(Q_3 - Q_2),\ \frac{\partial L}{\partial \lambda_3} = -(Q_4 - Q_3),$$

$$Q_1 = Q_2 = Q_3 = Q_4 = \sqrt{\frac{\left(\begin{array}{c} 2n\left[D_1(Sv_1+n(F_1+U_1L_0))+2D_2(Sv_2+n(F_2+U_2L_0))+\right. \\ \left. 2D_3(Sv_3+n(F_3+U_3L_0))+D_4(S_{v_4}+n(F_4+U_4L_0))\right] \end{array}\right)}{\left(\begin{array}{c} \left[(n-2)\left(1-\frac{D_4}{R_1}\right)h_{v_1}+h_{v_1}+h_{b_1}+\frac{P_1I_1}{1+I_4t}\right]+ \\ 2\left[(n-2)\left(1-\frac{D_3}{R_2}\right)h_{v_2}+h_{v_2}+h_{b_2}+\frac{P_2I_2}{1+I_3t}\right]+ \\ 2\left[(n-2)\left(1-\frac{D_2}{R_3}\right)h_{v_3}+h_{v_3}+h_{b_3}+\frac{P_3I_3}{1+I_2t}\right]+ \\ \left[(n-2)\left(1-\frac{D_1}{R_4}\right)h_{v_4}+h_{v_4}+h_{b_4}+\frac{P_4I_4}{1+I_1t}\right] \end{array}\right)}}.$$

Because the above solution $\tilde{Q} = (Q_1, Q_2, Q_3, Q_4)$ satisfies all inequality constraints, the procedure terminates with $\tilde{Q}$ as a local optimum solution to the problem. Since the above local optimum solution is the only one feasible solution of formula (4.2). So it is an optimum solution of the inventory model with fuzzy production quantity according to extension of the Lagrangean Method.

Let $Q_1 = Q_2 = Q_3 = Q_4 = \tilde{Q}^*$. Then the optimal fuzzy production quantity is $\tilde{Q}^* = (Q^*, Q^*, Q^*, Q^*)$

$$Q^* = \sqrt{\frac{\begin{pmatrix} 2n\Big[D_1(Sv_1+n(F_1+U_1L_0))+2D_2(Sv_2+n(F_2+U_2L_0))+ \\ 2D_3(Sv_3+n(F_3+U_3L_0))+D_4(S_{v_4}+n(F_4+U_4L_0))\Big] \end{pmatrix}}{\begin{pmatrix} (n-2)\left(1-\frac{D_4}{R_1}\right)hv_1+2(n-2)\left(1-\frac{D_3}{R_2}\right)hv_2+2(n-2)\left(1-\frac{D_2}{R_3}\right)hv_3+(n-2)\left(1-\frac{D_1}{R_4}\right)hv_4+ \\ (h_{v_1}+2h_{v_2}+2h_{v_3}+h_{v_4})+(h_{b_1}+2h_{b_2}+2h_{b_3}+h_{b_4})+ \\ \frac{P_1I_1}{1+I_4t}+2\frac{P_2I_2}{1+I_3t}+2\frac{P_3I_3}{1+I_2t}+\frac{P_4I_4}{1+I_1t} \end{pmatrix}}}.$$

5. Numerical Examples

To illustrate the results obtained in this paper, the proposed analytic solution method is applied to efficiency solve the following numerical example. Consider an inventory system with the following characteristics.

$D = 2700$, $R = 9000$, $h_b = 5.00$, $h_v = 2.00$, $U = 1400$, $S_v = 200$, $P = 10$, $F = 300$, $t = 0.25$, $I = 0.15$, $n = 2$, $Q^* = 1182.77$, $L_0 = 0.105$

$$JTC(n, Q, L_0) = 5144.71$$

In this example can be transferred into the fuzzy parameters as follows. Consider any problem in which an annual demand is more or less than 2700 units, production rate is more or less than 9000, unit stock-holding cost is more or less than 5.00 per item per year for the buyer, unit stock-holding cost is more or less than 2.00 cost per item per year for the vender. Order processing cost is more or less than 1400 cost per unit time for the buyer, set up cost is more or less than 200 per production run for the vendor, purchase price is more or less than 10 per units, Fixed transportation cost is more or less than 300 per shipment, permissible delay is more or less than 0.25 in settling amounts, Carrying cost is more or less than 0.15 per dollar per year. Determine the optimum integrated total cost? Here we represent the case of value, "more or less than Y" as the type of trapezoidal fuzzy number. Suppose Fuzzy annual demand is "more or less than 2700"

$$\tilde{D} = (D_1, D_2, D_3, D_4) = (2550, 2725, 2730, 2740).$$

Fuzzy production rate is "more or less than 9000"

$$\tilde{R} = (R_1, R_2, R_3, R_4) = (8850, 9025, 9030, 9040).$$

Fuzzy unit stock holding cost is "more or less than 5.00" per item per year for the buyer

$$\tilde{h}_b = (hb_1, hb2, hb3, hb4) = (4.8, 4.9, 5.1, 5.2).$$

Fuzzy unit stock holding cost is "more or less than 2.00" cost per item per year for the buyer

$$\tilde{h}_v = (hv1, hv2, hv3, hv4) = (1.8, 1.9, 2.1, 2.2).$$

Fuzzy order processing cost is "more or less than 1400" Cost unit per time for the buyer

$$\tilde{U} = (U_1, U_2, U_3, U_4) = (1380, 1390, 1410, 1420).$$

Fuzzy setup cost is "more or less than 200" per production run for the vendor

$$\tilde{S} = (S_{v_1}, S_{v_2}, S_{v_3}, S_{v_4}) = (180, 190, 210, 220).$$

Fuzzy purchase price is "more or less than 10" per units

$\tilde{P} = (P_1, P_2, P_3, P_4) = (9.8, 9.8, 10.1, 10.2).$

Fuzzy fixed transportation cost is "more or less than 300" per shipment

$\tilde{F} = (F_1, F_2, F_3, F_4) = (280, 290, 310, 320).$

Fuzzy carrying cost is "more or less than 0.15" per dollar per year

$\tilde{I} = (I_1, I_2, I_3, I_4) = (0.13, 0.14, 0.16, 0.17).$

Fuzzy production quantity

$\tilde{Q} = (Q_1, Q_2, Q_3, Q_4)$ with $0 < Q_1 \leq Q_2 \leq Q_3 \leq Q_4$.

Replace the above fuzzy parameter values into formula, we find the optimal fuzzy production quantity,

$\tilde{Q}^* = (1274.07, 1229.67, 1149.48, 1075.56),$
$\tilde{Q}^* = 1183.08.$

The minimization fuzzy total production inventory cost for both the vender and buyer is

$\tilde{J}TC(n, Q, L_0)^* = (4533.13, 4918.92, 5122.23, 5232.53).$

Conclusion

This paper presents two fuzzy models for an optimal integrated inventory model and minimizing the total expected cost of the buyer and the vendor under conditions of order processing time production and permissible delay in payments. In the first model demand production cost, purchase cost, annual demand, set up cost per production run for the vender, unit stock-holding cost per item per year for the vender and for the buyer, size of each shipment from the vender to the buyer, transportation cost, carrying cost represented by fuzzy number while Q is treated as a fixed constant. In the second model Q is also represented as a fuzzy number. For each fuzzy model; a method of defuzzification, graded mean integration representation is applied to find the estimate of total expected cost of the buyer and the vender in the fuzzy type and then corresponding optimal order netsize is derived to maximize the total profit.

Acknowledgements. The authors wish to thank the anonymous reviewers for their valuable suggestions.

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