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FP-soft set theory and its applications

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Abstract. In this work, we first introduce fuzzy parameterized (FP) soft sets and their related properties. We then propose a decision making method based on FP-soft set theory. We finally give an example which shows that the method can be successfully applied to the problems that contain uncertainties.

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1. Introduction

Many fields deal daily with the uncertain data that may not be successfully modeled by the classical mathematics. The probability theory, fuzzy sets [20], rough sets [16], and other mathematical tools are well-known and often useful approaches to describe uncertainty. However, all of these theories have their own difficulties which are pointed out in [15] by Molodtsov who then proposed a completely new approach for modeling vagueness and uncertainty, that is free from the difficulties. This so-called *soft set theory* has potential applications in many different fields. Maji *et al.* [12] firstly worked on detailed theoretical study of soft sets. After than, the properties and applications on the soft set theory have been studied by many authors (e.g. [3, 4, 5, 9, 13, 18, 22]). The algebraic structure of soft set theory has also been studied in more detail (e.g. [1, 2, 7, 10]). Many interesting applications of soft set theory have been expanded by embedding the ideas of fuzzy sets (e.g. [6, 8, 11, 14, 17, 19, 21]).

In this paper, in the next section, most of the fundamental definitions of the operations of soft sets are presented. In Section 3, we have defined fuzzy parameterized soft sets, in short written FP-soft sets, whose parameters sets are fuzzy sets and have improved several results. In Section 4, we have defined the fuzzy decision set of a FP-soft set to construct a decision method by which approximate functions of a soft set are combined to produce a single fuzzy set that can be used to evaluate each alternative. In Section 5, we have given an application that shows that these methods work successfully. In the final section, some concluding comments have been presented.

2. Soft sets

In this section, for subsequent discussions of this work, we have presented the basic definitions and results of soft set theory that may be found in earlier studies [4, 12, 15].

Throughout this work, U refers to an initial universe, E is a set of parameters, $P(U)$ is the power set of U, and $A \subseteq E$.

Definition 2.1. A soft set F_A on the universe U is defined by the set of ordered pairs

$$F_A = \{(x, f_A(x)) : x \in E, f_A(x) \in P(U)\}$$

where $f_A : E \to P(U)$ such that $f_A(x) = \varnothing$ if $x \notin A$.

Here, f_A is called the approximate function of the soft set F_A. The value of $f_A(x)$ is a set called *x-element* of the soft set for all $x \in E$. It is worth noting that the sets $f_A(x)$ may be arbitrary. Some of them may be empty, some may have nonempty intersection.

The subscript A in the notation f_A indicates that f_A is the approximate function of F_A. From now on, we will use $S(U)$ instead of the set of all soft sets over U.

Definition 2.2. Let $F_A \in S(U)$. If $f_A(x) = \varnothing$ for all $x \in E$, then F_A is called a empty set, denoted by F_Φ. $f_A(x) = \varnothing$ means that there is no element in U related to the parameter $x \in E$. Therefore, we do not display such elements in the soft sets, as it is meaningless to consider such parameters.

Definition 2.3. Let $F_A \in S(U)$. If $f_A(x) = U$ for all $x \in A$, then F_A is called A-universal set, denoted by $F_{\tilde{A}}$. If $A = E$, then the A-universal set is called universal soft set denoted by $F_{\tilde{E}}$.

Definition 2.4. Let $F_A, F_B \in S(U)$. Then, F_A is a soft subset of F_B, denoted by $F_A \widetilde{\subseteq} F_B$, if $f_A(x) \subseteq f_B(x)$ for every $x \in E$.

Definition 2.5. Let $F_A, F_B \in S(U)$. Then, F_A and F_B are soft equal, denoted by $F_A = F_B$, if and only if $f_A(x) = f_B(x)$ for every $x \in E$.

Definition 2.6. Let $F_A \in S(U)$. Then, complement of F_A, denoted by $F_A^{\tilde{c}}$, is a soft set defined by the approximate function

$$f_{A^{\tilde{c}}}(x) = U \setminus f_A(x).$$

Definition 2.7. Let $F_A, F_B \in S(U)$. Then, union of F_A and F_B, denoted by $F_A \widetilde{\cup} F_B$, is a soft set defined by the approximate function

$$f_{A \widetilde{\cup} B}(x) = f_A(x) \cup f_B(x).$$

Definition 2.8. Let $F_A, F_B \in S(U)$. Then, intersection of F_A and F_B, denoted by $F_A \widetilde{\cap} F_B$, is a soft set defined by the approximate function

$$f_{A \widetilde{\cap} B}(x) = f_A(x) \cap f_B(x).$$

Proposition 2.9. *Let* $F_A, F_B, F_C \in S(U)$. *Then*

(1) $F_A \tilde{\cup} F_A = F_A$, $F_A \tilde{\cap} F_A = F_A$.
(2) $F_A \tilde{\cup} F_\Phi = F_A$, $F_A \tilde{\cap} F_\Phi = F_\Phi$.
(3) $F_A \tilde{\cup} F_{\tilde{E}} = F_{\tilde{E}}$, $F_A \tilde{\cap} F_{\tilde{E}} = F_A$.
(4) $F_A \tilde{\cup} F_A^{\tilde{c}} = F_{\tilde{E}}$, $F_A \tilde{\cap} F_A^{\tilde{c}} = F_\Phi$.
(5) $F_A \tilde{\cup} F_B = F_B \tilde{\cup} F_A$, $F_A \tilde{\cap} F_B = F_B \tilde{\cap} F_A$.
(6) $(F_A \tilde{\cup} F_B) \tilde{\cup} F_C = F_A \tilde{\cup} (F_B \tilde{\cup} F_C)$, $(F_A \tilde{\cap} F_B) \tilde{\cap} F_C = F_A \tilde{\cap} (F_B \tilde{\cap} F_C)$.

3. FP-soft sets

In this section, we give definition of fuzzy parameterized soft sets (FP-soft sets) and their operations. In Section 2, the subsets of E were classical sets, denoted by the letter $A, B, C, ...$, but in this section, the subsets of E will be fuzzy denoted by the letter $X, Y, Z, ...$, to avoid confusion and complexity of the symbols.

Definition 3.1. Let U be an initial universe, $P(U)$ be the power set of U, E be the set of all parameters and X be a fuzzy set over E. An FP-soft set F_X on the universe U is defined by the set of ordered pairs

$$F_X = \{(\mu_X(x)/x, f_X(x)) : x \in E, f_X(x) \in P(U), \mu_X(x) \in [0,1]\},$$

where the function $f_X : E \to P(U)$ is called approximate function such that $f_X(x) = \varnothing$ if $\mu_X(x) = 0$, and the function $\mu_X : E \to [0,1]$ is called membership function of FP-soft set F_X. The value of $\mu_X(x)$ is the degree of importance of the parameter x, and depends on the decision maker's requirements.

Note that from now on the sets of all FP-soft sets over U will be denoted by $FPS(U)$.

Definition 3.2. Let $F_X \in FPS(U)$. If $f_X(x) = \varnothing$ for all $x \in E$, then F_X is called an X-empty FP-soft set, denoted by F_{Φ_X}.

If $X = \varnothing$, then F_X is called an empty FP-soft set, denoted by F_Φ.

Definition 3.3. Let $F_X \in FPS(U)$. If X is a crisp subset of E and $f_X(x) = U$ for all $x \in X$, then F_X is called X-universal FP-soft set, denoted by $F_{\tilde{X}}$.

If $X = E$, then the X-universal FP-soft set is called universal FP-soft set, denoted by $F_{\tilde{E}}$.

Example 3.4. Assume that $U = \{u_1, u_2, u_3, u_4, u_5\}$ is a universal set and $E = \{x_1, x_2, x_3, x_4\}$ is a set of parameters.

If $X = \{0.2/x_2, 0.5/x_3, 1/x_4\}$ and $f_X(x_2) = \{u_2, u_4\}$, $f_X(x_3) = \varnothing$, and $f_X(x_4) = U$, then $F_X = \{(0.2/x_2, \{u_2, u_4\}), (0.5/x_3, \varnothing), (1/x_4, U)\}$.

If $Y = \{0.3/x_2, 0.7/x_3\}$, $f_Y(x_2) = \varnothing$ and $f_Y(x_3) = \varnothing$, then $F_Y = F_{\Phi_Y}$.

If $Z = \varnothing$, then $F_Z = F_\Phi$.

If $T = \{1/x_1, 1/x_2\}$, $f_T(x_1) = U$, and $f_T(x_2) = U$, then $F_T = F_{\tilde{T}}$.

Definition 3.5. Let $F_X, F_Y \in FPS(U)$. Then, F_X is an FP-soft subset of F_Y, denoted by $F_X \tilde{\subseteq} F_Y$, if $\mu_X(x) \le \mu_Y(x)$ and $f_X(x) \subseteq f_Y(x)$ for all $x \in E$.

Remark 3.6. $F_X \tilde{\subseteq} F_Y$ does not imply that every element of F_X is an element of F_Y as in the definition of the classical subset.

For example, assume that $U = \{u_1, u_2, u_3, u_4\}$ is a universal set of objects and $E = \{x_1, x_2, x_3\}$ is a set of all the parameters. If $X = \{0.5/x_1\}$ and $Y = \{0.9/x_1, 0.1/x_3\}$, and $F_X = \{(0.5/x_1, \{u_2, u_4\})\}$, $F_Y = \{(0.9/x_1, \{u_2, u_3, u_4\}), (0.1/x_3, \{u_1, u_5\})\}$, then for all $x \in E$, $\mu_X(x) \leq \mu_Y(x)$ and $f_X(x) \subseteq f_Y(x)$ is valid. Hence, $F_X \tilde{\subseteq} F_Y$. It is clear that $(0.5/x_1, \{u_2, u_4\}) \in F_X$, but $(0.5/x_1, \{u_2, u_4\}) \notin F_Y$.

Proposition 3.7. *Let $F_X, F_Y \in FPS(U)$. Then*

(1) $F_X \tilde{\subseteq} F_{\tilde{E}}$.
(2) $F_\Phi \tilde{\subseteq} F_X$.
(3) $F_X \tilde{\subseteq} F_X$.
(4) $F_X \tilde{\subseteq} F_Y$ *and* $F_Y \tilde{\subseteq} F_Z \Rightarrow F_X \tilde{\subseteq} F_Z$.

Proof. They can be proved easily by using the approximate and membership functions of the FP-soft sets. □

Definition 3.8. Let $F_X, F_Y \in FPS(U)$. Then, F_X and F_Y are FP-soft equal, written as $F_X = F_Y$, if and only if $\mu_X(x) = \mu_Y(x)$ and $f_X(x) = f_Y(x)$ for all $x \in E$.

Proposition 3.9. *Let $F_X, F_Y, F_Z \in FPS(U)$. Then*

(1) $F_X = F_Y$ *and* $F_Y = F_Z \Rightarrow F_X = F_Z$.
(2) $F_X \tilde{\subseteq} F_Y$ *and* $F_Y \tilde{\subseteq} F_X \Leftrightarrow F_X = F_Y$.

Proof. The proofs are trivial. □

Definition 3.10. Let $F_X \in FPS(U)$. Then, complement F_X, denoted by $F_X^{\tilde{c}}$, is an FP-soft set defined by the approximate and membership functions as

$$\mu_{X^{\tilde{c}}}(x) = 1 - \mu_X(x) \text{ and } f_{X^{\tilde{c}}}(x) = U \setminus f_X(x).$$

Proposition 3.11. *Let $F_X \in FPS(U)$. Then,*

(1) $(F_X^{\tilde{c}})^{\tilde{c}} = F_X$.
(2) $F_\Phi^{\tilde{c}} = F_{\tilde{E}}$.

Proof. By using the approximate and membership functions of the FP-soft sets, the proofs can be straightforward. □

Definition 3.12. Let $F_X, F_Y \in FPS(U)$. Then, union F_X and F_Y, denoted by $F_X \tilde{\cup} F_Y$, is defined by

$$\mu_{X\tilde{\cup}Y}(x) = \max\{\mu_X(x), \mu_Y(x)\} \text{ and } f_{X\tilde{\cup}Y}(x) = f_X(x) \cup f_Y(x), \text{for all } x \in E.$$

Proposition 3.13. *Let $F_X, F_Y, F_Z \in FPS(U)$. Then*

(1) $F_X \tilde{\cup} F_X = F_X$.
(2) $F_X \tilde{\cup} F_\Phi = F_X$.
(3) $F_X \tilde{\cup} F_{\tilde{E}} = F_{\tilde{E}}$.
(4) $F_X \tilde{\cup} F_Y = F_Y \tilde{\cup} F_X$.
(5) $(F_X \tilde{\cup} F_Y) \tilde{\cup} F_Z = F_X \tilde{\cup} (F_Y \tilde{\cup} F_Z)$.

Proof. The proofs can be easily obtained from Definition 3.12. □

Definition 3.14. Let $F_X, F_Y \in FPS(U)$. Then, intersection of F_X and F_Y, denoted by $F_X \tilde{\cap} F_Y$, is an FP-soft sets defined by the approximate and membership functions

$$\mu_{X\tilde{\cap}Y}(x) = \min\{\mu_X(x), \mu_Y(x)\} \text{ and } f_{X\tilde{\cap}Y}(x) = f_X(x) \cap f_Y(x).$$

Proposition 3.15. *Let $F_X, F_Y, F_Z \in FPS(U)$. Then*

(1) $F_X \tilde{\cap} F_X = F_X$.
(2) $F_X \tilde{\cap} F_\Phi = F_\Phi$.
(3) $F_X \tilde{\cap} F_{\tilde{E}} = F_X$.
(4) $F_X \tilde{\cap} F_Y = F_Y \tilde{\cap} F_X$.
(5) $(F_X \tilde{\cap} F_Y) \tilde{\cap} F_Z = F_X \tilde{\cap} (F_Y \tilde{\cap} F_Z)$.

Proof. The proofs can be easily obtained from Definition 3.14. □

Remark 3.16. Let $F_X \in FPS(U)$. If $F_X \neq F_\Phi$ and $F_X \neq F_{\tilde{E}}$, then $F_X \tilde{\cup} F_X^{\tilde{c}} \neq F_{\tilde{E}}$ and $F_X \tilde{\cap} F_X^{\tilde{c}} \neq F_\Phi$.

Proposition 3.17. *Let $F_X, F_Y \in FPS(U)$. Then De Morgan's laws are valid*

(1) $(F_X \tilde{\cup} F_Y)^{\tilde{c}} = F_X^{\tilde{c}} \tilde{\cap} F_Y^{\tilde{c}}$.
(2) $(F_X \tilde{\cap} F_Y)^{\tilde{c}} = F_X^{\tilde{c}} \tilde{\cup} F_Y^{\tilde{c}}$.

Proof. For all $x \in E$,

$$\begin{aligned} (1)\quad \mu_{(X\tilde{\cup}Y)^{\tilde{c}}}(x) &= 1 - \mu_{X\tilde{\cup}Y}(x) \\ &= 1 - \max\{\mu_X(x), \mu_Y(x)\} \\ &= \min\{1 - \mu_X(x), 1 - \mu_Y(x)\} \\ &= \min\{\mu_{X^{\tilde{c}}}(x), \mu_{Y^{\tilde{c}}}(x)\} \\ &= \mu_{X^{\tilde{c}}\tilde{\cap}Y^{\tilde{c}}}(x) \end{aligned}$$

and

$$\begin{aligned} f_{(X\tilde{\cup}Y)^{\tilde{c}}}(x) &= U \setminus f_{X\tilde{\cup}Y}(x) \\ &= U \setminus (f_X(x) \cup f_Y(x)) \\ &= (U \setminus f_X(x)) \cap (U \setminus f_Y(x)) \\ &= f_{X^{\tilde{c}}}(x) \tilde{\cap} f_{Y^{\tilde{c}}}(x) \\ &= f_{X^{\tilde{c}}\tilde{\cap}Y^{\tilde{c}}}(x). \end{aligned}$$

Likewise, the proof of (2) can be made similarly. □

Proposition 3.18. *Let $F_X, F_Y, F_Z \in FPS(U)$. Then*

(1) $F_X \tilde{\cup} (F_Y \tilde{\cap} F_Z) = (F_X \tilde{\cup} F_Y) \tilde{\cap} (F_X \tilde{\cup} F_Z)$.
(2) $F_X \tilde{\cap} (F_Y \tilde{\cup} F_Z) = (F_X \tilde{\cap} F_Y) \tilde{\cup} (F_X \tilde{\cap} F_Z)$.

Proof. For all $x \in E$,

$$\begin{aligned} (1)\quad \mu_{X\tilde{\cup}(Y\tilde{\cap}Z)}(x) &= \max\{\mu_X(x), \mu_{Y\tilde{\cap}Z}(x)\} \\ &= \max\{\mu_X(x), \min\{\mu_Y(x), \mu_Z(x)\}\} \\ &= \min\{\max\{\mu_X(x), \mu_Y(x)\}, \max\{\mu_X(x), \mu_Z(x)\}\} \\ &= \min\{\mu_{X\tilde{\cup}Y}(x), \mu_{X\tilde{\cup}Z}(x)\} \\ &= \mu_{(X\tilde{\cup}Y)\tilde{\cap}(X\tilde{\cup}Z)}(x) \end{aligned}$$

and

$$\begin{aligned} f_{X\tilde{\cup}(Y\tilde{\cap}Z)}(x) &= f_X(x) \cup f_{Y\tilde{\cap}Z}(x) \\ &= f_X(x) \cup (f_Y(x) \cap f_Z(x)) \\ &= (f_X(x) \cup f_Y(x)) \cap (f_X(x) \cup f_Z(x)) \\ &= f_{X\tilde{\cup}Y}(x) \cap f_{X\tilde{\cup}Z}(x) \\ &= f_{(X\tilde{\cup}Y)\tilde{\cap}(X\tilde{\cup}Z)}(x). \end{aligned}$$

Likewise, the proof of (2) can be made in a similar way. □

4. Fuzzy decision set of an FP-soft set

In this section, we define fuzzy decision set of an FP-soft set to construct a decision method by which approximate functions of a soft set are combined to produce a single fuzzy set that can be used to evaluate each alternative.

Definition 4.1. Let $F_X \in FPS(U)$. Then a fuzzy decision set of F_X, denoted by F_X^d, is defined by

$$F_X^d = \{\mu_{F_X^d}(u)/u : u \in U\}$$

which is a fuzzy set over U, its membership function $\mu_{F_X^d}$ is defined by

$$\mu_{F_X^d} : U \to [0,1], \quad \mu_{F_X^d}(u) = \frac{1}{|supp(X)|} \sum_{x \in supp(X)} \mu_X(x)\chi_{f_X(x)}(u)$$

where $supp(X)$ is the support set of X, $f_X(x)$ is the crisp subset determined by the parameter x and

$$\chi_{f_X(x)}(u) = \begin{cases} 1, & u \in f_X(x), \\ 0, & u \notin f_X(x). \end{cases}$$

5. Applications

Once a fuzzy decision set of an FP-soft set has been arrived at, it may be necessary to choose the best single alternative from the alternatives. Therefore, we can make a decision by the following algorithm.

Step 1: Construct a soft set F_X over U,
Step 2: Compute the fuzzy decision set F_X^d,
Step 3: Select the largest membership grade $\max \mu_{F_X^d}(u)$.

Let us consider the following example to illustrate the idea.

Example 5.1. Let us assume that some one goes to an automobile showroom to buy an automobile. There are eight alternatives $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\}$. There may be five parameters $E = \{e_1, e_2, e_3, e_4, e_5\}$ to evaluate the automobiles. For $i = 1, 2, 3, 4, 5$, the parameters e_i stand for "beautiful", "expensive", "large", "equipped with conveniences", "high speedy", respectively. He/She considers only four parameters "expensive", "large", "equipped with conveniences" and "high speedy" which are important with degree 0.8, 0.3, 0.5 and 0.6, respectively. That is, the subset of parameters is $X = \{0.8/e_2, 0.3/e_3, 0.5/e_4, 0.6/e_5\}$. Now we can find a suitable automobile to buy.

Step 1: After a serious discussion, He/She evaluates the alternative from point of view of the goals and the constraint according to a chosen subset X of E to constructs an FP-soft set,

$$F_X = \Big\{(0.8/e_2, \{u_2, u_5, u_7\}), (0.3/e_3, \{u_1, u_2, u_3, u_4, u_5, u_8\}), (0.5/e_4, \{u_1, u_2, u_4, u_7, u_8\}), (0.6/e_5, \{u_1, u_3, u_7, u_8\})\Big\}.$$

Step 2: The fuzzy decision set of F_X can be found as,

$$F_X^d = \{0.35/u_1, 0.40/u_2, 0.22/u_3, 0.20/u_4, 0.27/u_5, 0.47/u_7, 0.35/u_8\}.$$

Step 3: Finally, the largest membership grade can be chosen by

$$\max \mu_{F_X^d}(u) = 0.47$$

which means that the candidate u_7 has the largest membership grade, hence it is selected.

Note that if there are more then one largest membership grades, then it better to reset the degree of parameters.

6. Conclusion

In this paper, we first defined FP-soft sets and their operations. We then presented the decision methods on the FP-soft set theory. Finally, we provided an application that demonstrated that this method can successfully work. It can be applied to many fields to the problems that contain uncertainty, and would be beneficial to extend the proposed method to subsequent studies. However, the approach should be more comprehensive in the future to solve the related problems.

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