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Multi level inventory management decisions with transportation cost consideration in fuzzy environment

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Abstract. This paper preambles decentralized ordering model with fuzzy parameters for fuzzy replenishment order quantity. The fuzzy total cost of this model under the fuzzy arithmetical operations of function principle is stipulated. Our ultimate aim is to find optimal solution of this model by using graded mean integration representation method for defuzzifying fuzzy total cost and by using extension of Lagrangian method for solving inequality constraint problem. In addition, when the fuzzy parameters (fuzzy fixed cost, fuzzy variable purchase cost, fuzzy demand quantity, fuzzy travelling distance, fuzzy carrying charge, fuzzy safety factor, fuzzy variation of the lead time, fuzzy fixed cost of transportation, fuzzy variable cost) are all crisp real numbers. The optimal solutions of our stated model can be utilized to meet ancient decentralized ordering model.

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1. Introduction

The decentralized ordering model is a constructive extension of the classic optimal economic order quantity model. Several attempts have been made to extend the EOQ model to different conditions. For this purpose, a few authors incorporated transportation costs into the lot size determination analysis. Burwell et al [3] developed a model for determining the reseller's lot size and price assuming that there are freight and all unit quantity discount break points in the pricing schedule offered by the supplier. Gupta [9] considered a situation in which a fixed cost is incurred for a transport node such as a truck that has a fixed load capacity. He developed a model to determine the optimal lot size, which minimizes the sum of the inventory holding, ordering and transportation cost. Zhao et al [16] introduced the problem of evaluating the optimal ordering quantity in a supplier-customer model by considering the transportation cost.

In [1], Alireza Madadi, Mary E. Kurz and Jalal Ashayeri addressed specific inventory management decisions with transportation cost consideration in a multi-level environment consisting of a supplier-warehouse-retailer. A two-level supply chain consisting of a warehouse (distribution centre) and N retailers is considered in our model.

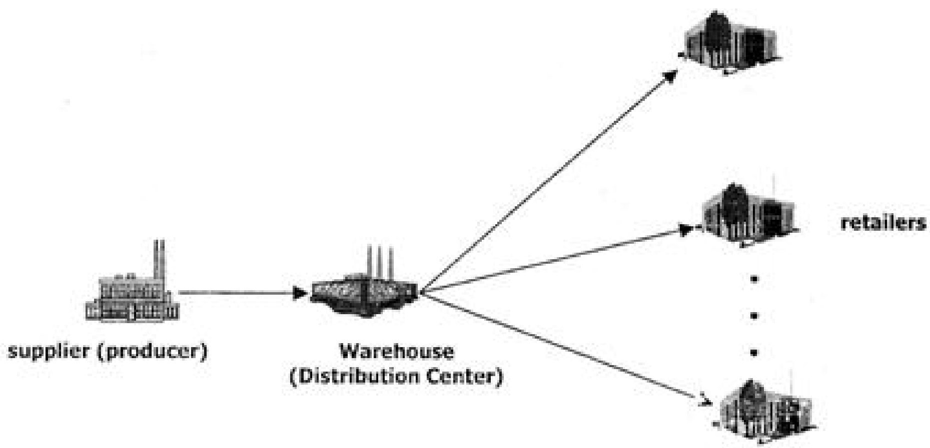

FIGURE 1. The structure of the model

Each retailer and the warehouse have a set of control parameters that affects the performance of other components. Each retailer's costs include transportation cost, cost of replenishment, carrying cost and cost of stock-out. We adopt the model of Daganzo [8] for one-to-many distribution model transportation cost and Burns et al. [2] for distribution strategy to minimize transportation cost.

In the decentralized ordering model each individual level tries to optimize its own total cost. The optimal lot size formula is

$$Q_j^* = \sqrt{\frac{2D_j(A_j + \alpha_w + t_w d_{w_j})}{V_j r_j}}$$

Inventory control of goods or products is a very important part of logistic systems, common to all economic sectors such as agriculture, industry, trade and business. Very large costs are incurred as a result of replenishment actions, shortages and the use of managerial and clerical time in making and routinely implementing inventory management decisions. Thus, properly designed decision rules, based on mathematical modeling, can lead to substantial benefits. The major problem in inventory control can be summed up by the two fundamental questions (i) when should a replenishment order be placed? And (ii) how much should be ordered? In the real world, the parameters and variables in inventory model may be almost uncertain datum. In 1987, Park [12] used fuzzy set concept to treat the inventory problem with fuzzy inventory cost under arithmetic operations of Extension Principle. In 1996, Chen et al. [5] introduced backorder fuzzy inventory model under Function Principle. In [7], Chen and Hsieh discussed fuzzy inventory model for crisp order quantity or for fuzzy order quantity with generalized trapezoidal fuzzy number.

In the crisp inventory models, all the parameters in the total cost are known and have definite values without ambiguity, as well as the real variable of the total cost is positive. But in the reality, it is not so sure. Hence it is needed to consider the fuzzy inventory models. In order to simplify the calculation of trapezoidal fuzzy number, we use Chen's Function Principle [4] instead of Extension Principle to calculate the fuzzy total replenishment inventory cost of our proposed model. Function Principle is proposed as the fuzzy arithmetical operations of fuzzy numbers in 1985.

Also the principle is turnout that it does not change the type of membership function under fuzzy arithmetical operations of fuzzy number. In the fuzzy sense, it is reasonable to discuss the grade of each point of support set of fuzzy number for representing fuzzy number. Therefore, Chen and Hsieh's Graded Mean Integration Representation method [6] adopted grade as the important degree of each point of support set of generalized fuzzy number. With this reason, we use it to defuzzify the trapezoidal fuzzy total cost of each retailer. In fuzzy decentralized ordering model for crisp replenishment order quantity, the first derivative of fuzzy total cost of each retailer is used to solve the optimal replenishment order quantity. Furthermore, the algorithm of Extension of the Lagrangian method [15] is used to solve inequality constrains in fuzzy decentralized ordering model for replenishment order quantity. Moreover, we consider an example of an organisation for fuzzy replenishment order quantity.

Economic lot size models have been studied extensively since Harris [10] presented the famous EOQ formula in 1913. Five years later, the economic production quantity (EPQ) inventory model was proposed by Taft [14]. However, in recent years, both academicians and researchers have shown an increasing level of interest in finding alternatives ways to solve inventory models. In [11] Hsieh introduced the fuzzy production inventory models in which fuzzy parameters and fuzzy production quantity or fuzzy order quantity are all trapezoidal fuzzy numbers.

In this paper, we introduce the fuzzy decentralized ordering models in which fuzzy parameters and fuzzy replenishment order quantity are all trapezoidal fuzzy numbers.

1.1. Notations used.

N	Number of retailers
j	Retailer index $(j = 1, 2 \ldots, N)$
$\tilde{Q}_j$	Fuzzy replenishment order quantity in units of retailer j
$\tilde{A}_j$	Fuzzy fixed cost of order at the retailers (not per unit)($)
$\tilde{V}_j$	Fuzzy variable purchase cost of item at retailer j ($)
S_j	reorder point at retailer j
$\tilde{D}_j$	Fuzzy Demand quantity (yearly) observed by retailer j

$\tilde{d}_{wj}$	Fuzzy travelling distance from warehouse to retailer j (km)
$\tilde{r}_j$	Fuzzy carrying charge in % of unit value at retailer j (per year)
$\tilde{K}_j$	Fuzzy safety factor at retailer j
$\tilde{\sigma}_{jLwj}$	Fuzzy variation of the lead time from warehouse to retailer j
$\tilde{\alpha}_w$	Fuzzy fixed cost of transportation per order from warehouse to retailer j (small value for all retailers)
$\tilde{t}_w$	Fuzzy variable cost of transportation from warehouse to retailer j
$\tilde{T}C_j(Q_j)$	Fuzzy total cost for retailer j
$P\left(\tilde{T}C_j(Q_j)\right)$	Defuzzified value of fuzzy total cost $P\left(\tilde{T}C_j(Q_j)\right)$ for retailer j
$\tilde{Q}_j^*$	Fuzzy optimal value of Q_j

1.2. **Assumptions.**

(i) A continuous review (S, Q) policy for all of the retailers.
(ii) Initial inventory position is assumed to be zero.
(iii) The inventory position at each retailer decreases, an amount Q_j will be ordered.
(iv) All of the retailers use a similar service level say P_1 [13] that determines the probability of no stock-out per order cycle.
(v) Lost sales are not considered in the model.
(vi) The resulting inventory position will be strictly larger than S_j and smaller than or equal to $S_j + Q_j$.
(vii) Lead-time demand is normally distributed with average μ_{jLwj} and variance σ^2_{jLwj} at each retailer j.

2. Methodology

2.1. **Fuzzy Numbers.** Any fuzzy subset of the real line R, whose membership function $\mu_{\tilde{A}}$ satisfies the following conditions, is a generalized fuzzy number $\tilde{A}$

(1) $\mu_{\tilde{A}}(x)$ is a continuous mapping from R to the closed interval [0, 1].
(2) $\mu_{\tilde{A}}(x) = 0$, $-\infty < x \le a_1$,
(3) $\mu_{\tilde{A}}(x) = L(x)$ is strictly increasing on $[a_1, a_2]$,
(4) $\mu_{\tilde{A}}(x) = w_A$, $a_2 \le x \le a_3$,
(5) $\mu_{\tilde{A}}(x)$= R(x) is strictly decreasing on $[a_3, a_4]$,
(6) $\mu_{\tilde{A}}(x) = 0$, $a_4 \le x < \infty$,where $0 < w_A \le 1$, and a_1, a_2, a_3 and a_4 are real numbers. Also this type of generalized fuzzy number be denoted as $\tilde{A} = (a_1, a_2, a_3, a_4; w_A)_{LR}$. When $w_A = 1$, it can be simplified as $\tilde{A} = (a_1, a_2, a_3, a_4)_{LR}$.

2.2. **Trapezoidal Fuzzy Number.** The fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$, where $a_1 < a_2 < a_3 < a_4$ and defined on R is called the trapezoidal fuzzy number, if the membership function $\tilde{A}$ is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0; & x < a_1 \text{ or } x > a_4 \\ \dfrac{(x-a_1)}{(a_2-a_1)}; & a_1 \le x < a_2 \\ 1; & a_2 \le x < a_3 \\ \dfrac{(x-a_4)}{(a_3-a_4)}; & a_3 \le x \le a_4 \end{cases}$$

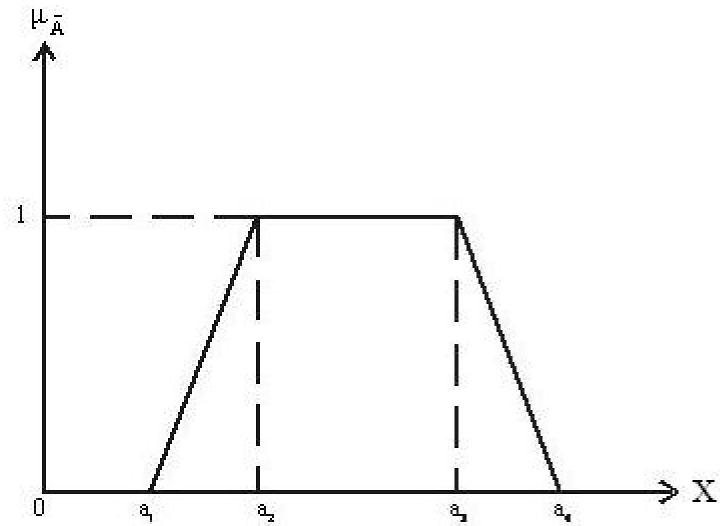

FIGURE 2.

2.3. **The Function Principle.** The function principle was introduced by Chen [4] to treat fuzzy arithmetical operations. This principle is used for the operation of addition, multiplication, subtraction and division of fuzzy numbers.

Suppose $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ are two trapezoidal fuzzy numbers. Then

(1) The addition of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$, where a_1, a_2, a_3, a_4, b_1, b_2, b_3 and b_4 are any real numbers.

(2) The multiplication of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A} \otimes \tilde{B} = (c_1, c_2, c_3, c_4)$, where $T = \{a_1b_1, a_1b_4, a_4b_1, a_4b_4\}$, $T_1 = \{a_2b_2, a_2b_3, a_3b_2, a_3b_3\}$, $c_1 = \min T$, $c_2 = \min T_1$, $c_3 = \max T_1$, $c_4 = \max T$. If a_1, a_2, a_3, a_4, b_1, b_2, b_3 and b_4 are all non zero positive real numbers, then $\tilde{A} \otimes \tilde{B} = \{a_1b_1, a_2b_2, a_3b_3, a_4b_4\}$

(3) $-\tilde{B} = (-b_4, -b_3, -b_2, -b_1)$, then the subtraction of $\tilde{A}$ and $\tilde{B}$ is

$$\tilde{A} \ominus \tilde{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1),$$

where a_1, a_2, a_3, a_4, b_1, b_2, b_3 and b_4 are any real numbers.

(4) $\frac{1}{\tilde{B}} = \tilde{B}^{-1} = \left(\frac{1}{b_4}, \frac{1}{b_3}, \frac{1}{b_2}, \frac{1}{b_1}\right)$, where b_1, b_2, b_3 and b_4 are all positive real numbers. If a_1, a_2, a_3, a_4, b_1, b_2, b_3 and b_4 are all non-zero positive real numbers, then the division of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A} \oslash \tilde{B} = \left(\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1}\right)$

(5) Let $k \in R$ then

$$k \otimes \tilde{A} = \begin{cases} (ka_1, ka_2, ka_3, ka_4), & \text{if } k \geq 0 \\ (ka_4, ka_3, ka_2, ka_1), & \text{if } k < 0 \end{cases}$$

2.4. **Graded Mean Integration Representation Method.** If $\tilde{A} = (a_1, a_2, a_3, a_4; w_A)_{LR}$ is a generalized fuzzy number then the defuzzified value $P\left(\tilde{A}\right)$ by graded mean integration representation method is given by

$$P\left(\tilde{A}\right) = \int_0^{w_A} h\left(\frac{L^{-1}(h) + R^{-1}(h)}{2}\right)\mathrm{d}h \bigg/ \int_0^{w_A} h\mathrm{d}h \quad \text{with } 0 < h \leq w_A \text{ and } 0 < w_A \leq 1.$$

If $\tilde{A} = (a_1, a_2, a_3, a_4)$ is a trapezoidal number, then the graded mean integration representation of $\tilde{A}$ by above formula is

$$P\left(\tilde{A}\right) = \int_0^1 h\left(\frac{a_1 + a_4 + (a_2 - a_1 - a_4 + a_3)h}{2}\right)\mathrm{d}h \bigg/ \int_0^1 h\mathrm{d}h$$

$$P\left(\tilde{A}\right) = \frac{a_1 + 2a_2 + 2a_3 + a_4}{6}$$

2.5. **Extension of the Lagrangian Method.** In [15], Taha discussed how to solve the optimum solution of non-linear programming problem with equality constraints by using Lagrangian Method and showed how the Lagrangian Method may be extended to solve inequality constraints. The general idea of extending the lagrangian procedure is that if the unconstrained optimum of problem does not satisfy all the constraints, the constrained optimum must occur at a boundary point of the solution space.

Suppose that the problem is given by
Minimize $y = f(x)$
Subject to $g_i(x) \geq 0$, $i = 1, 2, \ldots, m$.

The non-negativity constraints $x \geq 0$, if any are included in the m constraints. Then the procedure of extension of the Lagrangian Method involves the following steps.

Step (1): Solve the unconstrained problem. Minimize $y = f(x)$. If the resulting optimum satisfies all the constraints, stop because all constraints are redundant. Otherwise, set $k = 1$ and go to Step 2.

Step (2): Activate any k constraints (i.e., convert them into equality) and optimize $f(x)$ subject to the k active constraints by the Lagrangian Method. If the resulting solution is feasible with respect to the remaining constraints, stop; it is a local optimum. Otherwise, activate another set of k-constraints and repeat the step. If all sets of active constraints taken k at a time are considered without encountering a feasible solution, go to Step 3.

Step (3): If $k = m$, stop : no feasible solution exists. Otherwise, set $k = k + 1$ and go to Step 2.

3. Fuzzy Mathematical Model

Suppose $\tilde{A} = (a_1, a_2, a_3, a_4)$, $\tilde{D}_j = (d_{j_1}, d_{j_2}, d_{j_3}, d_{j_4})$, $\tilde{V}_j = (v_{j_1}, v_{j_2}, v_{j_3}, v_{j_4})$, $\tilde{\alpha}_w = (\alpha_{w_1}, \alpha_{w_2}, \alpha_{w_3}, \alpha_{w_4})$, $\tilde{r}_j = (r_{j_1}, r_{j_2}, r_{j_3}, r_{j_4})$, $\tilde{d}_{w_j} = (d_{wj_1}, d_{wj_2}, d_{wj_3}, d_{wj_4})$, $\tilde{k}_j = (k_{j_1}, k_{j_2}, k_{j_3}, k_{j_4})$, $\tilde{\sigma}_{jLw_j} = (\sigma_{jLw_{j_1}}, \sigma_{jLw_{j_2}}, \sigma_{jLw_{j_3}}, \sigma_{jLw_{j_4}})$, $\tilde{t}_w = (t_{w_1}, t_{w_2}, t_{w_3}, t_{w_4})$, $\tilde{Q}_j = (q_{j_1}, q_{j_2}, q_{j_3}, q_{j_4})$ are non negative trapezoidal fuzzy numbers based on the above assumptions, fuzzy model can be represented as

$$\begin{aligned}\tilde{TC}_j(Q_j) = &\left(\tilde{A}_j \otimes \tilde{D}_j\right) \oslash \tilde{Q}_j + \left[\left(\tilde{Q}_j \oslash 2\right) \oplus \left(\tilde{K}_j \otimes \tilde{\sigma}_{jLwj}\right)\right] \\ &\otimes \tilde{v}_j \otimes \tilde{r}_j \oplus \left(\tilde{\alpha}_w \oplus \left(\tilde{t}_w \otimes \tilde{d}_{wj}\right)\right) \otimes \tilde{D}_j \oslash \tilde{Q}_j\end{aligned}$$

$$\begin{aligned}\tilde{T}C_j(Q_j) = \Bigg\{ &\frac{a_{j_1} d_{j_1}}{q_{j_4}} + \left(\frac{q_{j_1}}{2} + k_{j_1}\sigma_{jLwj_1}\right) v_{j_1} r_{j_1} + (\alpha_{w_1} + t_{w_1} d_{wj_1}) \frac{d_{j_1}}{q_{j_4}}, \\ &\frac{a_{j_2} d_{j_2}}{q_{j_3}} + \left(\frac{q_{j_2}}{2} + k_{j_2}\sigma_{jLwj_2}\right) v_{j_2} r_{j_2} + (\alpha_{w_2} + t_{w_2} d_{wj_2}) \frac{d_{j_2}}{q_{j_3}}, \\ &\frac{a_{j_3} d_{j_3}}{q_{j_2}} + \left(\frac{q_{j_3}}{2} + k_{j_3}\sigma_{jLwj_3}\right) v_{j_3} r_{j_3} + (\alpha_{w_3} + t_{w_3} d_{wj_3}) \frac{d_{j_3}}{q_{j_2}}, \\ &\frac{a_{j_4} d_{j_4}}{q_{j_1}} + \left(\frac{q_{j_4}}{2} + k_{j_4}\sigma_{jLwj_4}\right) v_{j_4} r_{j_4} + (\alpha_{w_4} + t_{w_4} d_{wj_4}) \frac{d_{j_4}}{q_{j_1}} \Bigg\}\end{aligned}$$

By Graded Mean Integration, solve the unconstrained problem
Minimize

$$\begin{aligned}P\left(\tilde{T}C_j(Q_j)\right) = \frac{1}{6}\Bigg\{ &\frac{a_{j_1} d_{j_1}}{q_{j_4}} + \left(\frac{q_{j_1}}{2} + k_{j_1}\sigma_{jLwj_1}\right) v_{j_1} r_{j_1} + (\alpha_{w_1} + t_{w_1} d_{wj_1}) \frac{d_{j_1}}{q_{j_4}} \\ &+ \frac{2a_{j_2} d_{j_2}}{q_{j_3}} + 2\left(\frac{q_{j_2}}{2} + k_{j_2}\sigma_{jLwj_2}\right) v_{j_2} r_{j_2} + (\alpha_{w_2} + t_{w_2} d_{wj_2}) \frac{d_{j_2}}{q_{j_3}} \\ &+ \frac{2a_{j_3} d_{j_3}}{q_{j_2}} + 2\left(\frac{q_{j_3}}{2} + k_{j_3}\sigma_{jLwj_3}\right) v_{j_3} r_{j_3} + (\alpha_{w_3} + t_{w_3} d_{wj_3}) \frac{d_{j_3}}{q_{j_2}} \\ &+ \frac{a_{j_4} d_{j_4}}{q_{j_1}} + \left(\frac{q_{j_4}}{2} + k_{j_4}\sigma_{jLwj_4}\right) v_{j_4} r_{j_4} + (\alpha_{w_4} + t_{w_4} d_{wj_4}) \frac{d_{j_4}}{q_{j_1}} \Bigg\}\end{aligned}$$

with $0 < q_{j_1} \le q_{j_2} \le q_{j_3} \le q_{j_4}$.
Differentiate $P\left(\tilde{T}C_j(Q_j)\right)$ partially with respect to $q_{j_1}, q_{j_2}, q_{j_3}$ and q_{j_4}:

$$\frac{\partial P}{\partial q_{j_1}} = 0 \;\Rightarrow\; q_{j_1} = \sqrt{\frac{2d_{j_4}\left(a_{j_4} + \alpha_{w_4} + t_{w_4} d_{wj_4}\right)}{v_{j_1} r_{j_1}}},$$

$$\frac{\partial P}{\partial q_{j_2}} = 0 \;\Rightarrow\; q_{j_2} = \sqrt{\frac{4d_{j_3}\left(a_{j_3} + \alpha_{w_3} + t_{w_3} d_{wj_3}\right)}{2v_{j_2} r_{j_2}}},$$

$$\frac{\partial P}{\partial q_{j_3}} = 0 \;\Rightarrow\; q_{j_3} = \sqrt{\frac{4d_{j_2}\left(a_{j_2} + \alpha_{w_2} + t_{w_2} d_{wj_2}\right)}{2v_{j_3} r_{j_3}}}$$

and

$$\frac{\partial P}{\partial q_{j_4}} = 0 \;\Rightarrow\; q_{j_4} = \sqrt{\frac{2d_{j_1}\left(a_{j_1} + \alpha_{w_1} + t_{w_1} d_{wj_1}\right)}{v_{j_4} r_{j_4}}}.$$

The above show that with $q_{j_1} > q_{j_2} > q_{j_3} > q_{j_4}$, it does not satisfy the constraint with $0 < q_{j_1} \leq q_{j_2} \leq q_{j_3} \leq q_{j_4}$, set $k = 1$ and go to Step 2.

Convert the inequality constraint into equality constraint $q_{j_2} - q_{j_1} = 0$ and optimize $P\left(\tilde{T}C_j\left(Q_j\right)\right)$ subject to $q_{j_2} - q_{j_1} = 0$ by the Lagrangian Method. We have Lagrangian function is $L(q_{j_1}, q_{j_2}, q_{j_3}, q_{j_4}, \lambda) = P\left(\tilde{T}C_j\left(Q_j\right)\right) - \lambda(q_{j_2} - q_{j_1})$. Let all the partial derivatives equal to zero and solve $q_{j_1}, q_{j_2}, q_{j_3}$ and q_{j_4}, then we get

$$q_{j_1} = q_{j_2} = \sqrt{\frac{2\left\{2d_{j_3}\left(a_{j_3} + \alpha_{w_3} + t_{w_3} d_{wj_3}\right) + d_{j_4}\left(a_{j_4} + \alpha_{w_4} + t_{w_4} d_{wj_4}\right)\right\}}{v_{j_1} r_{j_1} + 2v_{j_2} r_{j_2}}}$$

$$q_{j_3} = \sqrt{\frac{4d_{j_2}\left(a_{j_2} + \alpha_{w_2} + t_{w_2} d_{wj_2}\right)}{2v_{j_3} r_{j_3}}}, \quad \text{and} \quad q_{j_4} = \sqrt{\frac{2d_{j_1}\left(a_{j_1} + \alpha_{w_1} + t_{w_1} d_{wj_1}\right)}{v_{j_4} r_{j_4}}}$$

The above show that $q_{j_3} > q_{j_4}$, it does not satisfy the constraint $0 < q_{j_1} \leq q_{j_2} \leq q_{j_3} \leq q_{j_4}$, it is not a local optimum. Set $k = 2$ and go to Step 3.

Step 3: Convert the inequality constraint into equality constraint $q_{j_2} - q_{j_1} = 0$ and $q_{j_3} - q_{j_2} = 0$. We optimize $P\left(\tilde{T}C_j\left(Q_j\right)\right)$ subject to $q_{j_2} - q_{j_1} = 0$ and $q_{j_3} - q_{j_2} = 0$ by the Lagrangian Method. Then the Lagrangian function is

$$L(q_{j_1}, q_{j_2}, q_{j_3}, q_{j_4}, \lambda_1, \lambda_2) = P\left(\tilde{T}C_j\left(Q_j\right)\right) - \lambda_1(q_{j_2} - q_{j_1}) - \lambda_2(q_{j_3} - q_{j_2}).$$

Let all the partial derivatives equal to zero and solve $q_{j_1}, q_{j_2}, q_{j_3}$ and q_{j_4}, then we get

$$q_{j_1} = q_{j_2} = q_{j_3} = \sqrt{\frac{2(2X + 2Y + 2Z)}{v_{j_1} r_{j_1} + 2v_{j_2} r_{j_2} + 2v_{j_3} r_{j_3}}} \quad \text{and} \quad q_{j_4} = \sqrt{\frac{2W}{v_{j_4} r_{j_4}}}$$

where

$$W = d_{j_1}\left(a_{j_1} + \alpha_{w_1} + t_{w_1} d_{wj_1}\right), \qquad X = d_{j_2}\left(a_{j_2} + \alpha_{w_2} + t_{w_2} d_{wj_2}\right),$$
$$Y = d_{j_3}\left(a_{j_3} + \alpha_{w_3} + t_{w_3} d_{wj_3}\right) \qquad \text{and} \quad Z = d_{j_4}\left(a_{j_4} + \alpha_{w_4} + t_{w_4} d_{wj_4}\right).$$

The above result show that $q_{j_1} > q_{j_4}$ does not satisfy the constraint $0 < q_{j_1} \leq q_{j_2} \leq q_{j_3} \leq q_{j_4}$, it is not a local optimum. Set $k = 3$ and go to Step 4.

Step 4: Convert the inequality constraint into equality constraints $q_{j_2} - q_{j_1} = 0$, $q_{j_3} - q_{j_2} = 0$ and $q_{j_4} - q_{j_3} = 0$. We optimize $P\left(\tilde{T}C_j\left(Q_j\right)\right)$ subject to $q_{j_2} - q_{j_1} = 0$,

$q_{j_3} - q_{j_2} = 0$ and $q_{j_4} - q_{j_3} = 0$ by the Lagrangian Method. The Lagrangian function is

$$\begin{aligned} &L(q_{j_1}, q_{j_2}, q_{j_3}, q_{j_4}, \lambda_1, \lambda_2, \lambda_3) \\ &\quad = P\left(\tilde{T}C_j\left(Q_j\right)\right) - \lambda_1(q_{j_2} - q_{j_1}) - \lambda_2(q_{j_3} - q_{j_2}) - \lambda_3(q_{j_4} - q_{j_3}). \end{aligned}$$

Let all the partial derivatives equal to zero and solve $q_{j_1}, q_{j_2}, q_{j_3}$ and q_{j_4}, then we get

$$q_{j_1} = q_{j_2} = q_{j_3} = q_{j_4} = \sqrt{\frac{2(W + 2X + 2Y + Z)}{v_{j_1}r_{j_1} + 2v_{j_2}r_{j_2} + 2v_{j_3}r_{j_3} + v_{j_4}r_{j_4}}} \tag{3.1}$$

Because the above solution $\tilde{Q}_j = (q_{j_1}, q_{j_2}, q_{j_3}, q_{j_4})$ satisfies all inequality constraints, the procedure terminates with $\tilde{Q}_j$ as a local optimum solution to the problem.

Since the above local optimum solution is the only one feasible solution of graded mean integration formula, so it is an optimum solution of the inventory model with fuzzy replenishment order quantity according to extension of the Lagrangian Method.

Let $q_{j_1} = q_{j_2} = q_{j_3} = q_{j_4} = q_j$. Then the optimal fuzzy replenishment order quantity is $\tilde{Q}_j^* = \left(q_j^*, q_j^*, q_j^*, q_j^*\right)$, where

$$q_j^* = \sqrt{\frac{2(W + 2X + 2Y + Z)}{v_{j_1}r_{j_1} + 2v_{j_2}r_{j_2} + 2v_{j_3}r_{j_3} + v_{j_4}r_{j_4}}} \tag{3.2}$$

4. Numerical Example

The purchase manager of an organisation has collected the following data for one of the A-class items:

Fuzzy fixed cost of order (not per unit, only in dollars) at the retailer j:

$$\tilde{A}_j = (95, 100, 100, 105)$$

Fuzzy variable purchase cost of item (in dollars) at retailer j:

$$\tilde{V}_j = (85.5, 90, 90, 94.5)$$

Fuzzy Demand quantity (yearly) observed by retailer j:

$$\tilde{D}_j = (814.15, 857, 857, 899.85)$$

Fuzzy fixed cost of transportation per order (in dollars) from warehouse to retailer j:

$$\tilde{\alpha}_w = (95, 100, 100, 105)$$

Fuzzy travelling distance (in km) from warehouse to retailer j:

$$\tilde{d}_{wj} = (14.25, 15, 15, 15.75)$$

Fuzzy variable cost of transportation from warehouse to retailer j :

$$\tilde{t}_w = (14.25, 15, 15, 15.75)$$

Fuzzy carrying charge in % of unit value (per year) at retailer j:

$$\tilde{r}_j = (0.95, 1, 1, 1.05)$$

Fuzzy safety factor at retailer j:

$$\tilde{K}_j = (1.558, 1.64, 1.64, 1.772)$$

Fuzzy variation of the lead time from warehouse to retailer j:

$$\tilde{\sigma}_{jLwj} = (2.775, 2.9, 2.9, 3.045)$$

Determine the Economic lot size.

Solution. Here we can use a general rule for trapezoidal fuzzy numbers as
"Value of X" $= (0.95X, X, X, 1.05X)$
Fuzzy replenishment order quantity $\tilde{Q}_j = (q_{j_1}, q_{j_2}, q_{j_3}, q_{j_4})$ with
$0 < q_{j_1} \leq q_{j_2} \leq q_{j_3} \leq q_{j_4}$.

Formula (3.1) is an optimum solution of the inventory model with fuzzy replenishment order quantity.

By extension of the Lagrangian Method.

Let $q_{j_1} = q_{j_2} = q_{j_3} = q_{j_4} = q_j$, then the optimal fuzzy replenishment order quantity is $\tilde{Q}_j^* = (\ q_j^*, q_j^*, q_j^*, q_j^*)$

Substitute the given fuzzy parameter values in formula (3.2), we obtain the optimal fuzzy replenishment order quantity $\tilde{Q}_j^* = (90.00, 90.00, 90.00, 90.00)$.

5. Conclusion

In the fuzzy environment, it may be possible to discuss the fuzzy decentralized ordering model with trapezoidal fuzzy numbers for fuzzy replacements order quantity. In addition, we find that the optimal fuzzy replacement quantity $\tilde{Q}_j^* = \left(q_j^*, q_j^*, q_j^*, q_j^*\right)$ is the special type of trapezoidal fuzzy number. Hence the optimal solution of our stipulated model can be effective to meet the conventional decentralized ordering model. Thus this fuzzy decentralized ordering model is practical and very useful in the real world. The model presented here can be extended to include a multi-item multi-level inventory model or true costs of transportation, like environmental costs, or costs of return flow due to lack of demand (excess inventory) or customer returns.

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